Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

# DETERMINING THE SIGN OF $\Delta_{31}$ BY FUTURE LONG-BASELINE AND REACTOR EXPERIMENTS

BIPIN SINGH KORANGA, SOUMYA MAHAPATRA and S. UMA SANKAR

*Department of Physics, Indian institute of Technology Bombay, Mumbai 400076, India*

Received 22 February 2009; Accepted 2 July 2009
Online 7 October 2009

We study the determination of neutrino mass hierarchy through neutrino experiments foreseen in the next ten years. The T2K neutrino oscillation experiment will start in 2009. In the experiment, the high intensity $\nu_\mu$ beam from JHF is directed to Super-Kamiokande (SK) detector 295 km away. The $NO\nu A$ (off axes neutrino oscillation) experiment is being planned, with the $\nu_\mu$ beam from Fermi-Lab directed to a site 610 km away, which is 0, 7 and 14 milliradian off-axes. Both above experiments will measure $\nu_\mu \to \nu_e$ oscillation probability. The Double-CHOOZ experiment under construction will detect $\overline{\nu_e}$s emitted by nuclear reactors both through a near detector (150 m) and a far detector (1.05 km) to measure $\overline{\nu_e} \to \overline{\nu_e}$ survival probability. In this paper, we outline a procedure to determine the sign of $\Delta_{31}$ from the simulated data of the above experiments.

PACS numbers: 14.60.Lm, 13.15.+g UDC 539.123

Keywords: neutrino experiment, mass hierarchy, neutrino oscillation experiments, sign of $\Delta_{31}$

## 1. *Introduction*

Recent advance in neutrino physics observation, mainly in astrophysical observation, suggest the existence of a tiny neutrino mass. The experiment and observation have shown evidences for neutrino oscillation. The solar neutrino deficit has been observed long ago [1 – 4]. The atmospheric neutrino anomaly has indicated [5 – 9] that neutrinos are massive and that there is mixing in the lepton sector. This has currently been almost confirmed by KamLAND [10]. Since the lepton mixing does exist, there occurs CP violation effect in the lepton sector. Several physicists have considered whether we can see CP violation effects in the lepton sector through long-baseline oscillation experiments. The neutrino oscillation probabilities, generally, depend on six parameters, two independent mass-squared differences, $\Delta_{21}$

and $\Delta_{31}$, three mixing angles, $\theta_{12}$, $\theta_{23}$ and $\theta_{13}$ and one CP violating phase $\delta$. Two mixing angles are large ($\theta_{12}$, $\theta_{23}$) and one is small ($\theta_{13}$), and two mass–square differences, $\Delta_{ij} = m_j^2 - m_j^2$, where $m_{ij}$ are the neutrino masses,

$$\Delta_{21} = \Delta_{\rm solar} \tag{1}$$

$$\Delta_{31} = \Delta_{\rm atmo} \tag{2}$$

The sign of $\Delta_{31}$and of $\theta_{23}$, when $\theta_{23} \neq 0$, can not be determine from the existing data. For the mass-square difference, there are two possibilities, $\Delta_{31} > 0$ or $\Delta_{31} < 0$, that correspond to two different neutrino mass orders, normal mass hierarchy, $m_1 < m_2 < m_3$ ($\Delta_{31} > 0$), and inverted hierarchy, $m_1 > m_2 > m_3(\Delta_{31} < 0)$. The angles $\theta_{12}$ and $\theta_{23}$ represent the neutrino mixing angles corresponding to solar and atmospheric neutrino oscillation. Much progress has been made in considerations how to determine the values of the three mixing angles. From the measurement of the neutrino survival probability $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_e$ in the atmospheric flux, one mixing angle is near $\frac{\pi}{4}$ and one is small [11], and from the $\nu_e \to \nu_e$ survival probability in the solar flux, the mixing angle is either large (LMA) or small (SMA) from the solar solution [12]. Nothing is known about CP violating phase. In this paper, we consider the determination of the sign of $\Delta_{31}$ by three different baseline experiments (T2K, $NO\nu A$ and Double CHOOZ). The purpose of this paper is to determine the sign of $\Delta_{31}$ by $\chi^2$ analysis. Section 2 describes the mixing angles and mass differences. Section 3 describes the mass hierarchy effect in $\nu_\mu \to \nu_e$ oscillation probability. In Sec. 4. the determination of the sign of $\Delta_{31}$ by $\chi^2$ analysis is considered. Section 5 summarizes the results and presents conclusions.

## 2. *Mixing angles and neutrino mass-squared differences*

The first evidence of the neutrino oscillations is the observation of zenith-angle dependence of the atmospheric neutrino defect [13], the $\nu_\mu \to \nu_\mu$ transition with the mass difference and the mixing

$$\Delta_{31} = (1-2) \times 10^{-3}\,{\rm eV}^2, \quad \sin^2 2\theta_{23} = 1.0\,. \tag{3}$$

The second evidence is the solar neutrino deficit [14], which is consistent with $\nu_\mu \to \nu_\tau/\nu_e$ transition. The SNO experiments [15] are consistent with the standard solar model [16] and strongly suggest the LMA solution,

$$\Delta_{21} = 7 \times 10^{-5}\,{\rm eV}^2, \quad \sin^2 2\theta_{12} = 0.8\,. \tag{4}$$

Solar neutrino experiments (Super-K, GALLEX, SAGE, SNO and GNO) show that neutrino oscillations provide the most elegant explanation of all data [17],

$$\Delta_{\rm solar} = 7^{+5}_{-1.3} \times 10^{-5}\,{\rm eV}^2, \tag{5}$$

$$\tan^2\theta_{\text{solar}} = 0.4^{+0.14}_{-0.1}\,. \tag{6}$$

Atmospheric neutrino experiments (Kamiokande, Super-K) also show that neutrino oscillation. The most excellent fit to all data is [17].

$$\Delta_{\text{atmo}} = 2.0^{+1.0}_{-0.92}\times 10^{-3}\,\text{eV}^2, \tag{7}$$

$$\sin^2 2\theta_{\text{atmo}} = 0.4^{+0.14}_{-0.10}\,. \tag{8}$$

The CHOOZ reactor experiments [18] give the upper bounds of the third mixing angle $\theta_{13}$,

$$\sin^2\theta_{13} < 0.20 \qquad \text{for} \qquad |\Delta_{31}| = 2.0\times 10^{-3}\,\text{eV}^2, \tag{9}$$

$$\sin^2\theta_{13} < 0.16 \qquad \text{for} \qquad |\Delta_{31}| = 2.5\times 10^{-3}\,\text{eV}^2, \tag{10}$$

$$\sin^2\theta_{13} < 0.14 \qquad \text{for} \qquad |\Delta_{31}| = 3.0\times 10^{-3}\,\text{eV}^2, \tag{11}$$

at the 90 % CL. The CP phase $\delta$ has not been constrained. Future neutrino experiments plan to measure the neutrino oscillation parameters precisely.

## 3. *Mass hierarchy effect in neutrino oscillation probability*

Let us briefly recall our present knowledge of neutrino oscillation parameters. There are three flavors of neutrinos and they mix to form three mass eigensates. This mixing is given by

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \tag{12}$$

where mixing matrix $U$ is parametrized [19] as

$$U = R(\theta_{23})\;\Pi R(\theta_{13})\;\Pi^* R(\theta_{12})\,. \tag{13}$$

$\Pi$ is a diagonal matrix containing the CP violating phase $\delta$ and $R(\theta_{ij})$ has the form of rotation matrices. The mass eigenstates $\nu_i$ have eigenvalues $m_i$. Neutrino oscillation probabilities depend on the two mass-squared differences $\Delta_{21} = m_2^2 - m_1^2$, $\Delta_{31} = m_3^2 - m_1^2$, the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and the CP violating phase $\delta$. Solar neutrino data and KamLAND experiment determine $\Delta_{21}$ and $\theta_{12}$. Atmospheric neutrino data and K2K and MINOS experiments determine $|\Delta_{31}|$ and $\theta_{23}$. CHOOZ experiment and solar neutrino data constrain $\theta_{13}$ to be small. There is no information at present on the CP phase $\delta$. The future experiments are expected to measure $\theta_{13}$ and determine the sign of $\Delta_{31}$ and the magnitude of CP violation, in addition to improving the precision of known neutrino oscillation parameters.

Double CHOOZ experiment is a reactor-based experiment dedicated to measurement of $\theta_{13}$. In this experiment systematic errors are minimised by having identical near and far detectors at distances 150 m and 1050 m from the sources, respectively. This experiment can measure non-zero value of $\theta_{13}$ if $\sin^2 2\theta_{13} \geq 0.05$ [20]. Daya Bay reactor experiment will have a similar sensitivity [21]. In the T2K experiment, the high intensity of $\nu_\mu$ beam from JPARC accelerator is directed to SK detector 295 km away. The detector is 2° off-axis from the beam, which leads to the neutrino flux peaking at lower energy. T2K is a very high statistics experiment that is expected to start taking data in 2009. The neutrino flux is about 100 times the flux of K2K. The number of $\nu_\mu$ charged-current events expected, in the case of no oscillation, is about 3100 per year. This experiment will improve the precision of $\Delta_{31}$ and $\theta_{23}$ by measuring the muon neutrino survival probability $P(\nu_\mu \to \nu_e)$. It can also measure $\theta_{13}$ through the measurement of $P_{\mu e}$. The $NO\nu A$ is also an accelerator-based experiment, which uses $\nu_\mu$ beam from Fermilab 810 km away. The detecting material in this experiment is a scintillator which gives it an excellent electron detection capability. Thus $NO\nu A$ can make a precise determination of $P_{\mu e}$,$NO\nu A$. It is expected to start taking data in 2011 and will also be placed at an off-axis location. Because of the longer distance, the in flux $NO\nu A$ experiment will be peaked at higher energy compared to that of T2K. The matter term, which is proportional to the neutrino energy, causes a 25% change in $P_{\mu e}$, whereas the change in $P_{\mu e}$ of T2K is only about 10% [22]. If $\Delta_{31}$ positive, $P_{\mu e}$ increases, whereas if $\Delta_{31}$ is negative it decreases. Below we describe a procedure by which the sign of $\Delta_{31}$ can be determined using the data from Double CHOOZ, T2K and $NO\nu A$ experiments. We will compute the smallest value of $\theta_{13}$ for which the sign of $\Delta_{31}$ can be determined independently of the CP phase $\delta$.

## 4. *Mass hierarchy effect in $P^m_{\mu e-}$ oscillation probability*

### 4.1. *$P^m_{\mu e}$ oscillation probability with $\Delta_{21} = 0$*

The neutrino oscillation probability, $\nu_\mu \to \nu_e$ in long-baseline experiments is modified by the propagation of neutrino through the matter of earth's crust [23]. It increases the oscillation probability for neutrinos if $\Delta_{31}$is positive and decreases it $\Delta_{31}$ is negative. The reverse is true for anti-neutrinos. Here we consider a method of determining the sign of $\Delta_{31}$ using $\nu_\mu$ beams only.

In three-flavor mixing, the $\nu_\mu \to \nu_e$ oscillation probability is given by

$$P_{\mu e} = \sin^2\theta_{23}\ \sin^2 2\theta_{13}\ \sin^2\left(\frac{1.27\Delta_{31}L}{E}\right), \tag{14}$$

where $\Delta_{31}$ is in eV$^2$, the baseline $L$ is in km and the neutrino energy $E$ is in GeV. In the above equation, we made the approximation of setting $\Delta_{21} = 0$, which made it independent of $\theta_{13}$ and the CP phase $\delta$. In long-baseline experiments, the neutrinos propagate through earth's crust which has constant density of about 3g/cm$^3$. The oscillation probability modified by the matter effect is given by

$$P^m_{\mu e} = \sin^2\theta_{23}\ \sin^2 2\theta^m_{13}\ \sin^2\left(\frac{1.27\Delta^m_{31}L}{E}\right), \tag{15}$$

where

$$\sin 2\theta^m_{13} = \frac{\Delta_{31}\sin 2\theta_{13}}{\Delta^m_{31}}, \tag{16}$$

$$\Delta^m_{31} = \sqrt{(\Delta_{31}\cos 2\theta_{13} - A)^2 + (\Delta_{31}\sin 2\theta_{13})^2}\,. \tag{17}$$

Here $A$ is the matter term and is given by

$$A = 2\sqrt{2}G_F NeE = 0.76\times 10^{-4}\rho\ (\mathrm{in\,g/cm^3})E_\nu\ (\mathrm{in\,GeV}). \tag{18}$$

From the expression for $P_{\mu e}$ in three flavor oscillations, we can compute the magnitude of the terms dependent on $\Delta_{21}$. It turns out that as the CP violating phase $\delta$ varies from $-\pi$ to $\pi$, $P_{\mu e}$ changes by about 25%. Therefore, setting $\Delta_{21} = 0$ is not a good approximation for analyzing matter effects in long-baseline experiments.

### *4.2.* $P^m_{\mu e}$ *oscillation probability with* $\Delta_{21} \neq 0$

The exact expression for $P_{\mu e}$ with matter effects is very complicated. The expression derived using a perturbation expansion with $\theta_{13}$ and $\alpha = \frac{\Delta_{21}}{\Delta_{31}}$ as a small parameters works very well for baselines up to 1000 km [24, 25]. Carrying out the perturbation expansion to the second order in the small parameters, the following analytic formula for $\nu_\mu \to \nu_e$ is obtained with the assumption of constant matter density

$$P^m_{\mu e} = \sin^2 2\theta_{23}\frac{\sin^2 2\theta_{13}}{(A_1-1)^2}\sin^2((A_1-1)\Delta)$$

$$\pm\frac{\alpha\sin\delta\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}}{A_1(1-A_1)}\sin(\Delta)\sin(A_1\Delta)\sin((1-A_1)\Delta)$$

$$+\frac{\alpha\cos\delta\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}}{A_1(1-A_1)}\sin(\Delta)\cos(A_1\Delta)\sin((1-A_1)\Delta)$$

$$\frac{\alpha^2\cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2(A_1\Delta)}{A_1^2}, \tag{19}$$

where $\alpha = \Delta_{21}/\Delta_{31}$, $\Delta = \Delta_{31}L/4E$, $A_1 = 2\sqrt{2}G_F NeE/\Delta_{31}$, $G_F$ is the Fermi coupling constant and $n_e$ is the electron density in earth's crust. We see the above expression depends on three unknown quantities $\theta_{13}$, sign of $\Delta_{31}$ and the CP phase $\delta$.

There are two kinds of degeneracies inherent in the three flavor expressions for $P^m_{\mu e}$. The first one occurs due to the following reason. Since $\theta_{13}$ is unknown, the $P^m_{\mu e}$ for positive $\Delta_{31}$ and smaller $\theta_{13}$ can be essentially the same as the $P^m_{\mu e}$ for negative $\Delta_{31}$ and larger $\theta_{13}$ This is illustrated in Figs. 1 and 2. Precise determination of $\theta_{13}$ by Double CHOOZ can eliminate this degeneracy. There is a further degeneracy involving the CP phase $\delta$. At present, there is no experimental information on this phase. Note that Double CHOOZ is completely insensitive to $\delta$. For a given long baseline experiment, it is possible to find two values of the CP phase, $\delta^+$ and $\delta^-$, such that $P^m_{\mu e}(+\Delta_{31}, \delta^+) = P^m_{\mu e}(-\Delta_{31}, \delta^-)$, with all other oscillation parameters

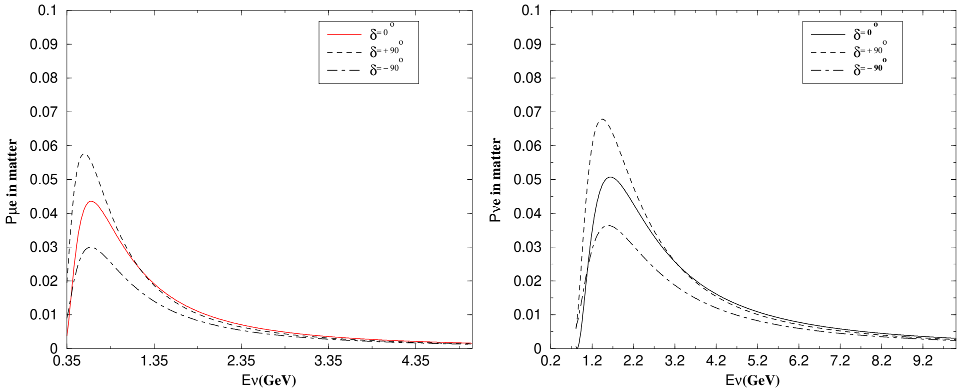


*Fig. 1. $P_{\mu e}$ oscillations probabilities vs. $E$ for $\Delta_{21} = 2.5 \times 10^{-3}$eV$^2$, $\theta_{13} = 8°$, $L = 295$ km and $L = 810$ km. The middle line is $P^m_{\mu e}(\delta = 0°)$, the upper line is $P^m_{\mu e}(\delta = +90°)$ and the lower line is $P^m_{\mu e}(\delta = -90°)$.*

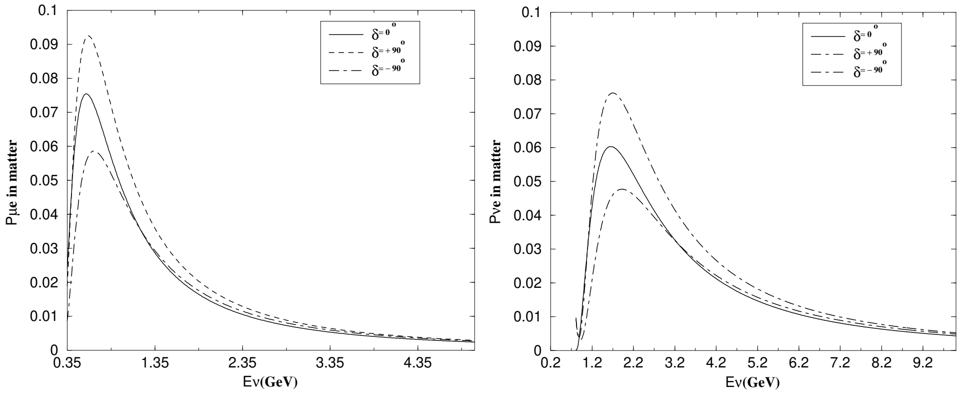


*Fig. 2. $P_{\mu e}$ oscillations probabilities vs. $E$ for $-2.5 \times 10^{-3}$eV$^2$, $\theta_{13} = 12°$, $L = 295$ km and $L = 810$ km. The middle line is $P^m_{\mu e}(\delta = 0°)$, the upper line is $P^m_{\mu e}(\delta = +90°)$ and the lower line is $P^m_{\mu e}(\delta = -90°)$.*

fixed, including $\theta_{13}$. This is illustrated in Figs. 3 and 4. However, the above degeneracy can occur for only one baseline length at a time. In Fig. 3, $P^m_{\mu e}$ for T2K is essentially the same for both signs of $\Delta_{31}$, but $P^m_{\mu e}$ can distinguish between the two signs of $\Delta_{31}$. In Fig. 4, the situation between T2K and $NO\nu A$ is reversed. If we have data from two long baseline experiments with different baselines, then we can resolve the above degeneracy independent of the $\delta$ and the sign of $\Delta_{31}$.

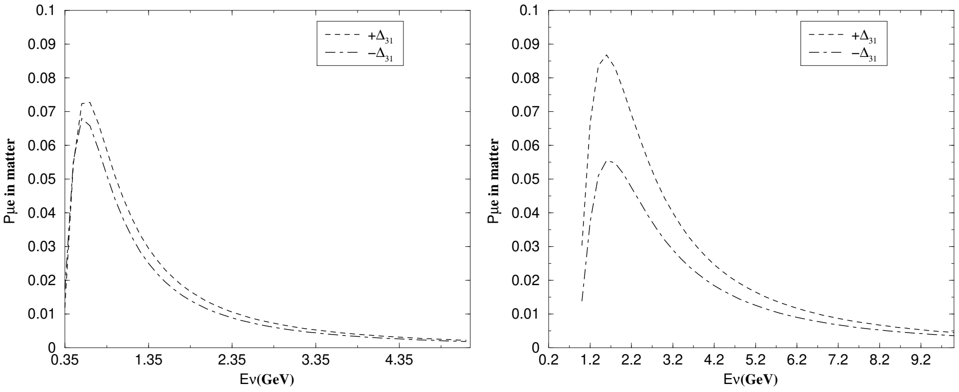


*Fig. 3. $P_{\mu e}$ oscillations probabilities vs. $E$ for $\Delta_{21} = 2.5 \times 10^{-3}\mathrm{eV}^2$, $\theta_{13} = 10°$, $L = 295$ km and $L = 810$ km. The dashed line is $P^m_{\mu e}(\delta = 30°)$ with $\Delta_{31}$ positive and the dot-dashed line is $P^m_{\mu e}(\delta = 75°)$ with $\Delta_{31}$ negative.*

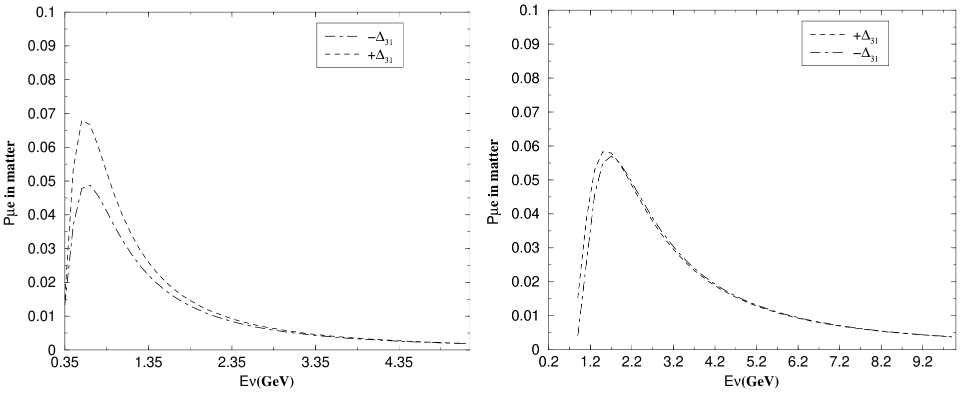


*Fig. 4. $P_{\mu e}$ oscillations probabilities vs. $E$ for $\Delta_{21} = 2.5 \times 10^{-3}\mathrm{eV}^2$, $\theta_{13} = 10°$, $L = 295$ km and $L = 810$ km. The dashed line is $P^m_{\mu e}(\delta = -90°)$ with $\Delta_{31}$ positive and the dot-dashed line is $P^m_{\mu e}(\delta = 90°)$ with $\Delta_{31}$ negative.*

The following method may be used to test the sign of $\Delta_{31}$. From the experiments, we will get three pieces of data from three different neutrino oscillation experiments (T2K, $NO\nu A$, Double CHOOZ). $N$(T2K), $N(NO\nu A)$ and $N$(Double

CHOOZ) are the expected numbers of neutrino events from these three different experiments. We tested whether the hypothesis of positive or negative $\Delta_{31}$ fits the data better. These numbers will be a function of $\theta_{13}$ and $\delta$ which is yet unknown. We compute $P^m_{\mu e}$ numerically by diagonalizing the matter-dependent mass-squared matrix for each energy bin. In the next section, we discuss the testing of $\Delta_{31}$ sign by $\chi^2$ analysis.

## 5. *Determine the $\Delta_{31}$ sign by $\chi^2$ analysis*

We take the combined data from the Double CHOOZ [26], T2K [27] and *NOνA* [28] experiments to resolve the sign of $\Delta_{31}$ problem. This solution depends crucially on matter effects which in turn depend on $\theta_{13}$. If $\theta_{13}$ is unmeasurably small, it is extremely difficult to determine the sign of $\Delta_{31}$. Here we address the question: what is the smallest value of $\theta_{13}$ for which the sign of $\Delta_{31}$ can be resolved using the data from the above three experiments, independent of the value of $\delta$.

Since there are no data yet from any of these three experiments, we simulate data for each experiment. In our calculation we fix the values of the following neutrino parameters: $\Delta_{21} = 8.0 \times 10^{-5}\,\mathrm{eV}^2$, $\theta_{12} = 34°$ and $\theta_{23} = 45°$. First we take $|\Delta_{31}| = 2.5 \times 10^{-3}\,\mathrm{eV}^2$. The presently allowed range for $\theta_{13}$ is 0° to 15° and that for the CP phase $\delta$ is $-180°$ to 180°. We pick the true value for $\theta_{13}$ from its allowed range and similarly for $\delta$.. We call these values $\theta_{13}^{\mathrm{true}}$ and $\delta^{\mathrm{true}}$. We take $\Delta_{31}$to be positive and compute the expected number of events in each bin of each experiment for $\theta_{13}^{\mathrm{true}}$ and $\delta^{\mathrm{true}}$. We smear the computed event distributions in energy using the energy resolution functions estimated by the respective collaborations. We call the data obtained after the energy smearing to be our simulated data, which consist of 92 data points $N_p^{\mathrm{simu}}$, $p = 1$, 92. Now we take $\Delta_{31}$ to be negative but keep $\Delta_{31}$ the same. We choose test values for $\theta_{13}$ and $\delta$ which we call $\theta_{13}^{\mathrm{test}}$ and $\delta^{\mathrm{test}}$. With these as inputs, we compute theoretical values for the number of events in each bin of each experiment. Thus we get 92 theoretical expectations, $N_p^{\mathrm{test}}, p = 1$, 92. We compute $\chi^2$ between the simulated data and the theoretical values

$$\chi^2(\delta^{\mathrm{test}}, \theta_{13}^{\mathrm{test}}) = \sum_{p=1}^{92} \frac{(N_p^{\mathrm{simu}} - N_p^{\mathrm{th}})^2}{\sigma_p^2} \,. \tag{20}$$

We assume, $p = 1$ to 28 are data of Double CHOOZ, $p = 29$ to 46 are data of T2K and $p = 47$ to 92 are data of *NOνA* experiments. $\sigma_p$ is the error in $N_p^{\mathrm{simu}}$. It is the square root of the sum of squares of statistical and systematic errors. In calculating the statistical error, the background contribution to it is taken into account. Following the above procedure, we compute a set of $\chi^2(\delta^{\mathrm{test}}, \theta_{13}^{\mathrm{test}})$ for all allowed values of $\theta_{13}^{\mathrm{test}}$ and $\delta^{\mathrm{test}}$. Since $N_p^{\mathrm{simu}}$ and $N_p^{\mathrm{test}}$ are calculated using different signs of $\Delta_{31}$, we expect $\chi^2$ in Eq. (20) to be large. But $\chi^2$ is a function of $\theta_{13}^{\mathrm{test}}$ and $\delta^{\mathrm{test}}$. Because of the parameter degeneracies discussed in Sec. 4, it is possible to have small $\chi^2$ for $\theta_{13}^{\mathrm{test}} \neq \theta_{13}^{\mathrm{true}}$ and $\delta^{\mathrm{test}} \neq \delta^{\mathrm{true}}$even if $\Delta_{31}$ values have opposite signs in the calculation of $N_p^{\mathrm{simu}}$ and $N_p^{\mathrm{test}}$. In particular, we require $\theta_{13}^{\mathrm{true}}$ to be large enough such that Double CHOOZ will be able to measure its value. If the minimum of $\chi^2(\delta^{\rm test}, \theta_{13}^{\rm test})$ is greater than 4.0, then the two signs of $\Delta_{31}$ are distinguishable at 95% CL for the given values of $\theta_{13}^{\rm true}$ and $\delta^{\rm true}$. If the minimum $\chi^2(\delta^{\rm test}, \theta_{13}^{\rm test})$ is less than 4.0, then the two signs of $\Delta_{31}$ can not be distinguished at 95% CL for the given values of $\theta_{13}^{\rm true}$ and $\delta^{\rm true}$. We repeat the calculation for other values of $\theta_{13}^{\rm true}$ and $\delta^{\rm true}$. We look for values of $\theta_{13}^{\rm true}$ for which the minimum of $\chi^2(\delta^{\rm test}, \theta_{13}^{\rm test})$ is greater than 4.0 for all allowed values of $\delta^{\rm true}$. The minimum of $\theta_{13}^{\rm true}$, for which the above condition is satisfied, is the smallest value of $\theta_{13}$ for which the sign of $\Delta_{31}$ and hence the neutrino mass hierarchy, can be determined irrespective of the value of the CP phase $\delta$.

We assume the Double CHOOZ data [26] are divided into 28 bins. The measurements of the near detector give us the unoscillated neutrino event rate in each bin. The expected measurement in the far detector, for each bin, is given by

$$\frac{dN^{\rm far}}{dE_\nu} = \frac{dN^{\rm near}}{dE_\nu} \times \left(\frac{L^{\rm near}}{L^{\rm far}}\right)^2 P\{(\overline{\nu_e} \to \overline{\nu_e})\}, |\Delta_{31}|, \theta_{13}^{\rm true}, L^{\rm far})\}\,. \tag{21}$$

For the Double CHOOZ experiment, the expected error in the energy measurement is much smaller than the bin size. Therefore, the energy resolution can be taken to be a Dirac delta function. Thus the simulated number of events per bin is given by the above equation.

We see that the T2K data [27] are divided into 18 bins. The expected electron-neutrino event rate, in each bin, is given by

$$\frac{dN_e}{dE_\nu} = \frac{dN_\mu}{dE_\nu} P_{\mu e}^m(+\Delta_{31}, \theta_{13}^{\rm true}, \delta^{\rm true}, L_{T2K})\,. \tag{22}$$

The T2K collaboration estimates the error in reconstructing the neutrino energy to be 100 MeV. We take the energy resolution function $R(E_\nu, E_{\rm meas})$ to be a Gaussian with $\sigma = 100$MeV. We obtain the smeared event rate per bin by

$$\left.\frac{dN_e}{dE_{\rm meas}}\right|_{\rm sim} = \sum \frac{dN_e}{dE_\nu} R(E_\nu, E_{\rm meas}) dE_\nu\,. \tag{23}$$

Finally, we take the $NO\nu A$ [28] data are divided into 46 bins, for each of the off-axis locations 0 mrd, 7 mrd and 14 mrd. We consider one off-axis location at a time. As in the case of T2K, the expected electron-neutrino event rate, in each bin, is given by

$$\frac{dN_e}{dE_\nu} = \frac{dN_\mu}{dE_\nu} P_{\mu e}^m(+\Delta_{31}, \theta_{13}^{\rm true}, \delta^{\rm true}, L_{NO\nu A}). \tag{24}$$

Again, as in the case of T2K, we obtain the smeared event distribution by means of a Gaussian resolution function with $\sigma = 100$ MeV. We assume that both T2K and $NO\nu A$ will run only in neutrino mode for five years. In computing the numbers for $NO\nu A$, we consider various different possibilities, low energy beam with various different off-axis angles and also medium energy beam with various different off-axis angles. The theoretical expectation values are calculated using Eqs. (21), (22) and (24) with $-\Delta_{31}$, $\theta_{13}^{\text{test}}$ and $\delta^{\text{test}}$ as neutrino parameters. Note that no smearing is done in calculating the theoretical expectation values for event numbers.

## 6. *Summary*

In this paper we study the neutrino mass hierarchy. Our results are displayed in Figs. (5), (6) and (7). In each figure, we give a plot of $\chi^2_{\text{min}}$ in the $\theta_{13}^{\text{true}} - \delta^{\text{true}}$ plane. The star symbol represents $\chi^2_{\text{min}} < 4.0$, the square represents $4.0 < \chi^2_{\text{min}} < 9.0$, the triangle represents $9.0 < \chi^2_{\text{min}} < 16.0$ and the circle represents $\chi^2_{\text{min}} > 16.0$. In each figure, the left panel is generated assuming that $NO\nu A$ will run in the low-energy option and right panel is generated assuming high-energy option. Figure (5) corresponds to 0 mrd off-axis location of $NO\nu A$, Fig. (6) corresponds to 7 mrd off-axis location and Fig. (7) corresponds to 14 mrd off-axis location. From the $\chi^2$ analysis, we calculate the minimum value of $\theta_{13}$ for which the sign of $\Delta_{31}$ can be resolved at 95% CL. For $|\Delta_{31}| = 2.5 \times 10^{-3}\,\text{eV}^2$, the low energy option with 0 mrd and 7 mrd off-axis location seem to have the best resolving ability. We repeated our calculation for other allowed values of $|\Delta_{31}|$. In Table 1, we compute the minimum value of $\theta_{13}^{\text{true}}$ for which the sign of $\Delta_{31}$ could be resolved at 95% CL, independent of the CP phase. We consider the 0 mrd, 7 mrd and 14 mrd off-axis angles of $NO\nu A$ experiment for different values of $\Delta_{31}$.

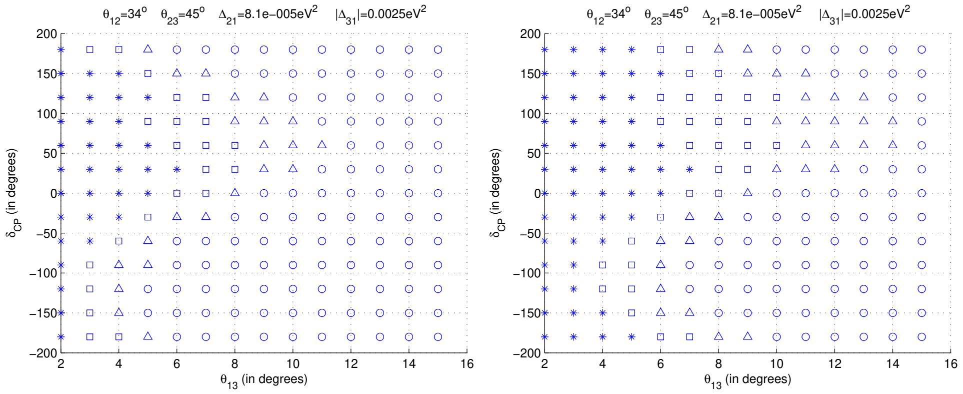


*Fig. 5. Plots of $\chi^2_{\text{min}}$ in $\theta_{13}^{\text{true}} - \delta^{\text{true}}$ plane, 0* mrd *off-axis location with low energy (left) and medium energy (right) options for $NO\nu A$ are assumed.* $|\Delta_{31}| = 2.5 \times 10^{-3}\text{eV}^2$. *The symbols are explained in the text.*

From the table, we see that the minimum value of $\theta_{13}^{\text{true}}$ for which the sign of $\Delta_{31}$ could be resolved at 95% CL independent of the CP phase. This minimum $\theta_{13}^{\text{true}}$ is the same for 0 mrd and 7 mrd off-axis angles of the low energy option of $NO\nu A$.

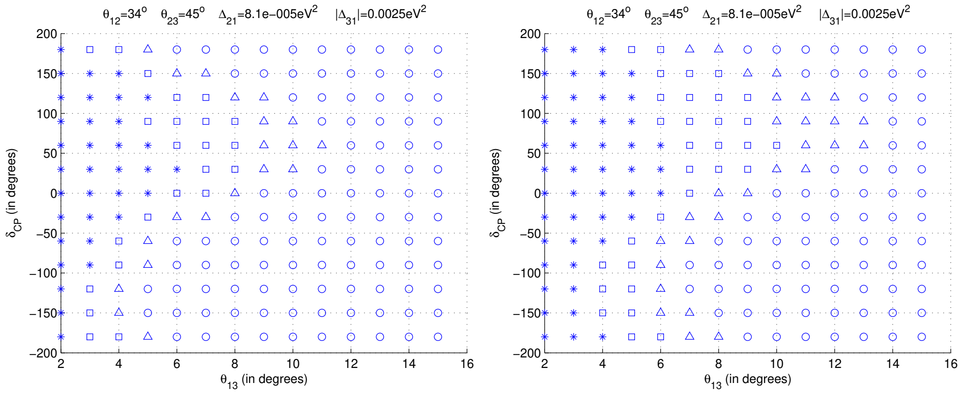


*Fig. 6. Plots of $\chi^2_{\rm min}$ in $\theta_{13}^{\rm true} - \delta^{\rm true}$ plane, 7* mrd *off-axis location with low energy (left) and medium energy (right) options for NO$\nu$A are assumed.* $|\Delta_{31}| = 2.5 \times 10^{-3}{\rm eV}^2$*. The symbols are explained in the text.*

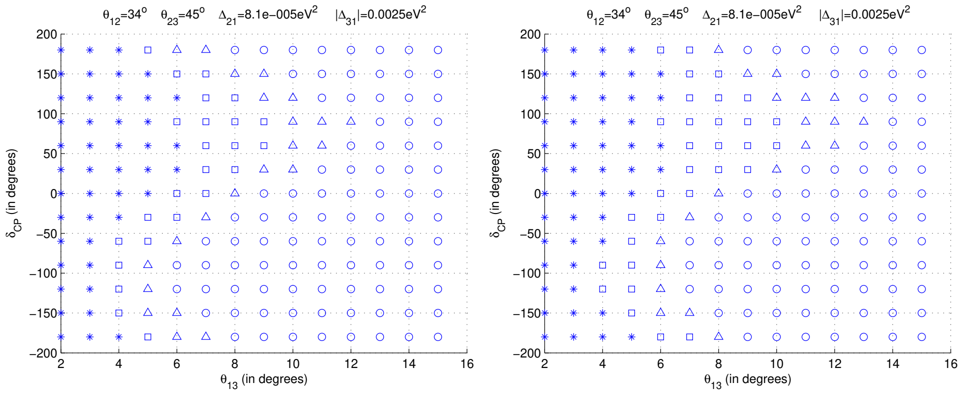


*Fig. 7. Plots of $\chi^2_{\rm min}$ in $\theta_{13}^{\rm true} - \delta^{\rm true}$ plane, 14* mrd *off-axis location with low energy (left) and medium energy (right) options for NO$\nu$A are assumed.* $|\Delta_{31}| = 2.5 \times 10^{-3}{\rm eV}^2$*. The symbols are explained in the text.*

The results are a little worse for the medium-energy option of $NO\nu A$. Determining the type of neutrino mass hierarchy, whether normal or inverted, constitutes one of the fundamental question in neutrino physics. Future long-baseline experiments aim at addressing this fundamental issue, but suffer typically from degeneracies with other neutrino parameters, namely $\theta_{13}$ and $\delta$. The presence of such degeneracies limit the sensitivity to the type of hierarchy. Many earlier studies focused on the determination of the sign of $\Delta_{31}$ by using the data of neutrinos and antineutrinos from more then one experiment [29−33]. In the present paper, we study the possibility of solving the neutrino mass hierarchy using only neutrino data of

*TABLE 1. Minimum value of $\theta_{13}^{\mathrm{true}}$, for which the sign of $\Delta_{31}$ could be resolved at 95% CL, independent of CP phase.*

| $\lvert\Delta_{31}\rvert$ | Minimum value of $\theta_{13}^{\mathrm{true}}$ |
|---|---|
| $1.5 \times 10^{-3}\mathrm{eV}^2$ | $15°$ |
| $2.0 \times 10^{-3}\mathrm{eV}^2$ | $9°$ |
| $2.5 \times 10^{-3}\mathrm{eV}^2$ | $7°$ |
| $3.0 \times 10^{-3}\mathrm{eV}^2$ | $4°$ |
| $3.5 \times 10^{-3}\mathrm{eV}^2$ | $4°$ |
| $4.0 \times 10^{-3}\mathrm{eV}^2$ | $4°$ |

long-baseline experiments T2K and $NO\nu A$ and data from Double CHOOZ. We determined, for each allowed value of $|\Delta_{31}|$, the minimum value of $\theta_{13}$ for which the sign of $\Delta_{31}$ could be resolved, independent of the value of the CP phase. If $|\Delta_{31}| = 0.0025\,\mathrm{eV}^2$, we can rule out the wrong neutrino mass hierarchy at 95 % CL, for the whole range $\delta^{\mathrm{true}} = -180° - 180°$, if $\theta_{13}^{\mathrm{true}} \geq 7.0°$. For larger values of $|\Delta_{31}|$, it is less then 0.002 eV$^2$ and the neutrino mass hierarchy can not be resolved by the data of the above three experiments for any of the allowed values of $\theta_{13}$.

References

[1] R. Davis et al., Phys. Rev. Lett. **20** (1968) 1205.

[2] GALLEX Collaboration, W. Hampal et al., Phys. Lett. B **447** (1999) 127.

[3] GALLEX Collaboration, P. Anselmann et al., Nucl. Phys. Proc. Suppl. **38** (1995) 68.

[4] SAGE Collaboration, J. N. Abdurashitor et al., Nucl. Phys. Proc. Suppl. **118** (2003) 39.

[5] Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **82** (1999) 1810.

[6] Homstake Collaboration, B. T. Clevel et al., Astrophys. J. **496** (1998) 505.

[7] Kamiokande Collaboration, K. S. Hirata et al., Phys. Lett. B **205** (1998) 416.

[8] IMB Collaboration, D. Casper et al., Phys. Rev. Lett. **66** (1991) 2561.

[9] MACRO Collaboration, M. Ambrosio et al., Phys. Lett. B **434** (1998) 451.

[10] KamLAND Collaboration, T. Araki et al., Phys. Rev. Lett. 94 (2005) 081801.

[11] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. Rev. Lett. **45** (1973) 652.

[12] V. Barger, K. Whisnant and R.J.N Phillips, Phys. Rev. Lett. 45 (1980) 2084.

[13] Super-Kamiokande Collaboration, Y. Ashie et al., Phys. Rev. D **71** (2005) 112005.

[14] M. Freund, P. Hunder and M. Lindner, Nucl. Phys. B **615** (2001) 331.

[15] A. Cervera et al., Nucl. Phys. B **579** (2000) 17.

[16] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. **81** (1998) 1562.

[17] W. Hampel, Phys. Lett. B **447** (1999) 127.
[18] CHOOZ Collaboration, C. Apollonio et al., Eur. Phy. J. **27** (2003) 33.
[19] Review of Particle Physics, J. Phys. G **33** (2006) 156.
[20] Double CHOOZ Collaboration, F. Ardellier et al., hep-ex/0606025.
[21] Jun Cao, Nucl. Phys. Proc. Suppl. **155** (2006) 229.
[22] M. Narayan and S. Uma Sankar, Mod. Phys. Lett. A **18** (2003) 569.
[23] L. Wolfenstein, Phys. Rev. D **17** (1978) 2369.
[24] M. Freund, P. Hunder and M. Lindner, Nucl. Phys. B **615** (2001) 331.
[25] H. Minakata et al., Phys. Rev. D **57** (1998) 4403.
[26] Double CHOOZ Collaboration, F. Ardellier et al., arXiv:hep-ex/0405032.
[27] T2K Collaboration, Y. Itow, F. Ardellier et al., arXiv:hep-ex/0106019.
[28] $NO\nu A$ Collaboration, D. S. Ayres et al., arXiv:hep-ex/0503053.
[29] H. Minakata, H. Nunokawa and S. J. Parke, Phys. Rev. D **68** (2003) 3.
[30] V. Barger, D. Marfatia and K. Whisnant, Phys. Lett. B **560** (2003) 75.
[31] M. Narayan and S. Uma Sankar, Mod. Phys. Lett. A **16** (2001) 1881.
[32] M. Ishitsuka et al., Nucl. Phys. Proc. Suppl. **155** (2006) 169.
[33] H. Miniakata et al., Phys. Rev. D **74** (2006) 053008.

## ODREĐIVANJE PREDZNAKA $\Delta_{31}$ BUDUĆIM UDALJENIM REAKTORSKIM MJERENJIMA

Proučavamo određivanje redosljeda neutrinskih masa predviđenim mjerenjima tijekom sljedećih deset godina. Mjerenje neutrinskih oscilacija T2K započinje u 2009. U tom se mjerenju snažan snop $\nu_\mu$ iz JHF usmjerava u Super-Kamiokande (SK) detektor udaljen 295 km. Eksperiment $NO\nu A$ (neutrinske oscilacije izvan osi snopa) priprema se sa snopom $\nu_\mu$ iz Fermi-Lab-a na udaljenosti 610 km i kutovima 0, 7 i 14 miliradijana izvan osi snopa. Oba ova mjerenja određuju oscilatorne vjerojatnosti $\nu_\mu \rightarrow \nu_e$. U gradnji je Double-CHOOZ laboratorij u kojem će se mjeriti $\overline{\nu_e}$ neutrini emitirani iz nuklearnih reaktora u blizom (150 m) i dalekom (1.05 km) detektoru radi određivanja vjerojatnosti "preživljavanja" $\overline{\nu_e} \rightarrow \overline{\nu_e}$. U ovom se radu raspravlja metoda određivanja predznaka $\Delta_{31}$ simuliranim podacima iz gornjih mjerenja.