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DIRAC FUNDAMENTAL QUANTIZATION OF GAUGE THEORIES IS THE NATURAL WAY OF REFERENCE FRAMES IN MODERN PHYSICS

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We analyse two known principal approaches to the quantization of physical models. They are the Faddeev-Popov (FP) "heuristic" approach, based on fixing a gauge in the FP path integrals formalism, and the "fundamental" approach of Dirac based on the constraint-shell reduction of Hamiltonians with deleting of unphysical variables. The relativistically invariant FP "heuristic" approach deals with a small class of problems associated with S-matrices squared considering on-shell quantum fields. On the other hand, the "fundamental" quantization approach of Dirac involves the manifest relativistic covariance of quantum fields that survive the constraint-shell reduction of Hamiltonians. One can apply this approach to a broader class of problems than by studying S-matrices. Investigations of various bound states in QED and QCD are examples of such applications. In the present study, with the example of the Dirac "fundamental" quantization of the Minkowskian non-Abelian Higgs model (studied in its historical retrospective), we show obvious advantages of this quantization approach. The arguments in favour of the Dirac fundamental quantization of a physical model will be presented as a way of Einstein and Galilei relativity in modern physics.

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1. Introduction

The modern gauge physics developed in such a way that the quantization approach by Feynman [1], referred to as *heuristic*, became the main approach to the end of the 60-ies. Calculating radiation corrections to scattering processes, Feynman has elucidated that scattering amplitudes of elementary particles in the perturbation theory do not depend on a reference frame and choice of the gauge[1]. Utilizing this fact, it is possible to alter the QED Lagrangian turning it into a gauge model without constraints. The reference-frame independence (we shall refer to it as the S-invariance in the present study, recently also named *the relativistic invariance*), while *gauge fixing* came to a formal procedure of choice of gauge covariant field variables.

Indeed, an imperceptible substitution of the sense of notions in the method of gauge (G)-covariant and S-invariant heuristic quantization [1] has occurred. An alternative approach to the quantization of gauge (non-Abelian) theories is known [1]. In this approach of Dirac [3], finding the S-covariant and G-invariant solutions to *constraint equations* was proposed. The name *fundamental quantization* to this quantization method was devised by Schwinger [4].

Briefly, the strategy of the fundamental quantization approach [3] is the following.

1. One would utilize the constraint equations and G-invariance in order to remove unphysical variables (degrees of freedom) and construct G-invariant nonlocal functionals of gauge fields, the so-called *Dirac variables* [3]. In particular, it was demonstrated in Ref. [3] by utilizing Dirac variables that solving the QED equations in the class of mentioned nonlocal functionals of gauge fields involves the Coulomb (radiative) gauge for electromagnetic fields.

2. One would also prove the S-covariance on the level of Poincaré generators for G-invariant observables. One of the first proofs is due to Zumino [5]. The dependence of the G-invariant observables on the chosen reference frame parameters is called the *implicit relativistic covariance*.

3. Finally, one would construct the S-covariant S-matrix in terms of G-invariant observables.

This program regarding QED was stated in the review [6]. One can discover that series of well-known facts and conclusions of QED was interpreted therein not as it is customary in modern literature. For instance, the Coulomb field is the precise consequence of solving of one of the classical equations, (*the Gauss law*), but on no account of the large-mass approximation. Herewith the action functional of QED taking in the Coulomb gauge is the "one-to-one" consequence of solving the Gauss law in terms of G-invariant Dirac variables [3] and not (only) the result of choice of a gauge. As an example, a proton and electron in a relativistic atom form this atom due to the Coulomb field transformed into the appropriate Lorentz reference frame and not as a result of an interaction described by additional Feynman diagrams.

After this brief analysis of the fundamental quantization approach [3, 6] it becomes obvious that complete substitution of this approach by the "heuristic" [1] one is not realistic. It would be necessary to prove the relativistic covariance on the level of Poincaré generators for G-invariant observables, if the result of computations for scattering amplitudes is S-invariant, i.e. does not depend on a reference frame. Also the question is what G-invariant observables are necessary if one can utilize various

[1]Indeed, as we shall discuss below, repeating the arguments [2], only scattering amplitudes squared are relativistically invariant.

variables, including those for solving problems constructing the unitary perturbation theory and proving the renormalizibility of the Standard model. Formulating and solving these actual problems implemented in the framework of the heuristic quantization led to the situation when this quantization approach became, in fact, the only method associated with solving of problems in the modern field theory. One forgets, however, that the application the sphere of the heuristic quantization is restricted strictly to the problems of scattering of elementary particles (quantum fields) where this quantization method has arisen [1]. For the needs of study of the bound-state physics, hadronisation and confinement in describing the quantum universe, the fundamental quantization [3, 6] is more adequate, as Schwinger [4] has predicted.

The present study is an attempt to compare in detail both quantization methods: the fundamental and heuristic ones, with regard to the important sphere of modern theoretical physics, the non-Abelian gauge theory (although some aspects of QED, the typical Abelian gauge theory, will be also the subject of our discussion).

The present article is organized as follows. In Sec. 2 we discuss in detail the fundamental and heuristic approaches to quantization of gauge theories. Herewith the Faddeev-Popov (FP) "heuristic" quantization method [7], involving the FP path integrals formalism, as the modern realization of the Feynman approach [1] in the sphere of gauge physics, will be investigated. The principal result of Sec. 2 will be the demonstration that the Feynman rules $(\mathrm{FR})^F$ of the FP path integrals formalism [7] for a gauge model (when a gauge F is fixed) coincide with the Feynman rules $(\mathrm{FR})^*$ of the fundamental quantization formalism [3] only for on-shell quantum fields described correctly by S-matrices. The latter statement may be treated as the *gauge equivalence* (or independence) theorem [8, 9]. On the other hand, because of the manifest relativistic covariance of the Green functions in gauge models quantized by Dirac [3], in which the constraint-shell reduction of appropriate Hamiltonians is performed, various *spurious* Feynman diagrams (SD) [10, 11] appear in those models. As a result, on the level of the heuristic FP quantization [7], the appearance of spurious Feynman diagrams in constraint-shell gauge theories implies, for the on-shell of quantum fields, the modification of the *gauge equivalence* theorem [8, 9] in such a way that the Feynman rules for SD would be added to the Feynman rules $(\mathrm{FR})^F$ for relativistic covariant Green functions. However, when asymptotical states contain composite fields (say, hadronic bound states off-shell) or collective (vacuum) excitations, the *gauge equivalence* theorem [8, 9] between the FP path integrals formalism [7] and Dirac fundamental quantization method [3] becomes very problematic, and one can be sure only in the above adding of the Feynman rules for SD when such states are in question. Violating the gauge equivalence theorem [8, 9] in this case does not mean the gauge non-invariance and relativistic non-covariance. It reflects only the non-equivalence of the different definitions of sources in Feynman and FP path integrals because of nontrivial boundary conditions and residual interactions forming asymptotical composites or collective states.

In Sec. 3, with the example of the Dirac fundamental quantization [3] of the Minkowskian non-Abelian Higgs model (studied in its historical retrospective), we demonstrate obvious advantages of this quantization approach in comparison with the Feynman-FP "heuristic" quantization method [1, 7], when the topologically nontrivial dynamics is taken into account.

In Sec. 4 we discuss the future perspectives of development of the Minkowskian non-Abelian Higgs model quantized by Dirac. It will be argued in favour of the "discrete" vacuum geometry as that justifying various effects associated with the Dirac fundamental quantization [3] of that model.

2. Comparison of the heuristic and fundamental quantization schemes

The essence of the heuristic FP approach [7] to quantization of gauge theories, logically continuing the Feynman method [1], is fixing a gauge (say, $F(A) = 0$) by the so-called *Faddeev trick*: a gauge is fixed in an unique way within a range of the appropriate gauge group (to within the *Gribov ambiguity* in specifying the transverse gauges [12, 13, 14, 15] in non-Abelian gauge theories).

It will be now appropriately to recall some features of the FP heuristic quantization of non-Abelian gauge theories, the important part of modern gauge physics (QCD, the electroweak and standard models). The Gribov ambiguity in non-Abelian gauge theories, considering in the transverse (Landau) gauge [15] $\partial_\mu A_\mu = 0$, comes to FP path integrals regular (nonzero) out of the *Gribov horizon* $\partial\Omega$ [13, 14, 15]. This horizon may be defined [13, 14, 15] as the boundary of the *Gribov region* (in the coordinate space) where the FP operator [15]

$$\Delta_{\rm FP} \equiv \partial_\mu(\partial_\mu \cdot + [A_\mu, \cdot]) \qquad (1)$$

is nonnegative[2].

Thus in non-Abelian gauge theories, in which the transverse Landau gauge $\partial_\mu A_\mu = 0$ is fixed, the appropriate FP path integrals become singular over the light cone $p^2 \equiv -\partial_\mu \partial^\mu$ coinciding with the Gribov horizon $\partial\Omega$ [13, 14, 15].

Finally, the (non-Abelian) FP path integrals for gauge models involving gauge fields A and fermionic ones, ψ and $\bar\psi$, are given by [10]

$$Z^{FP}[s^F, \bar s^F, J^F] = \int \prod_\mu DA^F_\mu D\psi^F D\bar\psi^F \Delta^F_{FP} \delta(F(A^F)) e^{iW[A^F, \psi^F \bar\psi^F] + S^F}, \qquad (2)$$

[2] In general [15], there is a countable number of Gribov regions, C_0, $C_1, \ldots$, in an (Euclidian) non-Abelian gauge theory where the Landau gauge $\partial_\mu A_\mu = 0$ is not taken. Herewith subscript indices $0, 1, \ldots$ denote the numbers of zeros of the FP operator $\Delta_{\rm FP}$ in the appropriate Gribov region.

But with taking the transverse Landau gauge, only the one Gribov region, C_0, survives. The FP operator $\Delta_{\rm FP}$ is positive inside this region, but attains its (infinitely degenerated) zero on its boundary, the Gribov horizon $\partial\Omega$. Herewith it becomes evident that the Gribov horizon $\partial\Omega$ (in the coordinate representation) coincides with the light cone $p^2 \equiv -\partial_\mu\partial^\mu = 0$.

The said may serve as a (perhaps rough, but obvious) description for the Gribov ambiguity [12, 13, 14, 15] in non-Abelian gauge theories.

with $\Delta^F_{\rm FP} \equiv \det M_F$ being the FP operator for the gauge $F(A) = 0$ (in general, different from the transverse Landau gauge $\partial_\mu A_\mu = 0$) and

$$S^F = \int \mathrm{d}^4x \left(\bar{s}^F \psi^F + \bar{\psi}^F s^F + A^F_\mu J^\mu \right) \tag{3}$$

being the sources term[3].

Alternatively, the FP operator M_F may be specified [16, 17] in terms of the linear response of the gauge $F(A) = 0$ to a gauge transformation

$$F(e^\Omega (A + \partial) e^{-\Omega}) = M_F \Omega + O(\Omega^2).$$

The approach [7] to the heuristic quantization of gauge (non-Abelian) theories had, of course, series of its unquestionable services and successes: for instance, in constructing GUT, the universal model of gauge fields. In particular, with the aid of the heuristic approach [7], the renormalizability of GUT was proved.

But an essential shortcoming of the heuristic quantization method [7] was "throwing off" of the notion "reference frame" from gauge physics. This notion is simply not necessary in that the method dealing with scattering amplitudes of quantum fields on-shell[4]. Thus FP path integrals induced by the "heuristic" quantization approach [7] do not depend on anyone's choice of reference frames.

It may be verified that in calculations of elements of S-matrices inherent in the on-shell gauge models, the following obvious identity [19] for the appropriate Feynman rules (FR) takes place

$$(\mathrm{FR})^F = (\mathrm{FR})^* \quad (\text{for } S - \text{matrices}) \tag{4}$$

[3] It will be also well-timed to cite here the explicit expression for the FP determinant M_F in a (non-Abelian) gauge theory. It is [16]

$$\det M_F \quad \sim \quad \int [\mathrm{d}\mathbf{c}][\mathrm{d}\mathbf{c}^\dagger] \exp \left\{ \mathrm{i} \int \mathrm{d}^4x \mathrm{d}^4y \sum_{a,b} c^\dagger_a(x)\ M^{ab}_F(x,y) c_b(y) \right\},$$

with $\mathbf{c}$ and $\mathbf{c}^\dagger$ being, respectively, FP ghost and anti-ghost fields.

The FP determinant $\det M_F$ implies, for instance, the FP ghost action functional $S_{\rm FPG}$ [16],

$$S_{\rm FPG} = \frac{1}{g} \int \mathrm{d}^4x \sum_{a,b} c^\dagger_a(x) \partial^\mu [\delta_{ab} \partial_\mu - g \epsilon_{abc} A^c_\mu] c_b(x),$$

contributing obligatory to the total (non-Abelian) action as the Lorentz covariant gauge $\partial_\mu A^\mu$ is set.

[4] We recommend our readers §2 to Chapter 3 in the monograph [18] where the FP integral for the "exact" YM theory, involving the manifest unbroken $SU(2)$ symmetry and only gauge fields, was derived utilising the properties of the appropriate S-matrix. Indeed, the heuristic quantization approach [7] involves the manifest relativistic invariance of *local* scattering amplitudes squared, $|S_{fi}|^2$, with f and i being, respectively, the final and initial states of colliding particles. However the scattering amplitudes S are, indeed, manifestly relativistically covariant (see e.g. §20.4 in [2]), and this implies their manifest unitarity. On the other hand, probabilities of scattering processes, that would be, doubtless, relativistically invariant values, always involve scattering amplitudes squared.

when the gauge F is fixed.

The expression $(\mathrm{FR})^*$, on the right-hand side of (4), is referred to the Feynman rules in the considered gauge model upon performing the constraint-shell reduction of that model, involving ruling out of the unphysical (manifestly gauge covariant) field variables. This statement may be treated as the *gauge equivalence* (or independence) theorem [8, 9, 19].

But the diapason of problems solved in modern theoretical (in particular, gauge) physics is not restricted to the scattering processes of on-shell quantum fields. Among such problems, one can point out the problem of (asymptotically) bound and collective vacuum states. These are patterns of composite quantum fields that are off-shell of elementary particles. It turns out that the presence of such states in a quantum-field theory (QFT) may violate the gauge equivalence theorem [8, 9, 19], at least it becomes quite problematic in this case. On the other hand, the constraint-shell (Hamiltonian) reduction of a gauge theory implies ruling out of the unphysical fields variables, i.e. describing this gauge theory in terms of only the gauge invariant physical (observable) fields. In the so-called *particular* gauge theories (for instance, in the terminology [20]), examples of which are four-dimensional QED, the YM theory and QCD (i.e. Abelian as well as non-Abelian gauge models), involving the singular Hessian matrix

$$M_{ab} = \frac{\partial^2 L}{\partial \dot{q}^a \partial \dot{q}^b} \tag{5}$$

(with L being the Lagrangian of the considered gauge theory, q^i being the appropriate degrees of freedom and $\dot{q}^i$ being their time derivatives), the removal of unphysical degrees of freedom is associated, in the first place, with ruling out of the temporal components A_0 of gauge fields. In turn, it is associated with the zero canonical momenta $\partial L/\partial \dot{q}^0$ conjugate to the fields A_0 in the particular gauge theories

$$\partial L/\partial \dot{A}_0 \equiv 0.$$

Thus temporal components A_0 of gauge fields are, indeed, non-dynamical degrees of freedom in particular theories, the quantization of which contradicts the Heisenberg uncertainty principle.

Dirac [3], and after him other authors of the first classical studies in quantization of gauge fields, for instance [21, 22], eliminated temporal components of gauge fields by gauge transformations. The typical expression for such gauge transformations is [23]

$$v^T(\mathbf{x},t)(A_0 + \partial_0)(v^T)^{-1}(\mathbf{x},t) = 0. \tag{6}$$

This equation may be treated as that specifying the gauge matrices $v^T(\mathbf{x},t)$. This, in turn, allows to write down the gauge transformations for spatial components of gauge fields [17] (say, in a non-Abelian gauge theory)

$$\hat{A}_i^D(\mathbf{x},t) := v^T(\mathbf{x},t)(\hat{A}_i + \partial_i)(v^T)^{-1}(\mathbf{x},t); \quad \hat{A}_i = g\frac{\tau^a}{2\mathrm{i}}A_{ai}. \tag{7}$$

It is easy to check that the functionals $\hat{A}_i^D(\mathbf{x},t)$ specified in such a way are gauge invariant and transverse fields

$$\partial_i \hat{A}_i^D(\mathbf{x},t) = 0; \quad u(\mathbf{x},t)\hat{A}_i^D(\mathbf{x},t)u(\mathbf{x},t)^{-1} = \hat{A}_i^D(\mathbf{x},t) \tag{8}$$

for gauge matrices $u(\mathbf{x},t)$.

Following Dirac [3], we shall refer to the functionals $\hat{A}_i^D(\mathbf{x},t)$ as to the *Dirac variables*. The Dirac variables $\hat{A}_i^D$ may be derived by resolving the Gauss law constraint

$$\partial W/\partial A_0 = 0 \tag{9}$$

(with W being the action functional of the considered gauge theory).

Solving Eq. (9) [10], one expresses temporal components A_0 of gauge fields A through their spatial components; by that the nondynamical components A_0 are indeed ruled out from the appropriate Hamiltonians. Thus the reduction of particular gauge theories occurs over the surfaces of the appropriate Gauss-law constraints. Only upon expressing temporal components A_0 of gauge fields A through their spatial components one can perform gauge transformations (7) in order to turn spatial components $\hat{A}_i$ of gauge fields into gauge-invariant and transverse Dirac variables $\hat{A}_i^D$ [17]. Thus, formally, temporal components A_0 of these fields become zero. By that the Gauss law constraint (9) acquires the form [10]

$$\partial_0 \left(\partial_i A_i^D(\mathbf{x},t)\right) \equiv 0.$$

For further detailed study of the "technology" getting Dirac variables, in particular gauge theories, we recommend the articles [10, 11, 24] (four-dimensional constraint-shell QED involving electronic currents) and [17, 25] (the Minkowskian non-Abelian Gauss law constraint-shell model involving vacuum BPS monopole solutions; we shall discuss it briefly also in the next section).

Dirac variables prove to be manifestly relativistically covariant. Relativistic properties of the Dirac variables in gauge theories were investigated in the papers [21] (with the reference to the unpublished note by von Neumann), and then this work was continued by I. V. Polubarinov in his review [6].

These investigations displayed that there exist such relativistic transformations of Dirac variables that maintain transverse gauges of fields. More precisely, Dirac variables $\hat{A}^{(0)D}$ observed in a rest reference frame $\eta_\mu^0 = (1,0,0,0)$ in the Minkowski space-time (thus $\partial_i \hat{A}_i^{(0)D} = 0$), in a moving reference frame

$$\eta' = \eta_0 + \delta_L^0 \eta_0 \tag{10}$$

are also transverse, but now regarding the new reference frame η' [10, 23]

$$\partial_\mu \hat{A}_\mu^{D\prime} = 0.$$

In particular, $A_0(\eta^0) = A_0(\eta') = 0$, i.e. the Dirac removal (6) [3, 23] of temporal components of gauge fields, is transferred from the rest to the moving reference frame. In this consideration [6, 10, 21], δ_L^0 are ordinary total Lorentz transformations of coordinates, involving appropriate transformations of fields (bosonic and fermionic). When one transforms fields entering the gauge theory into Dirac variables[5] in a rest reference frame η_0 and then goes over to a moving reference frame η', Dirac variables $\hat{A}^D$, ψ^D, ϕ^D suffer relativistic transformations consisting of two terms [10, 11].

The first term is the response of Dirac variables on ordinary total Lorentz transformations of coordinates (Lorentz busts)

$$x'_k = x_k + \epsilon_k t, \quad t' = t + \epsilon_k x_k, \quad |\epsilon_k| \ll 1.$$

The second term corresponds to the "gauge" Lorentz transformations $\Lambda(x)$ of Dirac variables $\hat{A}^D$, ψ^D, ϕ^D [10, 11]

$$\Lambda(x) \sim \epsilon_k \dot{A}_k^D(x) \Delta^{-1},$$

with

$$\frac{1}{\Delta} f(x) = -\frac{1}{4\pi} \int \mathrm{d}^3 y \frac{f(y)}{|\mathbf{x} - \mathbf{y}|}$$

for any continuous function $f(x)$. Thus any relativistic transformation for Dirac variables may be represented as the sum of two enumerated terms. For instance [10],

$$A_k^D[A_i \quad + \quad \delta_L^0 A] - A_k^D[A] = \delta_L^0 A_k^D + \partial_k \Lambda, \tag{11}$$

$$\psi^D[A \quad + \quad \delta_L^0 A, \psi + \delta_L^0 \psi] - \psi^D[A, \psi] = \delta_L^0 \psi^D + \mathrm{i}e\Lambda(x')\psi^D. \tag{12}$$

Relativistic transformations of Dirac variables of the (11), (12) type imply immediately definite relativistic transformations of Green functions inherent in the constraint-shell (Gauss-shell) gauge theories. For example [11], in the four-dimensional constraint-shell QED the electronic Green function

$$G(p) = G_0(p) + G_0(p)\Sigma(p)G_0(p) + O(\alpha^4), \quad G_0(p) = [p_\mu \gamma^\mu - m]^{-1},$$

with $\Sigma(p)$ being the electronic self-energy, proves to be relativistic covariant under the "gauge" Lorentz transformations $\Lambda(x)$. This, in turn, is mathematically equivalent to the complete Lorentz invariance of the electronic self-energy $\Sigma(p)$ [11]

$$\delta_L^{\mathrm{tot}} \Sigma(p) = (\delta_L^0 + \delta_\Lambda)\Sigma(p) = 0.$$

[5]It may be demonstrated [10, 11, 19, 24] that the transformations (7), turning gauge fields A into Dirac variables $\hat{A}^D$, imply the $\psi^D = v^T(\mathbf{x}, t)\psi$ transformations for fermionic fields ψ and $\phi^D = v^T(\mathbf{x}, t)\phi$ transformations for spin 0 fields: to latter ones belong, for instance, Higgs vacuum BPS monopole solutions investigated in the recent papers [17, 25].

The relativistic covariance of Green functions inherent in constraint-shell gauge theories implies the appearance of various *spurious* Feynman diagrams (SD) in those theories [10, 11]. SD are generated [19] by gauge factors $v^T(\boldsymbol{x}, t)$. On the level of the heuristic FP quantization [7], the appearance of spurious Feynman diagrams in constraint-shell gauge theories implies, on-shell of quantum fields, the modification of the gauge equivalence theorem [8, 9, 19]

$$(\mathrm{FR})^F + (\mathrm{SD}) \equiv (\mathrm{FR})^* \quad \text{(for Green functions)}$$

as a consequence of the independence of FP path integrals (19) on the choice of a reference frame. When, however, asymptotical states contain composite fields (say, hadronic bound states off-shell) or collective (vacuum) excitations, the gauge equivalence theorem (4) [8, 9, 19] becomes problematic, and one may be sure only in the identity

$$(\mathrm{FR})^F + (\mathrm{SD}) \equiv (\mathrm{FR})^* \quad \text{(for } S\text{-matrices with composite fields).} \tag{13}$$

Violating the gauge equivalence theorem [8, 9, 19] in this case does not mean the gauge non-invariance and relativistic non-covariance. It reflects only the non-equivalence of the different definitions of sources in Feynman and FP path integrals because of nontrivial boundary conditions and residual interactions forming asymptotical composite or collective states.

More exactly, with the transverse gauge $F(A) = 0$ fixed (for instance, in the Landau gauge $\partial_\mu A_\mu = 0$ in non-Abelian gauge models), the source term S^F, (3), in the given FP path integral (2) is on-shell of quantum fields. In this case, in the fermionic sector of the considered gauge theory, written down in terms of the FP path integral [7] (foreseeing herewith no constraint-shell reduction), takes place the current conservation law $\partial_0 j_0^F = \partial_i j_i^F$, coinciding mathematically with the one in the Gauss-shell reduced *equivalent unconstrained system* (EUS), $\partial_0 j_0^D = \partial_i j_i^D$ for Dirac variables taking on-shell.

But the current conservation law $\partial_0 j_0^D = \partial_i j_i^D$, derived from the *classical* equations for the fermionic fields, is destroyed for bound states off-shell, i.e. for "dressed" fermions (and moreover, these bound states are "outside the competence" of the heuristic FP method [7]). In this context, the notion "gauge" also concerns the gauge of sources in FP path integrals (2), but not only the choice of definite Feynman rules (that follows from (13)). Since gaugeless (G-invariant) quantization schemes take into account explicitly the whole physical information from (Gauss law) constraints, it is advantageous to use such G-invariant and relativistic (S) covariant approach to describe composite or collective states.

The above sketched quantization scheme by Dirac [3] (often referred to as *fundamental quantization by Dirac* [10, 17, 25]) is the pattern of such G-invariant and S-covariant quantization schemes. As we have made sure above, the Dirac fundamental quantization scheme [3, 6, 10, 11, 21, 24] involves the quantization procedure only for variables remaining on the constraint-shell reduction of appropriate Hamiltonians and spontaneous violation of initial gauge symmetries (when these take place).

Now it will be relevant to cite the explicit form of Feynman path integrals [1, 10, 17, 25], attached to the concrete reference frame (say, the rest reference frame $l^{(0)}$) and written down in terms of the constraint-shell reduced action functionals (EUS) W^*, i.e. in terms of Dirac variables [10]

$$Z^*_{l^{(0)}}[s, \bar{s}^*, J^*] \ = \ \int \prod_j DA^D_j D\psi^D D\bar{\psi}^D \mathrm{e}^{\mathrm{i}W^*[A^D,\psi^D,\bar{\psi}^D]+\mathrm{i}S^*}, \tag{14}$$

including the external sources term

$$S^* \ = \ \int \mathrm{d}^4x \left(\bar{s}^*\psi^D + \bar{\psi}^D s^* + J^*_i A^{Di}\right). \tag{15}$$

The important property of Feynman path integrals is their manifest relativistic covariance [5, 19] with respect to the Heisenberg-Pauli relativistic transformations (10) of the chosen (rest) reference frame η_0 [6, 10, 21] maintaining the transverse gauge of fields. This may be written down as

$$Z_{L\eta_0}[s^*, \bar{s}^*, J^*] = Z_{\eta_0}[L\ s^*, L\ \bar{s}^*, L\ J^*]. \tag{16}$$

To pass then from the Feynman path integral of the form (14) to the FP one, (2), given in the fixed gauge $F(A) = 0$, one would [10]:

∗) replace the variables;

∗∗) replace the sources.

The change of variables is fulfilled by the Dirac factors v^T; for example,

$$A^D_k[A^F] = v^T[A^F](A^F_k + \partial_k)(v^T[A^F])^{-1}; \tag{17}$$

$$\psi^D[A^F] = v^T[A^F]\psi. \tag{18}$$

This change is associated with the countable number of additional degrees of freedom and FP determinant det M_F of the transition to new variables of integration. These degrees may be removed (to within the Gribov ambiguity in non-Abelian gauge theories [12, 13, 14, 15]) by the additional constraint $F(A) = 0$. Thus the *constraint-shell* functional $Z^*_{l^{(0)}}$ (14) takes the equivalent form of the FP path integral [10, 25]

$$Z^*[s^*, \bar{s}^*, J^*] \ = \ \int \prod_\mu DA^F_\mu D\psi^F D\bar{\psi}^F M_F \delta(F(A^F)) \mathrm{e}^{\mathrm{i}W[A^F,\psi^F,\bar{\psi}^F]+S^*}, \tag{19}$$

where now all gauge factors $v^T[A^F]$ are concentrated in the source term [10, 24][6]

$$S^* \ = \ \int \mathrm{d}^4x \ \left(v[A^F]\bar{s}^*\psi^F + \bar{\psi}^F(v[A^F])^{-1}s^* + J^*_i A^*_i[A^F]\right). \tag{20}$$

[6]Indeed [24],

$$\bar{s}^* = \bar{s}^F v^T[A^F]; \quad s^* = (v^T)^{-1}[A^F]\ s^F; \quad \bar{\psi}^D = \bar{\psi}^F \cdot (v^T)^{-1}[A^F].$$

Finally, the removal of the gauge (Dirac) factors $v^T[A^F]$ by the replacement of gauge fields $A^D \Longrightarrow A$, accompanied by the change of sources (the step $**$)),

$$S^* \;\Rightarrow\; S^F = \int \mathrm{d}^4x \left(\bar{s}^F\psi^F + \bar{\psi}^F s^F + A^F_\mu J^\mu\right), \tag{21}$$

restores the initial FP path integral (2) in the considered gauge theory. Such a replacement is made with the only purpose to remove the dependence of the path integrals on a reference frame and initial data. But losing the dependence of a gauge model on any reference frame is often fraught with serious problems for such a gauge model. So, for instance, Schwinger in his paper [4] warned that gauges independent of a reference frame may be physically inadequate to the fundamental operator quantization [3]; i.e. they may distort the spectrum of the original system[7].

The situation with asymptotical bound and collective vacuum states, as discussed in Sec. 2 and involving violating the gauge equivalence theorem [8, 9, 19], confirms this warning by Schwinger.

3. *Dirac fundamental quantization of Minkowskian non-Abelian gauge models*

In this section we give a short historical retrospective of the development of the Dirac fundamental quantization method [3] in the Minkowskian non-Abelian gauge theory. The role of collective vacuum excitations (involving various vacuum rotary effects) in constructing a consistent non-Abelian (Minkowskian) gauge model was considered for the first time in the paper [26].

The case of collective vacuum excitations is just one of cases (13) when the *gauge equivalence* theorem [8, 9, 19] regarding the "heuristic" FP [7] and Dirac fundamental [3] quantization approaches is violated. In the paper [26], it was assumed that in the (Minkowskian) non-Abelian models possessing the strong coupling (YM, QCD), collective vacuum degrees of freedom and long-range correlations of local excitations are possible, similar to those taking place in the liquid helium theory [27]. Moreover, drawing further a parallel between the (Minkowskian) non-Abelian models and liquid helium theory [27], it was concluded about the manifest superfluid properties of the physical vacuum in Minkowskian gauge models (indeed [28], such a conclusion is correct only for a narrow class of Minkowskian gauge models involving vacuum BPS monopole solutions [12, 29, 30] when the initial gauge symmetries are violated and Higgs modes appear as the sign of such breakdown; we shall discuss this below). In [26] it was demonstrated that the manifest superfluid properties of the Minkowskian non-Abelian physical vacuum in such models are quite compatible with the Dirac fundamental quantization [3] involving fixing the Coulomb (transverse) gauge for fields. As a result, non-Abelian gauge fields were transformed into (topologically degenerated) Dirac variables satisfying the Coulomb

[7] "We reject all Lorentz gauge formulations as unsuited to the role of providing the fundamental operator quantization" [4].

gauge: G-invariant and S-covariant simultaneously [10, 11]. The Gribov ambiguity [12, 13, 14, 15] in specifying non-Abelian (transverse) gauge fields induces in the Minkowskian non-Abelian theory the appropriate second-order differential equation in partial derivatives (*the Gribov ambiguity equation* [10, 17, 25]) imposed onto the Higgs field Φ; this equation proves to be responsible for the superfluid properties of the Minkowskian non-Abelian physical (topologically degenerated) vacuum quantized in the Dirac fundamental scheme [3].

This method of constructing the Dirac variables turns the appropriate Gauss law constraint into a homogeneous equation of the form [26]

$$(D^2)^{ab}\Phi_b = 0 \tag{22}$$

(with D being the [covariant] derivative), involving the nontrivial collective vacuum dynamics (more exactly, collective rotations of the Minkowskian non-Abelian vacuum). In the paper [26] it was postulated that the existence of a dynamical variable (denoted as $c(t)$ in [26]) is responsible for this collective vacuum dynamics. The nature of this variable was explained. The possibility to express $c(t)$ through the integer degree of the map (Pontruagin number) by multiplying it by

$$\int\limits_{n(t_{\mathrm{in}})}^{n(t_{\mathrm{out}})} \mathrm{d}t$$

was demonstrated (herewith it may be set [31] $t_{\mathrm{in,\,out}} = \pm T/2$, while interpretion of $c(t)$ as a noninteger degree of map becomes transparent). The necessity to take account of group-theoretical properties of the considered Minkowskian non-Abelian model is the basis for such form of the dynamical cooperative variable $c(t)$ (as well as of other dynamical variables that this model implicates).

This allowed to write down explicitly the term in the YM Lagrangian describing the collective vacuum rotations [26]

$$L_{\mathrm{coop}} = \left[\int \mathrm{d}^3x (D_i\Phi)^2\right] \frac{1}{2}\dot{c}^2(t). \tag{23}$$

The similar nature of the collective vacuum rotations in Minkowskian non-Abelian models and quantum vortices and in a liquid helium specimen was noted (see e.g. §§ 30 − 31 in Ref. [32]). It was shown in Ref. [26] that the collective vacuum rotations (involving the appropriate rotary term L_{coop} (23) in the YM Lagrangian) may be expressed in terms of the Higgs vacuum modes Φ^a, setting the transverse vacuum "electric" field $D_\mu E^\mu = 0$. The connection between the zero modes $Z^a \sim \dot{c}(t)\Phi^a$ of the YM Gauss law constraint and this transverse vacuum "electric" field E was ascertained.

Additionally, it was demonstrated that the *purely real* and simultaneously discrete energy-momentum spectrum

$$P \sim 2\pi k + \theta; \quad k \in \mathbf{Z}; \tag{24}$$

corresponds to the collective vacuum rotations in the Minkowskian non-Abelian theory. This purely real and discrete energy-momentum spectrum is the alternative to the *complex* topological momentum

$$P_{\mathcal{N}} = 2\pi k \pm 8\pi \mathrm{i}/g^2 \equiv 2\pi k + \theta \tag{25}$$

proper (as it was demonstrated in Refs. [26, 33] and then repeated in Ref. [19] (see also Ref. [34]) to the Euclidian θ-vacuum. This result [19, 26, 33] means, as it is easy to see, that topologically degenerated instanton solutions inherent in the Euclidian YM model [12, 16, 35] are purely gauge, i.e. *unphysical and unobservable*, fields. The additional argument in favour of the latter assertion was made recently in Ref. [17]. It was noted that the θ-vacuum plane wave function [26]

$$\Psi_0[A] = \exp(\mathrm{i}P_{\mathcal{N}} X[A]), \tag{26}$$

corresponding to the zero energy $\epsilon = 0$ of an instanton [16, 35] (with $X[A]$ being the winding number functional taking integers), is specified wrongly with the minus sign before $P_{\mathcal{N}}$ in (26). This implies that it is impossible to give the correct probability description of the instanton θ-vacuum [12, 16, 35][8]; that is why the latter one refers to unobservable, i.e. unphysical, values.

In Ref. [19] (see also [36]), the result (25) was referred to as the so-called no-go theorem, the presence of unphysical solutions in the Euclidian instanton YM (non-Abelian) theory [35].

Later on, in Ref. [31], it was explained the common property of cyclical motions (to which belong also the collective vacuum rotations inside the Minkowskian non-Abelian vacuum described in Ref. [26]) that all they possess the discrete energy-momentum spectrum, similar to that described above. This can serve as a definition of the Minkowskian θ-vacuum, somewhat alternative to that of Ref. [36], given for the θ-vacuum in the Euclidian non-Abelian theory [35] involving instantons (the arguments [36] were then repeated in Refs. [19, 26]). The discrete energy-momentum spectra P of cyclical motions found [31] to be, firstly, a purely quantum effect, disappearing in the semi-classical limit $\hbar \to 0$ and, secondly, such motions cannot vanish until $\theta \neq 0$.

Really, in the $\hbar$ terms, the discrete energy-momentum spectra P of cyclical motions may be expressed as [31]

$$P = \hbar \, \frac{2\pi k + \theta}{L},$$

with L being the length of the whole closed line along which a physical material point (say, physical field) moves. Thus when $\theta \neq 0$, the momentum P attains its nonzero minimum $P_{\min} = \hbar\theta/L$ as $k = 0$.

[8] since the Hilbert space of (topologically degenerated) instanton states becomes non-separable in this case.

This is the display of the so-called *Josephson effect* [37] for superconductors included in an electric circuit. The essence of this effect is just in the persistent cyclical motion of a quantum "train" that cannot stop until $\theta \neq 0$ [31].

For the Minkowskian non-Abelian physical vacuum, such Josephson effect comes to the vacuum (transverse) "electric" field E, proving to be a definite function of the appropriate rotary energy-momentum spectrum P. More exactly, $E = f(k, \hbar, \theta)$. This means that E also attains its nonzero minimum value $E_{\min}$ as $k = 0$ and $\theta \neq 0$.

The dependence of $E_{\min}$ on the Planck constant $\hbar$ (this was noted for the first time in Ref. [31]) is connected with the claim for the strong interaction coupling constant to be, indeed, dimensionless; that gives $g^2/(\hbar c)^2$ in the lowest-order of the perturbation theory. In this case [31], the collective rotations term in the non-Abelian action functional proves to be directly proportional to the Planck constant $\hbar$ and disappearing in the (semi)classical limit $\hbar \to 0$ (see below).

As we have already discussed in the previous Section, the general principles for constructing constraint-shell (Gauss-shell) gauge models were stated in Refs. [11, 24]. These general principles (with some corrections, for instance, replacing ∂ by the [covariant] derivative D) may be spread from the four-dimensional constraint-shell QCD to the (Minkowskian) non-Abelian gauge models (including that involving Higgs and fermionic modes and violating initial symmetries groups). A remarkable feature of the constraint-shell reduction of gauge models proves to be the appearance of the current-current instantaneous interaction terms in EUS Hamiltonians. For comparison, in the four-dimensional constraint-shell QCD, the current-current instantaneous interaction term in the appropriate Gauss-shell reduced Lagrangian density $\mathcal{L}^D(x)$ is read as [11]

$$\frac{1}{2} j_0^D \frac{1}{\Delta} j_0^D \tag{27}$$

and implicates G-invariant currents

$$j_\mu^D = e\bar{\psi}^D \gamma_\mu \psi^D.$$

In the (Minkowskian) non-Abelian constraint-shell QCD, the analogy of (27) the "potential" term [38, 39] will be

$$\frac{1}{2} \int\limits_{V_0} \mathrm{d}^3x \mathrm{d}^3y j^b_{\mathrm{tot},(0)}(\mathbf{x}) G_{bc}(\mathbf{x}, \mathbf{y}) j^c_{\mathrm{tot},(0)}(\mathbf{y}). \tag{28}$$

in the constraint-shell reduced QCD Hamiltonian.

This "potential" term involves [38, 39] the topologically trivial and G-invariant total currents

$$j^a_{\mathrm{tot},(0)} = g\bar{\psi}^I (\lambda^a/2)\gamma_0 \psi^I + \epsilon^{abc} \tilde{E}_i^{Tb} \tilde{A}^i_{Tc(0)}, \tag{29}$$

involving fermionic topologically trivial Dirac variables ψ^I, $\bar{\psi}^I$.

In this equation, the transverse "electric" tension $D^i \tilde{E}_i^{Ta} = 0$, belonging to the excitation spectrum of (Minkowskian) constraint-shell QCD, can be expressed [26] through the topologically trivial gauge potentials $\tilde{A}^i_{a(0)}$ (which can be chosen to be also transverse: $D_i \tilde{A}^i_{Ta(0)} = 0$; for instance, the Dirac variables (8))

$$\tilde{E}_i^{(T)a} = (\delta_{ij} - D_i \frac{1}{D^2})^{ab} \partial_0 \tilde{A}_{ib}. \tag{30}$$

The Green function $G_{ab}(\mathbf{x}, \mathbf{y})$ of the Gauss law constraint [10, 17, 25]

$$D_i^{cd}(A) D^i_{db}(\Phi^{(0)}) \tilde{\sigma}^b = j^c_{\mathrm{tot}(0)} \tag{31}$$

enters the "potential term" (28).

The longitudinal "electric" field $\tilde{\sigma}^a$ has the form [10]

$$\sigma^a[A^T, E^T] = (\frac{1}{D_i(A)\partial^i})^{ac} \epsilon_{cbd} A_k^{Tb} E^{Tkd} \tag{32}$$

and involves transverse fields A^T and E^T. This equation reflects the manifest non-linear nature of non-Abelian gauge models of the YM type.

Speaking about Minkowskian constraint-shell QCD, one should note the support of the infrared quark confinement in that model. It turned out that the infrared quark confinement in Minkowskian constraint-shell QCD has actually topological origins.

In Refs. [11, 24], the interference of topological Gribov multipliers [13] in the gluonic and fermionic Green functions in all orders of the perturbation theory was demonstrated. More exactly, the stationary gauge multipliers of the typical form $v^{T(n)}(\mathbf{x})$, depending explicitly on topologies $n \in \mathbf{Z}$, enter *topological Dirac variables* in non-Abelian gauge models

$$v^{T(n)}(\mathbf{x}) = v^{T(n)}(\mathbf{x}, t)|_{t=t_0}. \tag{33}$$

In non-Abelian gauge theories, matrices $v^{T(n)}(\mathbf{x}, t)$ may be found easily, satisfying the Cauchy condition (33) and Eq. (6) [23], specifying the (topological) Dirac variables (7) in these theories quantized by Dirac [3].

As it was shown in Ref. [17, 26],

$$v^{T(n)}(t, \mathbf{x}) = v^{T(n)}(\mathbf{x}) T \exp\{\int_{t_0}^{t} \left[\frac{1}{D^2} \partial_0 D_k \hat{A}^k\right] \, \mathrm{d}\bar{t} \, \}, \tag{34}$$

where the symbol T stands for the time ordering of the matrices under the exponent sign.

Following Ref. [17], one notes the exponential expression in (34) as $U^D[A]$; this expression may be rewritten [17] as

$$U^D[A] = \exp\{\frac{1}{D^2} D_k \hat{A}^k\}. \tag{35}$$

This mechanism [24] of the infrared (at the spatial infinity $|\mathbf{x}| \to \infty$) (*destructive*) interference of Gribov stationary multipliers $v^{T(n)}(\mathbf{x}, t)$ in the gluonic and fermionic Green functions in all the orders of the perturbation theory leads to the following results.

Firstly, one claims [19, 24, 31]

$$v^{T(n)}(\mathbf{x}) \to \pm 1, \quad \text{as} \ \ |\mathbf{x}| \to \infty, \tag{36}$$

This claim imposed onto the Gribov stationary multipliers $v^{T(n)}(\mathbf{x}, t)$ at the spatial infinity is quite natural and legitimate.

So in the Euclidian instanton model [35], the similar spatial asymptotic of gauge matrices is equivalent to disappearing instantons at the "four-dimensional" infinity $|x| \to \infty$ (as it was noted, for instance, by Dashen et al. [35])

$$A_\mu(x) \to 0, \quad |x| \to \infty. \tag{37}$$

As a consequence, the Pontruagin degree of the map [12],

$$n(g) = \frac{1}{24\pi^2} \int\limits_{S^3} \text{tr } \{(g^{-1}(x) dg(x))[g^{-1}(x) dg(x) \wedge g^{-1}(x) dg(x)]\} \mathrm{d}x,$$

involving gauge matrices $g(x)$, takes integer values.

Indeed, disappearing gauge fields (37) at the "four-dimensional" infinity have the universal nature for the Euclidian as well as for the Minkowskian space-time. In particular, the boundary condition (36) [19, 24, 31], imposed onto the Gribov stationary multipliers $v^{T(n)}(\mathbf{x}, t)$ at the spatial infinity in the Minkowskian gauge model (quantized by Dirac [3]), is quite correct.

Secondly, as was shown in Ref. [24], in the lowest order of the perturbation theory, averaging (quark) Green functions over all topologically nontrivial field configurations (including vacuum monopole ones, us discussed below) results in [11, 24]

$$G(\mathbf{x}, \mathbf{y}) = \frac{\delta}{\delta s^*(x)} \frac{\delta}{\delta \bar{s}^*(y)} Z_{\text{conf}}(s^*, \bar{s}^*, J^*)|_{s^* = \bar{s}^* = 0} = G_0(x - y) f(\mathbf{x}, \mathbf{y}), \tag{38}$$

with $G_0(x-y)$ being the (one-particle) quark propagator in the perturbation theory and

$$f(\mathbf{x}, \mathbf{y}) = \lim_{|\mathbf{x}| \to \infty, \ |\mathbf{y}| \to \infty} \lim_{L \to \infty} (1/L) \sum_{n=-L/2}^{n=L/2} v^{(n)}(\mathbf{x}) v^{(n)}(-\mathbf{y}). \tag{39}$$

The origin of the generating functional $Z_{\rm conf}(s^*, \bar{s}^*, J^*)$, entering (38), is following. It comes from the standard FP integral (2) [7] in which one fixes the transverse gauge for (YM) fields A [10, 26]

$$D_i(A)\partial_0 A^i = 0, \tag{40}$$

turning these fields into (topological) Dirac variables of the (7) type.

In this case [20], the FP operator M_F takes the form $\Delta_{\rm FP}$ [14, 15], (1)

$$\hat{\Delta} = -\partial_i D^i(A) = -(\partial_i^2 + \partial_i \ {\rm ad}(A^i)) \tag{41}$$

with

$${\rm ad}(A)X \equiv [A, X]$$

for an element X of the appropriate Lee algebra. Such form of the FP operator is mathematically equivalent to (1) [14, 15] when setting $A_0 = 0$ for temporal components of gauge fields. In particular, it becomes correct for the removal (6) [3, 23] of these components in the Dirac fundamental quantization scheme.

Upon fixing the gauge (40), the FP integral (2) takes the form [24]

$$Z_{R,T}(s^*, \bar{s}^*, J^*) = \int DA_i^* D\psi^* {\rm d}\bar{\psi}^* \ \det \hat{\Delta} \ \delta(\int\limits_{t_0}^{t} {\rm d}\bar{t} D_i(A)\partial_0 A^i)$$

$$\times \exp\{{\rm i} \int\limits_{-T/2}^{T/2} {\rm d}t \int\limits_{|\mathbf{x}|\leq R} d^3x [\mathcal{L}^I(A^*, \psi^*) + \bar{s}^*\psi^* + \bar{\psi}^* s^* + J_i^{*a} A_a^{i*}]\}. \tag{42}$$

It includes the Lagrangian density $\mathcal{L}^I$ [11] corresponding to the constraint-shell action of the Minkowskian non-Abelian theory (Minkowskian QCD) taking on the surface of the Gauss law constraint (9); here R is a large real number, and one can assume that $R \to \infty$.

Thus in the case (42) of the constraint-shell Minkowskian non-Abelian theory, when the transverse gauge (40) is fixed, turning gauge fields into (topological) Dirac variables A^*, this FP integral depends formally on these Dirac variables and also on ψ^*, $\bar{\psi}^*$. Then the generating functional $Z_{\rm conf}(s^*, \bar{s}^*, J^*)$, entering Eq. (38) [24] specifying quark Green functions in Minkowskian QCD involving topologically nontrivial (vacuum) configurations, may be derived from the FP integral (42) by its averaging over the Gribov topological degeneration [13] of initial data, i.e. over the set $\mathbf{Z}$ of integers

$$Z_{\rm conf}(s^*, \bar{s}^*, J^*) = \lim_{|\mathbf{x}|\to\infty, \ T\to\infty} \lim_{L\to\infty} \frac{1}{L} \sum_{n=-L/2}^{n=L/2} Z_{R,T}^I(s^*_{n,\phi_i}, \bar{s}^*_{n,\phi_i}, J^*_{n,\phi_i}), \tag{43}$$

with $Z^I_{R,T}(s^*_{n,\phi_i}, \bar{s}^*_{n,\phi_i}, J^*_{n,\phi_i})$ being the FP path integral (42) rewritten in terms of the Gribov exponential multipliers $v^{T(0)}(\mathbf{x})$[9].

The variation of $Z_{\rm conf}(s^*, \bar{s}^*, J^*)$ by the sources,

$$(\prod_{\alpha=1}^{3} \frac{\delta}{\delta s^*_{n,\phi_\alpha}})(\prod_{\beta=1}^{3} \frac{\delta}{\delta \bar{s}^*_{n,\phi_\beta}})(\prod_{\gamma=1}^{3} \frac{\delta}{\delta J^*_{n,\phi_\gamma}}),$$

involving the appropriate Euler angles $\phi_\alpha(x_\alpha)$ ($\alpha = 1, 2, 3$; x_α, y_α, z_α are the Cartesian coordinates) and topological charges $n \in \mathbf{Z}$, just results the Green functions of the (38) [24] type (in particular, to derive Eq. (38) for fermionic Green functions, it is necessary to omit the variation of $Z_{\rm conf}(s^*, \bar{s}^*, J^*)$ by gauge currents J^*_{n,ϕ_α}).

Returning to Eq. (39), note that always $f(\mathbf{x}, \mathbf{y}) = 1$ due to the spatial asymptotic (36) for the Gribov topological multipliers $v^{T(n)}(\mathbf{x})$. This implies that only "small" (topologically trivial) Gribov exponential multipliers $v^{T(0)}(\mathbf{x})$ contribute in $f(\mathbf{x}, \mathbf{y}) = 1$ and, therefore, in the (one-particle) quark Green function (38). A similar reasoning [24] remains correct also for multi-particle quark and gluonic Green functions in all orders of the perturbation theory.

Just above described only "small" surviving Gribov exponential multipliers $v^{T(0)}(\mathbf{x})$ in Green functions in all orders of the perturbation theory was referred to as the (infrared) *topological confinement* in the series of papers (for instance, in the review [10], multiply cited in the present study).

The new stage in the development of the Minkowskian non-Abelian model quantized in the fundamental scheme by Dirac [3] began in the second half of the 90-ies and continues to date. It is connected with the papers [10, 17, 23, 25, 39]. These papers were devoted to the Dirac fundamental quantization [3] of the Minkowskian non-Abelian theory involving the spontaneous breakdown (say, $SU(2) \to U(1)$) of the initial gauge symmetry and appearing Higgs modes (we shall refer to such theory as to the *Minkowskian Higgs non-Abelian model*).

Now we consider some points of the recent investigations [10, 17, 23, 25, 39] (especially those representing new results in comparison with the "old" research of the Dirac fundamental quantization of the Minkowskian non-Abelian model).

A. *Vacuum BPS monopoles.*

The idea to utilize vacuum BPS monopole solutions [12, 29, 30] for describing Minkowskian Higgs models quantized in the fundamental scheme by Dirac [3] was proposed, probably, already in the paper [19]. In the recent papers [10, 17, 25, 39], this idea becomes the basic one, while in the work [23] the spatial asymptotic

[9]It is correct due to the manifest G-invariance of the constraint-shell theory. For the same reason, the constraint-shell Lagrangian density $\mathcal{L}^I$, entering the FP integral (42), may be expressed solely in terms of topologically trivial Gribov exponential multipliers $v^{T(0)}(\mathbf{x})$ [24].

of vacuum BPS monopole solutions in the shape of Wu-Yang monopoles [40] was studied[10].

Unlike the Wu-Yang monopoles [40] (as analysed above briefly), diverging as $1/r$ at the origin of coordinates, YM vacuum BPS monopole solutions [12, 29, 30] $A_i^a(t,\mathbf{x}) \equiv \Phi_i^{aBPS}(\mathbf{x})$ (in denotations [17]) are regular in the whole spatial volume. Thus a good approximation of Wu-Yang monopoles [40] by YM vacuum BPS monopoles [12, 29, 30] is on hand.

By the way, note that the Euclidian YM instanton solutions A_i^a [35] are also singular at the origin of coordinates (for instance, this was demonstrated in the monograph [12], in §Φ23).

As to the Higgs vacuum BPS monopole solutions $\Phi^a(\mathbf{x})$ [17], they diverge at the spatial infinity although they are regular at the origin of coordinates (like YM BPS monopoles $\Phi_i^{aBPS}(\mathbf{x})$)[11].

The important new step in research of vacuum BPS monopole solutions undertaken in resent papers [17, 25], in comparison with "classical" issues [12, 29, 30], is assuming (topologically degenerated) BPS monopole solutions (in the YM and Higgs sectors of the Minkowskian non-Abelian Higgs model) depending explicitly on the effective Higgs mass $m/\sqrt{\lambda}$ taken in the *Bogomol'nyi-Prasad-Sommerfeld* (BPS) limit [12, 17, 25, 29, 30]

$$m \to 0; \quad \lambda \to 0$$

[10] Wu-Yang monopoles [40] are solutions to the classical equation of motion

$$D_k^{ab}(\Phi_i)F_a^{bk}(\Phi_i) = 0$$

of the "pure" (Minkowskian) YM theory (without Higgs fields). The solution to this classical equation is the "magnetic" tension F_a^{bk} taking the form

$$B^{ia}(\Phi_i) = \frac{x^a x^i}{g r^4}.$$

Thus such "magnetic" tension diverges at the origin of coordinates $r \to 0$, while the spatial YM components

$$\hat{\Phi}_i = -\mathrm{i}\frac{\tau^a}{2}\epsilon_{iak}\frac{x^k}{r^2}f^{\mathrm{WY}}(r),$$

with $f^{\mathrm{WY}}(r) = \pm 1$, correspond to Wu-Yang monopoles [40] with topological charges ± 1, respectively.

Indeed, the above classical equation of motion implies the following equation for the function $f(r)$ [10, 17]:

$$D_k^{ab}(\Phi_i)F_a^{bk}(\Phi_i) = 0 \Longrightarrow \frac{\mathrm{d}^2 f}{\mathrm{d}r^2} + \frac{f(f^2-1)}{r^2} = 0.$$

$f(r) = \pm 1$ just gives Wu-Yang monopoles [40] with topological charges ± 1: $f(r) = f^{\mathrm{WY}}(r)$, while the solution $f(r) \equiv f^{\mathrm{PT}} = 0$ corresponds to the naive unstable perturbation theory, involving the asymptotic freedom formula [41].

[11] Meanwhile, as shown in Ref. [29], the vacuum "magnetic" field $\mathbf{B}$ corresponding to the vacuum YM BPS monopole solutions $\Phi_i^{aBPS}(\mathbf{x})$ diverges as $1/r^2$ at the origin of coordinates, and in this, as the author of the present work recognizes, is a definite problem requiring a solution. Really, in this ultraviolet region of the momentum space, gluons and quarks would be asymptotically free [16, 41], but the $1/r^2$ behaviour of the vacuum "magnetic" field $\mathbf{B}$ hinder this.

for the Higgs mass m and Higgs selfinteraction λ, respectively.

More exactly, this dependence of the vacuum BPS monopole solutions on the effective Higgs mass $m/\sqrt{\lambda}$ becomes dependent on the value ϵ introduced as [17, 25]

$$\frac{1}{\epsilon} \equiv \frac{gm}{\sqrt{\lambda}} \neq 0. \tag{44}$$

Since the YM coupling constant g and Higgs selfinteraction constant λ are dimensionless, ϵ has the dimension of distance. It may be treated as the effective size of the vacuum BPS monopoles and proves to be inversely proportional to the spatial volume V occupied by the (YM- Higgs) field configuration, as was shown in Ref. [17, 25]

$$\frac{1}{\epsilon} = \frac{gm}{\sqrt{\lambda}} \sim \frac{g^2 < B^2 > V}{4\pi}, \tag{45}$$

with $< B^2 >$ being the vacuum expectation value of the "magnetic" field $\mathbf{B}$ set by the Bogomol'nyi equation [12, 17, 25, 29, 30]

$$\mathbf{B} = \pm D\Phi. \tag{46}$$

Thus one can speak that the effective size ϵ of vacuum BPS monopoles is a function of the distance r with the inversely proportional dependence

$$\epsilon(r) \equiv f(r) \sim O(r^{-3}).$$

There is an important physical meaning of the effective size ϵ of the vacuum BPS monopoles and the effective Higgs mass $m/\sqrt{\lambda}$. As follows from (45), the values are some functions of the distance r, and this gives the possibility to utilize them as scale parameters describing renormalization group (RG) properties of the Minkowskian non-Abelian Higgs model (quantized by Dirac [3]). For instance, the effective Higgs mass $m/\sqrt{\lambda}$ may be treated as a *Wegner mass* [42, 43]. The possibility to apply the Bogomol'nyi equation (46) for the Dirac fundamental quantization [3] of the Minkowskian Higgs model was pointed out already in the paper [19].

In recent articles [17, 25, 28], the relation of the Bogomol'nyi equation (46) and vacuum BPS monopole solutions with superfluid properties of the Minkowskian Higgs model quantized by Dirac was noted (as we have pointed out above, such superfluid properties of that model were assumed already in the paper [26][12]). The physical non-Abelian vacuum specified by YM and Higgs vacuum BPS monopoles is described by the Bogomol'nyi equation (46) as a potential superfluid liquid similar to the superfluid component in a liquid helium II specimen [27][13].

On the other hand, although manifest superfluid properties of the Minkowskian Higgs model are proper only at utilizing BPS monopole solutions [12, 17, 25, 29, 30]

[12] This work summarized a series of results [44].

[13] More precisely, one can trace easily a transparent parallel between the vacuum "magnetic" field $\mathbf{B}$ set by the Bogomol'nyi equation (46) and the velocity $\mathbf{v}_s$ [45] of the superfluid motion in

for describing the appropriate (physical) vacuum, the Bogomol'nyi equation (46) is associated, indeed, with the FP "heuristic" quantization [7] of that model. As it was demonstrated for instance in Ref. [12] (in §Φ11), the Bogomol'nyi equation (46) can be derived without solving the YM Gauss law constraint (9), but only evaluating the *Bogomol'nyi bound* [12, 17, 25]

$$E_{\min} = 4\pi \mathbf{m}\frac{a}{g}, \qquad a \equiv \frac{m}{\sqrt{\lambda}} \tag{47}$$

(where $\mathbf{m}$ is the magnetic charge) of the (YM-Higgs) field configuration involving vacuum BPS monopole solutions taken in the BPS limit.

By applying the Dirac fundamental quantization scheme [3] to the Minkowskian non-Abelian Higgs model implicating BPS monopole solutions, the potentiality and superfluidity proper to the physical vacuum of that model are set [46] by the Gribov ambiguity equation, coinciding mathematically with (22) (we have already discussed this at the beginning of the present section). In this case, the relation between the Bogomol'nyi equation (46) and the Gribov ambiguity equation (having the form (22)) is accomplished via the Bianchi identity $D\ B = 0$.

In the papers [10, 17, 25, 39] the solution to the Gribov ambiguity equation was found in the form of the so-called *Gribov phase*

$$\hat{\Phi}_0(r) = -i\pi \frac{\tau^a x_a}{r} f_{01}^{BPS}(r), \quad f_{01}^{BPS}(r) = \left[\frac{1}{\tanh(r/\epsilon)} - \frac{\epsilon}{r}\right]. \tag{48}$$

It is the $U(1) \to SU(2)$ isoscalar "made" of vacuum Higgs BPS monopole solutions.

This allowed to write down explicitly the Gribov topological multipliers $v^{T(n)}(\mathbf{x})$, (33), through the Gribov phase (48)

$$v^{T(n)}(\mathbf{x}) = \exp[n\hat{\Phi}_0(\mathbf{x})]. \tag{49}$$

Thus the immediate relation between the Dirac fundamental quantization [3] of the Minkowskian non-Abelian Higgs model involving vacuum BPS monopole solutions [17, 25] (this quantization comes to topological Dirac variables (7), G-invariant and taking in the transverse gauge (8), as solutions to the YM Gauss law constraint (9)), the Gribov ambiguity equation and the Bogomol'nyi equation (46) (responsible for manifest superfluid properties of that model) was ascertained.

Additionally, the function $f_{01}^{BPS}(r)$ entering the expression for the Gribov phase (48) has the spatial asymptotic

$$f_{01}^{BPS}(r) \to 1 \quad \text{as } r \to \infty,$$

a liquid helium II specimen

$$\mathbf{v}_s = \frac{\hbar}{m}\nabla\Phi(t,\mathbf{r}),$$

with m being the mass of a helium atom and $\Phi(t,\mathbf{r})$ being the phase of the helium Bose condensate wave function $\Xi(t,\mathbf{r})$

$$\Xi(t,\mathbf{r}) = \sqrt{n_0(t,\mathbf{r})}\ \mathrm{e}^{\mathrm{i}\Phi(t,\mathbf{r})},$$

where $n_0(t,\mathbf{r})$ is the number of particles in this helium Bose condensate.

as shown in the paper [39].

Such spatial asymptotic [39] of $f_{01}^{BPS}(r)$, in a good agreement with the boundary condition (36), should be imposed onto Gribov topological multipliers $v^{T(n)}(\mathbf{x})$ at the spatial infinity in order to ensure the infrared topological confinement [24] of topologically nontrivial multipliers $v^{T(n)}(\mathbf{x})$ with $n \neq 0$ in fermionic and gluonic Green functions in all the orders of the perturbation theory.

B. *Specific character of the Josephson effect in the Minkowskian non-Abelian Higgs model.*

In recent papers [10, 17, 23, 25, 39, 41], the following specific features of the Josephson effect proceeding in the Minkowskian non-Abelian Higgs model quantized by Dirac [3] were noted. As demonstrated in Ref. [23], the main manifestation of the Josephson effect [31, 37] in the Minkowskian non-Abelian Higgs model quantized by Dirac is the minimum vacuum "electric" field $\mathbf{E}$ never vanishing if $\theta \neq 0$

$$(E_i^a)_{\min} = \theta \frac{\alpha_s}{4\pi^2 \epsilon} B_i^a; \quad -\pi \leq \theta \leq \pi \tag{50}$$

(with $\alpha_s \equiv g^2/4\pi$).

Such minimum value of the vacuum "electric" field $\mathbf{E}$ corresponds to trivial topologies $k = 0$, while generally [23],

$$F_{i0}^a \equiv E_i^a = \dot{c}(t)\ (D_i(\Phi_k^{(0)})\ \Phi_{(0)})^a = P_c \frac{\alpha_s}{4\pi^2 \epsilon} B_i^a(\Phi_{(0)}) = (2\pi k + \theta) \frac{\alpha_s}{4\pi^2 \epsilon} B_i^a(\Phi_{(0)}). \tag{51}$$

This equation for the vacuum "electric" field $\mathbf{E}$ contains (topologically trivial) Higgs vacuum BPS monopoles $\Phi_{(0)}^a$, whereas the (covariant) derivative $D_i^a(\Phi_k^{(0)})$ is taken in the background of (topologically trivial) YM BPS monopoles $\Phi_k^{a(0)}$.

In the papers [17, 25], vacuum "electric" fields $\mathbf{E}$ were referred to as vacuum "electric" monopoles. Their actual form

$$F_{i0}^a \equiv E_i^a = \dot{c}(t) D_i^{ac}(\Phi_k^{(0)}) \Phi_{(0)c}(\mathbf{x}), \qquad E_i^a \sim D_i^{ac} Z_c \tag{52}$$

was elucidated already in the work [26].

Herewith Eq. (51) [23] for vacuum "electric" monopoles (52) follows immediately from the rotary Lagrangian (23) [10, 17, 19, 23, 25, 26, 39] recast into the action functional [23]

$$W_{\text{coop}} = \int d^4x \frac{1}{2} (F_{0i}^c)^2 = \int dt \frac{\dot{c}^2(t) I}{2}, \tag{53}$$

with

$$I = \int_V d^3x (D_i^{ac}(\Phi_k) \Phi_{(0)c})^2 = \frac{4\pi^2 \epsilon}{\alpha_s} = \frac{4\pi^2}{\alpha_s^2} \frac{1}{V < B^2 >} \tag{54}$$

being the *rotary momentum* of the Minkowskian (YM-Higgs) vacuum. Since actually [31, 34]

$$\alpha_s = \frac{g^2}{4\pi(\hbar c)^2} \, ,$$

it becomes obvious that the rotary momentum I and, therefore, the action functional (53), prove to be directly proportional to the Planch constant squared $\hbar^2$. Thus, in the (semi)classical limit $\hbar \to 0$, collective rotations of the discussed physical BPS monopole vacuum disapear, as it was already noted.

One obtains directly from (53) that

$$P_c \equiv \frac{\partial W_{\rm coop}}{\partial \dot{c}} = \dot{c} I = 2\pi k + \theta. \tag{55}$$

This confirms the general Eq. (24) assumed in [26] for the Minkowskian non-Abelian Higgs theory quantized by Dirac [3] (now in the concrete case of vacuum BPS monopole solutions). In other words, vacuum BPS monopole solutions involve the purely real energy-momentum spectrum of collective rotations associated with the topological dynamical variable $c(t)$.

The important point of Eq. (54) [23] for the rotary momentum I of the Minkowskian (YM-Higgs) vacuum is its direct proportionality to the effective BPS monopole size ϵ, (45). Thus the contribution of the collective (YM-Higgs) vacuum rotations in the total action of the Minkowskian (Gauss-shell) non-Abelian Higgs theory quantized by Dirac is suppressed in the infinite spatial volume limit $V \to \infty$. On the other hand, the presence in (54) of the vacuum expectation value $< B^2 >$ for the "magnetic" field $\mathbf{B}$ is the direct trace of the vacuum BPS monopole solutions associated with the Bogomol'nyi equation (46).

As noted in Ref. [23], the minimum never vanishing (until $\theta \neq 0$) vacuum "electric" field $\mathbf{E}_{\rm min}$ (50), and the (constraint-shell) action functional $W_{\rm coop}$ (53), describing the collective rotations of the physical Minkowskian (YM-Higgs) vacuum, are a specific display of the general Josephson effect [31] in the Minkowskian non-Abelian Higgs model quantized by Dirac. This effect now comes to the "persistent field motion" around the "cylinder" of the effective diameter $\sim \epsilon$, (45). Moreover, repeating the arguments [31] regarding the Josephson effect, it was shown in [23] that

$$\Psi_c(c+1) = {\rm e}^{{\rm i}\theta} \Psi_c(c). \tag{56}$$

This equation reflects the periodicity condition that should be imposed onto the wave function Ψ_c of the physical Minkowskian (YM-Higgs) vacuum at shifts of the topological dynamical variable $c(t)$ on integers $n \in \mathbf{Z}$

$$c(t) \to c(t) + n.$$

Herewith the quantum-mechanical meaning of Eq. (56) is quite transparent: equal probabilities to detect different topologies in the Minkowskian (YM-Higgs) vacuum quantized by Dirac [3]. Then the purely real energy-momentum spectrum (55), (24) of the mechanical rotator (53) can be read also from the periodicity constraint (56). Thus the field theoretical analogy of the Josephson effect [31] was obtained in [23] for the Minkowskian Gauss-shell non-Abelian Higgs theory quantized by Dirac [3].

Coleman et al. [47] were the first who guessed an effect similar to (50) in $\mathrm{QED}_{(1+1)}$, but from a classical point of view.

The quantum treatment of the Josephson effect in (Minkowskian) $\mathrm{QED}_{(1+1)}$ was discussed in the papers [31, 48, 49],[14] and we recommend our readers Refs. [31, 48, 49] for a detailed study of the topic "Minkowskian $\mathrm{QED}_{(1+1)}$".

The explicit expressions for the rotary momentum I and the momentum P_c proper to the physical (YM-Higgs) vacuum quantized by Dirac [3] obtained in the recent papers [10, 17, 23, 25, 39] allowed to derive, in Refs. [17, 25], the form of the vacuum (Bose condensation) Hamiltonian $H_{\rm cond}$ written down over the YM Gauss law constraint (9) surface

$$H_{\rm cond} = \frac{2\pi}{g^2\epsilon}[P_c^2(\frac{g^2}{8\pi^2})^2 + 1]. \tag{57}$$

This Hamiltonian contains the "electric" and "magnetic" contributions.

The "electric" contribution to the constraint-shell Bose condensation Hamiltonian (57) [17, 25] is determined by the rotary action functional (53) (associated with vacuum "electric" monopoles (51) − (52)), while the "magnetic" contribution in this Hamiltonian is [23]

$$\frac{1}{2}\int\limits_{\epsilon}^{\infty} \mathrm{d}^3x[B_i^a(\Phi_k)]^2 \equiv \frac{1}{2}V < B^2 >= \frac{1}{2\alpha_s}\int\limits_{\epsilon}^{\infty}\frac{\mathrm{d}r}{r^2} \sim \frac{1}{2}\frac{1}{\alpha_s\epsilon} = 2\pi\frac{gm}{g^2\sqrt{\lambda}} = \frac{2\pi}{g^2\epsilon}. \tag{58}$$

[14] In particular, it was demonstrated in Ref. [48] that the Josephson effect in (Minkowskian) $\mathrm{QED}_{(1+1)}$ comes to circular motions of topologically degenerated gauge fields $A_1^{(n)}(x,t)$ around the circle S^1 of the infinite radius. Herewith such closed trajectories of infinite radii are the result identifying [48] points

$$A_1^{(n)}(x,t) = \exp(\mathrm{i}\Lambda^{(n)}(x))(A_1(x,t) + \mathrm{i}\frac{\partial_1}{e})\exp(-\mathrm{i}\Lambda^{(n)}(x)), \quad n \in \mathbf{Z},$$

in the $\mathrm{QED}_{(1+1)}$ configuration space $\{A_1(x,t)\}$ at the spatial infinity. Here $\Lambda^{(n)}(x)$ are $U(1)$ gauge matrices possessing the spatial asymptotic [31, 48]

$$\Lambda^{(n)}(x) = \hbar\ 2\pi n\frac{x}{\pm R}$$

(with R standing for the spatial infinity). The immediate manifestation of the Josephson effect in $\mathrm{QED}_{(1+1)}$ [48, 49], involving identifying points, at the spatial infinity, in the field configuration $\{A_1(x,t)\}$, is the existence of the vacuum electric field

$$G_{10} = \dot{c}(t)\frac{2\pi}{e} = e(\frac{\theta}{2\pi} + k),$$

that again never vanishes until $\theta \neq 0$ (this equation was derived by Coleman et al. [47])

This "magnetic" contribution is associated with the Bogomol'nyi equation (46).

The remarkable feature of the constraint-shell Bose condensation Hamiltonian (57) is its manifest Poincaré (in particular, CP) invariance stipulated by the topologically momentum squared, P_c^2, entering this Hamiltonian.

This result for the Bose condensation Hamiltonian (57) in the Minkowskian non-Abelian Higgs model quantized by Dirac [3] is an alternative to the so-called θ-term [16] arising in the effective Lagrangian of the Euclidian instanton non-Abelian theory [35],

$$\mathcal{L}_{\rm eff} = \mathcal{L} + \frac{g^2\theta}{16\pi^2}\ {\rm tr}\ (F^a_{\mu\nu}\tilde{F}^{\mu\nu}). \tag{59}$$

This effective Lagrangian of the Euclidian instanton non-Abelian theory [35] is directly proportional to the pseudomentum θ, and this determines the Poincaré (CP) covariance of the Euclidian instanton non-Abelian theory, worsening its renormalization properties.

This Poincaré (CP) covariance of the Euclidian instanton effective Lagrangian (59) [16] is the essence of the instanton CP-problem, and may be avoided in the Minkowskian non-Abelian Higgs model quantized by Dirac (as we see this with the example of the Poincaré invariant Bose condensation Hamiltonian (57) [17, 25] of that model.

Generally speaking, the manifestly Poincaré invariance of the constraint-shell vacuum Hamiltonian (57) in the Minkowskian non-Abelian Higgs model quantized by Dirac [3] is somewhat a paradoxical thing in the light of the S (relativistic) covariance [10] (11), (12) of topological Dirac variables (7).

Indeed, the manifest Poincaré invariance of the constraint-shell vacuum Hamiltonian (57) is due absorbing [16] Gribov topological multipliers $v^{T(n)}(\mathbf{x})$ in the G-invariant YM tension tensor squared $(F^a_{\mu\nu})^2$.

C. *Rising "golden section" potential of the instantaneous interaction.*

As demonstrated in Refs. [10, 17, 23, 25, 39], in the YM BPS monopole background (turning into the Wu-Yang monopole background [40] at the spatial infinity), the Green function $G_{ab}(\mathbf{x},\mathbf{y})$ entering the "potential" term (28) [38, 39] in the constraint-shell Hamiltonian of the Minkowskian non-Abelian Higgs model (quantized by Dirac [3]) may be decomposed into the complete set of orthogonal vectors in the colours space

$$G^{ab}(\mathbf{x},\mathbf{y}) = [n^a(x)n^b(y)V_0(z) + \sum_{\alpha=1,2} e^a_\alpha(x)e^{b\alpha}(y)V_1(z)]; \quad (z=|\mathbf{x}-\mathbf{y}|). \tag{60}$$

This equation involves two instantaneous interaction potentials: $V_0(z)$ and $V_1(z)$.

The first of the potentials, $V_0(z)$, proves to be the Coulomb-type potential

$$V_0(|\mathbf{x}-\mathbf{y}|) = -1/4\pi\ |\mathbf{x}-\mathbf{y}|^{-1} + c_0, \tag{61}$$

where c_0 is a constant.

The second potential, $V_1(z)$, is the so-called "golden section" potential

$$V_1(|\mathbf{x}-\mathbf{y}|) = -d_1|\mathbf{x}-\mathbf{y}|^{-1.618} + c_1|\mathbf{x}-\mathbf{y}|^{0.618}, \qquad (62)$$

involving constants d_1 and c_1[15].

The "golden section" potential (62) (unlike the Coulomb potential, (61)) implies the rearrangement of the naive perturbation series and the spontaneous breakdown of the chiral symmetry. In turn, this involves the constituent gluonic mass in the Feynman diagrams: this mass changes the asymptotic freedom formula [41] in the region of low transferred momenta. Thus the coupling constant $\alpha_{QCD}(q^2 \sim 0)$ becomes finite. The "golden section" potential (62) can be also considered as an origin of "hadronization" of quarks and gluons in QCD [10, 38, 50, 51].

D. *Solving the $U(1)$-problem.*

The Dirac fundamental quantization [3] of the Minkowskian non-Abelian Higgs model may be adapted to solving the $U(1)$-problem, i.e. finding the η'-meson mass near to modern experimental data[16].

As demonstrated in recent papers [10, 17, 23, 25, 39], the way to solve the $U(1)$-problem in the Minkowskian non-Abelian Higgs model quantized by Dirac is associated with the manifest rotary properties of the appropriate physical vacuum involving YM and Higgs BPS monopole solutions. The principal result obtained in the works [10, 17, 23, 25, 39] regarding the solving of the $U(1)$-problem in the Minkowskian non-Abelian Higgs model quantized by Dirac is the following.

The η'-meson mass $m_{\eta'}$ proves to be inversely proportional to $\sqrt{I}$, where the rotary momentum I of the physical Minkowskian (YM-Higgs) vacuum is given by Eq. (54) [23]

$$m_{\eta'} \sim 1/\sqrt{I}.$$

More precisely,

$$m^2_{\eta'} \sim \frac{C^2_\eta}{IV} = \frac{N_f^2}{F_\pi^2}\frac{\alpha_s^2 < B^2 >}{2\pi^3}, \qquad (63)$$

involving a constant $C_\eta = (N_f/F_\pi)\sqrt{2/\pi}$, where F_π is the pionic decay constant and N_f the number of flavours in the considered Minkowskian non-Abelian Higgs model.

[15] Specifying constants d_1, c_0 and c_1, entering the potentials V_1 and V_0, respectively, is, indeed, a very important thing. These constants can depend, for instance, on a flavours mass scale m_f or the temperature T of surroundings about the system of quantum fields that the investigated Minkowskian non-Abelian Higgs model includes. The author is grateful personally to Prof. D. Ebert who has drawn his attention to the necessity to select correctly constants entering expressions for instantaneous interaction potentials (this was during L. L. visit of Alexander von Humboldt University Berlin in August 2002).

[16] New experimental data for the η'-meson mass give $m_{\eta'} \sim 957,57$ MeV (see, e.g. the reference book [52]).

The explicit value (54) of the rotary momentum I of the physical Minkowskian (YM-Higgs) vacuum was substituted in this equation for the η'-meson mass $m_{\eta'}$. The result (63) for the η'-meson mass $m_{\eta'}$ is given in Refs. [10, 17, 23, 25, 39] for the Minkowskian non-Abelian Higgs model quantized by Dirac [3] and implemented vacuum BPS monopole solutions allow to estimate the vacuum expectation value of the appropriate "magnetic" field **B** (specified in that case via the Bogomol'nyi equation (46))

$$< B^2 >= \frac{2\pi^3 F_\pi^2 m_\eta{}^2}{N_f^2 \alpha_s^2} \tag{64}$$

by using estimated $\alpha_s(q^2 \sim 0) \sim 0.24$ [23, 50].

One can assert analysing the results obtained in Refs. [10, 17, 23, 25, 39] concerning the η'-meson mass and estimating the vacuum "magnetic" field **B** with $< B^2 > \neq 0$, that going over to the Dirac fundamental quantization scheme [3] from the "heuristic" one [7] when considering the Minkowskian non-Abelian Higgs model is quite justified by the realistic results near to the new experimental data (see, for instance, [52]).

In particular, the crucial role of the collective solid rotations (53), (54) [23] inside the physical Minkowskian (YM-Higgs) vacuum (they are a direct display of the Dirac fundamental quantization [3] of the Minkowskian non-Abelian Higgs theory) in Eq. (63) for the η'-meson mass and Eq. (64) for $< B^2 >$ is highly transparent and impressing[17].

[17]It is worth to recall here two alternative "answers" to the question about the mass of the η'-meson that were given based on the Euclidian non-Abelian theory [35] involving instantons.

It is, firstly, the "massless variant" given in the paper [53]. This variant was associated with maintaining the θ-angle dependence in the effective Lagrangian $\mathcal{L}_{\rm eff}$ [16], (59), in the Euclidian non-Abelian instanton QCD. In this case, the θ-angle is covariant under chiral rotations [16]

$$\theta \to \theta' = {\rm e}^{{\rm i}\alpha Q_5}\theta$$

(involving the axial charge $Q_5 = \psi\gamma_0\gamma_5\bar{\psi}$ and an arbitrary parameter α), and small oscillations around the given θ-angle correspond to a massless and unphysical fermion implying the *Kogut-Suskind pole* [53] in the appropriate propagator. The diametrically opposite answer, in comparison with [53], to the question about the mass of the η'-meson was given in the paper [54], resting on the analysis of *planar diagrams* for the strong interaction, in turn worked out in the paper [55], and the ABJ-anomalies theory [16, 56].

The principal idea of the work [54] was deleting the θ-angle dependence from the effective QCD Lagrangian in the Euclidian non-Abelian instanton theory [35].

As a result, the nonzero mass of the η'-meson was obtained in the work [54]. This was one of early approaches to solving the $U(1)$-problem in which arguments in favour of the mesonic mass were given.

Unfortunately, general shortcomings of the Euclidian non-Abelian instanton theory [35] (for instance, the purely imaginary energy-momentum spectrum $P_{\mathcal{N}}$ [19, 26, 33, 34], (25), at the zero eigenvalue $\epsilon = 0$ of the θ-vacuum energy) turn the Euclidian methods [53, 54] to specify the η'-meson mass into little effective ones. This forced to search for other ways to construct mesonic bound states than the ones [53, 54] proposed in the Euclidian non-Abelian theory [35].

In Refs. [10, 17, 23, 25, 39], just such "another way" to solve the $U(1)$-problem, based on the Minkowskian non-Abelian Higgs model quantized by Dirac [3] and involving the vacuum BPS monopole solutions was proposed.

E. *Fermionic rotary degrees of freedom in the Wu-Yang monopole background.*

A good analysis of the question about the place of fermionic (quark) degrees of freedom in the Minkowskian non-Abelian Higgs model quantized by Dirac [3] was made in the recent papers [23, 39].

For instance, as we have seen above, G-invariant fermionic currents [10]

$$j_\mu^{Ia} = g\bar{\psi}^I(\lambda^a/2)\gamma_\mu\psi^I, \tag{65}$$

belonging (as defined in Ref. [39]) to the excitation spectrum over the physical Minkowskian (YM-Higgs) vacuum involving the vacuum BPS monopole solutions, enter total G-invariant currents (29) [39], satisfying the Gauss law constraint (31) [17, 39].

New interesting properties acquire fermionic (quark) degrees of freedom ψ^I, $\bar{\psi}^I$ in Minkowskian constraint-shell QCD involving the spontaneous breakdown of the initial $SU(3)_{\rm col}$ gauge symmetry in the

$$SU(3)_{\rm col} \to SU(2)_{\rm col} \to U(1) \tag{66}$$

way. Actually, such Minkowskian constraint-shell QCD is the particular case of the Minkowskian non-Abelian Higgs models quantized by Dirac [3].

The only specific of Minkowskian constraint-shell QCD (in comparison with the constraint-shell Minkowskian (YM-Higgs) theory) is the presence of three Gell-Mann matrices λ^a, generators of $SU(2)_{\rm col}$ (just these matrices would enter G-invariant quark currents j_μ^{Ia} in Minkowskian constraint-shell QCD). In the constraint-shell Minkowskian (YM-Higgs) theory, involving the initial $SU(2)$ gauge symmetry, the Pauli matrices τ^a ($a = 1, 2, 3$) would replace the Gell-Mann λ^a ones.

The very interesting situation, implying many important consequences, takes place to be in Minkowskian constraint-shell QCD involving the spontaneous breakdown (66) of the initial $SU(3)_{\rm col}$ gauge symmetry when the antisymmetric Gell-Mann matrices

$$\lambda_2, \lambda_5, \lambda_7 \tag{67}$$

are chosen to be the generators of the $SU(2)_{\rm col}$ subgroup in (66), as it was done in Refs. [10, 23, 39].

As demonstrated in Ref. [23], the "magnetic" vacuum field $B^{ia}(\Phi_i)$ corresponding to Wu-Yang monopoles Φ_i [40] acquires the form

$$b_i^a = \frac{1}{g}\epsilon_{iak}\frac{n_k(\Omega)}{r}; \quad n_k(\Omega) = \frac{x^l\Omega_{lk}}{r}, \quad n_k(\Omega)n^k(\Omega) = 1; \tag{68}$$

in terms of the antisymmetric Gell-Mann matrices $\lambda_2, \lambda_5, \lambda_7$, (67), with Ω_{lk} being an orthogonal matrix in the colour space.

For the "antisymmetric" choice (67), we have

$$b_i \equiv \frac{g}{2i} b_{ia} \tau^a = g \frac{b_i^1 \lambda^2 + b_i^2 \lambda^5 + b_i^3 \lambda^7}{2i}; \qquad b_i^a = \frac{\epsilon^{aik} n^k}{gr} \quad (\tau_1 \equiv \lambda_2, \tau_2 \equiv \lambda_5, \tau_3 \equiv \lambda_7). \tag{69}$$

On the other hand, the important task that Minkowskian constraint-shell QCD is called to solve is getting spectra of mesonic and baryonic bound states. As we have noted in Section 2, the presence of such hadronic bound states in a gauge model violates the gauge equivalence theorem [8, 9, 19]. As in the case of collective vacuum excitations, this implies the identity (13), involving spurious Feynman diagrams (SD).

A detailed analysis how to apply the Dirac fundamental quantization method [3] to constructing hadronic bound states was performed in the papers [57, 58], and then such analysis was repeated in Ref. [10]. The base of the approach to constructing hadronic bound states that was proposed in [10, 57, 58] is the so-called *Markov-Yukawa prescription* [59], the essence of which is [10, 59] in separating *absolute*, $X_\mu = (x+y)_\mu/2$ and *relative*, $z_\mu = (x-y)_\mu$, coordinates, involving treatment of (mesonic) bound states as *bilocal fields*

$$\mathcal{M}(x,y) = \mathrm{e}^{\mathrm{i} M X_0} \psi(z_i) \delta(z_0). \tag{70}$$

The important feature of such bilocal fields is observing two particles (say, same quark q and antiquark $\bar{q}$) as a bound state at one and the same time.

This principle of the simultaneity has more profound mathematical meaning [10, 51] as the constraint of irreducible nonlocal representations of the Poincaré group for arbitrary bilocal field $\mathcal{M}(x,y) \equiv \mathcal{M}(z|X)$ is

$$z_\mu \frac{\partial}{\partial X_\mu} \mathcal{M}(z|X) = 0, \qquad \mathcal{M}(z|X) \equiv \mathcal{M}(x,y). \tag{71}$$

This constraint is not connected with the dynamics of interaction and the Eddington simultaneity[18].

Thus the constraint (71) results in the choice of the bound state relative coordinates z_μ to be orthogonal to its total momentum $\mathcal{P}_\mu \equiv -i \frac{\partial}{\partial X_\mu}$

$$(z^\perp)_\mu = z_\mu - \mathcal{P}_\mu \left(\frac{\mathcal{P} \cdot z}{\mathcal{P}^2} \right). \tag{72}$$

Moreover, at the point of the forming of the bound state with a definite total momentum $\mathcal{P}_\mu$, it is possible to choose the time axis $\eta_\mu = (1,0,0,0)$ to be parallel to its total momentum, $\eta_\mu \sim \mathcal{P}_\mu$.

Therefore, [10]

$$\eta_\mu \mathcal{M}(z|X) \sim \mathcal{P}_{A\mu} \mathcal{M}(z|X) = \frac{1}{\mathrm{i}} \frac{\partial}{\partial X_\mu} \mathcal{M}(X|z). \tag{73}$$

[18] "A proton yesterday and electron today do not make an atom" [60].

In the rest reference frame η_μ chosen in the (73) way, the instantaneous interaction between particles forming the given bilocal bound state $\mathcal{M}(X|z)$ takes the form [10]

$$W_I = \int \mathrm{d}^4x\mathrm{d}^4y \frac{1}{2} j_\eta^D(x) V_I(z^\perp) j_\eta^D(y) \delta(\eta \cdot z). \tag{74}$$

This equation involves manifestly G-invariant and S-covariant fermionic currents

$$j_\eta^D = e\bar{\psi}^D \not{\eta} \psi^D; \quad \not{\eta} \equiv \eta_\mu \gamma^\mu$$

(attached to the rest reference frame η_μ chosen in the (73) way and implicating fermionic Dirac variables ψ^D, $\bar{\psi}^D$). $V_I(z^\perp)$ is the instantaneous interaction potential between the particles forming the bilocal bound state $\mathcal{M}(X|z)$. The manifest S-covariance of the constraint-shell action functional (74) follows immediately from the transformation law (12) [10] for fermionic Dirac variables ψ^D, $\bar{\psi}^D$.

Incidentally, note that upon extracting G-invariant fermionic currents j_μ^{aI} (65) from the total ones [39] (29), it is possible to write down the constraint-shell action functional of the (74) type for the Minkowskian Higgs model with vacuum BPS monopole solutions describing the instantaneous interaction between these fermionic currents, attached to the rest reference frame η_μ (73) and involving the Green function $G_{ab}(\mathbf{x}, \mathbf{y})$ of the Gauss law constraint (31) [17, 25, 39]. It may, in turn, be decomposed in the (60) way, implicating the Coulomb type potential $V_0(z)$, (61), and the "golden section" one, $V_1(z)$, (62).

In the papers [10, 57, 58], the algorithm is given for the derivation of mesonic bound-state spectra utilizing the Markov-Yukawa prescription [59], as outlined above. Omitting details of this algorithm and referring our readers to Ref. [10, 57, 58] (with the literature cited therein) for the detailed acquaintance with the question, now note that the important step of this algorithm is solving of the Dirac equation for a fermion (quark) in the BPS (Wu-Yang) monopole background.

For the spontaneous breakdown of the initial $SU(3)_{\mathrm{col}}$ gauge symmetry in the (66) way, involving antisymmetric Gell-Mann matrices λ_2, λ_5, λ_7 as generators of the "intermediate" $SU(2)_{\mathrm{col}}$ gauge symmetry, this BPS (Wu-Yang) monopole background takes the form (68) [23]. To write down the Dirac equation for a quark in the BPS (Wu-Yang) monopole background, note that each fermionic (quark) field may be decomposed by the complete set of the generators of the Lee group $SU(2)_{col}$ (i.e. $\lambda_2, \lambda_5, \lambda_7$ in the considered case) completed by the unit matrix $\mathbf{1}$ [16]. This involves the following decomposition [23] of a quark field by the antisymmetric Gell-Mann matrices λ_2, λ_5, λ_7

$$\psi_\pm^{\alpha,\beta} = s_\pm \delta^{\alpha,\beta} + v_\pm^j \tau_j^{\alpha,\beta}, \tag{75}$$

involving some $SU(2)_{\mathrm{col}}$ isoscalar, $s_\pm$ and isovector, $v_\pm$, amplitudes. $+, -$ are spinor indices, α, β are $SU(2)_{\mathrm{col}}$ group space indices and

$$(\lambda_2, \lambda_5, \lambda_7) \equiv (\tau_1, \tau_2, \tau_3).$$

The mix of group and spinor indices generated by Eqs. (68), (69) for the BPS (Wu-Yang) monopole background allows then to derive, utilising the decomposition (75), of the system of differential equations in partial derivatives [23]

$$(\mp q_o + m)s_\mp \mp i(\partial_a + \frac{n_a}{r})v^a_\pm = 0; \tag{76}$$

$$(\mp q_o + m)v^a_\mp \mp \mathrm{i}(\partial^a - \frac{n^a}{r})s_\pm - \mathrm{i}\epsilon^{jab}\partial_j v^b_\pm = 0 \tag{77}$$

(implicating the mass m of a quark and its complete energy q_0), mathematically equivalent to the Dirac equation

$$\mathrm{i}\gamma_0\partial_0\psi + \gamma_j[\mathrm{i}\partial_j\psi + \frac{1}{2r}\tau_a\epsilon^{ajl}n_l\psi] - m\psi = 0 \tag{78}$$

for a quark in the BPS (Wu-Yang) monopole background.

The decomposition (75) [23] of a quark field implies [61] that $v^j_\pm\tau_j^{\alpha,\beta}$ is a three-dimensional axial vector in the colour space. Thus the spinor (quark) field $\psi^{\alpha,\beta}_\pm$ is transformed, with the "antisymmetric" choice $\lambda_2, \lambda_5, \lambda_7$, by the *reducible* representation of the $SU(2)_{col}$ group that is the direct sum of the identical representation **1** and three-dimensional axial vector representation, we denote as $\mathbf{3}_{ax}$.

A new situation, in comparison with the usual $SU(3)_{\rm col}$ theory in the Euclidian space E_4 [16], arises in this case. That theory was worked out by Greenberg [62] Han and Nambu [63, 64]; its goal was getting the hadronic wave functions (describing bound quark states) with the correct spin-statistic connection. To achieve this, the *irreducible* colour triplet (i.e. three additional degrees of freedom of quark colours, forming the polar vector in the $SU(3)_{\rm col}$ group space), was introduced. It was postulated that only colour singlets are physically observable states. So the task of the colours confinement was outlined.

The transition to the Minkowski space in Minkowskian constraint-shell QCD quantized by Dirac [3] and involving the (66) breakdown of the $SU(3)_{col}$ gauge symmetry, the antisymmetric Gell-Mann matrices λ_2, λ_5, λ_7 and BPS (Wu-Yang) physical background, allows to introduce the new, reducible, representation of the $SU(2)_{\rm col}$ group with axial colour vector and colour scalar.

In this situation, the question about the physical meaning of the axial colour vector $v^j_\pm\tau_j^{\alpha,\beta}$ is posed.

For instance, it may be assumed that the axial colour vector $v^j_\pm\tau_j^{\alpha,\beta}$ has the form $\mathbf{v}_1 = \mathbf{r}\times\mathbf{K}$, where $\mathbf{K}$ is the polar colour vector ($SU(2)_{\rm col}$ triplet). These quark rotary degrees of freedom correspond to rotations of fermions together with the gluonic BPS monopole vacuum described by the free rotator action (53) [23]. It is induced by vacuum "electric" monopoles (52). These vacuum "electric" fields are, apparently, the cause of the above fermionic rotary degrees of freedom (similar to rotary singlet terms in two-atomic molecules; see e.g. §82 in [65])[19].

[19] A good analysis of the Dirac system (76), (77) for isospinor fermionic fields (in the YM

More exactly, repeating the arguments of Ref. [31], one can “nominate the candidature” for the “interference term”

$$\sim Z^a j_{Ia0} \tag{79}$$

in the constraint-shell Lagrangian density of Minkowskian QCD quantized by Dirac [3] between the zero mode solution Z^a to the Gauss law constraint (9) (involving vacuum “electric” monopoles (52), generating the rotary action functional $W_{\rm coop}$, (53), for the physical Minkowskian non-Abelian BPS monopole vacuum) and the G-invariant quark current j_{Ia0} [10] (65) belonging to the excitation spectrum over this physical vacuum, as the source of fermionic rotary degrees of freedom $\mathbf{v}_1$.

The appearance of fermionic rotary degrees of freedom $\mathbf{v}_1$ in Minkowskian constraint-shell QCD alone, quantized by Dirac [3], confirms indirectly the existence of the BPS monopole background in that model (coming to the Wu-Yang one [40] at the spatial infinity). These fermionic rotary degrees of freedom testify in favour of nontrivial topological collective vacuum dynamics proper to the Dirac fundamental quantization [3] of Minkowskian constraint-shell QCD (the vacuum dynamics as described above).

4. *Discussion*

First of all note that the experimental detection of fermionic rotary degrees of freedom $\mathbf{v}_1$, as well as the “golden section” instantaneous interaction potential $V_1(z)$ (62) between quarks, can be a good confirmation of the Dirac fundamental quantization [3] of Minkowskian constraint-shell QCD involving physical BPS monopole vacuum possessing manifest superfluidity and various rotary effects. This should be equally valid as the results obtained in Refs. [10, 17, 23, 25, 39] concerning the η'-meson mass $m_{\eta'}$ (63).

The “theoretical plan” for further development of the Minkowskian Higgs model quantized by Dirac [3] may be associated, in the first place, with the following assumption called for to explain the nontrivial topological collective vacuum dynamics inherent in that model. This is the assumption about the “discrete group geometry” for the initial (say, $SU(2)$) and residual (say, $U(1)$) gauge groups in the Minkowskian Higgs model [67]. That assumption was made already in the work [26], where it was demonstrated that a gauge group G, involving (smooth) stationary transformations, say

$$A'_\mu(\mathbf{x},t) = v^{-1}(\mathbf{x})A_\mu(\mathbf{x},t)v(\mathbf{x}) + v(\mathbf{x})\partial_\mu v^{-1}(\mathbf{x}), \tag{80}$$

may always be factorised in the “discrete” way as

$$G \simeq G_0 \otimes \mathbf{Z} \equiv \widetilde{G}; \quad \mathbf{Z} = G/G_0. \tag{81}$$

theory) in the background field of a (BPS, Wu-Yang) monopole was carried out in the work [66]. In that work was also obtained the Dirac system alike (76), (77) [23], and also, by means of the decomposition of a fermionic field by the $SU(2)$ generators, the Pauli matrices.

Note that, in definition $\pi_1(G_0) = 0$, i.e. G_0 is [12] the *one-connected and topologically trivial* component in the generic $\widetilde{G}$ gauge group factorised in the (81) way.

Moreover, G_0 is the *maximal connected component* in G (in the terminology §§ T17, T20 in [12]): $\pi_0(G_0) = 0$.

That implies [12]

$$\pi_0[G_0 \otimes \mathbf{Z}] = \pi_0(\mathbf{Z}) = \mathbf{Z}.$$

This relation indicates transparently the discrete nature of the $\widetilde{G}$ group space.

More exactly, the $\widetilde{G}$ group space consists of different topological sectors (each with its proper topological number $n \in \mathbf{Z}$), separated by *domain walls*. Additionally, the factorisation (81) reflects the essence of Gribov topological "copying" [13] of "small" gauge transformations.

On the other hand, since (81) is only an isomorphism, there is a definite freedom in assuming that the gauge group G possesses a "continuous" or "discrete" geometry. In particular, in the Euclidian non-Abelian model [35] involving instantons, the "continuous" geometry should be assumed for the $SU(2)$ group space. It is associated with the absence of any nonzero mass scale in this model. The thing is that domain walls between different topological sectors would become infinitely wide in this case.

Generally speaking, the width of a domain wall is roughly proportional to the inverse of the lowest mass of all physical particles being present in the (gauge) model considered [68]. Thus domain walls are really infinite in the Euclidian instanton model [35]. In this case, any transitions [16] are impossible between vacua (say, $|n>$ and $|n+1>$) with different topological numbers since the latter ones belong to topological domains separated by infinitely wide walls.

In principle, a different situation is in the Minkowskian Higgs model quantized by Dirac [3] and involving vacuum gauge and Higgs BPS monopole solutions. In this model, a natural mass scale may be introduced. For instance, the effective Higgs mass $m/\sqrt{\lambda}$ may be treated as a mass scale, depending indeed on the distance r via Eq. (45) (because $V \sim r^3$). This creates objective prerequisites for utilizing the "discrete" representations of the $\widetilde{G}$ type [26] (81) for the initial, $SU(2)$, and residual, $U(1)$, gauge symmetry groups in the Minkowskian Higgs model quantized by Dirac [67]

$$\widetilde{S}U(2) \simeq G_0 \otimes \mathbf{Z}; \quad \widetilde{U}(1) \simeq U_0 \otimes \mathbf{Z}, \tag{82}$$

respectively.

As a result, the *degeneration space* (*vacuum manifold*)

$$R_{YM} \equiv SU(2)/U(1)$$

in this Minkowskian Higgs model acquires the "discrete" form

$$R_{YM} = \mathbf{Z} \otimes G_0/U_0. \tag{83}$$

Obviously, R_{YM} is the discrete space consisting of topological domains separated by domain walls.

If the Minkowskian Higgs model quantized by Dirac [3] involves vacuum gauge and Higgs BPS monopole solutions, it is quite naturally to suppose that the typical width of such domain walls is $\epsilon(r)$, with $\epsilon(r) \sim (m/\sqrt{\lambda})^{-1}(r)$ given by Eq. (45).

From Eq. (45) it can be concluded [67] that ϵ disappears in the infinite spatial volume limit $V \to \infty$, while it is maximal at the origin of the coordinates (it can be set $\epsilon(0) \to \infty$). This means, due to the reasoning [68], that walls between topological domains inside R_{YM} become infinitely wide, $O(\epsilon(0)) \to \infty$, at the origin of coordinates. The fact $\epsilon(\infty) \to 0$ is also meaningful. This implies actual merging of topological domains inside the vacuum manifold R_{YM} (83) at the spatial infinity. This merging of topological domains promotes the infrared topological confinement (destructive interference) of Gribov "large" multipliers $v^{T(n)}(\mathbf{x})$ in gluonic and quark Green functions in all orders of the perturbation theory (in the spirit of Ref. [24]). On the other hand, it becomes obvious that the effective Higgs mass $m/\sqrt{\Lambda}$ (as the value inversely proportional to ϵ) can be treated as a Wegner variable, disappearing in the ultraviolet fixed point (i.e. at the origin of coordinates) [42].

It may be demonstrated that the vacuum manifold R_{YM} in the Minkowskian Higgs model quantized by Dirac [3] in its "discrete" representation (83) possesses three kinds of topological defects. The first kind of topological defects are domain walls between different topological sectors of that Minkowskian Higgs model, us discussed above. The criterion of domain walls existing in a (gauge) model is a nonzero (for example, infinite) number π_0 of connection components in the appropriate degeneration space. In particular,

$$\pi_0(R_{YM}) = \mathbf{Z}$$

because of (82).

The next kind of topological defects inside the discrete YM vacuum manifold R_{YM} are *point hedgehog topological defects.* This type of topological defects comes to the vacuum "magnetic" field $\mathbf{B}$, generated by the Bogomol'myi equation (46), singular at the origin of coordinates, as shown in Ref. [29]. Actually, $|\mathbf{B}| \sim O(r^{-2})$. From the topological viewpoint, the criterion for point (hedgehog) topological defects to exist in a (gauge) theory is the nontrivial group π_2 of two-dimensional ways for the appropriate degeneration space (vacuum manifold).

Moreover, denoting by G the initial gauge symmetry group in the considered model and by H the residual one (then $R = G/H$ will be the vacuum manifold in that model), it may be proved [12] that always

$$\pi_2 R = \pi_1 H$$

(with $\pi_1 H$ being the fundamental group of one-dimensional ways in H), and if $\pi_1 H \neq 0$, point (hedgehog) topological defects exist in a (gauge) theory [12].

In particular, the topological relation

$$\pi_2(R_{YM}) = \pi_1 \widetilde{U}(1) = \mathbf{Z} \tag{84}$$

is the criterion for point (hedgehog) topological defects in the Minkowskian Higgs model quantized by Dirac [3].

Geometrically, point topological defects are concentrated in a coordinate region topologically equivalent to a two-sphere S^2 (in particular, point hedgehog topological defects are always concentrated in a two-sphere with its centre lying in the origin of coordinates). Just in such coordinate regions, the thermodynamic equilibrium (at a Curie point T_c in which the appropriate second-order phase transition occurs) corresponding to the minimum of the action functional set over the vacuum manifold R in a (gauge) model, is violated [12]. This violation involves singularities in order parameters. An example of such singularities order parameters found in (gauge) models with point topological defects is the $O(r^{-2})$ behaviour [29] of the vacuum "magnetic" field $\mathbf{B}$ in the Minkowskian Higgs model involving vacuum BPS monopole solutions.

In conclusion, the vacuum manifold R_{YM} contains a third kind of topological defects, *the thread topological defects.* The criterion for the existence of thread topological defects in a (gauge) theory is the topological relation [12]

$$\pi_1 R = \pi_0 H \neq 0. \tag{85}$$

In particular,

$$\pi_1(R_{YM}) = \pi_0 \ \widetilde{U}(1) = \mathbf{Z}. \tag{86}$$

Thus thread topological defects exist in the Minkowskian Higgs model quantized by Dirac [3] (implicating vacuum BPS monopole solutions) in which the "discrete" vacuum geometry (83) is assumed.

Geometrically, thread topological defects cause violation of the thermodynamic equilibrium along definite lines (for instance, rectilinear ones) in the given vacuum manifold. It may be demonstrated, repeating the arguments of Ref. [12], that thread topological defects possess the manifest S^1 topology (for instance, for "rectilinear" thread topological defects it is highly transparent).

Point (hedgehog) topological defects are always present in the Minkowskian Higgs model involving vacuum monopole solutions, irrespectively to the way in which the model is quantized: either this is the FP "heuristic" quantization scheme [7] or the Dirac fundamental one [3].

Besides the BPS monopoles [12, 29, 30] and Wu-Yang ones [40], granted a great attention in the present study, 't Hooft-Polyakov monopoles [69, 70] are also the very important kind of monopole solutions with which modern theoretical physics deals. Indeed, the analysed Minkowskian Higgs models involving vacuum monopole solutions and point (hedgehog) topological defects associated with these vacuum monopole solutions confine themselves within the FP "heuristic" quantization scheme [7].

On the other hand, it is sufficient to assume the "continuous", $\sim S^2$, vacuum geometry in the Minkowskian Higgs models [12, 29, 30, 40, 69, 70] with monopoles in order to quantize them in the "heuristic" [7] way. We have already discussed this with the example of vacuum BPS monopole solutions [12, 29, 30] in which the Bogomol'nyi equation (46) and the Bogomol'nyi bound $E_{\min}$ (47) were derived [12, 28] just assuming the continuous

$$SU(2)/U(1) \sim S^2$$

vacuum geometry and herewith without solving the YM Gauss law constraint (9)[20].

In the analysed Minkowskian Higgs models [12, 29, 30, 40, 69, 70] with monopoles, there is no nontrivial (topological) dynamics, since the physical content of the models is determined by *stationary* vacuum monopole solutions. Additionally, all "electric" tensions in the enumerated Minkowskian Higgs models are set identically in zero: $F^a_{0i} \equiv 0$. Thus assuming the "continuous", $\sim S^2$, vacuum geometry in the Minkowskian Higgs models [12, 29, 30, 40, 69, 70] with stationary vacuum (Higgs and YM) monopole solutions (setting additionally to zero of all "electric" tensions) ensures the lawfulness of the "heuristic" [7] quantization of the models[21].

Unlike the above discussed case, to justify the Dirac fundamental quantization [3] of the Minkowskian Higgs model, involving the collective vacuum rotations (53) [39], the "discrete" vacuum geometry of the (83) type should be supposed. More precisely, if the thread "rectilinear" topological defects are contained inside the vacuum manifold R_{YM} (83), this can explain the discrete vacuum rotary effect (53) occurring in the Minkowskian Higgs model quantized by Dirac. Just such rectilinear lines inside the vacuum manifold R_{YM} (that are, geometrically, cylinders of effective diameters $\sim \epsilon$), localized around the axis z of the chosen (rest) reference frame [68], are associated with the Josephson effect [31] in the Minkowskian Higgs model.

As we have ascertained above, the Josephson effect comes therein [23] to the "persistent field motion" around the above described rectilinear lines, with all ensuing physical consequences, including the real spectrum (55) of the appropriate topological momentum P_c, the never-vanishing (until $\theta \neq 0$) vacuum "electric" field $(E^a_i)_{\min}$ [23], (50), and the manifestly Poincaré invariant constraint-shell Hamiltonian $H_{\rm cond}$ [17, 25] (57) of the Bose condensation.

Investigating the Dirac fundamental quantization [3] of the Minkowskian Higgs model is not finished at present. So recently, in Ref. [67], it was demonstrated that

[20] Indeed, as it was explained in Refs. [17, 25, 46], the Bogomol'nyi equation (46) is compatible with the Dirac fundamental quantization [3] of the Minkowskian Higgs model with BPS monopoles. As discussed above, this connection between the Bogomol'nyi equation (46) and the Dirac fundamental quantization of the Minkowskian Higgs model is given via the Gribov ambiguity equation (having the form (22), to which the Bogomol'nyi equation (46) comes mathematically because of the Bianchi identity $D\,B = 0$.

[21] For instance [28], one can fix the *Weyl* gauge $A_0 = 0$ for temporal YM components in appropriate FP path integrals. This just results in $F^0_{0i} \sim 0$ if one deals with stationary monopole solutions in the analysed Minkowskian (Higgs) models

the first-order phase transition occurs in the constraint-shell Minkowskian Higgs model quantized by Dirac and involving vacuum BPS monopole solutions.

This first-order phase transition supplements the second-order one always taking place in the Minkowskian Higgs model and associated with the spontaneous breakdown of the initial gauge symmetry. The essence of the first-order phase transition occurring in the Minkowskian Higgs model quantized by Dirac and involving vacuum BPS monopole solutions is in coexisting collective vacuum rotations (described by the action functional (53) [23]) and superfluid potential motions (set in the Dirac fundamental scheme [3] by the Gribov ambiguity equation, coming to the Bogomol'nyi one (46)).

As it was demonstrated in Ref. [26], this first-order phase transition in the Minkowskian Higgs model quantized by Dirac claims that vacuum "magnetic" and "electric" fields, respectively **B** and **E**, are transverse

$$D\ B = D\ E = 0.$$

This condition, in turn, is mathematically equivalent to the system [26]

$$E \sim D\Phi; \quad B \sim D\Phi \tag{87}$$

of the first-order differential equations, involving Higgs vacuum BPS monopole modes Φ.

More exactly, acting by the (covariant) derivative D on the system (87), one turns the Bogomolnyi equation $B \sim D\Phi$ (second equation in this system) into the Gribov ambiguity equation, while the first equation in (87) comes then to the YM Gauss law constraint (22) at the constraint-shell reduction of the Minkowskian Higgs model in terms of the gauge invariant and transverse topological Dirac variables (7).

Thus assuming the "discrete" vacuum geometry of the (83) type appears to play a crucial role in the above assertion that first-order phase transition occurs in the Minkowskian Higgs model quantized by Dirac [3], as well as in explaining other phenomena taking place in that model.

The author whould also like to express his opinion about the further fate of gauge physics. In author's opinion, it seems to be connected with three things.

The first one is going over to the Minkowski space (from the Euclidian E_4 one). This would allow to avoid typical shortcomings inherent in Euclidian gauge theories (including the complex values (25) [19, 26, 33] of the topological momentum $P_{\mathcal{N}}$ of the θ-vacuum in the Euclidian instanton model [35]).

The second thing is the utilization of the vacuum BPS monopole solutions [12, 29, 30] in the development of the Minkowskian Higgs model. As we have seen in the course of our present discussion, this set manifest superfluid properties in that model, absent in other Minkowskian (Higgs) models with monopoles: for instance, in the Wu-Yang model [40] or in the 't Hooft-Polyakov model [69, 70].

The third thing is the Dirac fundamental quantization [3] of the Minkowskian Higgs model involving vacuum BPS monopole solutions, which gave perceptible results (for example, the η'-meson mass (63), near to the experimental data, or the rising "golden section" potential (62) of Refs. [10, 17, 23, 25, 39]).

Apart from the above, the described Minkowskian Higgs model quantized by Dirac (involving vacuum BPS monopole solutions and "discrete vacuum geometry" (83) [67]) gives the specific approach to the so-called *mass problem*. That problem was formulated as follows [71]. Experiment and computer simulations of the "pure" YM theory without other (quantum) fields suggest the existence of a "mass gap" in the solution to the quantum versions of the YM equations. But no proof of this property is known.

In the strict mathematical language, the mass gap problem can be expressed in the following way. Since the Hamiltonian H of a QFT is the element of the Lie algebra of the Poincaré group, and the appropriate vacuum vector Ω is Poincaré invariant, it is an eigenstate with zero energy, $H\Omega = 0$. The positive energy axiom (in absence of external negative potentials) asserts that in any QFT, the spectrum of H is supported in the region $[0, \infty\rangle$. In this terminology, a QFT has a *mass gap* if H has no spectrum in the interval $[0, \Delta\rangle$ for a $\Delta > 0$. The supremum of such Δ is called the mass m. Then the YM mass gap problem can be formulated mathematically [71] as *proving that for any compact simple gauge group G, the quantum YM theory on* R^4 *exists and has a mass gap* $\Delta > 0$. An important consequence of the existence of the mass gap is that for any positive constant $C < \Delta$ and for any local field operator $\mathcal{O}$ such that $\langle \Omega, \mathcal{O}\Omega \rangle = 0$, one has

$$|\langle \Omega, \mathcal{O}(x)\mathcal{O}(y)\Omega \rangle| \leq \exp(-C|x-y|)$$

if $|x-y|$ is sufficiently large (depending on C and $\mathcal{O}$).

The Minkowskian Higgs model quantized by Dirac, presented here, contains the Higgs (and fermionic) field modes. Thus this is somewhat another case than the case [72]. But the effective Higgs mass $m/\sqrt{\lambda}$ incorporated naturally in the Minkowskian Higgs model with vacuum BPS monopoles quantized by Dirac, becomes zero in the limit of "infinitely thick domain walls" inside the appropriate discrete vacuum manifold R_{YM} (83). It is just the ultraviolet region of the momentum space. On the other hand, in the limit of "infinitely thin domain walls" (it is just the infrared region of the momentum space), $m/\sqrt{\lambda}$ tends to a finite value [67] (the latter one can be treated as an infrared cut-off).

Thus the approach to the mass gap problem in the Minkowskian Higgs model quantized by Dirac, involving vacuum BPS monopole solution, BPS monopole solutions and "discrete vacuum geometry" can be reduced to solving renorm-group equations [16] implicating the Wegner variable $m/\sqrt{\lambda}$, that is a continuous function of the distance r. Of course, these renorm-group equations would be in agreement with the first-order phase transition occuring therein [67].

In general, in the author's opinion, the Dirac fundamental quantization [3] of gauge models seems to have a great perspectives in the future.

Really, the FP heuristic quantization method [7], fixing the gauges in FP integrals, has arisen at the end of 60-ies of the past century, and despite of all its advantages in solving the problems associated with scattering processes in gauge theories, supplanted utterly from modern theoretical physics the way of references frames and initial and boundary conditions, the historically arisen way in modern theoretical physics, associated with Einstein (special and general) relativity[22].

The FP heuristic quantization method [7] retains in gauge theories only the realm of physical laws bounded by the "absolutes", the S-invariants. But this approach is fit, as we have discussed above, only for solving the problems associated with scattering processes in gauge theories, leaving "overboard" other problems of modern theoretical physics, including construction of bound states in QED and QCD.

In the present study, with the example of the Minkowskian Higgs model quantized by Dirac [3], the author has attempted to attract attention of the readers to the dramatic situation that now arises in modern theoretical physics in connection with the introduction of the heuristic quantization method [7] and supplanting, by this method, the Dirac fundamental quantization scheme [3] (associated with the Hamiltonian approach to the quantization of gauge theories and attached to a definite reference frame).

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DIRACOVA TEMELJNA KVANTIZACIJA BAŽDARNIH TEORIJA JE PRIRODAN PRISTUP REFERENTNIM SUSTAVIMA U MODERNOJ FIZICI

Analiziramo dva osnovna pristupa kvantizaciji fizičkih modela. To su "heuristički" pristup Faddeeva i Popova (FP), zasnovan na utvrđivanju baždarenja u formalizmu staznih integrala, i "temeljni" Diracov pristup zasnovan na redukcijskom hamiltonijanu s ograničenjem na ljusku masa uz uklanjanje nefizičkih varijabla. Relativistički invarijantan FP "heuristički" pristup proučava malu klasu problema povezanih s kvadratima S-matrica, razmatrajući samo kvantna polja na ljusci energije. Nasuprot tome, "temeljni" Diracov pristup kvantizaciji sadrži izričito relativističku kovarijanciju kvantnih polja koja "preživi" redukciju hamiltonijana s ograničenjem na ljusku masa. Taj se pristup može primijeniti na širu klasu problema nego S-matrica. Istraživanja vezanih stanja u QED i QCD su primjeri tih primjena. U ovom radu, s primjerom Diracove "temeljne" kvantizacije relativističkog ne-Abelovog Higgsovog modela (proučavanog u povijesnom okviru) pokazujemo očigledne prednosti tog pristupa kvantizaciji. Tvrdnje u prilog Diracovoj temeljnoj kvantizaciji fizičkih modela predstavljamo kao Einsteinovu i Galilejevu relativnost u modernoj fizici.