Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

BIANCHI TYPE V IMPERFECT FLUID COSMOLOGICAL MODELS WITH HEAT FLOW

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Received 16 August 2008; Revised manuscript received 19 February 2008
Accepted 4 March 2009 Online 15 July 2009

We obtain two classes of exact solutions to the Einstein's field equations by applying a special law of variation of the Hubble's parameter for a Bianchi type V space-time in the presence of an imperfect fluid with both shear and bulk viscosities together with heat flow. The coefficient of the shear viscosity is determined on the basis of the physical assumption that it is proportional to the Hubble's expansion parameter, whereas the bulk viscosity coefficient is determined when the energy-density and the kinetic pressure satisfy the barotropic equation of state. These classes of solutions represent expanding cosmological models with and without a finite physical singularity. The physical and kinematical features of the models in two types of cosmologies are discussed.

PACS numbers: 98.80.Es; 98.80.-k UDC 52-423, 524.83
Keywords: Bianchi type V, imperfect fluid, shear viscosity, bulk viscosity, heat flow

1. *Introduction*

Astronomical observations of the large-scale distribution of galaxies in the universe show that the distribution of matter can be satisfactorily described by a perfect fluid. The adequacy of cosmological models with perfect fluid is no basis for expecting that it is equally suitable for describing its early stages of evolution. At the early stages of evolution of the universe, when radiation in the form of photon as well as neutrino decoupled, the matter behaved like a viscous fluid. Since viscosity counteracts the cosmological collapse, a different picture at the initial state of the universe may appear due to the dissipative processes caused by viscosity. A

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number of authors have presented exact solutions of Einstein's field equations by considering viscous fluid in isotropic as well as anisotropic cosmological models.

Misner [1, 2] studied the effect of viscosity on the evolution of the cosmological models and suggested that the strong dissipation due to the neutrino viscosity may considerably reduce the anisotropy of the black-body radiation. The viscosity mechanism in cosmology has explained the anomalously high entropy per baryon in the present universe [3, 4]. Murphy [5] developed a uniform cosmological model of the Friedmann type taking into account the effect of bulk viscosity. The solutions that he presented exhibit an interesting feature that the big-bang type singularity appears in the infinite past. A number of authors obtained exact solutions of Einstein's field equations by considering viscous fluid in isotropic as well as anisotropic cosmological models. Banerjee and Santosh [6, 7], Banerjee, Duttachoudhury and Sanyal [8], Dunn and Tupper [9], Coley and Tupper [10] etc. constructed and discussed cosmological models under the influence of both shear and bulk viscosity. Goener and Kowalewski [11] have given a method for obtaining irrotational anisotropic viscous fluid matter solutions of Bianchi type I with barotropic equation of state. Saha and Rikhvitsky [12] investigated the nature of cosmological solution for a spatially homogeneous Bianchi type I model in the presence of a cosmological term by taking into account dissipative process due to viscosity. Grøn [13] presented an excellent review of research works on viscous cosmological models and studied inflationary cosmological models of Bianchi type I with shear and non-linear bulk viscosities. Banerjee and Sanyal [14] presented an irrotational Bianchi type V cosmological model under the influence of both shear and bulk viscosity together with heat flow. Coley [15] investigated Bianchi type V spatially homogeneous imperfect fluid cosmological models which contain both viscosity and heat flow. Coley and Hoogan [16], while generalizing the work of Coley and Tupper [10], studied diagonal Bianchi type V imperfect fluid cosmological models with both viscosity and heat conduction and presented cosmological models with and without the cosmological term using the technique from dynamical system theory. Singh and Chaubey [17] investigated the evolution of a spatially homogeneous and anisotropic Bianchi type V cosmological model with viscous fluid and a cosmological term.

The Einstein's field equations are a coupled system of highly non-linear differential equations and we seek solutions to the field equations for their applications in cosmology and astrophysics. In order to find solutions of field equations, one has to make certain physical and mathematical assumptions at the cost of physics of the problem. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter as proposed by Berman [18]. In this paper, we obtain two classes of exact solutions to the Einstein's field equations for a Bianchi type V model filled with an imperfect fluid with both bulk and shear viscosities together with heat flow by applying the law of variation for Hubble's parameter. One class of solutions represents a cosmological model of the universe with power-law expansion having a big-bang singularity at a finite time, whereas the other class of solutions corresponds to an exponentially expanding model of the universe having singularity in the infinite past. The physical and kinematical behaviors of the models in the two types of cosmologies are studied.

2. Einstein's field equations and general expressions

The metric for the spatially homogeneous and anisotropic Bianchi type V cosmological model is

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx}\left(B^2 dy^2 + C^2 dz^2\right), \tag{1}$$

where $A(t)$, $B(t)$ and $C(t)$ are the cosmic scale factors and m is a constant.

The energy-momentum tensor of a viscous fluid with heat flow is given by

$$\begin{aligned} T_{\mu\nu} \;=\; & (\rho+\bar{p})\, u_\mu u_\nu - \bar{p} g_{\mu\nu} + \eta \delta^\beta_\nu \left[u_{\mu;\beta} + u_{\beta;\mu} - u_\mu u^\alpha u_{\beta;\alpha} - u_\beta u^\alpha u_{\mu;\alpha}\right] \\ & + h_\mu u_\nu + h_\nu u_\mu, \end{aligned} \tag{2}$$

where

$$\bar{p} = p - \left(\xi - \frac{2}{3}\eta\right)\theta, \tag{3}$$

and semi-colon denotes co-variant differentiation.

Here u_μ is the 4-velocity of the fluid, ρ its energy density, p the kinetic pressure, $\bar{p}$ is the effective pressure, $\xi(> 0)$ and $\eta(> 0)$ the co-efficients of bulk and shear viscosities, respectively, and h_μ is the heat flow vector satisfying $h^\mu u_\mu = 0$. We assume that the heat flow is in the x-direction only, then $h_\mu = (h_1, 0, 0, 0)$, h_1 being a function of time. In co-moving system of co-ordinates, we have $u^\mu = (0, 0, 0, 1)$. Expansion and shear scalars are defined by

$$\theta = u^\mu_{;\mu}, \tag{4}$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}, \tag{5}$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}P^\alpha_\nu + u_{\nu;\alpha}P^\alpha_\mu) - \frac{1}{3}\theta P_{\mu\nu}\,. \tag{6}$$

Here the projection tensor $P_{\mu\nu}$ is given by

$$P_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu\,. \tag{7}$$

The expansion scalar, which determines the volume behavior of the fluid, and the shear scalar for Bianchi type V metric (1), are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{B}}{B} = \frac{\dot{V}}{V}\,, \tag{8}$$

and

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{\theta^2}{6}\,, \tag{9}$$

where a dot denotes differentiation with respect to t. The average scale factor a of the metric (1) is defined as

$$a = (ABC)^{1/3}\,. \tag{10}$$

The volume scale factor V is given by

$$V = a^3 = ABC\,. \tag{11}$$

We also define the directional Hubble's parameters

$$H_1 = \dot{A}/A\,, \qquad H_2 = \dot{B}/B\,, \qquad H_3 = \dot{C}/C\,, \tag{12}$$

in the directions of x, y and z, respectively, and the average Hubble's parameter H as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\sum_{\mu=1}^{3} H_\mu = \frac{1}{3}\theta\,. \tag{13}$$

As the measure of the anisotropy, we take

$$A_m = \frac{1}{3}\sum_{\mu=1}^{3}\frac{(H_\mu - H)^2}{H^2}\,. \tag{14}$$

Also, the deceleration parameter q is defined by

$$q = -\frac{\ddot{a}a}{\dot{a}^2}\,. \tag{15}$$

In the system of units $8\pi G = 1$, the Einstein's field equations for a viscous fluid with heat flow, in view of Eqs. (2) and (3) for Bianchi type V space-time (1), are given as the following set of equations:

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = \left(-\bar{p} + 2\eta\frac{\dot{A}}{A}\right), \tag{16}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{m^2}{A^2} = \left(-\bar{p} + 2\eta\frac{\dot{B}}{B}\right), \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{B}}{B}-\frac{m^2}{A^2}=\left(-\bar{p}+2\eta\frac{\dot{C}}{C}\right), \tag{18}$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}\frac{\dot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{3m^2}{A^2}=\rho\,, \tag{19}$$

$$m\left(2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=h_1. \tag{20}$$

From the energy conservation law $T^{\mu}_{\nu;\mu}u^{\nu}=0$, we obtain

$$\dot{\rho}+(\rho+\bar{p})\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)-2\eta\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2\right]=\frac{2m}{A^2}h_1. \tag{21}$$

From the field equations (16) – (19), the effective pressure $\bar{p}$ and the energy density ρ, in terms of physical parameters, can be written as follows

$$\bar{p}=H^2\,(2q-1)-\sigma^2+\frac{m^2}{A^2}+\frac{2}{3}\eta\theta\,, \tag{22}$$

$$\rho=3H^2-\sigma^2-\frac{3m^2}{A^2}\,. \tag{23}$$

Here we follow the approach of Saha and Rikhvitsky [12], Singh and Chaubey [17] and Singh et al. [19] to solve the field equations (16) – (20). Subtracting Eqs. (16) and (17), Eqs. (17) and (18) and Eqs. (16) and (18), we get the following quadrature solutions of the field equations, respectively,

$$\frac{B}{A}=d_1\exp\left[k_1\int\frac{\exp\left(-2\int\eta\mathrm{d}t\right)}{a^3}\mathrm{d}t\right], \tag{24}$$

$$\frac{C}{B}=d_2\exp\left[k_2\int\frac{\exp\left(-2\int\eta\mathrm{d}t\right)}{a^3}\mathrm{d}t\right], \tag{25}$$

$$\frac{C}{A}=d_3\exp\left[k_3\int\frac{\exp\left(-2\int\eta\mathrm{d}t\right)}{a^3}\mathrm{d}t\right], \tag{26}$$

where d_1, d_2, d_3 and k_1, k_2, k_3 are constants of integration. From Eqs. (24) – (26), we find the quadrature solutions of the metric functions as follow

$$A=l_1a\exp\left[X_1\int\frac{\exp\left(-2\int\eta\mathrm{d}t\right)}{a^3}\mathrm{d}t\right], \tag{27}$$

$$B = l_2 a \exp\left[X_2 \int \frac{\exp\left(-2\int \eta \mathrm{d}t\right)}{a^3} \mathrm{d}t\right], \tag{28}$$

$$C = l_3 a \exp\left[X_3 \int \frac{\exp\left(-2\int \eta \mathrm{d}t\right)}{a^3} \mathrm{d}t\right], \tag{29}$$

where $l_1 = \sqrt[3]{({d_1}^2 d_2)^{-1}}$, $l_2 = \sqrt[3]{d_1 {d_2}^{-1}}$, $l_3 = \sqrt[3]{d_1 {d_2}^2}$, and
$X_1 = -(2k_1 + k_2)/3$, $X_2 = -(k_2 - k_1)/3$, $X_3 = (k_1 + 2k_2)/3$,

and these constants X_1, X_2, X_3 and l_1, l_2, l_3 satisfy the relations

$$X_1 + X_2 + X_3 = 0 \quad \text{and} \quad l_1 l_2 l_3 = 1\,. \tag{30}$$

Let us assume that the shear viscosity η is proportional to the expansion, i.e., $\eta \propto \theta = 3H$ [17]. Here we consider the value of η as

$$\eta = \eta_0 \theta = 3\eta_0 H\,, \tag{31}$$

which means that the nature of the shear viscosity is dependent upon the expansion of the universe.

We also make a certain physically valid assumption of the Hubble's parameter H as

$$H = l a^{-n}, \tag{32}$$

where $l > 0$ and $n(\geq 0)$ are constants. This type of relation, which gives a constant value of deceleration parameter, was initially considered by Bermam [18] and Berman and Gomide [20] for solving FRW cosmological models. The same concept of the constant deceleration parameter was used, later on, by many workers (see Singh and Kumar [21] and references therein) for solving Einstein's field equations in FRW models. Singh and Kumar [22] and Kumar and Singh [23] have further applied the same assumption of the law of variation for Hubble's parameter for solving the field equations in anisotropic Bianchi types I and II cosmological models in general relativity and in different scalar-tensor theories. Recently, Singh et al. [19, 24] have further extended this work to Bianchi type V perfect fluid cosmological models with and without heat flow in general relativity. In this paper, we solve the field equations of Bianchi type V imperfect viscous fluid with heat conduction by using relations (31) and (32).

From Eqs. (13) and (32), we obtain

$$\dot{a} = l a^{-n+1}, \tag{33}$$

$$\ddot{a} = -l^2 (n-1)\, a^{-2n+1}. \tag{34}$$

Using Eqs. (33) and (34) into Eq.(15), we get

$$q = n - 1\,. \tag{35}$$

It is clear here that the law of variation of Hubble's parameter gives a constant value of the deceleration parameter. The sign of q indicates the behavior of the model. The positive sign of $q(n > 1)$ corresponds to the decelerating model of the universe whereas the negative sign of $q(0 \leq n < 1)$ shows the inflation. Integrating Eq. (33), we find two values of the average scale factor as follows

$$a = (nlt + c_1)^{1/n}\,, \qquad n \neq 0\,, \tag{36}$$

and

$$a = c_2 \exp(lt)\,, \qquad n = 0\,, \tag{37}$$

where c_1 and c_2 are constants of integration.

In the next section, we solve Eqs. (27) – (29) with the help of the above two expressions of the average scale factor for $n \neq 0$ and $n = 0$ and obtain two classes of cosmological models having singularity at finite time or in the infinite past.

3. Exact solutions

3.1. Cosmological model with $n \neq 0$

Using the power-law solution of the average scale-factor obtained in Eq. (36), the generalized mean Hubble's parameter H, from Eq. (13), can be obtained as

$$H = l\,(nlt + c_1)^{-1}\,. \tag{38}$$

From Eqs. (31) and (38), the expression for the shear viscosity η is given by

$$\eta = 3\eta_0 l(nlt + c_1)^{-1}. \tag{39}$$

With the help of the values of the average scale factor a and the shear scalar η, obtained in Eqs. (36) and (39) respectively, the exact solutions of the metric functions (27) – (29) are obtained as

$$A(t) = l_1\,(nlt + c_1)^{1/n} \exp\left[\frac{X_1}{l(n-3-6\eta_0)}\,(nlt + c_1)^{(n-3-6\eta_0)/n}\right], \tag{40}$$

$$B(t) = l_2\,(nlt + c_1)^{1/n} \exp\left[\frac{X_2}{l(n-3-6\eta_0)}\,(nlt + c_1)^{(n-3-6\eta_0)/n}\right], \tag{41}$$

$$C(t) = l_3\,(nlt + c_1)^{1/n} \exp\left[\frac{X_3}{l(n-3-6\eta_0)}\,(nlt + c_1)^{(n-3-6\eta_0)/n}\right], \tag{42}$$

provided $n \neq (3+6\eta_0)$. The directional Hubble's parameters H_1, H_2 and H_3 can be written as

$$H_1 = l\,(nlt+c_1)^{-1} + X_1\,(nlt+c_1)^{-(3+6\eta_0)/n}\,, \tag{43}$$

$$H_2 = l\,(nlt+c_1)^{-1} + X_2\,(nlt+c_1)^{-(3+6\eta_0)/n}\,, \tag{44}$$

$$H_3 = l\,(nlt+c_1)^{-1} + X_3\,(nlt+c_1)^{-(3+6\eta_0)/n}\,. \tag{45}$$

The heat conduction h_1, in this case can be obtained as

$$h_1 = 3mX_1\,(nlt+c_1)^{-(3+6\eta_0)/n}\,. \tag{46}$$

The physical parameters θ,σ^2, Am and V are given by

$$\theta = 3l\,(nlt+c_1)^{-1}\,, \tag{47}$$

$$\sigma^2 = \frac{1}{2}\left({X_1}^2+{X_2}^2+{X_3}^2\right)(nlt+c_1)^{-(6+12\eta_0)/n}\,, \tag{48}$$

$$Am = \frac{n^2\left({X_1}^2+{X_2}^2+{X_3}^2\right)}{3}\,(nlt+c_1)^{2(n-3-6\eta_0)/n}\,, \tag{49}$$

$$V = (nlt+c_1)^{3/n}\,. \tag{50}$$

Using these expressions for H, σ^2, Am, η, q and θ, one obtains from Eqs. (22) and (23) the effective pressure and the energy density as

$$\begin{aligned} \bar{p} \;&=\; \frac{2nl^2}{(nlt+c_1)^2} - \frac{n^2l^2}{2}.\frac{\left({X_1}^2+{X_2}^2+{X_3}^2\right)}{(nlt+c_1)^{12/n}} \\ &\quad+\frac{m^2}{{l_1}^2}\frac{1}{(nlt+c_1)^{2/n}}.\exp\left[\frac{-2nX_1}{(n-6)}\,(nlt+c_1)^{(n-6)/n}\right], \end{aligned} \tag{51}$$

$$\begin{aligned} \rho \;&=\; \frac{3l^2}{(nlt+c_1)^2} - \frac{n^2l^2}{2}.\frac{\left({X_1}^2+{X_2}^2+{X_3}^2\right)}{(nlt+c_1)^{12/n}} \\ &\quad-\frac{3m^2}{{l_1}^2}\frac{1}{(nlt+c_1)^{2/n}}.\exp\left[\frac{-2nX_1}{(n-6)}\,(nlt+c_1)^{(n-6)/n}\right], \end{aligned} \tag{52}$$

provided $n \neq 6$.

To find the value of the bulk viscosity coefficient ξ, we assume that the energy density and the kinetic pressure satisfy the equation of state $p = \gamma\rho$, $0 \le \gamma \le 1$. Then, from Eqs. (3), (51) and (52), we obtain

$$\begin{aligned}\xi \;=\; & \frac{l(3\gamma-2n+3)}{3}(nlt+c_1)^{-1}+\frac{\left({X_1}^2+{X_2}^2+{X_3}^2\right)(1-\gamma)}{6l}(nlt+c_1)^{(n-6-12\eta_0)/n} \\ & -\frac{m^2(3\gamma+1)}{3{l_1}^2 l}(nlt+c_1)^{(n-2)/n}\exp\left[\frac{2X_1}{l(3+6\eta_0-n)}(nlt+c_1)^{(n-3-6\eta_0)/n}\right]. \qquad (53)\end{aligned}$$

It can be observed that, with the help of the above solutions, the energy conservation equation (21) is identically satisfied.

We now investigate the existence of a singularity of this model, which can be done by investigating the behaviors of the above physical parameters. We study the characteristics of each parameter obtained above, and find that at the point $t = t_1$, $t_1 = -c_1/nl$, the important parameters such as H, H_1, H_2, H_3, θ, σ^2, η, p and ρ are all infinite and the volume scalar vanishes. The bulk viscosity coefficient ξ also tends to infinity at $t = t_1$ if $n < 2$. This clearly indicates that the model has a finite physical singularity at $t = t_1$ from which it starts expanding with power-law expansion. Initially, the rate of expansion is infinite, but for the large time as $t \to \infty$, the expansion will completely be vanished. The quantities H, H_1, H_2, H_3, σ^2, η, p and ρ are well behaved for $t > t_1$ and will become zero for large time. The component h_1 of heat conduction vector is initially infinite and approaches zero as $t \to \infty$. The volume scalar of the model will also become infinite for the large time. The ratio σ^2/θ tends to zero as $t \to \infty$, which shows that the model is isotropic for large time. Thus, the universe is essentially an empty space-time for large t.

3.2. *Cosmological model with* $n = 0$

Using the exponential form of the average scale factor obtained in Eq. (37), the generalized mean Hubble's parameter H and the expansion scalar θ from Eq. (13) can be written as

$$H = l, \qquad \theta = 3l\,. \qquad (54)$$

From Eqs. (31) and (54), the shear viscosity η can be given as

$$\eta = 3l\eta_0\,. \qquad (55)$$

In this case, the solutions for A, B and C, from Eqs. (27)−(29), with the help of Eqs.(37) and (55), can be obtained as

$$A(t) = c_2 l_1 \exp\left[lt - \frac{X_1}{3l{c_2}^3(1+2\eta_0)}\exp\left\{-3l(1+2\eta_0)t\right\}\right], \qquad (56)$$

$$B(t) = c_2 l_2 \exp\left[lt - \frac{X_2}{3l{c_2}^3(1+2\eta_0)}\exp\left\{-3l(1+2\eta_0)t\right\}\right], \qquad (57)$$

$$C(t) = c_2 l_3 \exp\left[lt - \frac{X_3}{3lc_2{}^3(1+2\eta_0)} \exp\left\{-3l(1+2\eta_0)t\right\}\right]. \tag{58}$$

The directional Hubble's parameters, in this case, are given as follows:

$$H_1 = l + \frac{X_1}{c_2{}^3} \exp\left[-3l(1+2\eta_0)t\right], \tag{59}$$

$$H_2 = l + \frac{X_2}{c_2{}^3} \exp\left[-3l(1+2\eta_0)t\right], \tag{60}$$

$$H_3 = l + \frac{X_3}{c_2{}^3} \exp\left[-3l(1+2\eta_0)t\right]. \tag{61}$$

The heat conduction h_1 is obtained as

$$h_1 = \frac{3mX_1}{c_2{}^3} \exp\left[-3l(1+2\eta_0)t\right]. \tag{62}$$

The shear scalar, anisotropy parameter and volume scalar, in this case, are obtained as

$$\sigma^2 = \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{2c_2{}^6} \exp\left[-6l(1+2\eta_0)t\right], \tag{63}$$

$$Am = \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{3l^2 c_2{}^6} \exp\left(-6l(1+2\eta_0)t\right), \tag{64}$$

and

$$V = c_2{}^3 \exp\left(3lt\right). \tag{65}$$

The effective pressure $\bar{p}$ and the energy density ρ, in this case from Eqs. (22) and (23), are given by

$$\begin{aligned} \bar{p} &= \frac{m^2}{c_2{}^2 l_1{}^2} \exp\left[-2\left(lt - \frac{X_1}{6lc_2{}^3}\exp\left(-6lt\right)\right)\right] \\ &\quad - \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{2c_2{}^6} \exp\left(-12lt\right), \end{aligned} \tag{66}$$

$$\begin{aligned} \rho &= 3l^2 - \frac{3m^2}{c_2{}^2 l_1{}^2} \exp\left[-2\left(lt - \frac{X_1}{6lc_2{}^3}\exp\left(-6lt\right)\right)\right] \\ &\quad - \frac{\left(X_1{}^2 + X_2{}^2 + X_3{}^2\right)}{2c_2{}^6} \exp\left(-12lt\right). \end{aligned} \tag{67}$$

To determine ξ, we again assume that the energy density and the kinetic pressure satisfy the equation of state $p = \gamma\rho$, $0 \leq \gamma \leq 1$. Then, from Eqs.(3), (66) and (67), we obtain the coefficient of bulk viscosity as given by

$$\begin{aligned} \xi &= (\gamma+1)l + \frac{\left({X_1}^2 + {X_2}^2 + {X_3}^2\right)(1-\gamma)}{6l{c_2}^6} \exp\left\{-6l(1+2\eta_0)t\right\} \\ &\quad - \frac{m^2(3\gamma+1)}{3l{c_2}^2{l_1}^2} \exp\left[\frac{2X_1}{3l{c_2}^3(1+2\eta_0)} \exp\left\{-3l(1+2\eta_0)t\right\} - 2lt\right]. \end{aligned} \qquad (68)$$

It can be seen here that the energy conservation equation (21) is identically satisfied.

Thus, for this model, the bulk viscosity coefficient is time dependent and the shear viscosity coefficient is constant. The deceleration parameter $q = -1$ indicates that the universe represented by this set of solutions is inflationary. The spatial volume V tends to zero as $t \to -\infty$. At this epoch ρ, p, σ^2, ξ, h_1 are all infinite. Therefore, the universe has a physical singularity in the infinite past. This shows that the universe is infinitely old and has exponential inflationary phase. The directional Hubble's parameters are time-dependent, while the average Hubble's parameter is constant. The expansion scalar is constant throughout the time of evolution right from the beginning. The model is well behaved for $-\infty < t < \infty$. The physical quantities ρ, p, σ^2, ξ and h_1 are decreasing functions of time. The heat function h_1 dies out completely for $t \to \infty$. As $t \to \infty$, the spatial volume becomes infinite and the energy-density becomes constant. The ratio σ^2/θ tends to zero as $t \to \infty$ which shows that the model is isotropic for large time.

4. *Conclusions*

We have presented two classes of exact solutions to the Einstein's field equations by applying the law of variation of the Hubble's parameter for a Bianchi type V space-time filled with an imperfect fluid with both bulk and shear viscosities together together with heat flow. One class of solutions represents the model of the universe with power-law expansion and having a finite physical singularity. The other class of solutions represents an exponentially expanding model of the universe having the singularity in the infinite past. These models approach to isotropy for large time. The physical and kinematical parameters have also been studied. The models of the universe in the two types of cosmologies are new and compatible with the recent observations.

Acknowledgements

The authors are thankful to the referees for their suggestions and comments to improve the manuscript.

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NESAVRŠENI FLUIDNI KOZMOLOŠKI MODELI BIANCHIJEVOG TIPA V S TOKOM TOPLINE

Izvodimo dvije vrste točnih rješenja Einsteinovih jednadžbi polja primjenom posebne ovisnosti Hubbleovog parametra u Bianchijevom prostoru-vremenu tipa V, uz pretpostavku nesavršenog fluida i prisustvo posmične i volumne viskoznosti i izmjene topline. Koeficijent posmične viskoznosti se određuje pretpostavkom da je razmjeran Hubbleovom parametru širenja, a koeficijent volumne viskoznosti uvjetom barotropske ravnoteže gustoće energije i kinetičkog tlaka. Te vrste rješenja predstavljaju kozmološke modele sa širenjem svemira i s početnom fizičkom singularnošću ili bez nje. Raspravljaju se fizikalne i kinematske odlike dvaju modela.