

BIANCHI TYPE-I COSMOLOGICAL MODEL IN LYRA'S MANIFOLD

K. S. ADHAV, A. S. NIMKAR, M. R. UGALE and V. B. RAUT

Department of Mathematics, Sant Gadge Baba Amravati University, Amravati 444602, India, E-mail address: ati_ksadhav@yahoo.co.in

Received 15 July 2008; Revised manuscript received 27 February 2009

Accepted 3 March 2009 Online 31 May 2009

Bianchi type-I cosmological model in Lyra's manifold is obtained assuming a massless scalar field as the source of gravitational field. Some physical and geometrical properties of the model are discussed.

PACS numbers: 98.80.Jk, 98.80.Es

UDC 524.83

Keywords: cosmological model, Bianchi type-I, massless scalar field, Lyra's manifold

1. Introduction

Lyra [1] proposed a modification of the Riemannian geometry by introducing a gauge function into the structureless manifold, as a result of which the cosmological constant arises naturally from the geometry. This bears a remarkable resemblance to Weyl's [2] geometry. In subsequent investigations, Sen [3] and Sen and Dunn [4] formulated a new scalar-tensor theory of gravitation and constructed an analogue of the Einstein field equations based on Lyra's geometry. Halford [5] has shown that the scalar-tensor treatment based on Lyra's geometry predicts the same effects as in general relativity.

Several authors studied cosmological models within the framework of Lyra's geometry with a constant gauge vector in the time direction. Halford [6], Bhamra [7], Kalyanshetti and Waghmode [8], Reddy and Innaiah [9], Reddy and Venkateswarlu [10] are some of the authors who investigated various aspects of the four-dimensional cosmological models in Lyra's manifold. Beesham [11] considered FRW models with a time-dependent field. Singh and Singh [12–14] presented Bianchi type I and III and Kantowaski-Sach cosmological models with a time-dependent displacement field and made a comparative study of the Robertson-Walker models with a constant deceleration parameter based on Lyra's geometry.

Pradhan and Pandey [15] and Rahaman et al. [16, 17] studied some topolog-

ical defects within the framework of Lyra's geometry. Bhowmik and Rajput [18] obtained anisotropic Bianchi type cosmological models on the basis of the Lyra's geometry. Recently Reddy [19] examined the plane-symmetric cosmic strings in Lyra manifold. Adhav et al. [20] studied string cosmology in Brans-Dicke theory for Kasner-type metric.

The massless scalar field has an important role in the study of cosmological models. The massless scalar field in relativistic mechanics yields some significant results regarding both the singularities involved and the Mach's principle. Panigrahi and Sahu [21] studied micro- and macro-cosmological models in the presence of massless scalar-field interacting with a perfect fluid. Mohanty et al. [22] and Adhav et al. [23] studied the exact Bianchi type-I cosmological micromodel in modified theory of general relativity.

In this paper, we study the Bianchi type-I cosmological model in Lyra's manifold assuming a massless scalar field as the source of gravitational field.

2. Field equations and the cosmological model

We consider the Bianchi type-I metric

$$ds^2 = dt^2 - A^2 dx^2 - B^2 dy^2 - C^2 dz^2, \quad (1)$$

where A , B and C are functions of t only. This ensures that the model is spatially homogeneous.

The relativistic field equations in normal gauge in Lyra's manifold, as obtained by Sen [3] for a massless scalar field, read as

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi T_{ij}, \quad (2)$$

where ϕ_i is the time-dependant Lyra's displacement vector and the other symbols have their usual meaning as in Riemannian geometry. We have

$$\phi_i = (0, 0, 0, \beta). \quad (3)$$

The energy momentum tensor for massless scalar field distribution is given by

$$T_{ij} = V_{,i}V_{,j} - \frac{1}{2}g_{ij}V_kV^k, \quad (4)$$

together with

$$g_{ij}V^iV^j = 1. \quad (5)$$

In addition to above, the massless scalar field V satisfies the Klein-Gordon wave equation

$$g^{ij}V_{;ij} = 0. \quad (6)$$

Here the semicolon (;) denotes covariant differentiation. Now, with the help of Eqs. (4) and (5), the field Eqs. (2) and Eq. (6) for the metric (1) can be written as

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{3}{4} \beta^2 = -4\pi V_4^2, \quad (7)$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} + \frac{3}{4} \beta^2 = -4\pi V_4^2, \quad (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{3}{4} \beta^2 = -4\pi V_4^2, \quad (9)$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3}{4} \beta^2 = 4\pi V_4^2, \quad (10)$$

$$\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} \right) V_4 + V_{44} = 0. \quad (11)$$

The suffix '4' after A , B and C denotes ordinary differentiation w.r. to t . Adding Eqs. (7), (8) and (9) to 3-times Eq. (10), we get

$$\frac{(ABC)_{44}}{ABC} = 0,$$

which implies that

$$(ABC)_{44} = 0. \quad (12)$$

Since A , B and C are non-zero, simple integration of Eq. (12) yields

$$ABC = at + b, \quad (13)$$

where a and b are constants of integration. Further, (Kasner [24], Brevik and Pettersen [25], Cataldo et al. [26]) Eq. (13) can be written in the following explicit form

$$A = A_0(at + b)^{P_1}, \quad B = B_0(at + b)^{P_2}, \quad C = C_0(at + b)^{P_3}, \quad (14)$$

where the constants of integration P_1 , P_2 and P_3 , and A_0 , B_0 and C_0 satisfy

$$P_1 + P_2 + P_3 = 1, \quad P_1^2 + P_2^2 + P_3^2 = 1, \quad A_0 B_0 C_0 = 1. \quad (15)$$

With the help of Eq. (14), equation Eq. (1) becomes

$$ds^2 = dt^2 - A_0^2(at + b)^{2P_1} dx^2 - B_0^2(at + b)^{2P_2} dy^2 - C_0^2(at + b)^{2P_3} dz^2. \quad (16)$$

The metric can be transformed through a proper choice of coordinates into the form

$$ds^2 = dT^2 - T^{2P_1}dX^2 - T^{2P_2}dY^2 - T^{2P_3}dZ^2, \quad (17)$$

which is Bianchi type-I metric of the Kasner form. With the help of Eqs. (13) and (14), Eq. (11) reduces to

$$V = \frac{V_0}{a} \log(at + b), \quad (18)$$

where V_0 is a constant of integration. Hence we get

$$\beta^2 = \frac{4}{3} \left[\frac{a^2(P_1P_2 + P_1P_3 + P_2P_3) - 4\pi V_0^2}{(at + b)^2} \right]. \quad (19)$$

3. Some physical properties

The model (17) represents an exact cosmological model in Lyra's manifold when the source of gravitational field is a massless scalar field. The massless scalar field V is given by Eq. (18). The physical and kinematical parameters for the model (17) are

$$\text{spatial volume} = \sqrt{-g} = T, \quad (20)$$

$$\text{expansion scalar} \quad \theta = \frac{a}{3T}, \quad (21)$$

$$\text{shear scalar} \quad \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{a^2}{162T^2}, \quad (22)$$

$$\text{rotation tensor} \quad \omega_{ij} = \frac{1}{2} [g_{i4,j} - g_{j4,i}], \quad (23)$$

$$\omega^2 = \frac{1}{2} \omega_{ij} \omega^{ij} = 0. \quad (24)$$

The deceleration parameter q is given by (Feinstein et al. [27])

$$q = -3\theta^{-2} \left[\theta_{;\alpha} U^\alpha + \frac{1}{3} \theta^2 \right],$$

$$q = 8 > 0. \quad (25)$$

In this model, the spatial volume tends to infinity and the expansion scalar θ and shear scalar σ tend to zero as $T \rightarrow \infty$. Also, since $\lim_{T \rightarrow \infty} (\sigma/\theta) \neq 0$, the model does not approach isotropy for large value of T . The positive value of the deceleration parameter indicates that the model decelerates in the standard way.

4. Conclusion

In this paper, we have obtained Bianchi type-I cosmological model in the framework of Lyra's geometry, assuming a massless scalar field as the source of gravitational field. It is observed that the model is expanding, shearing, non-rotating and has no initial singularity. Our model throws some light on the understanding of structure formation of the universe in Lyra's manifold.

Acknowledgements

Authors are thankful to referees for their valuable comments which helped in the improvement of the standard of paper.

References

- [1] G. Lyra, *Math. Z.* **54** (1951) 52.
- [2] H. Weyl, *Sitzungsber. Preuss. Akad. Wiss.* **465** (1918).
- [3] D. K. Sen, *Z. Phys.* **149** (1957) 311.
- [4] D. K. Sen and K. A. Dunn, *J. Math. Phys.* **12** (1971) 578.
- [5] W. D. Halford, *J. Math. Phys.* **13** (1972) 1399.
- [6] W. D. Halford, *Austr. J. Phys.* **23** (1970) 863.
- [7] K. S. Bhamra, *Austr. J. Phys.* **27** (1974) 541.
- [8] S. B. Kalyanshetti and B. B. Waghmode, *Gen. Rel. Grav.* **14** (1982) 823.
- [9] D. R. K. Reddy and P. Innaiah, *Astrophys. Spa. Sci.* **114** (1985) 285.
- [10] D. R. K. Reddy and R. Venkateswarlu, *Astrophys. Space Sci.* **149** (1988) 287.
- [11] A. Beesham, *Austr. J. Phys.* **41** (1988) 833.
- [12] T. Singh and G. P. Singh, *J. Math. Phys.* **32** (1991) 2456.
- [13] T. Singh and G. P. Singh, *Int. J. Theor. Phys.* **31** (1992) 1433.
- [14] T. Singh and G. P. Singh, *Fortschr. Phys.* **41** (1993) 737.
- [15] A. Pradhan and H. R. Pandey, arXiv: gr-qc/0307038 (2003).
- [16] F. Rahaman, S. Chakraborty, M. Hossain, N. Begum and J. Beram, *Ind. J. Phys. B* **76** (2002a) 747.
- [17] F. Rahaman and J. Bera, *Int. J. Mod. Phys. D* **10** (2002b) 729.
- [18] B. B. Bhowmik and A. Rajput, *Pramana J. Phys.* **62**, 6 (2004) 1187.
- [19] D. R. K. Reddy, *Astrophys. Space Sci.* **300** (2005) 381.
- [20] K. S. Adhav, A. S. Nimkar and R. P. Holey, *Int. J. Theor. Phys.* **46** (2007) 2396.
- [21] U. K. Panigrahi and R. C. Sahu, *Theo. Appl. Mech.* **30**, 3 (2003) 163.
- [22] G. Mohanty, U. K. Panigrahi and R. C. Sahu, *Astrophys. Space Sci.* **281** (2002) 633.
- [23] K. S. Adhav, A. S. Nimkar and M. V. Dawande, *Int. J. Theor. Phys.* **47** (2008) 3201.
- [24] E. Kasner, *Am. J. Math.* **43** (1921) 217.
- [25] I. Brevik and S. V. Pettersen, *Phy. Rev. D* **56** (1997) 3322.

- [26] M. Cataldo, S. D. Campo and P. Salgado, arXiv:gr-qc/0011027v1 (2000).
- [27] A. Feinstein, J. Ibanez and P. Labarge, J. Math. Phys. **36** (1995) 4962.

BIANCHIJEV KOZMOLOŠKI MODEL TIP I U LYRINOJ
MNOGOSTRUKOSTI

Razvili smo Bianchijev kozmološki model tipa I u Lyrinoj mnogostrukosti kada je izvor gravitacijskog polja bezmaseno skalarno polje. Raspravljaju se neka fizička i geometrijska svojstva modela.