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LRS BIANCHI TYPE II STRING COSMOLOGICAL MODELS FOR PERFECT FLUID DISTRIBUTION IN GENERAL RELATIVITY

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The present study deals with locally rotationally symmetric (LRS) Bianchi type II string cosmological models with perfect fluid distribution of matter. We consider two cases: (i) $\rho+\lambda = 0$ and (ii) $\rho-\lambda = 0$, where ρ and λ are the rest energy density and the tension density of the string cloud, respectively. The physical properties of the models is discussed.

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1. *Introduction*

In recent years, there has been a considerable interest in string cosmology. Cosmic strings are considered as topologically stable objects which presumably existed during a phase transition in the early universe (Kibble [1]). They play an important role in the study of the early universe. They arose during the phase transition after the big bang explosion as the temperature decreased below some critical temperature as predicted by the grand unified theories (Zel'dovich et al. [2]; Kibble [1, 3]; Everett [4]; Vilenkin [5]). It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies (Zel'dovich [6]). The cosmic strings have stress-energy and couple to the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier [7] who considered the massive string to be formed by geometric strings with particle attached along its extension. Letelier [8] first used this idea in obtaining cosmological solutions in Bianchi I and Kantowski-Sachs space-times. Stachel [9] also studied massive string.

Bali et al. [10 – 16] obtained Bianchi types I, III and IX string cosmological models in general relativity. Yadav et al. [17] studied some Bianchi type I viscous fluid string cosmological models with magnetic field. Recently, Wang [18 – 21] also discussed LRS Bianchi type I and Bianchi type III cosmological models for a cloud string with bulk viscosity. Recently Yadav, Pradhan and Rai [22] obtained the integrability of cosmic string in Bianchi type III space-time in the presence of bulk viscous fluid by applying a new technique.

The present day Universe is satisfactorily described by homogeneous and isotropic models given by the FRW space-time. The Universe on a smaller scale is neither homogeneous nor isotropic nor do we expect the Universe in its early stages to have had these properties. Homogeneous and anisotropic cosmological models have been widely studied in the framework of general relativity in the search of a realistic picture of the Universe in its early stages. Although these are more restricted than the inhomogeneous models which explain a number of observed phenomena quite satisfactorily. Bianchi type-II space-time has a fundamental role in constructing cosmological models suitable for describing the early stages of evolution of Universe. Asseo and Sol [23] emphasized the importance of Bianchi type-II universe. A spatially homogeneous Bianchi model necessarily has a three-dimensional group, which acts simply transitively on space-like three-dimensional orbits. Here we confine ourselves to a locally rotationally-symmetric (LRS) model of Bianchi type-II. This model is characterized by three metric functions $R_1(t)$, $R_2(t)$ and $R_3(t)$ such that $R_1 = R_2 \neq R_3$. The metric functions are functions of time only. (For non-LRS Bianchi metrics we have $R_1 \neq R_2 \neq R_3$). For LRS Bianchi type -II metric, Einstein's field equations reduce, in the case of perfect fluid distribution of matter, to three nonlinear differential equations.

Roy and Banerjee [24] dealt with LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Recently, Wang [25] studied the Letelier model in the context of LRS Bianchi type-II space-time. In this Letter, we investigate LRS Bianchi type-II string cosmological models for perfect fluid distribution under two conditions: (i) $\rho + \lambda = 0$ and (ii) $\rho = \lambda$.

2. *Metric and field equations*

We consider the LRS Bianchi type II metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + (B\mathrm{d}x + Bz\mathrm{d}y)^2 + A^2\mathrm{d}y^2 + A^2\mathrm{d}z^2, \tag{1}$$

where A and B are functions of t only. The energy-momentum tensor for a cloud of strings with perfect fluid distribution is taken as

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \lambda x_i x^j, \tag{2}$$

where v_i and x_i satisfy condition

$$v^i v_i = -x^i x_i = -1, \;\; v^i x_i = 0 \,, \tag{3}$$

p is the isotropic pressure, ρ is the proper energy density for cloud strings with particles attached to them, λ is the string tension density, v^i the four-velocity of the particles and x^i is a unit space-like vector representing the direction of the string. In a co-moving coordinate system, we have

$$v^i = (0,0,0,1), \;\; x^i = \left(\frac{1}{B}, 0, 0, 0\right). \tag{4}$$

If the particle density of the configuration is denoted by ρ_p, then we have

$$\rho = \rho_p + \lambda \,. \tag{5}$$

The Einstein's field equations

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \,, \tag{6}$$

for the line-element (1) lead to

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = p - \lambda \,, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = p \,, \tag{8}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = -\rho \,, \tag{9}$$

where an overdot stands for the first and double overdot for the second derivative with respect to t. The particle density ρ_p is given by

$$\rho_p = \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{3\dot{A}\dot{B}}{AB} - \frac{1}{4}\frac{B^2}{A^4} \,. \tag{10}$$

3. Solutions of the field equations

The field equations (7) − (9) are a system of three equations with five unknown parameters A, B, p, ρ and λ. Two additional constraints relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion (θ) in the model is proportional to the shear (σ). This condition leads to

$$A = B^m, \tag{11}$$

where m is the proportionality constant. Eqs. (7) – (9) lead to

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} + \frac{2\dot{A}}{A}\right) - \frac{5}{4}\frac{B^2}{A^4} = -(\lambda + \rho)\,. \tag{12}$$

From Eqs. (11) and (12), we obtain

$$(m-1)\frac{\ddot{B}}{B} + 3m^2\frac{\dot{B}^2}{B^2} = \frac{5}{4}B^{(2-4m)} - (\lambda + \rho)\,. \tag{13}$$

In order to overcome the under-determinacy we have here because of the five unknowns involved in three independent field equations, we consider the following two cases:

(I) $\rho + \lambda = 0$, i.e. the sum of rest energy density and tension density for cloud of strings vanish (Reddy [26, 27], Mohanty et al. [28]) and

(II) $\rho_p = 0$. This corresponds to the state equation for a cloud of massless geometric (Nambu) strings given by $\rho = \lambda$.

3.1. Case I:

In this case

$$\lambda + \rho = 0\,. \tag{14}$$

From (13) and (14), we obtain

$$2\ddot{B} + \left(\frac{6m^2}{m-1}\right)\frac{\dot{B}^2}{B} = \frac{5}{2(m-1)}B^{(3-4m)}. \tag{15}$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \mathrm{d}f/\mathrm{d}B$. Then

$$\frac{\mathrm{d}(f^2)}{\mathrm{d}B} + \frac{6m^2}{(m-1)}\frac{f^2}{B} = \frac{5}{2(m-1)}B^{(3-4m)}. \tag{16}$$

Eq. (16), after integration, reduces to

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \sqrt{\frac{5B^{4(1-m)}}{4(m^2+4m-2)} + c_1 B^{6m^2/(1-m)}}, \tag{17}$$

where c_1 is an integrating constant.
Hence the metric (1) reduces to

$$\mathrm{d}s^2 = -\left[\frac{\mathrm{d}B^2}{\frac{5B^{4(1-m)}}{4(m^2+4m-2)} + c_1 B^{6m^2/(1-m)}}\right] + B^2\mathrm{d}x^2$$

$$+ [B^2z^2 + B^{2m}]\mathrm{d}y^2 + 2B^2z\,\mathrm{d}x\,\mathrm{d}y + B^{2m}\mathrm{d}z^2. \tag{18}$$

After making suitable transformation of coordinates, i.e. $B = \tau$, metric (18) reduces to

$$\mathrm{d}s^2 = -\left[\frac{\mathrm{d}\tau^2}{\frac{5\tau^{4(1-m)}}{4(m^2+4m-2)} + c_1\tau^{6m^2/(1-m)}}\right] + \tau^2\mathrm{d}x^2$$

$$+ [\tau^2 z^2 + \tau^{2m}]\mathrm{d}y^2 + 2\tau^2 z\,\mathrm{d}x\,\mathrm{d}y + \tau^{2m}\mathrm{d}z^2. \tag{19}$$

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (19) are given by

$$p = \frac{(4m^2+m-3)}{(m^2+4m-2)}\tau^{2(1-2m)} + c_1\tau^{6m^2/(1-m)}\left[\frac{1}{\tau^2} + \frac{m^2(1+m)}{(1-m)}\right], \tag{20}$$

$$\rho = -\lambda = \frac{(3m^2-3m-1)}{2(m^2+4m-2)}\tau^{2(1-2m)} + c_1\tau^{2(3m^2+m-1)/(1-m)}, \tag{21}$$

$$\rho_p = \frac{(3m^2-3m-1)}{(m^2+4m-2)}\tau^{2(1-2m)} + 2c_1\tau^{2(3m^2+m-1)/(1-m)}, \tag{22}$$

$$\theta = \frac{(2m+1)}{\tau}\left[\frac{5}{4(m^2+4m-2)}\tau^{4(1-m)} + c_1\tau^{m^2/(1-m)}\right]^{\frac{1}{2}}, \tag{23}$$

$$\sigma^2 = \frac{(m-1)^2}{3\tau^2}\left[\frac{5}{4(m^2+4m-2)}\tau^{4(1-m)} + c_1\tau^{m^2/(1-m)}\right], \tag{24}$$

$$V^3 = \tau^{(2m+1)}. \tag{25}$$

From Eqs. (23) and (24), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} = \text{constant}. \tag{26}$$

If we choose a suitable value of constant m, i.e. for $m \geq \frac{4}{3}$, we find that the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. The model (19) starts with a big bang at $\tau = 0$. The expansion in the model decreases as time increases. The expansion in the model stops at $\tau = \infty$. When $\tau \to 0$ then $\rho \to \infty$, $\lambda \to \infty$. When $\tau \to \infty$ then $\rho \to 0$, $\lambda \to 0$. Also $p \to \infty$ when $\tau \to 0$ and $p \to 0$ when $\tau \to \infty$. Since $\sigma/\theta =$ constant, the model does not approach isotropy in general.

According to Refs. [1, 29], when $\rho_p/ \mid \lambda \mid > 1$, in the process of evolution, the universe is dominated by massive strings, and when $\rho_p/ \mid \lambda \mid < 1$, the universe is dominated by the strings. In this case, from Eqs. (21) and (22), we obtain

$$\frac{\rho_p}{\mid \lambda \mid} = 2 > 1. \tag{27}$$

Thus, in our model, the universe is dominated by massive strings throughout the whole process of evolution.

3.2. *Case II:*

In this case

$$\lambda - \rho = 0. \tag{28}$$

Eqs. (7)−(9) lead to

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{3}{4}\frac{B^2}{A^4} = \rho - \lambda. \tag{29}$$

From Eqs. (28) and (29), we obtain

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} - \frac{3}{4}\frac{B^2}{A^4} = 0. \tag{30}$$

Using (11) in (30) leads to

$$2\ddot{B} + \left(\frac{2m(m-2)}{m-1}\right)\frac{\dot{B}^2}{B} = \frac{3}{2(m-1)}B^{(3-4m)}. \tag{31}$$

Let $\dot{B} = f(B)$ which implies that $\ddot{B} = ff'$, where $f' = \mathrm{d}f/\mathrm{d}B$. Hence (31) reduces to

$$\frac{d}{dB}(f^2) + \frac{2m(m-2)}{(m-1)}\frac{f^2}{B} = \frac{3}{2(m-1)}B^{(3-4m)}. \tag{32}$$

Eq. (32), after integration, reduces to

$$\frac{dB}{dt} = \sqrt{-\frac{3}{4(m^2+2)}B^{4(1-m)} + c_2 B^{2m(m-4)/(1-m)}}. \tag{33}$$

where c_2 is an integrating constant.
After using suitable transformation of coordinates i.e ($B = \tau$), metric (1) reduces to

$$\mathrm{d}s^2 = -\left[\frac{\mathrm{d}\tau^2}{-\frac{3}{4(m^2+2)}\tau^{4(1-m)} + c_2\tau^{2m(m-4)/(1-m)}}\right] + \tau^2\mathrm{d}x^2$$
$$+ [\tau^2 z^2 + \tau^{2m}]\mathrm{d}y^2 + 2\tau^2 z\,\mathrm{d}x\,\mathrm{d}y + \tau^{2m}\mathrm{d}z^2. \tag{34}$$

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p, the scalar of expansion (θ), the shear σ) and the proper volume (V^3) for the model (34) are given by

$$p = \frac{(m^2-1)}{(m^2+2)}\tau^{2(1-2m)} - c_2 m \tau^{2(m^2-3m-1)/(1-m)}, \tag{35}$$

$$\rho = \lambda = \frac{(2m^2+3m+1)}{2(m^2+2)}\tau^{2(1-2m)} - m(m+2)c_2\tau^{2(m^2-3m-1)/(1-m)}, \tag{36}$$

$$\rho_p = \rho - \lambda = 0, \tag{37}$$

$$\theta = \frac{(2m+1)}{\tau}\left[-\frac{3}{4(m^2+2)}\tau^{4(1-m)} + c_2\tau^{2m(m-4)/(1-m)}\right]^{\frac{1}{2}}, \tag{38}$$

$$\sigma^2 = \frac{(m-1)^2}{3\tau^2}\left[-\frac{3}{4(m^2+2)}\tau^{4(1-m)} + c_2\tau^{2m(m-4)/(1-m)}\right]. \tag{39}$$

From Eqs. (38) and (39), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(m-1)^2}{3(2m+1)^2} = \text{constant}. \tag{40}$$

If we choose suitable values of constants m and c_2, i.e. for $m > 3$, $c_2 < 0$, we find that the energy conditions $\rho \geq 0$, $\rho_p \geq 0$ are satisfied. Otherwise, the physical behaviour of the model (34) is similar to that of the model (19).

From Eqs. (35) and (36), we obtain

$$\frac{\rho_p}{\mid \lambda \mid} = 0. \tag{41}$$

Hence, in this case the strings dominate over the particles.

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OPĆE-RELATIVISTIČKI BIANCHIJEVI LRS KOZMOLOŠKI MODELI TIPA II SA STRUNAMA I PERFEKTNOM RASPODJELOM TEKUĆINE

Proučavamo lokalno-rotacijski simetrične (LRS) Bianchijeve kozmološke modele tipa II sa strunama za perfektnu tekućinsku raspodjelu mase. Razmatramo dva slučaja: $\rho + \lambda = 0$ i (ii) $\rho - \lambda = 0$, gdje su ρ i λ gustoća energije mirovanja odn. gustoća napetosti oblaka struna. Opisujemo također fizikalna svojstva tih modela.