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LETTER TO THE EDITOR

ACCELERATED MAGNETIC GAUSS-BONNET COSMOLOGY

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Gauss-Bonnet cosmology with magnetic field and non-minimal coupling is investigated and some features are discussed.

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Recently, the cosmological matter field called *quintessence* (shortly Q-matter/scalar field), that filled the universe and was shown to give rise to an accelerated expansion, stems from the observations of extra-galactic supernovae of type Ia (SNeIa) [1] based upon measurements of luminosity-redshift relation up to about $z \approx 1$ and independently supported by the cosmic microwave background radiation (CMBR) [2] and large scale structure (LSS) [3]. This Q-matter with negative pressure (dark energy) contributes up to $\Omega_{\rm DE} \approx 0.60 - 0.70$ to the critical density of the present universe and obeys the equation of state parameter $w = p/\rho < -1/1$ where p and ρ are, respectively, the pressure and energy density and accounts for the missing energy if one really believes in inflation theory in all its aspects predicting $\Omega = 1$. Note that the flatness of the universe is one of the main predictions of inflation. Thus the universe is nearly geometrically flat, and structures grown out of a primordial linear spectrum of nearly Gaussian and scale-invariant energy-density perturbations. Starting from a wide set of initial conditions in the early universe, the scalar field is able to converge to the present accelerated regime, thus alleviating the fine tuning. Moreover, analysis of recent observational data also supports $w \leq -1$ strongly [4−7]. The critical value $w = -1$ corresponds to the phantom barrier. In other words, dark energy is an observationally challenging problem. The

Q-matter can behave like a cosmological constant by combining positive energy density with negative pressure. However, the energy scale related to the cosmological constant is approximately 122 orders of magnitude smaller than the energy of the vacuum originated at the Planck time (the cosmological constant problem).

Despite the fact that a dark-matter component is generically predicted in supersymmetry (SUSY), the dark-energy component poses severe questions. A consistent dark-energy candidate should explain why the present amount is so small with respect to the fundamental scale (a fine tuning of 123 orders of magnitude if compared to the Planck energy scale), and why it is comparable with the critical density today (the cosmological coincidence problem). At the present epoch, much work has been done and a new class of cosmological models has been proposed to solve the quintessence problem. Most popular candidates include the ΛCDM model [8] consisting a mixture of cosmological constant Λ and CDM or WIMPS composed of weakly interacting massive particles which must be relics of a grand unified phase of the Universe, quintessence with a very shallow many-forms potential (self-interacting, slowly varying scalar field) [9], K-essence [10], viscous fluid [11], Chaplygin gas which obeys the equation of state $p = -B\rho$ [12, 13], generalized Chaplygin gas model (GCGM) which mimics both dark matter and dark energy and obeys the equation of state $p = -B\rho^{\alpha}$ or the modified equation of state $p = A\rho - B\rho^{\alpha},\ A >),\ 0 \le \alpha \le 1$ which represents a universe evolving from a radiation dominated era to the model ΛCDM [14, 15], Brans-Dicke (BD) pressureless solutions (the scalar field that accounts for the dark energy component is non-minimally coupled to gravity and, besides, is a part of the gravity sector) [16 − 18], decaying Higgs fields [19], dilaton field of string theories with gaugino condensation [20], tachyon as a dark energy source [21, 22], etc. Most of these models require many constraints and fine tuning of parameters for different types of potentials which model quintessence. What most of these models do in general is to fine tune the overall scale of the dark energy potential to be of order of the present critical density of the universe $\rho_{c\,0} = 3H_0^2/(8\pi G) = 8.1h^2 \times 10^{-47}$ GeV^4, where $H_0 = 100\,h$ km s^{-1} Mpc^{-1} is the Hubble constant, $h = 0.72 \pm 0.08$ and G is the gravitational constant.

There exist phenomenological attempts to explain the evidence for cosmic acceleration entirely in terms of a modified gravity theory. Some nice alternative scalar theories include the string-inspired dilaton gravities, a time-varying energy density, dilaton from string theory, supersymmetric exotic particles, massive neutrinos, holographic dark energy, $N = 8$ and $N = 2$ supergravity, M/string theory, phantom energy and the higher-derivative theories with an additional quadratic scalar curvature like R^n, R^{-1}, R^2, R^3, R^4, $R_{\mu\nu}R^{\mu\nu}$, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, $\Box R$, $R\Box R$, TR$(1/R)$, $fR)$, ln$R$ multidimensional R^{-1} theory [23] which may mimic the effects of DE on the Hubble flow, Gauss-Bonnet cosmology [24], non-minimal coupling theories in all their aspects and forms [25], tachyonic quintessence inflation models [26], holographic dark energy density models [27], the introduction of extra-dimensions which may provide a possible solution to the flatness and horizon problem by producing large amount of entropy during contraction process, as compared to the standard inflationary scenario [28], Lyra's geometrical manifold [29], etc. These scalar-tensor theories of gravity revealed interesting consequences and have potential to provide a linkage between the accelerated expansion of the universe and fundamental physics, although the acceleration cannot be explained by the standard model of particle physics and classical general relativity. Certainly, various predictions of a consistent theory of modified gravity should be tested. All these tests may suggest further modifications giving true and realistic cosmological description. Despite the number of efforts and theories, it seems that we are still far away from the theoretical understanding of dark energy and its astrophysical origin. The choice of possibilities reflects the undisputable fact that the true nature of the DE has not been convincingly explained yet.

In this work, the flat Friedmann-Robertson-Walker (FRW) model with metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\big[\mathrm{d}r^2 + r^2\big(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\big)\big], \tag{1}$$

where $a(t)$ is the scale curvature will be explored within the framework of Gauss-Bonnet (GB) curvature action. For instance, non-singular cosmology may occur in the study of string-induced gravity if the GB coupling with scalar field is taken into account. The present paper is devoted to the study of the role of GB theory non-minimal coupling to the scalar field ϕ and the Ricci curvature R as $\xi R\phi^2/2$, where ξ is the non-minimal coupling parameter. Moreover, we believe that the magnetic fields have a significant role at the galactic, intergalactic and cosmological scales. According to Melvin, the cosmological solution for dust and electromagnetic field shows the important role played by the magnetic field. Remember that in the early universe, matter was highly ionized and coupled to field. Latter during cooling, as a result of expansion of ions combined to form neutral matter [30]. The cosmological models with magnetic field have been discussed by a number of authors in general relativity. But to our knowledge, there has not been any work in literature where GB action has been considered to study a cosmological model with magnetic field and non-minimal coupling simultaneously. So it is interesting to study non-minimal cosmological model in the presence of a magnetic field within the framework of GB action.

The starting action of the theory is

$$S_{\mathrm{Total}} = \int \sqrt{-g}\,\mathrm{d}^4x \left(\frac{R}{2}\left(\frac{1}{\kappa^2} - \xi\phi^2\right) - \frac{1}{2}\omega\partial_\mu\phi\partial^\mu\phi - V(\phi)\right.$$
$$\left. + f(\phi)\Big(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\Big)\right) + S_{\mathrm{matter}} + S_{\mathrm{EM}}\,, \tag{2}$$

where ξ is a dimensionless coupling constant equal to 1/6 in conformal coupling, $\kappa^2 = 8\pi G$, G is the gravitational coupling constant, g is the metric, R is the scalar curvature, $\omega = 1$ for the canonical scalar, ϕ is the scalar field, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ is the GB invariant, S_{matter} is the action for ordinary matter and S_{EM} is the electromagnetic action. $V(\phi)$ and $f(\phi)$ are given by the power-law functions

$f(\phi) = f_0\phi^m$, $V(\phi) = V_0\phi^m$, with the constant parameters f_0 and V_0. m and n are real numbers.

The action (2) yields the following gravitational field equation [24, 25]

$$\left(\frac{1}{\kappa^2} - \xi\phi^2\right)\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right) - \omega\left(\frac{1}{2}\partial^\mu\phi\partial^\nu\phi - \frac{1}{4}g^{\mu\nu}\partial_\rho\phi\partial^\rho\phi\right)$$

$$-\frac{1}{2}g^{\mu\nu}\Big[-V(\phi) + f(\phi)\Big(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\Big)\Big]$$

$$+2f(\phi)RR^{\mu\nu} - 4f(\phi)R^{\mu}{}_{\sigma}R^{\nu\rho} + 2f(\phi)R^{\mu\rho\sigma\tau}R^{\nu}_{\rho\sigma\tau} - 4f(\phi)R^{\mu\rho\sigma\nu}R_{\rho\sigma}$$

$$-2(\nabla^\mu\nabla^\nu f(\phi))R + 2g^{\mu\nu}(\nabla^2 f(\phi))R + 4(\nabla_\rho\nabla^\nu f(\phi))R^{\nu\rho} + 4(\nabla_\rho\nabla^\nu f(\phi))R^{\mu\rho}$$

$$-4(\nabla^2 f(\phi))R^{\mu\nu} - 4g^{\mu\nu}(\nabla_\rho\nabla_\sigma f(\phi))R^{\rho\sigma} + 4(\nabla_\rho\nabla_\sigma f(\phi))R^{\mu\rho\nu\sigma}$$

$$-\xi(g^{\mu\nu}\Box - \nabla^\mu\nabla^\nu)\phi^2 = T^{\mu\nu}_{\text{matter}} + T^{\mu\nu}_{\text{EM}}\,, \tag{3}$$

where the operator

$$\Box = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}\,g^{\mu\nu}\frac{\partial}{\partial x^\nu}\right) \tag{4}$$

denotes the d'Alembertian, ∇^ν is the contra-covariant derivative, $\mu, \nu = 0,\ 1,\ 2,\ 3$ and $g^{\mu\nu}$ is the metric tensor. The stress-energy momentum tensor for a perfect fluid and the magnetic tensor are:

$$T^{\mu\nu}_{\text{matter}} = (p+\rho)u^\mu u^\nu + pg^{\mu\nu}\,, \tag{5}$$

$$T^{\mu\nu}_{\text{EM}} = \frac{1}{4\pi}\Big(F^\mu_\delta F^\delta_\nu - \frac{1}{4}F^{\delta\rho}_{\delta\rho}\delta^\mu_\nu\Big)\,, \tag{6}$$

respectively. p and ρ are the pressure and density of the cosmic fluid, F_{ij} is the electromagnetic field tensor, $T = g^{ij}T_{ij}$ is the trace of the stress-energy tensor. The effective gravitational constant in the theory is

$$\kappa^2_{\text{effective}} = \frac{\kappa^2}{1 - \kappa^2\xi\phi^2}\,. \tag{7}$$

The Maxwell equations $F_{[\mu\nu,\rho]} = 0$ and $\Big[F^{\mu\nu}\sqrt{-g}\Big]_{,\mu} = 0$ give [30, 31]

$$E^0_0 = E^1_1 = -E^2_2 = -E^3_3 = -\frac{A^2}{8\pi a^4}\,, \tag{8}$$

which are the components of the stress energy tensor for the electromagnetic field. A is a constant to be identified as the only non-zero component of the electromagnetic field tensor T_{23}, being that since the magnetic field is assumed in the x-direction. The (t,t) component in the action has the form

$$\frac{\omega}{2}\dot{\phi}^2 + 6\xi H\phi\dot{\phi} - \frac{3H^2}{\kappa^2}\left(1-\kappa^2\xi\phi^2\right) + V(\phi) - 24\dot{\phi}\,\frac{\mathrm{d}f}{\mathrm{d}\phi}\,H^3 + \rho_m + \frac{A^2}{8\pi a^4} = 0\,, \quad (9)$$

where $H = \dot{a}/a$ is the Hubble parameter and ρ_m is the matter density. By varying the action over the scalar field ϕ, we obtain

$$\omega(\ddot{\phi} + 3H\dot{\phi}) + \frac{\mathrm{d}V}{\mathrm{d}\phi} - 24\frac{\mathrm{d}f}{\mathrm{d}\phi}(\dot{H}H^2 + H^4) + \xi R\phi = 0\,. \quad (10)$$

A characteristic feature is the presence of the last term GA^2/a^4 that behaves like radiation. It arises from the electromagnetic field in the theory. A similar situation appears also in brane cosmology. We assume that (latetime behavior) the scale factor behaves as $a = a_0 t^q$ (power-law), a_0 is a positive constant and q is assumed to be positive to correspond to expanding universe. Moreover, we propose the power-law behavior of the scalar field $\phi = \phi_0 t^p$, where p is a real constant. It is easy to check from Eq. (6) that a consistent relation is obtained if the scalar curvature decays like $R = R_0 t^{-2}$ with the constant parameter R_0. Consequently, Eq. (10) takes the form

$$\omega\Big(\phi_0 p(p-1)t^{p-2} + 3pq\phi_0 t^{p-2}\Big) + nV_0\phi_0 t^{p(n-1)}$$

$$-24mf_0\phi_0^{m-1}t^{p(m-1)-4}(q^4 - q^3) + \xi R_0\phi_0 t^{p-2} = 0\,. \quad (11)$$

Thus $n = 2(p-1)/p$ and $m = 2(p+1)/p$. Further, Eq. (9) gives

$$\frac{\omega}{2}\phi_0^2 p^2 t^{2p-2} + 6\xi q\phi_0\phi_0 p t^{2p-2} - \frac{3q^2}{t^2\kappa^2} + 3q^2\xi\phi_0^2 t^{2p-2}$$

$$+V_0\phi_0^n t^{2p-2} - 24pq^3 mf_0\phi_0^m\, t^{p(m-1)+p-4} + \rho_m + \frac{A^2 t^{-4q}}{8\pi a_0^4} = 0\,, \quad (12)$$

from which a desirable consistent solution is obtained if the gravitational coupling constant behaves like $\kappa^2 = \kappa_0^2 t^{-2p}$ for some constant parameter κ_0^2, and the density of matter varies like $\rho_m = \rho_m^0 t^{2p-2}$ for some constant parameter ρ_m^0 augmented by the constraints $p + 2q = 1$, which is due to the presence of the magnetic term. Consequently, Eq. (12) takes for the canonical scalar field ($\omega = 1$) form

$$2(2-9\xi)q^2 + 4q(3\xi-1) + \phi_0^{-2}\left(2V_0\phi_0^n - 6q^2\kappa_0^{-2} - 48pq^3 mf_0\phi_0^m + 2\rho_m^0 + \frac{A^2}{4\pi a_0^4}\right) = 0\,. \quad (13)$$

A desirable solution which corresponds to $\phi_0 \neq 0$ is

$$2(2-9\xi)q^2 + 4q(3\xi - 1) + 1 = 0\,, \tag{14}$$

and

$$2V_0\phi_0^n - 6q^2\kappa_0^{-2} - 48pq^3 m f_0 \phi_0^m + 2\rho_m^0 + \frac{A^2}{4\pi a_0^4} = 0\,. \tag{15}$$

Equation (14) gives the following possible positive root

$$q = \frac{-2(3\xi-1) + \sqrt{6\xi(6\xi-1)}}{2(2-9\xi)}\,, \quad \xi \neq 2/9\,. \tag{16}$$

In the absence of the non-minimal coupling parameter, e.g. $\xi = 0$, $q = 1/2$. For $\xi = 1/6$, which corresponds to the conformal coupling case widely used in quantum field theory in curved spaces and in string theories (the value of ξ that is an infrared fixed point of the renomalization group theory and as required by the Einstein equivalence principle), $q = 1$ and thus the scale factor evolves uniformly as $a(t) = a_0 t$, while the scalar field decays with time as $\phi = \phi_0 t^{-1}$, the gravitational coupling constant increases with time like $\kappa^2 = \kappa_0^2 t^2$ and the density of matter decays like $\rho_m = \rho_m^0 t^{-4}$. An accelerated expansion may occur for $q > 1$ ($n < 4,\ m < 0$) or $1/6 < \xi < 2/9$. This range is interesting for superstring theories [32]. Equation (11) gives

$$\xi R_0 = 24 m f_0 \phi_0^{m-2}(q^4 - q^3) - \omega q(1-2q) - nV_0\,. \tag{17}$$

However, recently some interest has been given to the possibility that the spatial curvature of the universe be non-negligible, both for theoretical and observational reasons. A positive spatial curvature could, for instance, mimic a phantom regime. Another interesting point is that CMB experiments, even when combined with different astronomical data, present a tendency for some small positive spatial curvature of the universe, a result that can be checked soon using other observations [33]. Thus, for $V_0 \ll 1$, $R_0 > 0$, for $f_0 < 0$, $f(\phi) < 0$. Note that at the heterotic string one-loop level $f(\phi) \propto \phi - \pi e^{\phi}/3 + 4\sum_{n=1} \ln(1 - \exp(-2n\pi\exp(\phi))) + \ln 2 < 0$. It is worth mentioning that there exist many experimental limits on the time variation of the gravitational constant: radar ranging data to the Viking landers on Mars [34], lunar laser ranging experiments [35, 36], measurements of the masses of young and old neutron stars in binary pulsars [37]. In general, it was recently observed that for late times, a modified cosmology with varying κ^2 is in accordance with the observed values of the cosmological parameters [38 – 42]. We hope that the idea used in the present work may be useful to understand inflationary cosmology and related issues in early universe. Further consequences are under consideration.

In summary, we argue that the magnetic field and non-minimal coupling play a crucial role in cosmology with Gauss-Bonnet curvature corrections. However, one needs to investigated whether such theories are indeed compatible with all the current cosmological observations, big-bang nucleosynthesis, and solar system constraints in its weak-field limit. This may provide a new window into the extra-dimensional physics which we intend to further investigate.

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UBRZANA MAGNETSKA GAUSS-BONNETOVA KOZMOLOGIJA

Istražuju se i raspravljaju neke odlike Gauss-Bonnetove kozmologije s magnetskim poljem i neminimalnim vezanjem.