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A NEW CLASS OF BIANCHI TYPE I VISCOUS FLUID UNIVERSE WITH A TIME DEPENDENT COSMOLOGICAL TERM

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A new class of Bianchi type I viscous-fluid cosmological models with a variable cosmological constant are investigated in which the expansion is considered only in two directions, i.e. one of the Hubble parameter ($H_1 = A_4/A$) is zero. We have considered four cases: (i) coefficients of bulk (ξ) and shear (η) viscosity are taken as constant, (ii) ξ and η are considered to be inversely dependent on time, (iii) Λ is taken as inverse square of t and (iv) ξ and η are considered as proportional to scale of expansion in the model. The cosmological constant Λ is found to be positive and is a decreasing function of time which is supported by results from recent supernovae Ia observations. Some physical and geometric properties of the models are also discussed.

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1. Introduction

In general relativity, the cosmological constant Λ may be regarded as a measure of the energy density of the vacuum, and can in principle lead to the avoidance of the big-bang singularity which is characteristic of other Friedmann-Robertson-Walker (FRW) models. However, the rather simplistic properties of the vacuum

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that follow from the usual form of Einstein's equations can be made more realistic if that theory is extended, which in general leads to a variable Λ. Models with a dynamic cosmological term $\Lambda(t)$ are becoming popular as they solve the cosmological constant problem in a natural way. There are significant observational evidence for the detection of the Einstein's cosmological constant Λ or a component of material content of the universe that varies slowly with time and space to act like Λ. Recent cosmological observations by High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [1], Perlmutter et al. [2] Riess et al. [3], Schmidt et al. [4]) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda/(G\hbar/c^3) \approx 10^{-123}$. These observations on the magnitude and red-shift of type Ia supernovae suggest that our universe may be an accelerating function of the cosmological density in the form of the cosmological Λ-term. Earlier studies on this topic are contained in the work of Zeldovich [5], Weinberg [6], Dolgov [7], Bertolami [8], Ratra and Peebles [9], Carroll, Press and Turner [10]. Some of the recent discussions on the cosmological constant "problem" and consequences on cosmology with a time-varying cosmological constant have been discussed by Dolgov [11], Tsagas and Maartens [12], Sahni and Starobinsky [13], Peebles [14], Padmanabhan [15], Vishwakarma [16], Saha [17], Saha and Shikin [18] and Pradhan et al. [19]. This motivated us to study the cosmological models in which Λ varies with time.

The distribution of matter can be satisfactorily described by a perfect fluid due to the large scale distribution of galaxies in our universe. However, the observed physical phenomena, such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest the analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. Misner [20] has studied the effect of viscosity on the evolution of cosmological models. The role of viscosity in cosmology has been investigated by Weinberg [21]. Nightingale [22], Heller and Klimek [23] have obtained a viscous universes without the initial singularity. The model studied by Murphy [24] possessed an interesting feature in which big-bang type of singularity of infinite space-time curvature does not appear to be a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. Roy and Prakash [25] have investigated plane-symmetric cosmological models representing a viscous fluid with free gravitational field of non-degenerate Petrov type I in which coefficient of shear viscosity is constant. Bali and Jain [26] have obtained some expanding and shearing viscous fluid cosmological models in which coefficient of shear viscosity is proportional to the rate of expansion in the model and the free gravitational field is Petrov type D and non-degenerate. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity (Padmanabhan and Chitre [27], Johri and Sudarshan [28], Maartens [29], Zimdahl [30], Pradhan et al. [31], Kalyani and Singh [32], Singh et al. [33], Pradhan et al. [34], Saha [35], Saha and Rikhvitsky [36]). This motivated the study of cosmological bulk viscous fluid model.

In general relativity, spatially homogeneous space-times either belong to Bianchi type or to Kantowaski-Sachs models and interpreted as cosmological models [37]. The current astronomical observations are consistent with an isotropic universe. Spatially homogeneous and isotropic universes can be well described by Friedmann-Robertson-Walker (FRW) models [38, 39]. However, the FRW model has the disadvantage of being unstable near the singularity [40] and it fails to describe the early universe. Anisotropic cosmological models play an important role in the understanding of the universe on the galactic scale. Therefore spatially homogeneous and anisotropic Bianchi type-I models are undertaken to understand the universe at its early stage of evolution.

Recently Pradhan and Pandey [41] have obtained some Bianchi type I viscous fluid cosmological models with a variable cosmological constant. Bali and Jain [42] have investigated Bianchi type I viscous fluid cosmological models. We have revisited both above solutions. Here we wish to approach this subject from a different prospective, and present a new class of exact solutions wherein Λ can change in a manner that is in agreement with observations. We shall focus on the problem with varying cosmological constant in the presence of bulk and shear viscous fluid in an expanding universe. The plan of the paper is as follows. The metric and the field equations are presented in Section 2. In Section 3, we deal with the solution of the field equations in the presence of viscous fluid. In Section 3.1, we consider the coefficient of bulk viscosity (ξ) and shear viscosity (η) as constant, whereas in Section 3.2, ξ and η are taken as inversely proportional to the time t. In Section 3.3, we deal the problem by considering Λ as to be proportional to the inverse square of time. In Section 3.4, we consider ξ and η as proportional to the scale of expansion in the model. In Section 4, we have given the concluding remarks.

2. *The metric and field equations*

We consider the Bianchi type I metric in the form

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}x^2 + B^2\mathrm{d}y^2 + C^2\mathrm{d}z^2, \tag{1}$$

where B and C are functions of t alone. This metric depicts the case when one of the Hubble parameters (here $H_1 = A_4/A$) is zero, i. e. the expansion is only in two directions. The kinematic parameters are then related as $\theta = -3\sigma_1^1$. This condition leads to the metric (1).

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read

$$R_i^j - \frac{1}{2}Rg_i^j + \Lambda g_i^j = -8\pi T_i^j, \tag{2}$$

where R_i^j is the Ricci tensor, $R = g^{ij}R_{ij}$ is the Ricci scalar, and T_i^j is the stress energy-tensor in the presence of bulk stress, given by Landau and Lifshitz [43]

$$T_i^j = (\rho + p)v_i v^j + p g_i^j - \eta \left(v_{i;}^j + v^j{}_{;i} + v^j v^l v_{i;l} + v_i v^l v^j{}_{;l} \right)$$

$$- \left(\xi - \frac{2}{3}\eta \right) \ \theta (g^j{}_i + v_i v^j). \tag{3}$$

Here ρ, p, η and ξ are the energy density, isotropic pressure, coefficient of shear viscosity and bulk viscous coefficient, respectively, and v_i is the flow vector satisfying the relations

$$g_{ij} v^i v^j = -1. \tag{4}$$

The semicolon (;) indicates covariant differentiation. We choose the coordinates to be comoving, so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1. \tag{5}$$

The Einstein's field equations (2) for the line element (1) has been set up as

$$-8\pi \left[p - \left(\xi - \frac{2}{3}\eta \right) \theta \right] = \frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \Lambda, \tag{6}$$

$$-8\pi \left[p - 2\eta \frac{B_4}{B} - \left(\xi - \frac{2}{3}\eta \right) \theta \right] = \frac{C_{44}}{C} + \Lambda, \tag{7}$$

$$-8\pi \left[p - 2\eta \frac{C_4}{C} - \left(\xi - \frac{2}{3}\eta \right) \theta \right] = \frac{B_{44}}{B} + \Lambda, \tag{8}$$

$$8\pi\rho = \frac{B_4 C_4}{BC} + \Lambda, \tag{9}$$

where the suffix 4 at the symbols A and B denotes ordinary differentiation with respect to t and θ is the scalar of expansion given by

$$\theta = v^i{}_{;i}. \tag{10}$$

3. *Solution of the field equations*

We have revisited the solutions obtained by Bali and Jain [42] and Pradhan and Pandey [41] and have obtained a new class of solutions. Equations (6) – (9) are four independent equations in seven unknowns B, C, ρ, p, η, ξ and Λ. For the complete determinacy of the system, we need three extra conditions. The research on exact solutions is based on some physically reasonable restrictions used to simplify the Einstein equations. From Eqs. (6) – (8), we obtain

$$\frac{B_{44}}{B} + \frac{B_4 C_4}{BC} = -16\pi\eta \frac{B_4}{B}, \tag{11}$$

and

$$\frac{B_{44}}{B} - \frac{C_{44}}{C} = -16\pi\eta \left(\frac{B_4}{B} - \frac{C_4}{C} \right). \tag{12}$$

Integrating Eqs. (11) and (12), we obtain

$$B_4 C = \alpha\, \mathrm{e}^{-16\pi \int \eta \mathrm{d}t}, \tag{13}$$

and

$$B_4 C - C_4 B = k\, \mathrm{e}^{-16\pi \int \eta \mathrm{d}t}, \tag{14}$$

respectively. Here α and k are integrating constants.

From Eqs. (13) and (14), we get

$$B^{(1-k/\alpha)} = C, \tag{15}$$

which proves that the relation between B and C is independent of the physical conditions. Putting the value of C from Eq. (15) in (13) and then by integrating the result, leads to

$$B = \left[k_2 \{ \alpha \int (\, \mathrm{e}^{-16\pi \int \eta \mathrm{d}t}) \mathrm{d}t + k_1 \} \right]^{1/k_2}, \tag{16}$$

where $k_2 = 2 - \frac{k}{\alpha}$ and k_1 is an integration constant.

From Eqs. (15) and (16), we obtain

$$C = \left[k_2 \{ \alpha \int (\, \mathrm{e}^{-16\pi \int \eta \mathrm{d}t}) \mathrm{d}t + k_1 \} \right]^{(k_2-1)/k_2}. \tag{17}$$

Subtracting (8) from (6) gives

$$16\pi\eta \frac{C_4}{C} = \frac{C_{44}}{C} + \frac{B_4 C_4}{BC}. \tag{18}$$

From Eqs. (17) and (18), we obtain

$$32\pi\eta (k_2 - 1) \frac{B_4}{B} = 0. \tag{19}$$

Here we have three choices, i.e., either $\eta = 0$ or $B_4 = 0$ or $k_2 - 1 = 0$. We observe that the first two choices do not give physically relevant models of the universe, so we do not consider these cases. Now we have only the third choice, i.e., $k_2 - 1 = 0$. Substituting $k_2 = 1$ in Eqs. (16) and (17), we obtain

$$B = \alpha \int (\,\mathrm{e}^{-16\pi \int \eta \mathrm{d}t})\mathrm{d}t + k_1, \tag{20}$$

$$C = 1. \tag{21}$$

For the above values of B and C, Eqs. (6) − (9) reduce to only two independent equations given by

$$8\pi \Big[p - \left(\xi - \frac{2}{3}\eta\right)\theta\Big] = 16\pi\eta \frac{\alpha\,\mathrm{e}^{-16\pi \int \eta \mathrm{d}t}}{\Big[\alpha \int (\,\mathrm{e}^{-16\pi \int \eta \mathrm{d}t})\mathrm{d}t + k_1\Big]} - \Lambda, \tag{22}$$

and

$$8\pi\rho = \Lambda. \tag{23}$$

In this case the scalar of expansion θ is obtained as

$$\theta = \frac{\alpha\,\mathrm{e}^{-16\pi \int \eta \mathrm{d}t}}{\Big[\alpha \int (\,\mathrm{e}^{-16\pi \int \eta \mathrm{d}t})\mathrm{d}t + k_1\Big]}. \tag{24}$$

Eqs. (22) and (24) lead to

$$p = \left(\xi + \frac{4}{3}\eta\right)\theta - \frac{\Lambda}{8\pi}. \tag{25}$$

Therefore, we have to construct our cosmological models according to Eqs. (23), (24) and (25).

Here we consider four cases. Here we consider four cases. It is remarkable to mention here that we consider specific assumptions on the values of parameters η and ξ in each of the following models I−IV. These assumptions lead to analytically solvable models.

3.1. Models I

Let us consider that $\eta = \eta_0$ and $\xi = \xi_0$, where η_0 and ξ_0 are constants.

This implies that $\int \eta \mathrm{d}t = \eta_0 t + \eta'$, where η' is an integrating constant. Since η' is ineffective, so we take $\eta' = 0$. Hence we obtain

$$\int \eta \mathrm{d}t = \eta_0 t. \tag{26}$$

Using Eqs. (24) and (26) in (25), we obtain

$$p = \left(\xi + \frac{4}{3}\eta\right)\left[\frac{\alpha\,\mathrm{e}^{-16\pi\eta_0 t}}{-\left(\alpha/(16\pi\eta_0)\right)\mathrm{e}^{-16\pi\eta_0 t} + k_1}\right] - \frac{\Lambda}{8\pi}. \tag{27}$$

In order to completely determine the system, we choose a barotropic equation of state

$$p = \gamma\rho, \quad 0 \le \gamma \le 1. \tag{28}$$

Equations (23) and (28) lead to

$$p = \frac{\gamma\Lambda}{8\pi}. \tag{29}$$

Eliminating p between Eqs. (27) and (29), we have

$$\Lambda = \frac{128\pi^2\eta_0\left(\xi_0 + \frac{4}{3}\eta_0\right)}{(1+\gamma)}\frac{\alpha}{\left[k_1\,\mathrm{e}^{16\pi\eta_0 t} - \alpha\right]}. \tag{30}$$

From Eq. (30), we observe that for $k_1 > \alpha$, the cosmological term Λ will be positive at all times. It is also observed that the cosmological constant is a decaying function of time and it approaches a small value at late time (i.e., the present epoch). This is in agreement with the recent results from supernovae Ia observations (Garnavich et al. [1], Perlmutter et al. [2], Riess et al. [3], Schmidt et al. [4]) predicting a bound on the contribution of the vacuum energy to the total energy density of the universe. Thus, with our approach, we obtain a physically relevant decay law for the cosmological term unlike other investigations where ad hoc laws were used to arrive at a mathematical expression for the decaying vacuum energy. From Eqs. (28) and (29), we also see that ρ and p are always positive. Moreover, as $t \to \infty$, all the physical quantities Λ, ρ, p vanish. The model represents an expanding , shearing but non-rotating universe in general. The model explodes with a big bang at $t = 0$ and the expansion in the model stops at $t = \infty$. When $\alpha = 0$ then $\theta = 0$, which implies that $\eta = 0$. Therefore, viscosity is due to the expansion in the model.

3.2. Models II

In general, η and ξ are not constant throughout the fluid, so that η and ξ can not be taken always constant, specially when the universe is expanding. Since, in general, η and ξ depend on temperature (T) and pressure (p), it is reasonable to consider

$$\eta = \frac{1}{\eta_0 + t} \quad \text{and} \quad \xi = \frac{1}{\xi_0 + t},$$

because in expansion, temperature and pressure decrease as time increases.

In this case, the values of the physical quantities p, ρ, Λ and θ are obtained as

$$p = \left(\frac{1}{t+\xi_0} + \frac{4}{3}\frac{1}{t+\eta_0}\right)\frac{\alpha}{(t+\eta_0)[\alpha\ln(t+\eta_0) + k_0]}, \tag{31}$$

$$\rho = \frac{\alpha}{(1+\gamma)} \frac{(7t + 4\xi_0 + 3\eta_0)}{(t+\xi_0)(t+\eta_0)^2[\alpha \ln(t+\eta_0) + k_0]}, \tag{32}$$

$$\Lambda = \frac{8\pi\alpha}{(1+\gamma)} \frac{(7t + 4\xi_0 + 3\eta_0)}{(t+\xi_0)(t+\eta_0)^2[\alpha \ln(t+\eta_0) + k_0]}, \tag{33}$$

$$\theta = \frac{\alpha}{(t+\eta_0)[\alpha \ln(t+\eta_0) + k_0]}. \tag{34}$$

From Eqs. (31), (32) and (33), we observe that $\rho > 0$, $p > 0$, $\Lambda > 0$ for all times. We also observe that when $t \to \infty$, ρ, p and Λ becomes zero. So this fits the best with the physical conditions.

We also conclude the following:

(i) η and ξ tend to zero when the cosmos expands to its ultimate scale (i.e., $t \to \infty$).

(ii) η and ξ have the maximum values when the cosmos has been at its most condensed stages.

(iii) ρ and Λ also have their maximum values at $t = 0$.

From Eq. (33), we again notice that the cosmological constant is a decreasing function of time. This is in agreement with the recent results from supernovae Ia, as discussed in the previous section.

3.3. Models III

In this case, Λ has been taken as the inverse square of time, i.e., we consider

$$\Lambda = \frac{1}{t^2 + \Lambda_0}, \tag{35}$$

where Λ_0 is an arbitrary constant. Using Eq. (35) in Eqs. (23) and (25), we obtain

$$\rho = \frac{1}{8\pi(t^2 + \Lambda_0)}, \tag{36}$$

$$p = \frac{\alpha(7t + 4\xi_0 + 3\eta_0)}{(t+\xi_0)(t+\eta_0)^2[\alpha \ln(t+\eta_0) + k_0]} - \frac{1}{8\pi(t^2+\Lambda_0)}. \tag{37}$$

From Eq. (36), we observe that $\rho > 0$ at all times.

From Eq. (37), we obtain when $t = 0$

$$p = \frac{\alpha(4\xi_0 + 3\eta_0)}{\xi_0^2 \eta_0^2(\alpha \ln \eta_0 + k_0)} - \frac{1}{8\pi\Lambda_0}. \tag{38}$$

We shall discuss the problem in the following ways:

(i) if

$$\frac{\alpha(4\xi_0 + 3\eta_0)}{\xi_0^2\eta_0^2(\alpha \ln \eta_0 + k_0)} < \frac{1}{8\pi\Lambda_0},$$

then we observe $p < 0$ for all times.

(ii) if

$$\frac{\alpha(4\xi_0 + 3\eta_0)}{\xi_0^2\eta_0^2(\alpha \ln \eta_0 + k_0)} > \frac{1}{8\pi\Lambda_0},$$

then this implies that although $p > 0$ at $t = 0$ but it will not remain so for all times, and as t increases, pressure will have negative values. This is because of the fact that the first term in the left hand side in relation (ii) decreases much faster than the second term as t increases. Although, the pressure becomes negative, it tends to zero ultimately as $t \to \infty$ which is physically acceptable.

3.4. Models IV

In this case, we consider $\eta = \eta_0\theta$ and $\xi = \xi_0\theta$, where η_0 and ξ_0 are constants.

Putting these values in Eqs. (24) and (25), we obtain

$$\theta = \frac{1}{(1 + 16\pi\eta_0)t + \delta}, \tag{39}$$

$$p = \frac{(\xi_0 + \eta_0)}{[(1 + 16\pi\eta_0)t + \delta]^2} - \frac{1}{8\pi\Lambda}, \tag{40}$$

where δ is an integrating constant.

For complete a determinacy of the system. let us consider

$$\Lambda = \frac{1}{(t + \Lambda_0)^2}, \tag{41}$$

where Λ_0 is an arbitrary constant.

Using Eq. (41) in Eqs. (23) and (40) leads to

$$\rho = \frac{1}{8\pi(t + \Lambda)^2}, \tag{42}$$

$$p = \frac{(\xi_0 + \eta_0)}{[(1 + 16\pi\eta_0)t + \delta]^2} - \frac{1}{8\pi(t + \Lambda_0)^2}. \tag{43}$$

From Eq. (42), we observe that ρ is always positive, whereas from Eq. (43), we see that $p > 0$ for all times when

$$\frac{\delta - \Lambda_0\sqrt{8\pi(\xi_0 + \eta_0)}}{\sqrt{8\pi(\xi_0 + \eta_0)} - (16\pi\eta_0 + 1)} > 0.$$

4. Conclusions

A new class of Bianchi type I anisotropic cosmological models with a viscous fluid as the source of matter and with a time-dependent cosmological term are obtained. Generally, the models are expanding, shearing and non-rotating. These models are new and different from those models obtained by Pradhan and Pandey [41] and Bali and Jain [26] in which free gravitational field was assumed to be Petrov type D and non-degenerate of Marder [44].

The cosmological term in all models given in Sections 3.1, 3.2, 3.3 and 3.4 are found to be a decreasing function of time and they all approach a small positive value as the time increases (i.e., the present epoch). The values of cosmological "constant" for these models are found to be small and positive what is supported by the results from recent supernovae Ia observations recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [1], Perlmutter et al. [2], Riess et al. [3], Schmidt et al. [4]).

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NOVA VRSTA VISKOZNOG TEKUĆEG SVEMIRA BIANCHIJEVOG TIPA I S VREMENSKI-PROMJENLJIVIM KOZMOLOŠKIM ČLANOM

Istražujemo novu vrstu kozmoloških modela viskoznog tekućeg svemira Bianchijevog tipa I s promjenljivom kozmološkom konstantom, u kojima se širenje razmatra samo u dva smjera, tj., jedan od Hubbleovih parametara, $H_1 = A_4/A$, jednak je nula. Razmatramo četiri slučaja: (i) volumni i viskozno-smični koeficijenti, (ξ) i (η), su stalni, (ii) (ξ) i (η) su inverzno razmjerni vremenu, (iii) Λ je razmjeran inverznom kvadratu vremena i (iv) ξ i η razmjerni su ljestvici širenja svemira u modelu. Nalazimo da je kozmološka konstanta pozitivna i opadajuća funkcija vremena, što je u skladu s nedavnim ishodima opažanja supernova Ia. Raspravljaju se također neka fizikalna i geometrijska svojstva tih modela.