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WHERE IS THE GLUEBALL?

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The processes $\pi\pi \to \pi\pi$, $K\bar{K}$ and $\eta\eta$, with $I^G J^{PC} = 0^+0^{++}$, are analysed by means of simultaneous coupled-channel analysis of all available data. The investigation is focussed on the properties of the $f_0(665)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ states with the aim to determine their quark-gluonic content. The analysis supports the existence of $f_0(665)$ as a very broad resonance. It suggests further to see the $f_0(980)$ state as predominantly the $\eta\eta$ bound state. The quark content of other states is inferred and $f_0(1500)$ appears as a mixed state with dominant glueball component.

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1. *Introduction*

The existence of glueballs is one of the yet unproven predictions of the Standard Model of particle physics. It describes the strong, weak and electromagnetic fundamental forces and fundamental constituents of matter of which quarks and gluons are responsible for strong interactions. According to the Quantum chromodynamics (QCD), the theory behind the Standard Model, quarks and gluons are the basic building blocks of all strongly interacting elementary particles that make up all known matter. While baryons are made of three quarks, mesons are made of quark-antiquark pairs. Free single quarks are not observed. This is the consequence of the confinement, the peculiar characteristic of strong interactions mediated by gluons. However, it follows from QCD that apart from elementary particles made of quarks bound by gluons, there should exist particles made of gluons only, without quarks. These glueballs are the subject of much theoretical as well as experimental investigation. Theory tries to calculate their possible masses and other properties while experiments should be able to see them in the high-energy collision experiments.

On the side of the theory, there are two different nonperturbative methods of QCD calculations, which are able to give us some hints of the glueball masses. On the side of experiment, it is expected that the first observable glueballs would be those with the vacuum quantum numbers. This is because their theoretically calculated masses should be somewhere within the interval of the present day experimentally known states. The proper identification of these resonant states from the scattering experiments is, therefore, very important. This is why the scalar-meson states of vacuum quantum numbers are playing so important role nowadays. However, there are problems on both sides.

Theoretically, the calculated masses of scalar glueballs give very different hints concerning the mass of the lightest glueball. QCD lattice simulations predict that the lightest glueball should have mass around 1650 MeV [1], while the QCD sum rules predict the lightest glueball as a narrow state with a mass below 900 MeV [2]. The experimental identification of resonant states in scattering experiments is, therefore, important, but experimental situation is also quite obscure. These resonances are not narrow and well separated ones. Moreover, they have masses at energies where new scattering or decay channels are being opened and, therefore, the simple pole identification by means of Breit-Wigner form identifying a resonance from the nearest pole in the scattering matrix at an unphysical sheet of the complex energy plane is unreliable, and the many-channels formalism has to be applied. It is, therefore, very important to know the topology of scattering matrix of such coupled channels. The general outline of such method of data analysis has been proposed and successfully applied in Ref. [3].

2. *Where to search for the glueball?*

As mentioned above, it is commonly agreed that the lightest glueball should have the vacuum quantum numbers. It is, therefore, wise to search it among the isoscalar scalar f_0 mesons, namely in the coupled scattering processes $\pi \to \pi$, $K\bar{K}$ and $\eta\eta$, with, in the channel with quantum numbers $I^G J^{PC} = 0^+0^{++}$.

The most convenient for the data analysis is to use the S-matrix because of its extraordinary physical characteristics. The S-matrix describes the scattering processes at relativistic energies from initial to final states by its matrix elements $\langle f|S|i\rangle = S_{fi}$. So in the case of three coupled channels, we have

$$S_{11} \sim \pi\pi \to \pi\pi, \qquad S_{22} \sim K\bar{K} \to K\bar{K}, \qquad S_{33} \sim \eta\eta \to \eta\eta,$$

$$S_{12} \sim \pi\pi \to K\bar{K} \qquad S_{13} \sim \pi\pi \to \eta\eta.$$

The S-matrix is suitable for the high-energy calculations because it is Lorenz invariant. It means that for Lorenz transformation L, there is a unitary operator U in the Hilbert space that

$$\langle f|S|i\rangle = \langle f|U^* S U|i\rangle = \langle Lf|S|Li\rangle.$$

The S-matrix is simply related to the experiment by the scattering matrix T

$$S = 1 + \mathrm{i}k^{1/2} T k^{1/2}, \tag{1}$$

where k is the diagonal matrix of the corresponding channel momenta

$$k_i = \frac{1}{2}(s - 4m_i^2)^{1/2}, \quad i = 1,\, 2,\, 3\ (\pi,\, K,\, \eta), \tag{2}$$

and s is the total energy square [4].

The S-matrix can be easily decomposed into the partial waves to describe the interaction with a particular orbital momentum quantum number. To search for the glueball with vacuum quantum numbers, we confine our analysis to the s-partial wave. The S-matrix partial wave elements are functions of the total energy of two incoming interacting π mesons. In relativistic calculations, we work with the square of the total energy

$$s = (2E)^2 = 4(m_\pi^2 + k_1^2), \tag{3}$$

where we use the units with light speed $c = 1$.

The square root in the channel momenta leads to the (+/-) ambiguity of the S-matrix elements and to the so-called kinematical cuts in the complex s-plane, starting at the energy thresholds, i.e. s_i-values when individual momenta k are equal to zero. In the case of three coupled channels, we have three cuts alongside the real axis starting from

$$s_i = 4m_i^2, \quad i = 1,\, 2,\, 3\ (\pi,\, K,\, \eta)$$

to infinity. To remove the ambiguity, the next (called unphysical) Riemann sheets are attached and adjoined to the first one (physical) alongside the cuts. Each momentum below the threshold is imaginary and the sign of it changes by going to the next adjoined unphysical sheet. This serves for the numeration of the Riemann sheets of the whole complex surface.

In the one-channel case, we have only ($\pi\pi \to \pi\pi$) and the two Riemann sheets are numbered according to the $\mathrm{Sign}(\mathrm{Im}k_1) = +, -$, as I and II. But as more channels are coupled together, each new channel doubles them, and for N channels we are left with the surface consisting of 2^N Riemann sheets. In the two-channel case, the surfaces are numbered as I, II, III, IV according to

$$\mathrm{Sign}(\mathrm{Im}k_1, \mathrm{Im}k_2) = (++, -+, --, +-)\,.$$

In the three-channel case, the numbering follows the same rule

$$\mathrm{Sign}(\mathrm{Im}k_1, \mathrm{Im}k_2, \mathrm{Im}k_3) =$$

$$\begin{pmatrix} +++ & -++ & --+ & +-+ & +-- & --- & -+- & ++- \\ \mathrm{I} & \mathrm{II} & \mathrm{III} & \mathrm{IV} & \mathrm{V} & \mathrm{VI} & \mathrm{VII} & \mathrm{VIII} \end{pmatrix}$$

3. *Analytic continuation*

The outstanding features of the S-matrix can be exploited for analytical continuation of the S-matrix elements to each of the attached Riemann sheets. These features are properties like symmetry

$$S_{ij} = S_{ji} \tag{4}$$

real analyticity

$$S_{ij}(s*) = S^*_{ij}(s) \tag{5}$$

and unitarity

$$SS^\dagger = S^\dagger S = 1 \tag{6}$$

where * means complex conjugation and † means hermitian conjugation.

These S-matrix properties are related to the very powerful fundamental physical concepts. Symmetry (Eq. (4)) holds due to the *time reversal* invariance of strong interactions. The real analyticity Eq. (5) is related to the concept of *causality*. It guaranties that all zeros or singularities of the S-matrix in the complex s-variable plane are on the real axis or at the complex conjugate positions. The S-matrix Eq. (6) holds because of the scattering *probability conservation* and causes that the S-matrix is multivalued in the s-variable and has the right-hand cuts [4]. At the same time, Eq. (6) defines the analytical continuation of the S-matrix to the unphysical sheets.

Derived from the so-called first principles, these S-matrix properties make of the S-matrix the useful and powerful tool in direct analysis of experimental data. This is particularly advantageous in the analysis of several overlapping resonances, because the simplified approach based on the use of simple Breit-Wigner resonance cannot be applied or leads to the model-dependent results. We have formulated the computational method useful for the global analysis of several coupled-channel processes with overlapping resonances, using the analytical continuation of the S-matrix elements and it was successfully applied to various experimental data analyses [3].

The unitarity can be used to find the analytical continuation of all S-matrix elements. It is possible to express them on unphysical sheets through their values on the physical sheet as can be seen from Table 1.

One can see from Table 1 how the analytical continuation determines that singularities and zeros corresponding to one process propagate to other coupled processes. This gives a hint that all of them can be simultaneously described by a simple parameterization. Resonance is then characterized by clusters of poles and zeros of the S-matrix in various sheets. In the absence of channel coupling, the zeros and poles are on top of each other in different Riemann sheets of the complex s-variable. The amount of their relative position shift on different sheets reflects the

TABLE 1. The 3×3 S-matrix elements analytically continued from the first Riemann sheet to 8 adjacent Riemann sheets. There D_{kl} denotes the minor of the element S_{kl}, so $D_{kk} = S_{ll}S_{mm} - S_{lm}^2$, and $D_{kl} = S_{kl}S_{mm} - S_{km}S_{lm}$ for $k, l, m = 1, 2, 3$, cyclically. The S matrix is symmetric due to the time reversal.

I	II	III	IV	V	VI	VII	VIII
S_{11}	$1/S_{11}$	S_{22}/D_{33}	D_{33}/S_{22}	$\det S/D_{11}$	$D_{11}/\det S$	S_{33}/D_{22}	D_{22}/S_{33}
S_{12}	$\mathrm{i}S_{12}/S_{11}$	$-S_{12}/D_{33}$	$\mathrm{i}S_{12}/S_{22}$	$\mathrm{i}D_{12}/D_{11}$	$-D_{12}/\det S$	$\mathrm{i}D_{12}/D_{22}$	D_{12}/S_{33}
S_{22}	D_{33}/S_{11}	S_{11}/D_{33}	$1/S_{22}$	S_{33}/D_{11}	$D_{22}/\det S$	$\det S/D_{22}$	D_{11}/S_{33}
S_{13}	$\mathrm{i}S_{13}/S_{11}$	$-\mathrm{i}D_{13}/D_{33}$	$-D_{13}/S_{22}$	$-\mathrm{i}D_{13}/D_{11}$	$D_{13}/\det S$	$-S_{13}/D_{22}$	$\mathrm{i}S_{13}/S_{33}$
S_{23}	D_{23}/S_{11}	$\mathrm{i}D_{23}/D_{33}$	$\mathrm{i}S_{23}/S_{22}$	$-S_{23}/D_{11}$	$-D_{23}/\det S$	$\mathrm{i}D_{23}/D_{22}$	$\mathrm{i}S_{23}/S_{33}$
S_{33}	D_{22}/S_{11}	$\det S/D_{33}$	D_{11}/S_{22}	S_{22}/D_{11}	$D_{33}/\det S$	S_{11}/D_{22}	$1/S_{33}$

strength of coupling of coupled channels. The characteristic appearances of various zero-pole clusters of different resonant states determine their properties and their quark and gluon structure.

For two coupled channels, i.e. for the coupled processes $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$ and $K\bar{K} \to K\bar{K}$ in our case, there are two cuts on the real s axis starting at branch points $s = 4m_\pi^2$ and $s = 4m_K^2$. The 2×2 S-matrix element's analytical continuations to Riemann sheets II, III and IV is expressed by the first 4 rows and 4 columns of Table 1. One can see that the zero due to the resonance in S_{11} will appear as the 2$^{\mathrm{nd}}$ sheet pole at the other two coupled processes. As $D_{33} = S_{11}S_{22} - S_{12}^2$, it will appear as a pole also on the 3$^{\mathrm{rd}}$ sheet at coupled processes, but at the shifted position. The magnitude of the shift depends on the value of S_{12}, and in the absence of the coupling ($S_{12} = 0$), there is no shift of poles relative to the zero on the 1$^{\mathrm{st}}$ sheet.

Thus one can see that by starting from the resonance zeros on the 1$^{\mathrm{st}}$ Riemann sheet, the resonance representations in terms of poles and zeros on the full Riemann surface is obtained. In the two-coupled channel case, one can distinguish three types of resonances generated by the 1$^{\mathrm{st}}$ sheet's zeros. They can be caused by zeros of S_{11}, of S_{22} or zeros of both of them. In the three-channel case, such classification gives more possible combinations causing resonance propagation to coupled channels [5].

4. Data analysis

For the analysis, it is convenient to eliminate the s-variable branch-point singularities at threshold energies of binary processes by means of conformal mapping $z = (k_1 + k_2)/(m_K^2 - m_\pi^2)^{1/2}$ and to map the four Riemann sheets into one z-plane. The k_i for $i = 1, 2$ are the channel momenta. Further step in the simplification of analysis is the use of the Le Couteur-Newton relations [6] expressing the S-matrix elements as ratios of just one real analytic function

$$S_{11} = \frac{d(-z^{-1})}{d(z)}, \quad S_{22} = \frac{d(z^{-1})}{d(z)}, \quad \mathrm{Det}S = \frac{d(-z)}{d(z)}\,. \tag{7}$$

Here $d(z)$ is the Jost function. In z-variable, it is free of the right-hand cuts and other singularities, therefore has only zeros in the complex z-plane. However, to account for the background contribution from related cross channels, as well as the existing left-hand cuts approximated by zeros, it is convenient to include these zeros to $d(z)$, too [5].

The part of $d(z)$ corresponding to the resonances has then the simple form

$$d_{\mathrm{res}} = z^{-M} \prod_{n=1}^{M} (1 - z_N^* z)(1 + z_n z)\,, \tag{8}$$

where M is the number of pairs of the conjugate zeros. The z_n positions are free parameters in fitting the scattering data of all scattering matrix elements. With the means of unitarity, they are conveniently expressed through the phase shifts and inelasticities which are our input data known from experiments. In the two coupled channel case, $S_{11} = \eta \exp \delta_1$, $S_{12} = (1 - \eta^2)^{1/2} \exp(\delta_{12})$ and $S_{22} = \eta \exp \delta_2$, where the inelasticity η and $\pi\pi \to \pi\pi$ phase shift δ_1 and $\pi\pi \to K\bar{K}$ phase shift δ_{12} are known from the scattering experiments, and $\delta_{12} = \delta_1 + \delta_2$. The experimental data were taken from Refs. [25] and [26] of our paper [7].

For the reason of clarity, we skip the details of the three-channel analysis. Its mathematical formalism is only more complex but in essence it follows the same ideas. It is clear that in that case the coupling of $\pi\pi \to \pi\pi$, $\pi\pi \to K\bar{K}$ and $\pi\pi \to \eta\eta$ processes have to be taken into account, and also all other combined processes as described in Table 1. Unfortunately, the experimental measurements are providing us mainly with the data from the $\pi\pi$ scattering processes, and we have to be content with rather sparse data from the two production processes [8]. The scattering data about the remaining processes do not exist. In the analysis, we are taking into account the following resonances coupled to these processes: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. The experimental data from the $\pi\pi$ threshold energy up to 1.9 GeV, as far as they are available for the coupled processes, are used as input for the analysis.

5. *Results*

We shall not enter into technical details here, but rather we describe the results. In Table 2 we present results of our two-channel analysis of four f_0-isoscalar resonances listed in the Particle Data [9]. The coupling constants are calculated through the residues of amplitudes of the T-matrix Eq. (1).

We see that the $f_0(980)$ and $f_0(1370)$ are coupled more strongly to the $K\bar{K}$ than to $\pi\pi$, therefore, they have a dominant quark component. The $f_0(1500)$ and

TABLE 2. Coupling constants.

	$f_0(600)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$
$\gamma_{\pi\pi}$ (GeV)	0.65 ± 0.06	0.17 ± 0.05	0.12 ± 0.03	0.66 ± 0.11
$\gamma_{K\bar{K}}$ (GeV)	0.72 ± 0.10	0.45 ± 0.03	0.99 ± 0.05	0.67 ± 0.15

partially the $f_0(600)$ couplings to both channels are almost the same, which is an indication that there is a significant glueball component in them. The three-channel calculation reveals that $f_0(980)$ is the $\eta\eta$ bound state, i.e. the quark anti-quark state and not a true resonance.

Table 3 shows the masses and total widths of the scalar mesons - the f_0 resonances from the analysis of all three-channels and their comparison to the Particle Data [9]. Because the positions of the zero-pole clusters on various Riemann sheets are quite stable for different models, they are the best model independent way to describe resonances. While positions of individual resonances are quite stable, the calculations of corresponding masses and widths are very model-dependent. For the comparison with results of other papers, the masses and widths in Table 3 were calculated for the relativistic resonance amplitude [5].

TABLE 3. Masses and total widths (in MeV*).*

Resonance	$m_{\rm res}$	$\Gamma_{\rm tot}$	$m_{\rm res(PDG)}$	$\Gamma_{\rm (PDG)}$
$f_0(600)$	889	1190	$400-1200$	$600-1000$
$f_0(980)$	1006	64	980 ± 10	$40-100$
$f_0(1370)$	1386	156	$1200-1500$	$200-500$
$f_0(1500)$	1539	640	1507 ± 5	109 ± 7
$f_0(1710)$	1710	164	1714 ± 5	140 ± 10

We do not show errors of our calculated values, because they are strongly model-dependent. Derivation of $f_0(600)$ mass for the non-relativistic resonance form gives $m_{\rm res} = 570$ MeV by comparison.

6. Conclusions

The three-channel analysis reveals a mixing of states $f_0(1370)$ and $f_0(1710)$ with the wide states $f_0(600)$ and $f_0(1500)$. The inclusion of the $\eta\eta$ coupled channel does not shift the pole clusters found from two-channel analyses. The pole clusters are very good indication of the type of resonant states.

The existence of the broad $f_0(600)$ is confirmed. The absence of poles on the sheets VI and VII indicates that $f_0(980)$ is an $\eta\eta$ bound state. $f_0(1370)$ is coupled more strongly to $\eta\eta$ than to $\pi\pi$ and $K\bar{K}$. The $f_0(1500)$ resonance has a dominant glueball component and $f_0(1710)$ couples more strongly to $K\bar{K}$ than to the other two.

The mixing seems to play an important role since if the $f_0(1370)$ and the $f_0(1710)$ were the pure $S\bar{S}$ quark states, then their coupling to both $K\bar{K}$ and $\eta\eta$ would be similar.

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GDJE JE GLUONSKA LOPTA?

Analiziramo sve poznate podatke za procese $\pi\pi \to \pi\pi$, $K\bar{K}$ i $\eta\eta$, za koje je $I^G J^{PC} = 0^+0^{++}$, primjenjujući istovremenu analizu za vezana stanja. Usredotočili smo se na istraživanje svojstava stanja $f_0(665)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ i $f_0(1710)$ s ciljem određivanja njihovog kvark-gluonskog sastava. Analiza podržava postojanje vrlo široke rezonancije $f_0(665)$. Nadalje, ukazuje da je stanje $f_0(980)$ pretežno vezano stanje $\eta\eta$. Za druga stanja procjenjuje se kvarkovski sadržaj, a $f_0(1500)$ je, čini se, miješano stanje s gluonskom loptom kao većinskom sastavnicom.