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ANOMALOUS DIMENSIONS OF TWIST-2 OPERATORS AND POMERON IN $N = 4$ SUSY

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We review the BFKL approach to the Regge processes in QCD and show that in the multi-colour QCD the BKP equations for composite states of several Reggeized gluons are integrable. The relation between pomeron and graviton in $N = 4$ SUSY is investigated. The maximal transcendentality hypothesis gives us a possibility to calculate the three-loop anomalous dimension. Using also the asymptotic Bethe anzatz and a model for wrapping effects, we find the anomalous dimension for twist-2 operators in four loops, in an agreement with the BFKL and double-logarithmic predictions.

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1. Introduction

According to Ref. [1], the high-energy scattering amplitude in the leading logarithmic approximation (LLA) of QCD has the Regge-type form with the gluon Regge trajectory given below

$$j(t) = 1 + \omega(t)\,,\ \omega(-|q|^2) = -\frac{\alpha_c}{4\pi^2}\,N_c \int d^2k\,\frac{|q|^2}{|k|^2|q-k|^2} \approx -\frac{\alpha_c}{2\pi}\,\ln\frac{|q^2|}{\lambda^2}\,. \qquad (1)$$

In LLA, the gluons are produced in the multi-Regge kinematics. The elastic amplitude with the vacuum quantum numbers in the t-channel can be obtained with the use of the s-channel unitarity by summing over multi-gluon intermediate states [1]. In this approach, the pomeron appears as a composite state of two reggeised gluons. It is convenient to introduce the complex variables for the gluon transverse coordinates and momenta

$$\rho_k = x_k + \mathrm{i}y_k \, , \ \rho_k^* = x_k - \mathrm{i}y_k \, , \ p_k = \mathrm{i}\frac{\partial}{\partial \rho_k} \, , \ p_k^* = \mathrm{i}\frac{\partial}{\partial \rho_k^*} \, . \tag{2}$$

In the coordinate representation, the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation for the pomeron wave function can be written as follows [1]

$$E\,\Psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = H_{12}\,\Psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \, , \ \Delta = -\frac{\alpha_s N_c}{2\pi} \min E \, , \tag{3}$$

where Δ is the pomeron intercept. The BFKL Hamiltonian is presented below [2]

$$H_{12} = \ln |p_1 p_2|^2 + \frac{1}{p_1 p_2^*} \ln |\rho_{12}|^2 p_1 p_2^* + \frac{1}{p_1^* p_2} \ln |\rho_{12}|^2 p_1^* p_2 - 4\psi(1) \, , \tag{4}$$

where $\rho_{12} = \rho_1 - \rho_2$. The kinetic energy is proportional to the gluon Regge trajectories, and the potential energy $\sim \ln |\rho_{12}|^2$ is related to the product of two gluon production vertices.

The Hamiltonian is invariant under the Möbius transformation [3]

$$\rho_k \to \frac{a\rho_k + b}{c\rho_k + d} \, , \tag{5}$$

where a, b, c and d are complex numbers. The eigenvalues of the Casimir operators

$$M^2 = \left(\sum_{r=1}^{2} \boldsymbol{M}^{(r)} \right)^2 = \rho_{12}^2 \, p_1 \, p_2 \, , \ M^{*2} = (M^2)^* \tag{6}$$

are simple functions of the conformal weights

$$m = 1/2 + \mathrm{i}\nu + n/2 \, , \ \widetilde{m} = 1/2 + \mathrm{i}\nu - n/2 \tag{7}$$

for the principal series of unitary representations.

2. Integrability of the BFKL dynamics at $N_c \to \infty$

The Bartels-Kwiecinskii-Praszalowicz (BKP) Eq. [5] for the n-gluon composite state is simple at $N_c \to \infty$, and its Hamiltonian has the property of the holomorphic separability [4]

$$H = \frac{1}{2} \sum_k H_{k,k+1} = \frac{1}{2}(h + h*) \, , \ [h, h*] = 0 \, , \tag{8}$$

where the holomorphic Hamiltonian can be written as follows

$$h = \sum_k h_{k,k+1}\,,\ \ h_{12} = \ln(p_1 p_2) + \frac{1}{p_1}\,\ln\rho_{12}\ p_1 + \frac{1}{p_2}\,\ln\rho_{12}\ p_2 - 2\psi(1)\,, \qquad (9)$$

where $\psi(x) = (\ln\Gamma(x))'$. As a result, we obtain for Ψ the holomorphic factorization [4]

$$\Psi(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2, ..., \boldsymbol{\rho}_n) = \sum_{r,s} a_{r,s}\ \Psi_r(\rho_1, ..., \rho_n)\,\Psi_s(\rho_1^*, ..., \rho_n^*) \qquad (10)$$

and the duality symmetry [6]

$$\rho_{r,r+1} \to p_r \to \rho_{r-1,r}\,. \qquad (11)$$

Moreover, in the holomorphic and anti-holomorphic sectors, there are integrals of motion commuting among themselves and with h [2, 7]

$$q_r = \sum_{k_1<k_2<...<k_r} \rho_{k_1k_2}\rho_{k_2k_3}...\rho_{k_rk_1}\ p_{k_1}p_{k_2}...p_{k_r}\,,\ \ [q_r, h] = 0\,. \qquad (12)$$

The integrability of the BFKL dynamics [7] is related to the fact, that H coincides with the local Hamiltonian of the Heisenberg spin model [8].

In LLA, the pomeron intercept Δ is positive [1]

$$\Delta = 4\,\frac{\alpha_s}{\pi} N_c\ \ln 2 \qquad (13)$$

and the Froissart bound for the total cross-section is violated. To restore the s-channel unitarity of scattering amplitudes one can use the effective field theory for Reggeized gluons [9] – [11].

3. DGLAP and BFKL dynamics in $N = 4$ SUSY

The momenta $f_a(j, Q^2)$ of parton distributions satisfy the renormalization group equation with the anomalous dimension matrix γ_{ab}

$$\frac{\mathrm{d}}{\mathrm{d}\ln \mathrm{Q}^2} f_a(j, Q^2) = \sum_b \gamma_{ab}(j) f_b(j, Q^2)\,. \qquad (14)$$

They are proportional to matrix elements of the light-cone components of twist-2 operators

$$O^a = \tilde{n}^{\mu_1}...\tilde{n}^{\mu_j}\ O^a_{\mu_1,...,\mu_j}\ ,\ \ \tilde{O}^a = \tilde{n}^{\mu_1}...\tilde{n}^{\mu_j}\,\tilde{O}^a_{\mu_1,...,\mu_j}\,. \qquad (15)$$

The anomalous dimensions do not depend on other possible tensor projections

$$\tilde{n}^{\mu_1} ... \tilde{n}^{\mu_{1+\omega}} \, O^a_{\mu_1,...,\mu_{1+\omega},\sigma_1,...,\sigma_{|n|}} \, l_\perp^{\sigma_1} ... l_\perp^{\sigma_{|n|}} \, . \tag{16}$$

The solution of the BFKL equation due to its Möbius invariance is classified by the anomalous dimension $\gamma = \frac{1}{2} + i\nu$ and the conformal spin $|n|$ coinciding with the number of transverse indices of O. The eigenvalue of the BFKL kernel in the next-to-leading approximation is

$$\omega = \omega_0(n, \gamma) + 4 \, \hat{a}^2 \, \Delta(n,\gamma) \, , \; \hat{a} = g^2 N_c/(16\pi^2) \, . \tag{17}$$

In QCD, $\Delta(n,\gamma)$ is a non-analytic function of the conformal spin $|n|$ [12, 13]

$$\Delta_{QCD}(n,\gamma) = c_0 \delta_{n,0} + c_2 \delta_{n,2} + \textit{analytic terms} \, ,$$

but in $N = 4$ SUSY the Kronecker symbols are cancelled [13].

Moreover, in this model $\Delta(n,\gamma)$ is a hermitially separable function

$$\Delta(n,\gamma) = \phi(M) + \phi(M^*) - \frac{\rho(M) + \rho(M^*)}{2\hat{a}/\omega} \, , \; M = \gamma + \frac{|n|}{2} \, , \tag{18}$$

$$\rho(M) = \beta'(M) + \frac{1}{2}\zeta(2) \, , \; \beta'(z) = \frac{1}{4}\left[\Psi'\Big(\frac{z+1}{2}\Big) - \Psi'\Big(\frac{z}{2}\Big)\right]. \tag{19}$$

It is important, that here all contributions have the maximal trancendentality property [13].

$$\phi(M) = 3\zeta(3) + \Psi''(M) - 2\Phi(M) + 2\beta'(M)\Big(\Psi(1) - \Psi(M)\Big), \tag{20}$$

where

$$\Phi(M) = \sum_{k=0}^{\infty} \frac{\beta'(k+1)}{k+M} + \sum_{k=0}^{\infty} \frac{(-1)^k}{k+M} \left(\Psi'(k+1) \; - \; \frac{\Psi(k+1) - \Psi(1)}{k+M}\right). \tag{21}$$

For one-loop anomalous dimension matrix of the twist-2 operators in the case of $N = 4$ the calculations were performed in Ref. [14]. The eigenvalues of this matrix are expressed in terms of the universal anomalous dimension for the super-multiplet unifying all twist-2 operators

$$\gamma^{(0)}_{\rm uni}(j) = -4 S_1(j-2) \, , \; S_r(j) = \sum_{i=1}^{j} \frac{1}{i^r} \, . \tag{22}$$

Note, that the function $\gamma^{(0)}_{\rm uni}(j)$ has the maximal transcendentality property related to an integrability of evolution equations for matrix elements of quasi-partonic operators in $N = 4$ SUSY [14].

4. Pomeron - graviton interplay

The pomeron intercept in the $N = 4$ supersymmetric gauge theory was calculated for large coupling constants in Ref. [17]. Here we shall review the basic arguments of this paper. To begin with, one can simplify the eigenvalue for the BFKL kernel in the diffusion approximation as follows (see Ref. [12])

$$j = 2 - \Delta - D\nu^2\,,\ \gamma_{\text{uni}} = \frac{j}{2} + \mathrm{i}\nu\,, \tag{23}$$

assuming, that the parameter Δ is small at large $z = \alpha N_c/\pi$. Due to the energy-momentum conservation, we have $\gamma|_{j=2} = 0$ and, therefore, γ can be expressed only in terms of the parameter Δ

$$\gamma = (j-2)\left(\frac{1}{2} - \frac{1/\Delta}{1 + \sqrt{1 + (j-2)/\Delta}}\right). \tag{24}$$

On the other hand, with the use of the AdS/CFT correspondence [18], the BFKL equation in the diffusion approximation can be written as the graviton Regge trajectory

$$j = 2 + \frac{\alpha'}{2}\,t\,,\ t = E^2/R^2\,,\ \alpha' = \frac{R^2}{2}\,\Delta\,. \tag{25}$$

The behaviour of γ at $g \to \infty$, $j \to \infty$ was predicted yearlier [19]

$$\gamma_{|z\to\infty} = -\sqrt{j-2}\,\Delta^{-1/2}_{|j\to\infty} = \sqrt{\pi j}\,z^{1/4}\,. \tag{26}$$

Here $z = 4\hat{a}$. Therefore, we can calculate the pomeron intercept at large couplings [17] (see also Ref. [23])

$$j = 2 - \Delta\,,\ \Delta = \frac{1}{\pi}\,z^{-1/2}\,. \tag{27}$$

5. Anomalous dimensions for $N = 4$ in two and three loops

One can argue [13], that the perturbative expansion of the universal anomalous dimension

$$\gamma_{\text{uni}}(j) = \hat{a}\gamma^{(0)}_{\text{uni}}(j) + \hat{a}^2\gamma^{(1)}_{\text{uni}}(j) + \hat{a}^3\gamma^{(2)}_{\text{uni}}(j) + ... \tag{28}$$

contains in each order of the perturbation theory only special functions with the highest trancendentality. With this assumption from the known results in QCD, one can obtain for $N = 4$ SUSY [13]

$$\frac{1}{8}\,\gamma^{(1)}_{\text{uni}}(j+2) = 2S_1(j)\left(S_2(j) + S_{-2}(j)\right) - 2S_{-2,1}(j) + S_3(j) + S_{-3}(j)\,, \tag{29}$$

where the corresponding harmonic sums are given below

$$S_a(j) = \sum_{m=1}^{j} \frac{1}{m^a}\,, \;\; S_{a,b,c,\cdots}(j) = \sum_{m=1}^{j} \frac{1}{m^a}\, S_{b,c,\cdots}(m)\,, \tag{30}$$

$$S_{-a}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a},\; S_{-a,b,\cdots}(j) = \sum_{m=1}^{j} \frac{(-1)^m}{m^a}\, S_{b,\cdots}(m), \tag{31}$$

$$\overline{S}_{-a,b,c\cdots}(j) = (-1)^j S_{-a,b,\ldots}(j) + S_{-a,b,\cdots}(\infty)\Big(1-(-1)^j\Big). \tag{32}$$

This result was verified by direct calculations of the anomalous dimension matrix in two loops [15].

Later the three-loop anomalous dimension matrix for QCD was calculated [16], which allowed us to find the universal anomalous dimension in three loops for $N=4$ SUSY [17]

$$\frac{1}{32}\,\gamma^{(2)}_{\rm uni}(j+2) = 24\, S_{-2,1,1,1} - 12\;(S_{-3,1,1} + S_{-2,1,2} + S_{-2,2,1})$$
$$+6\;(S_{-4,1} + S_{-3,2} + S_{-2,3}) - 3\, S_{-5}$$
$$-2\, S_1^2\,(3\, S_{-3} + S_3 - 2\, S_{-2,1}) - S_2\,(S_{-3} + S_3 - 2\, S_{-2,1})$$
$$-S_1\left(8\bar S_{-4} + \bar S_{-2}^2 + 4 S_2 \bar S_{-2} + 2 S_2^2\right)$$
$$-S_1\left(3S_4 - 12\bar S_{-3,1} - 10\bar S_{-2,2} + 16\bar S_{-2,1,1}\right)\,. \tag{33}$$

6. *Relations between weak and strong coupling regimes*

The asymptotics of $\gamma_{\rm uni}$ for $N=4$ SUSY at $j-1=\omega\to 0$

$$\gamma^{N=4}_{\rm uni}(j) = \hat a\frac{4}{\omega} - 32\zeta_3\,\hat a^2 + 32\zeta_3\,\hat a^3\frac{1}{\omega} - \frac{16\hat a^4}{\omega^4}\left(32\zeta_3 + \frac{\pi^4}{9}\omega\right) + \ldots \tag{34}$$

is in an agreement with the predictions from the BFKL Eq. [13].

Near the negative even points $j+2r=\omega\to 0$, the anomalous dimension satisfies the equation

$$\omega\gamma_{\rm uni} = \gamma^2_{uni} + 16\hat a^2(S_2+\zeta_2 - S_1^2)$$
$$+4\,\hat a\left(1 - \omega S_1 - \omega^2(S_2+\zeta_2) + \gamma^2(S_2 + S_{-2})\right)\,,$$

resumming the double logarithmic terms $\sim \alpha/\omega^2$ and corrections to them.

Further, the universal anomalous dimension at large j

$$\gamma_{\rm uni}^{N=4} = a(z)\,\ln j\,,\;\; z = \frac{\alpha N_c}{\pi} = 4\hat{a} \tag{35}$$

can be found from our results up to three loops

$$a(z) = -z + \frac{\pi^2}{12}\,z^2 - \frac{11}{720}\pi^4 z^3 + \ldots\,. \tag{36}$$

It is remarkable that using the AdS/CFT correspondence [18] between the superstring model on the anti-de-Sitter space and the $N = 4$ supersymmetric Yang-Mills theory, A. Polyakov with collaborators found the coefficient $a(z)$ in the strong coupling limit [19]

$$\lim_{z\to\infty} a(z) = -z^{1/2} + \frac{3\ln 2}{4\pi} + \ldots\,. \tag{37}$$

In Ref. [15], the resummation of $a(z)$ in the form

$$\widetilde{a} = -z + \frac{\pi^2}{12}\,\widetilde{a}^2. \tag{38}$$

was suggested. The prediction of this equation

$$\widetilde{a} = -z + \frac{\pi^2}{12}\,z^2 - \frac{1}{72}\pi^4 z^3 + \ldots \tag{39}$$

is in a rather good agreement with $a(z)$ in three loops and with the strong coupling asymptotics. Our results agree also with the recent papers [20, 21], where $a(z)$ is constructed in all orders.

7. Beisert-Eden-Staudacher equation

One can rewrite the Eden-Staudacher integral equation [20] as a set of linear equations [22]

$$a_{n,\epsilon} = \sum_{n'=1}^{\infty} K_{n,n'}(\epsilon)\;(\delta_{n',1} - a_{n',\epsilon})\;,\;\; K_{n,n'}(\epsilon) =$$

$$2n\sum_{R=0}^{\infty}(-1)^R\,\frac{2^{-2R-n-n'}}{\epsilon^{2R+n+n'}}\zeta(2R+n+n')\,\frac{(2R+n+n'-1)!\,(2R+n+n')!}{R!\,(R+n)!\,(R+n')!\,(R+n+n')!}\,, \tag{40}$$

where

$$\zeta(l) = \sum_{k=1}^{\infty} k^{-l} \tag{41}$$

and the function $a(z)$ is expressed in terms of $a_{1,\epsilon}$

$$a(z) = \frac{2(1 - a_{1,\epsilon})}{\epsilon^2}\,,\ \epsilon = \frac{1}{g\sqrt{2}}\,. \tag{42}$$

We can easiliy prove, that the maximal transcedentality property for $a(z)$ is valid in all orders of the perturbation theory and the coefficients in front of the products of corresponding ζ-functions are integer numbers [22]. Moreover, $a(z)$ has an essential singularity at the point $g = \infty$.

It is possible to show [22] that the asymptotic behaviour of $a(z)$ in the case of the Beisert-Eden-Staudacher equation [21] is in an agreement with the AdS/CFT prediction [19]

$$\lim_{g\to\infty} \gamma_{sing} = \frac{2}{\epsilon}\,\frac{I_1(2\epsilon^{-1})}{I_0(2\epsilon^{-1})} \approx 2\sqrt{2}\,g\, - \frac{1}{2}\,. \tag{43}$$

8. Universal anomalous dimension in 4 loops

With the use of the asymptotic Bethe anzatz and the maximal transcendentality hypothesis, the anomalous dimension in four loops was calculated [24]

$$\frac{\gamma_4}{256} = \mathbf{4S_{-7}} + \mathbf{6S_7} + 2(S_{-3,1,3} + S_{-3,2,2} + S_{-3,3,1} + S_{-2,4,1})$$

$$+3(-S_{-2,5} + S_{-2,3,-2}) + 4(S_{-2,1,4} - S_{-2,-2,-2,1} - S_{-2,1,2,-2}$$

$$-S_{-2,2,1,-2} - S_{1,-2,1,3} - S_{1,-2,2,2} - S_{1,-2,3,1}) + \dots$$

$$-72S_{1,1,1,-4} - 80S_{1,1,-4,1} - \zeta(\mathbf{3})\mathbf{S_1}(\mathbf{S_3} - \mathbf{S_{-3}} + \mathbf{2S_{-2,1}})\,,$$

where the dots mean the dropped terms. The last term is the contribution of the dressing phase firstly calculated in Ref. [21]. The harmonic sums here depend on the parameter $M = j - 2$. They can be analytically continued to the complex values of M. It turns out that the first two terms in the expression for γ_4 lead to its singularity at $\omega = M + 1 \to 0$

$$\lim_{M\to -1} \gamma_4(M) = -\frac{512}{\omega^7}\,,$$

which contradicts the above BFKL prediction.

Therefore, the asymptotic Bethe anzatz is not correct and one should take into account the so-called wrapping effects. As an attempt to find a contribution from these effects, we can modify the last term in the expression for γ_4 corresponding to the dressing phase. The simplest modification conserving the transcendentality property and reproducing the BFKL prediction is given by the substitution of the factor $\zeta(3)$ in the dressing phase by the following linear combination of the harmonic sums

$$\zeta_3 \to \frac{47\zeta_3}{24} - \frac{S_{-3}}{4} + \frac{3S_{-2}S_1}{4} + \frac{3S_1S_2}{8} + \frac{3S_3}{8} + \frac{S_{-2,1}}{6} - \frac{17S_{2,1}}{24}\,.$$

It turns out that after this substitution, the anomalous dimension has correct singularity also at even negative points M predicted by the double-logarithmic resummation

$$\frac{1}{256}\,\gamma_4|_{j\to -2k+\omega} = \frac{5}{\omega^7} - 20\frac{S_1}{\omega^6} + \frac{24\,S_1^2 - 14(S_2+\zeta_2) + 4(S_2+S_{-2})}{\omega^5} + \dots\,.$$

Therefore, it is plausible that this substitution leads to the correct expression for γ_4.

9. Discussion

The high-energy theory in QCD is based on the fact that the gluons and quarks are reggeized. To solve the unitarization problem for the BFKL pomeron, one should use the effective action local in the particle rapidities. The Reggeon calculus in the form of a 2+1 field theory can be derived from this action. The next-to-leading corrections to the BFKL kernel were calculated in QCD and in $N = 4$ SUSY. In $N = 4$ SUSY, the eigenvalue of the kernel is expressed as a sum of the most complicated functions which could appear in this order and does not have the non-analytic terms. Using the hypothesis of the maximal transcendentality for the universal anomalous dimension of the twist-2 operators, we calculated it up to the third order. Our resummation procedure is in an agreement with the strong coupling predictions obtained from the AdS/CFT correspondence. In particular, we calculated the intercept of the BFKL pomeron in $N = 4$ SUSY and the cusp anomalous dimension at strong couplings. The anomalous dimension in 4-loops was also found with the use of the asymptotic Bethe anzatz, maximal transcendentality and predictions obtained from the BFKL and double-logarithmic resummations.

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ANOMALNE DIMENZIJE UVOJ-2 OPERATORA I POMERON U $N = 4$ SUSY

Dajemo pregled BFKL pristupa Reggeovim procesima u QCD i pokazujemo da su BKP jednadžbe za složena stanja više reggeiziranih gluona integrabilne u višebojnoj QCD. Istražuje se relacija pomerona i gravitona u $N = 4$ SUSY. Hipoteza maksimalne transcendentalnosti omogućila je izračunavanje anomalne dimenzije na nivou tri petlje. Uzevši također asimptotsku Betheovu postavku i model za zamotne učinke, nalazimo anomalnu dimenziju uvoj-2 operatora za četiri petlje, u skladu s BFKL i dvojno-logaritamskim predviđanjima.