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MEASUREMENTS OF Ke4 AND $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ DECAYS AT NA48/2

SILVIA GOY LÓPEZ

CERN, CH-1211, Geneva 23, Switzerland

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During 2003 and 2004, the NA48/2 experiment at the CERN SPS has collected a large sample of charged kaon decays. Using 2003 data, the form factors of the $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ (Ke4) have been evaluated, allowing the extraction of the $\pi\pi$ scattering lengths a_0^0 and a_0^2. From the 2003 plus most of the 2004 statistics, an independent determination of the same parameters has been made through the interpretation of an anomaly (cusp) in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution of the decay $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$, observed in the region around $M_{00} = 2m_+$, where m_+ is the charged pion mass.

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1. *Introduction*

The main goal of the NA48/2 experiment was the search for direct CP violation in charged kaons decays to three pions, both in $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$ [1] and $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ [2], but also other studies have been performed on the collected data sample. In particular, preliminary results on the extraction of form factors and $\pi\pi$ scattering lengths from $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ and $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays will be presented here.

At low energies, the S-wave scattering lengths for isospin $I = 0$ (a_0^0), and for isospin $I = 2$ (a_0^2) are the relevant parameters of the $\pi\pi$ scattering amplitude expansion in partial waves. Different theoretical approaches can be used to determine the values of these two constants. Using causality and analiticity properties of the scattering amplitude, and taking as input $\pi\pi$ scattering data at intermediate energies, numerical solutions of dispersion relations [3, 4] allow to obtain a_0^0 from a fit of the phase shift (δ) variation with $M_{\pi\pi}$. In this context, a_0^2 is given by a known function of a_0^0 ($a_0^2 = f_{\mathrm{DR}}(a_0^0)$). In particular, using Roy equations [3], a universal band (UB) can be defined in the a_0^0,a_0^2 plane, which delimits the allowed region of a_0^0,a_0^2 values compatible with the measurements realized at intermediate energy of the $\pi\pi$ scattering process. In the chiral perturbation theory (ChPT), additional constrains can be used to reduce the extension of this band [5]. In this framework, the value of a_0^0 is related to the size of the chiral condensate and it is predicted to be $a_0^0 = 0.220 \pm 0.005$, with $a_0^2 = -0.0444 \pm 0.0010$ given by $a_0^2 = f_{\mathrm{ChPT}}(a_0^0)$

In the past, the experimental values for a_0^0, a_0^2 have been obtained from the study of the $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ form factors. These provide a clean measurement, as $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ decays give access to the final state interaction of two pions at small energies in the absence of any other hadron. The Geneva-Saclay experiment in 1977 obtained a result for the scattering lengths based on 30000 $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ events [6]. More recently, the BNL E865 experiment [7] collected 390000 $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ decays, resulting in a measurement of a_0^0 to 10 % precision. In addition, the DIRAC collaboration has presented a result of $|a_0^0 - a_0^2|$ from measurement of the pionium lifetime [8].

The NA48/2 experiment provided determination of the $\pi\pi$ scattering lengths by two independent methods: the study of $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ form factors and the interpretation of the cusp effect in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decay.

2. *Beam line and detector*

The NA48/2 experiment used an upgraded version of the NA48 detector [9] with a changed beamline providing simultaneous and collinear K^+ and K^- beams. The primary beam consisted of 400 GeV protons impinging on a beryllium target. Its intensity was 5 $\times 10^{10}$ protons per cycle, with a 4.8 s flat spill and 16.2 s cycle.

Two sets of achromat units selected charged particles with momentum of (60$\pm$3) GeV. After traversing cleaning and final collimators, positive and negative kaon beams travelled through a 114 m vacuum tank terminated by a 0.3% X_0 Kevlar window. The transverse dimension of boths beams is 5mm RMS and they were superimposed with a precision of about 1 mm.

The main components of the NA48 detector were a magnetic spectrometer, a charged hodoscope, a liquid krypton electromagnetic calorimeter, a hadronic calorimeter and a muon detection system.

The magnetic spectrometer consisted of a magnet and four drift chambers. The magnet provided a transverse momentum kick of 120 MeV/c. The spatial resolution was 150 μm and the momentum resolution could be parameterised as $\sigma(p)/p \simeq (1.0 \oplus 0.044\, p)\%$ (p in [GeV/c]).

The timing for charged events was given by an hodoscope with a resolution of 250 ps.

The electromagnetic calorimeter was a 27 X_0 liquid krypton. The energy resolution is given by $\sigma(E)/E = 3.2\%/\sqrt{E} \oplus (9\%)/E \oplus 0.42\%$ (E in GeV). The time resolution was better than 300 ps.

The hadronic calorimeter consisted of two stacks of scintillator slabs separated by iron blocks. It had a total thickness of 7.2 interaction lengths. The muon counter consisted on three planes of plastic scintillators, each shielded by 80 cm iron walls.

3. *Ke4 decay*

The $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ decay can be fully described by a set of kinematic variables introduced by Cabbibo and Maksymowicz [10] (CM-variables). These are the invariant mass of the $\pi\pi$ system ($M_{\pi\pi}$), the invariant mass of the eν system ($M_{e\nu}$) and the three angles, θ_π,θ_e and ϕ depicted in Fig. 1.

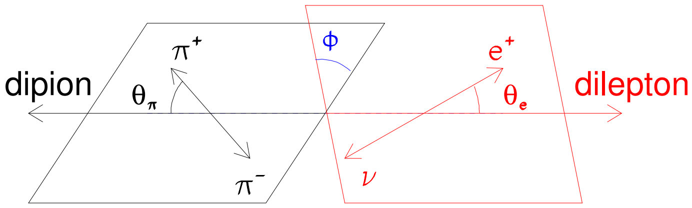

Fig. 1. Topology of the $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ decay showing the angle definitions.

The $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ matrix element is defined by means of three axial form factors F, G and R, and a vector form factor H, but the sensitivity to R is very small, as it is multiplied by the electron mass squared. Because of the small values of $S_\pi = {M_{\pi\pi}}^2$, a partial wave expansion of the form factors is restricted to s and p waves

$$F = F_s e^{i\delta_s} + F_p e^{i\delta_p} cos(\theta_\pi + ...), G = G_p e^{i\delta_g} + ..., H = H_p e^{i\delta_h} \tag{1}$$

Assuming all p-wave phases to be equal (δ_p=δ_g =δ_h), only four form factors are left: F_s, F_p, G_p and H_p, and a phase difference $\delta(q^2) = \delta_s - \delta_p$. These can be further expanded in terms of $q^2 = {M_{\pi\pi}}^2/4m_\pi^2 - 1$ and $S_e = {M_{e\nu}}^2$,

$$F_s = f_s(1 + f_s' q^2/f_s + f'' q^2/f_s + ...), F_p = (f_p + ...),$$
$$G_p = (g_p + g_p' q^2 + ...), H_p = (h_p + ...) \tag{2}$$

The $K^{\pm} \to \pi^+\pi^-e^{\pm}\nu$ decays were reconstructed requiring three charged tracks in the event, two of them with opposite charge and compatible with the pion hypothesis, and the other track compatible with an electron. Electron and pion identification is based on the relation between energy deposited in the electromagnetic calorimenter and the momentum measured at the drift chambers (the E/p ratio). The corresponding three-pion invariant mass is calculated, together with the total visible p_t with respect to the beam axis. The values of these variables must be outside an ellipse with semi-axes ± 20 MeV/c^2 and ± 35 MeV/c^2. The constrain given by the neutrino is used to solve the energy-momentum conservation equations and the solution with kaon momentum closest to 60 GeV is chosen.

The main background sources are $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$ with one pion either decaying into $\pi \to e\nu$ or being missidentified as an electron and $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ or $K^{\pm} \to \pi^{\pm}\pi^0$ with Dalitz decay of a π^0, where one electron is missidentified as a pion and photons are not detected. The total background level is of the order of 0.5 %, estimated using both the data and Monte Carlo simulation.

From the detected quantities, CM-variables can be reconstructed for every selected $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ event. The form factors F, G and H, and the phase shift δ are determined by fitting the data distributions of these variables against simulated Monte Carlo events.

In order to perform the fits, both data and simulated events are distributed in $10 \times 5 \times 5 \times 5 \times 12$ iso-populated bins in the $M_{\pi\pi}$, $M_{e\nu}$, $\cos(\theta_{\pi})$, $\cos(\theta_{e})$, ϕ plane. In each of the $M_{\pi\pi}$ bins, a log-likelihood estimator is defined, and form factors are assumed to be constant within the bin. Therefore, ten independent fits (one per $M_{\pi\pi}$ bin) of four parameters (F_p, G_p, H_p and δ) plus a free normalization constant (related to F_S) are perfomed in the four-dimensional space of the remaining CM-variables ($M_{\pi\pi}$, $M_{e\nu}$, $\cos(\theta_{\pi})$, $\cos(\theta_{e})$, ϕ). Different kaon charges are fitted separately, and the corresponding results are checked for consistency and then combined. Distribution of kinematic variables for data and for events simulated using the fitted form factor values are shown in Fig. 2.

The value of the phase difference δ is extracted from the measured asymmetry

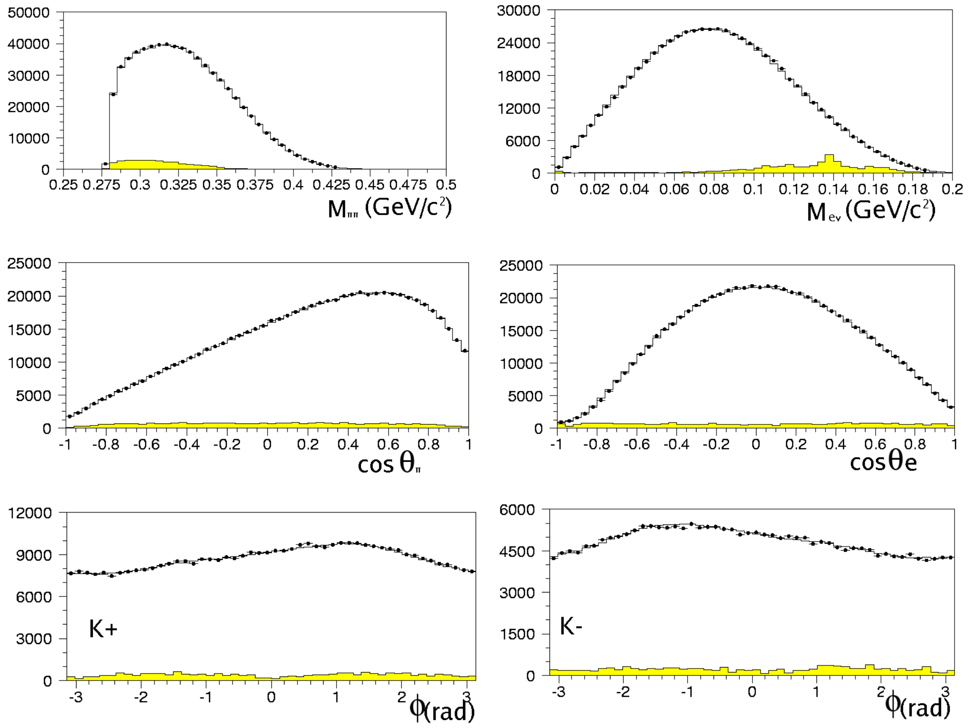

Fig. 2. Distribution of the five Cabibbo-Maksymowicz (CM) variables. ϕ distributions are shown separately for K^{+} and K^{-}. Data after background subtraction are shown as symbols with error bars, simulation using the fitted form factor values as histograms. The background contribution (calculated from data) is superimposed in the shaded area, increased by a factor of 10 to be visible on the linear scale.

of the ϕ distribution as a function of $M_{\pi\pi}$.

Once the values of F_p,G_p, H_p, δ and the normalization parameter have been extracted per $M_{\pi\pi}$ bin, the q^2 dependence of the form factors can be investigated. As no absolute normalization is available for the moment (no branching ratio measurement has been performed in these data), only the relative form factor variation with q^2 and with S_e can be extracted. All results are, therefore, given with respect to F_s(q=0), which is just a constant term. The F_s^2 variation with q^2 is obtained from the relative bin to bin normalization of data over simulated events. After the q^2 dependency of F_s^2 was extracted, a residual $M_{e\nu}$ variation was observed. Therefore, a two dimensional fit in $M_{\pi\pi}$-$M_{e\nu}$ plane was performed in order to get the final result for the slopes of the F_s expansion.

Once the normalization variation was taken into account, the variation of F_p, G_p and H_p with q^2 has also been studied. Results on the form factors variation with $M_{\pi\pi}$ are shown in Fig. 3.

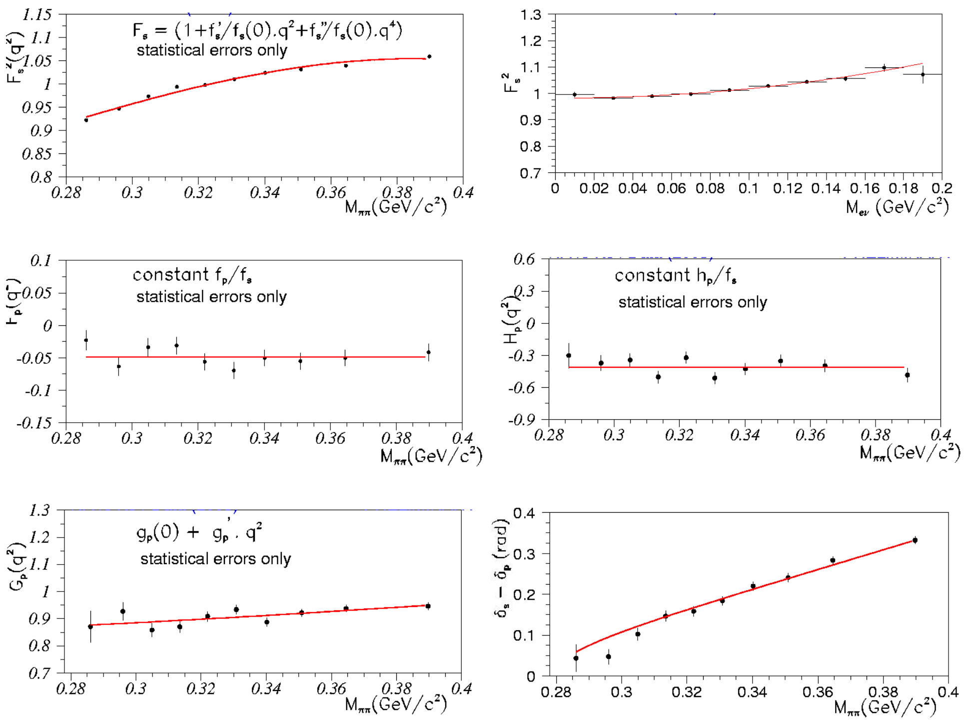

Fig. 3. Variation of the fitted form factors and phase shift with $M_{\pi\pi}$. Top row: F_s^2 projected on the $M_{\pi\pi}$ axis assuming no S_e dependence and residual dependence when projected on the $M_{e\nu}$ axis. Mid row: $F_p/F_s(0)$ and $G_p/F_s(0)$. Bottom row: $H_p/F_s(0)$ and phase shift δ. In the last plot, the line corresponds to a 1-parameter fit using the Universal Band center line constraint.

For the δ phase, the red (CHANGE - B/W printing!) curve shows the fit of the measured points to the center of the universal band. This function depends on only one parameter a_0^0, and its value can in this way be extracted from the data. A high sensitivity to the scattering length is achieved due to the good acceptance for high $M_{\pi\pi}$ values.

A variety of systematic checks have been perfomed. Two different methods of analysis, with different reconstruction, acceptance corrections and fitting procedures, were implemented, leading to compatible results. Effects of uncertainties in the beam simulation, background level, electron identification and implementation of radiative corrections have been studied. In addition, possible bin to bin correlations were investigated and taken into account in the final result. Results for the form factor slopes and for a_0^0 in the center of the univeral band are summarized in Table 1. All form factors are measured to between 5% and 15% accuracy, errors being dominated by statistics.

TABLE 1. Measured form factor slopes in Ke4 decays.

	Result	Statistical error	Systematic error
$f_s'/f_s =$	0.172	$\pm$ 0.009	$\pm$ 0.006
$f_s''/f_s =$	-0.090	$\pm$ 0.009	$\pm$ 0.007
$f_e'/f_s =$	0.081	$\pm$ 0.008	$\pm$ 0.009
$f_p/f_s =$	-0.048	$\pm$ 0.004	$\pm$ 0.004
$g_p/f_s =$	0.873	$\pm$ 0.013	$\pm$ 0.012
$g_p'/f_s =$	0.081	$\pm$ 0.022	$\pm$ 0.015
$h_p/f_s =$	-0.411	$\pm$ 0.019	$\pm$ 0.008
$a_0^0 =$	0.256	$\pm$ 0.006	$\pm$ 0.005

In addition to fitting the δ variation to the universal band, other methods can be used to relate δ measurements to a_0^0 and a_0^2. Taking into account ChPT constrains, data can be fitted to one parameter a_0^0 with $a_0^2 = f_{\mathrm{ChPT}}(a_0^0)$. A fit to two free parameters a_0^2 and a_0^0 can also be performed.

Results from NA48/2 and the Geneva-Saclay experiment are in agreement, however, E865 favor a slightly different region of the universal band.

Further improvements on the theoretical side have been recently developed. According to Ref. [11], the measured δ has to be corrected by isospin-symmetry-breaking effects before extracting a_0^0. After a preliminary correction for this effect is applied to the NA48/2 data, the a_0^0 and a_0^2 values decrease by $\sim$ 0.022 and $\sim$ 0.004, respectively, showing good consistency with ChPT predictions

4. *Cusp Effect*

In the analysis of 2003 NA48/2 data, a sudden slope variation in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution of $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays was observed in the region around $M_{00} = 2m_+$, where m_+ is the charged pion mass [12]. An interpretation of this effect was first given by Cabibbo [13]. Two amplitudes contribute to the $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays: direct emission, and charge exchange of $\pi^+\pi^-$ into $\pi^0\pi^0$ in the final state of $K^{\pm} \to \pi^{\pm}\pi^+\pi^-$. Direct emission is given by

$$M_0(u,v) = M_0(0,0)(1 + g_0 u/2 + h'u^2/2 + k'v^2/2), \tag{3}$$

where u and v are kinematic variables defined as $u = (s_3 - s_0)/m_+^2$, $v = (s_1 - s_2)/m_+^2$, $s_i = (P_K - P_i)^2$, $s_0 = (s_1 + s_2 + s_3)/3$, and P_K and P_i are the 4-momenta of kaon and pions (indices $i = 1$ and 2 correspond to the two even pions).

The matrix element of charge exchange is proportional to the a_x parameter, that in the limit of exact isospin symmetry is $a_x = a_0^0 - a_0^2$. These two amplitudes interfere destructively below threshold, so that a standard Dalitz plot parameterization shows a deficit in data before the cusp. This is the basic idea in the rescattering model at one-loop. A more complete formulation of the model includes all re-scattering processes at one-loop and two-loop level and has been used to extract NA48/2 result [14]. This includes more parameters than a_x and an isospin symmetry breaking correction has been applied in order to extract a_0^0 and a_0^2 from fits to data. The theoretical uncertainty of this model is about 5%, but radiative corrections could in principle be computed to 1% accuracy.

The study of this cusp effect has now been enlarged to 2003 plus 80 % of the 2004 data. A total of 59,624,170 $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ events were reconstructed requiring at least 4 clusters in the electromagnetic calorimeter, 15 cm away from the extrapolated impact possition of any reconstructed track, and 10 cm away from any other clusters. The value of M_{00} is computed using average vertex of the two π^0's. The resulting M_{00} distribution for this sample is shown in Fig. 4.

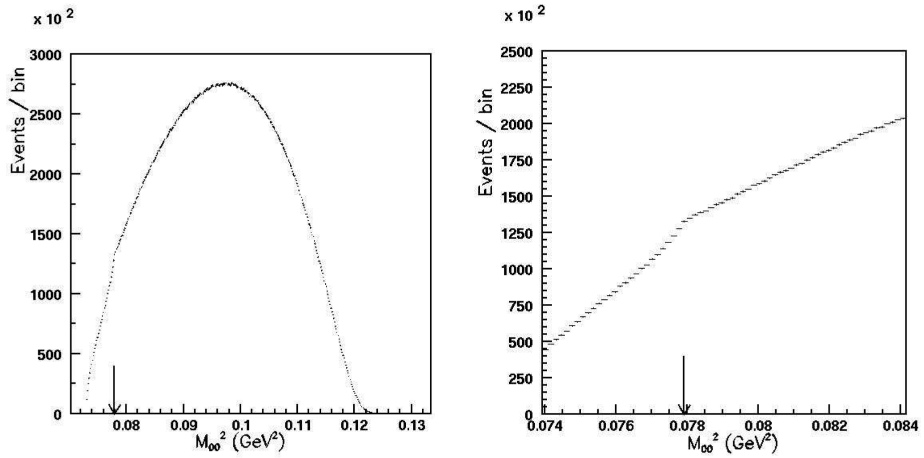

Fig. 4. M_{00} distribution in 2003 plus 2004 sample. A zoom in the cusp region clearly shows a slope variation.

A fit of the simulated spectral shape to the measured M_{00} distribution was performed through a χ^2 minimization.

The fitting function depends on five free parameters: a normalization constant, the form-factor slopes g and h' and the scattering lengths a_0^0- a_0^2 and a_0^2, with k' fixed to zero. For the final result, seven bins around the cusp were excluded from the fit in order to reduce sensitivity to Coulomb corrections and pionium contribution. The quality of the fit is shown in Fig. 5. The excess of events in the region around $2m_+$ can be interpreted as the pionium signature, yielding a rate of pionium formation $R = \Gamma(\pi^+ A_{2\pi})/\Gamma(K^{\pm} \to \pi^{\pm}\pi^+\pi^-) = (1.82 \pm 0.21) \times 10^{-5}$, somewhat higher than the theoretical prediction [15]. The k' slope was measured using the v projection of the data and fixing the measured values of a_0^0, a_0^2, g_0 and h'. Then the fit in M_{00} was repeated to take into account a k' different from zero.

A number of systematic effects have been investigated, like the analysis technique, acceptance determination, trigger efficiency, resolution effects, variation of fitting interval, electromagnetic shower simulation and the v-dependence of the amplitude. Results on Dalitz plot slopes are summarized in Table 2.

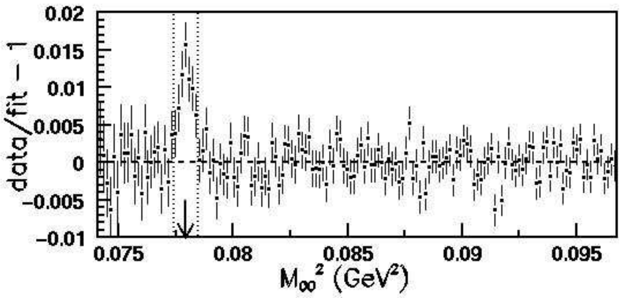

Fig. 5. Deviation of data spectrum from the fit result with statistical errors corresponding to 2003 plus 2004 sample.

TABLE 2. Measured Dalitz plot slopes in $K^{\pm} \to \pi^{\pm}\pi^0\pi^0$ decays.

	Result	Statistical error	Systematic error
g_0	0.649	± 0.003	± 0.004
h'	-0.048	± 0.007	± 0.005
k'	-0.0097	± 0.0003	± 0.0008

The scattering-length values extracted from the NA48/2 cusp effect are:

$$a_0^0 - a_0^2 = 0.261 \pm 0.006_{\mathrm{stat}} \pm 0.003_{\mathrm{syst}} \pm 0.001_{\mathrm{external}}$$

$$a_0^2 = -0.037 \pm 0.013_{\mathrm{stat}} \pm 0.009_{\mathrm{syst}} \pm 0.002_{\mathrm{external}}$$

The external error comes from the uncertainty on the branching ratios of charged kaon decays into three pions. The main contribution to the error is the 5% uncertainty in the theoretical model, that is not explicitly written above. This result is in good agreement with ChPT prediction, with the NA48/2 measurement from $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ decays (when isospin symmetry breaking corrections are considered) and with the results from the DIRAC collaboration.

5. *Conclusion*

From the NA48/2 2003 data, about 670000 Ke4 decays have been analyzed. A fit of the data distributions of the Cabibbo-Maksymowicz kinematic variables allows a precise measurement of the decay form factors F_s, F_p, G_p, H_p, which are expanded in powers of q^2, and of the phase difference $\delta = \delta_s - \delta_p$. Thanks to a sizeable acceptance at large $\pi\pi$ mass, a high sensitivity to the S-wave $\pi\pi$ scattering lengths a_0^0 and a_0^2 is achieved.

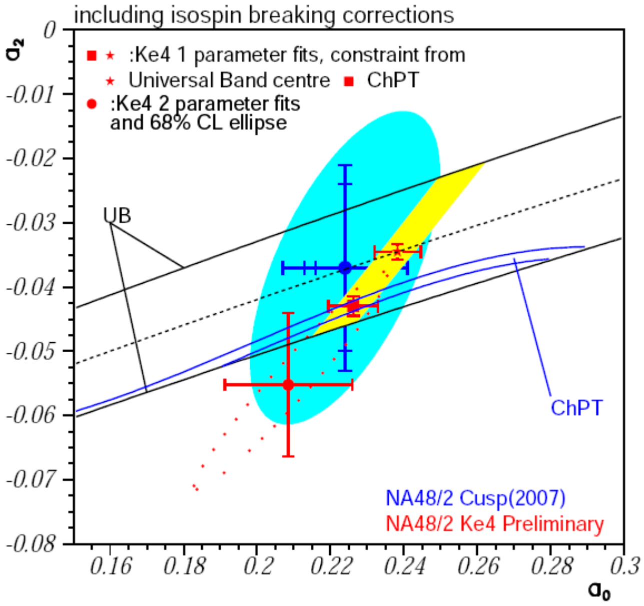

Fig. 6. Results for the $\pi\pi$ scattering lengths a_0^0 and a_0^2 (in m_{π^+} unit) from fits to the NA48/2 data using different theoretical assumptions. The black solid curves are the Universal Band boundaries (Roy equations), the narrow blue band is the restricted area using the ChPT constraint. The symbols correspond to the results of 1-parameter fits to the Ke4 data: the star is the value obtained at the center line of the Universal Band (dotted line), the square using the ChPT constraint. The error bars are 1 σ errors. The contour corresponds to 68% CL in a two-parameter fit. Isospin symmetry breaking corrections are considered. The blue point shows the cusp result, and the blue area its theoretical error assumed to be gaussian.

Using 2003 and 80% of the 2004 data sample, 59.6×10^6 $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays have been reconstructed. A measurement of the scattering lengths has been obtained through the interpretation of a cusp in the $\pi^0\pi^0$ invariant mass (M_{00}) distribution of this decay. This anomaly was observed in the region around $M_{00} = 2m_+$, where m_+ is the charged pion mass. In addition, a non-zero quadratic term on the v variable has been found on the $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ Dalitz plot.

Applying isosping breaking corrections, the extracted values of the scattering lengths from $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ and from the cusp on $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ are fully compatible, as shown in Fig. 6. In addition, NA48/2 results are in agreement with the measurement from the DIRAC experiment and with ChPT predictions.

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MJERENJE RASPADA Ke4 I $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ NA NA48/2

Tijekom 2003. i 2004., u eksperimentu NA48/2 na SPS-u u CERN-u, sakupili smo velik uzorak raspada nabijenih kaona. Na osnovi podataka iz 2003. odredili smo faktore oblika $K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu$ (Ke4) i na toj osnovi izveli duljine $\pi\pi$ raspršenja a_0^0 i a_0^2. Na osnovi podataka iz 2003. i većeg dijela podataka iz 2004. izveli smo neovisne vrijednosti tih parametara tumačenjem nepravilnosti (grbe) u raspodjeli invarijantne mase $\pi^0\pi^0$ (M_{00}) u raspadu $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$, koja se opaža oko $M_{00} = 2m_+$, gdje je m_+ masa nabijenog piona.