Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

SUM RULES FOR HADRON PHOTOPRODUCTION ON HADRONS AND PHOTON

E. BARTOŠ[a], S. DUBNIČKA[a], A. Z. DUBNIČKOVÁ[b] and E. A. KURAEV[c]

[a]*Institute of Physics, Slovak Academy of Sciences, Bratislava, Slovak Republic*

[b]*Department of Theoretical Physics, Comenius University, Bratislava, Slovak Republic*

[c]*Laboratory of Theoretical Physics, JINR, Dubna, Russia*

Received 23 October 2007; Accepted 20 February 2008
Online 18 June 2008

Sum rules are derived relating mean-squared charge radii in the case of pseudoscalar meson, Dirac mean-squared radii and anomalous magnetic moments in the case of $(1/2)^+$ baryon, and finally sum of ratios of the fourth power of quark charges to the squared quark masses in the case of photon, with convergent integrals over the hadron photoproduction total cross-sections on pseudoscalar mesons, octet baryons and over $\gamma\gamma \to 2\,jets$ cross-section, respectively. As a byproduct, chains of inequalities for corresponding total photoproduction cross-sections on meson and baryons in finite energy region, to be valid on an average, are also derived.

PACS numbers: 11.55.Hx, 13.40.Gp, 13.60.Hb UDC 539.122, 539.126

Keywords: sum rules, pseudoscalar meson nonet, $(1/2)^+$ baryon octet, photon, total photoproduction cross-section

1. *Introduction*

Under the sum rules one commonly understands the expressions relating some physical quantities with other physical quantities. There exist various approaches for the derivation of a complex of sum rules as it is fully demonstrated e.g. in Ref. [1]. In this paper we are concerned with the sum rules for hadron photoproduction on hadrons and photon.

Historically, only one attempt was made, by Kurt Gottfried [2], to derive a sum rule for hadron photoproduction, specially on proton target, relating the proton mean-squared charge radius $\langle r^2_{Ep}\rangle$ and the proton magnetic moment $\mu_p = 1 + \kappa_p$ to the integral over the total hadron photoproduction cross-section on proton, considering very high-energy electron-proton scattering and the non-relativistic quark model of hadrons. However, the corresponding integral is divergent and the Gottfried sum rule practically cannot be satisfied.

We are interested in a derivation of hadron photoproduction sum rules, relating static properties of pseudoscalar meson nonet, $(1/2)^+$ ground state octet baryons and quarks with the convergent integral over the total cross-sections of hadron photoproduction on pseudoscalar mesons, on octet baryons and over $\gamma\gamma \to 2\,jets$ cross-section, respectively, either by exploiting analytic properties of "the retarded forward Compton scattering amplitude on hadron" [3] $\tilde{A}(s_1, \mathbf{q})$ in s_1-plane (such amplitude represents only a class of diagrams in which the initial state photon is first absorbed by a hadron line and then emitted by the scattered hadron) for meson [4] and baryon [5] targets, or by its explicit calculation in quark loops approximation for the case of photon [6] target. The variable s_1 is the c.m. energy squared of the virtual Compton scattering process and $\mathbf{q}$ is defined by the transversal part $q_\perp = (0, 0, \mathbf{q})$ of the virtual photon four-momentum q.

This new approach gives sum rules with convergent integrals. Especially, the proton-neutron sum rule [7] was tested by using existing data and it is fulfilled with high precision. The latter fact gives us confidence to all other sum rules derived by a similar approach.

2. q^2 dependent meson sum rules

The q^2-dependent meson sum rules are derived by investigating the analytic properties of the retarded Compton scattering amplitude $\tilde{A}^h(s_1, \mathbf{q})$ in the s_1-plane as presented in Fig. 1a, then defining the integral I over the path C (for more detail see Ref. [8]) in the s_1-plane

$$I = \int\limits_C \mathrm{d}s_1 \frac{p_1^\mu p_1^\nu}{s^2} \left(\tilde{A}^h_{\mu\nu}(s_1, \mathbf{q}) - \tilde{A}^{h'}_{\mu\nu}(s_1, \mathbf{q}) \right) \tag{1}$$

from the gauge-invariant light-cone projection $p_1^\mu p_1^\nu \tilde{A}^h_{\mu\nu}(s_1, \mathbf{q})$ of the amplitude $\tilde{A}^h(s_1, \mathbf{q})$, and once closing the contour C to upper half-plane and another one to lower half-plane (see Fig. 1b).

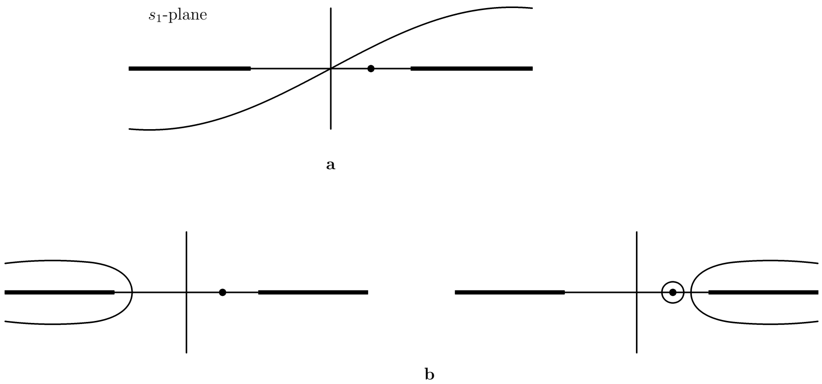

Fig. 1. Sum rule interpretation in s_1 plane.

As a result the following sum rule appears

$$\pi(Res^{h'} - Res^{h}) = \mathbf{q}^2 \int_{r.h.}^{\infty} \frac{\mathrm{d}s_1}{s_1^2}[Im\,\tilde{A}^h(s_1,\mathbf{q}) - Im\,\tilde{A}^{h'}(s_1,\mathbf{q})]. \qquad (2)$$

The left-hand cut contributions expressed by an integral over the difference $[Im\,\tilde{A}^h(s_1,\mathbf{q}) - Im\,\tilde{A}^{h'}(s_1,\mathbf{q})]$ are assumed to be mutually annulated.

Now, one has to take into account the corresponding residuum of the intermediate state pole (see Fig. 1).

As the electromagnetic structure of mesons is described by one charge form factor, the residuum takes the form

$$Res^{(M)} = 2\pi\alpha F_M^2(-\mathbf{q^2}), \qquad (3)$$

where the averaging over the initial photon spin is performed.

Then, substituting (3) into (2) and taking into account [4]

$$\Big(\frac{\mathrm{d}\sigma^{e^-h\to e^-X}(s,\mathbf{q})}{\mathrm{d}^2\mathbf{q}} - \frac{\mathrm{d}\sigma^{e^-h'\to e^-X'}(s,\mathbf{q})}{\mathrm{d}^2\mathbf{q}}\Big) =$$

$$= \quad \frac{\alpha}{4\pi^2\mathbf{q^2}} \int_{s_1^{thr}}^{\infty} \frac{\mathrm{d}s_1}{s_1^2}[Im\,\tilde{A}^h(s_1,\mathbf{q}) - Im\,\tilde{A}^{h'}(s_1,\mathbf{q})]. \qquad (4)$$

with $\mathrm{d}^2\mathbf{q} = \pi\mathrm{d}\mathbf{q^2}$, one comes to the q^2-dependent meson sum rule

$$[F_{P'}^2(-\mathbf{q^2}) - F_{P'}^2(0)] - [F_P^2(-\mathbf{q^2}) - F_P^2(0)] =$$

$$= \quad \frac{2}{\pi\alpha^2}(\mathbf{q^2})^2\Big(\frac{\mathrm{d}\sigma^{e^-P\to e^-X}}{\mathrm{d}\mathbf{q}^2} - \frac{\mathrm{d}\sigma^{e^-P'\to e^-X'}}{\mathrm{d}\mathbf{q}^2}\Big), \qquad (5)$$

where the left-hand side was re-normalized in order to separate the pure strong interactions from electromagnetic ones.

3. *Universal sum rule for total hadron photoproduction cross-sections on mesons*

Now, employing the Weizsäcker-Williams like relation [4]

$$\mathbf{q}^2\Big(\frac{\mathrm{d}\sigma^{e^-P\to e^-X}}{\mathrm{d}\mathbf{q}^2} - \frac{\mathrm{d}\sigma^{e^-P'\to e^-X'}}{\mathrm{d}\mathbf{q}^2}\Big)_{|\mathbf{q}^2\to 0} = \qquad (6)$$

$$= \quad \frac{\alpha}{\pi}\int_{s_1^{th}}^{\infty}\frac{\mathrm{d}s_1}{s_1}[\sigma_{\mathrm{tot}}^{\gamma P\to X}(s_1) - \sigma_{\mathrm{tot}}^{\gamma P'\to X'}(s_1)]$$

for mesons, taking a derivative according to $\mathbf{q^2}$ of both sides in q^2-dependent meson sum rule for $\mathbf{q^2} \to 0$, and using the laboratory reference frame by $s_1 = 2m_B\omega$, one comes to the new universal meson sum rule relating meson mean-square charge radii to the integral over a difference of the corresponding total photoproduction cross-sections on mesons

$$\frac{1}{3}(\langle r^2_{P'}\rangle - \langle r^2_P\rangle) = \frac{2}{\pi^2\alpha}\int\limits_{\omega_P}^{\infty}\frac{\mathrm{d}\omega}{\omega}\big[\sigma_{\mathrm{tot}}^{\gamma P\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma P'\to X}(\omega)\big], \tag{7}$$

in which just a mutual cancellation of the rise of the latter cross-sections for $\omega \to \infty$ is achieved.

4. *Application to various pairs of mesons*

According to the SU(3) classification of existing hadrons, the following ground state pseudoscalar meson nonet π^-, π^0, π^+, K^-, $\bar{K}^0$, K^0, K^+, η, η' is known to exist. However, in consequence of the CPT invariance, for the meson electromagnetic form factors $F_P(-\mathbf{q^2})$ the following relation holds

$$F_P(-\mathbf{q^2}) = -F_{\bar{P}}(-\mathbf{q^2}), \tag{8}$$

where $\bar{P}$ means antiparticle.

Since π_0, η and η' are true neutral particles, their electromagnetic form factors are zero due to Eq. (8) in the whole region of definition and, therefore, we exclude them from further considerations.

If one considers pairs of particle-antiparticle like $\pi^\pm$, $K^\pm$ and K^0, $\bar{K}^0$, the left-hand side of (5) is equal zero owing to the relation (8), and we also exclude the pairs $\pi^\pm$, $K^\pm$ and K^0, $\bar{K}^0$ from further considerations.

If one considers pairs of the iso-doublet of kaons K^+, K^0 and $K^-, \bar{K}^0$, the following Cabibbo-Radicati–like sum rules [9] for kaons can be written

$$\frac{1}{6}\pi^2\alpha\langle r^2_{K^+}\rangle = \int\limits_{\omega_{th}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma K^+\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma K^0\to X}(\omega)\right] \tag{9}$$

$$\frac{1}{6}\pi^2\alpha(-1)\langle r^2_{K^-}\rangle = \int\limits_{\omega_{th}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma K^-\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma \bar{K}^0\to X}(\omega)\right], \tag{10}$$

in which holds the relation $\langle r^2_{K^+}\rangle = -\langle r^2_{K^-}\rangle$ for kaon mean-squared charge radii (it follows directly from (8)), and divergence of the integrals, due to an increase of the total cross-sections $\sigma_{\mathrm{tot}}^{\gamma K^\pm\to X}(\omega)$ for large values of ω, is removed by the increase of total cross-sections $\sigma_{\rm tot}^{\gamma K^0 \to X}(\omega)$ and $\sigma_{\rm tot}^{\gamma \bar{K}^0 \to X}(\omega)$, respectively. If besides the latter, also the relations

$$\sigma_{\rm tot}^{\gamma K^0 \to X}(\omega) \equiv \sigma_{\rm tot}^{\gamma \bar{K}^0 \to X}(\omega) \tag{11}$$
$$\sigma_{\rm tot}^{\gamma K^+ \to X}(\omega) \equiv \sigma_{\rm tot}^{\gamma K^- \to X}(\omega),$$

(which follow from C invariance of the electromagnetic interactions) are taken into account, one can see the sum rule (10), as well as all other possible sum rules obtained by combinations $K^+\bar{K}^0, K^-K^0$, already included in (9).

The last possibility is a consideration of a pair of mesons taken from the isomultiplet of pions and the isomultiplet of kaons leading to the following sum rules

$$\frac{1}{6}\pi^2\alpha[(\pm 1)\langle r_{\pi^\pm}^2\rangle - (\pm 1)\langle r_{K^\pm}^2\rangle] = \int\limits_{\omega_{th}}^{\infty} \frac{{\rm d}\omega}{\omega}\left[\sigma_{\rm tot}^{\gamma\pi^\pm \to X}(\omega) - \sigma_{\rm tot}^{\gamma K^\pm \to X}(\omega)\right], \tag{12}$$

$$\frac{1}{6}\pi^2\alpha(\pm 1)\langle r_{\pi^\pm}^2\rangle = \int\limits_{\omega_{th}}^{\infty} \frac{{\rm d}\omega}{\omega}\left[\sigma_{\rm tot}^{\gamma\pi^\pm \to X}(\omega) - \sigma_{\rm tot}^{\gamma K^0 \to X}(\omega)\right]. \tag{13}$$

Now taking the experimental values [10]

$$(\pm 1)\langle r_{\pi^\pm}^2\rangle = (+0.4516 \pm 0.0108)\ {\rm fm}^2,$$
$$(\pm 1)\langle r_{K^\pm}^2\rangle = (+0.3136 \pm 0.0347)\ {\rm fm}^2.$$

one comes to the conclusion that in average

$$[\sigma_{\rm tot}^{\gamma\pi^\pm \to X}(\omega) - \sigma_{\rm tot}^{\gamma K^\pm \to X}(\omega)], > 0 \tag{14}$$
$$[\sigma_{\rm tot}^{\gamma K^- \to X}(\omega) - \sigma_{\rm tot}^{\gamma \bar{K}^0 \to X}(\omega)] > 0,$$

from which the following chain of inequalities for finite values of ω in average follows

$$\sigma_{\rm tot}^{\gamma\pi^\pm \to X}(\omega) > \sigma_{\rm tot}^{\gamma K^\pm \to X}(\omega) > \sigma_{\rm tot}^{\gamma \bar{K}^0 \to X}(\omega) > 0. \tag{15}$$

Subtracting (9) or (10) from the relation (13), the sum rule (12) is obtained, what demonstrates a mutual consistency of all considered sum rules.

5. q^2 dependent octet barion sum rule

Similarly to the meson case, as the electromagnetic structure of octet baryons is described by Dirac and Pauli form factors, the residuum takes the form

$$Res^B = 2\pi\alpha \left(F_{1B}^2 + \frac{\mathbf{q^2}}{4m_B^2} F_{2B}^2 \right), \tag{16}$$

where an averaging over the initial baryon and photon spins is performed. Finally, one obtains the q^2-dependent baryon sum rule in the form [5]

$$\begin{aligned} & [F_{1B'}^2(-\mathbf{q^2}) - F_{1B'}^2(0)] - [F_{1B}^2(-\mathbf{q^2}) - F_{1B}^2(0)] + \\ + \quad & \mathbf{q^2} \left[\frac{F_{2B'}^2(-\mathbf{q^2})}{4m_{B'}^2} - \frac{F_{2B}^2(-\mathbf{q^2})}{4m_B^2} \right] = \\ = \quad & \frac{2}{\pi\alpha^2} (\mathbf{q^2})^2 \left(\frac{\mathrm{d}\sigma^{e^- B \to e^- X}}{\mathrm{d}\mathbf{q}^2} - \frac{\mathrm{d}\sigma^{e^- B' \to e^- X}}{\mathrm{d}\mathbf{q}^2} \right), \end{aligned} \tag{17}$$

where the left-hand side was again re-normalized in order to separate the pure strong interactions from electromagnetic ones.

6. Universal sum rule for total hadron photoproduction cross-sections on barions

Using in (17) the Weizsäcker-Williams relation for baryons, taking a derivative according to $\mathbf{q}^2$ of both sides in q^2-dependent baryon sum rule for $\mathbf{q^2} \to 0$ and using the laboratory reference frame by $s_1 = 2m_B\omega$, one comes to the new universal baryon sum rule

$$\begin{aligned} & \frac{1}{3}\big[F_{1B}(0)\langle r_{1B}^2\rangle - F_{1B'}(0)\langle r_{1B'}^2\rangle\big] - \left[\frac{\kappa_B^2}{4m_B^2} - \frac{\kappa_{B'}^2}{4m_{B'}^2} \right] = \\ = \quad & \frac{2}{\pi^2\alpha} \int_{\omega_B}^{\infty} \frac{\mathrm{d}\omega}{\omega} \left[\sigma_{\mathrm{tot}}^{\gamma B \to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma B' \to X}(\omega) \right], \end{aligned} \tag{18}$$

relating Dirac baryon mean-square radii $\langle r_{1B}^2\rangle$ and baryon anomalous magnetic moments κ_B to the convergent integral, in which a mutual cancellation of the rise of the corresponding total cross-sections for $\omega \to \infty$ is achieved.

7. Application to various pairs of octet barions

According to the SU(3) classification of existing hadrons, the following members of the ground state $(1/2)^+$ baryon octet (p, n, Λ^0, Σ^+, Σ^0, Σ^-, Ξ^0, Ξ^-) are known. As a result, by using the universal expression (18), one can write down 28 different sum rules for total cross-sections of hadron photoproduction on ground state $(1/2)^+$ octet baryons.

In order to evaluate their left hand sides and to draw out some phenomenological consequences, one needs the reliable values of Dirac baryon mean-square radii $\langle r_{1B}^2 \rangle$ and baryon anomalous magnetic moments κ_B.

The latter are known (except for Σ^0, which is found from the well known relation $\kappa_{\Sigma^+} + \kappa_{\Sigma^-} = 2\kappa_{\Sigma^0}$) experimentally [10]. However, to calculate $\langle r_{1B}^2 \rangle$ by means of the difference of the baryon electric mean-square radius $\langle r_{EB}^2 \rangle$ and Foldy term, well known for all ground state octet baryons from the experimental information on the magnetic moments given by Review of Particle Physics [10]

$$\langle r_{1B}^2 \rangle = \langle r_{EB}^2 \rangle - \frac{3\kappa_B}{2m_B^2}, \tag{19}$$

we are in the need of the reliable values of $\langle r_{EB}^2 \rangle$. They are known experimentally only for the proton, neutron and Σ^--hyperon.

Fortunately, recent results [11] to fourth order in relativistic baryon chiral perturbation theory (giving predictions for the Σ^- charge radius and the Λ–Σ^0 transition moment in excellent agreement with the available experimental information) solve our problem completely.

Calculating the left-hand sides of all sum rules, one finds

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)\right] = 2.0415\,\mathrm{mb}, \;\Rightarrow\; \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega) > \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega) \tag{20}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^+}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)\right] = 2.0825\,\mathrm{mb}, \;\Rightarrow\; \sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) > \sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega) \tag{21}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^+}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)\right] = 4.2654\,\mathrm{mb}, \;\Rightarrow\; \sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) > \sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega) \tag{22}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega) - \sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)\right] = 2.1829\,\mathrm{mb}, \;\Rightarrow\; \sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega) > \sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega) \tag{23}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Xi^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=1.5921\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega) \tag{24}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)\right]=1.6673\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega) \tag{25}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)\right]=-0.4158\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) \tag{26}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)\right]=1.6667\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega) \tag{27}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)\right]=3.8496\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega) \tag{28}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)\right]=1.7259\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega) \tag{29}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_p}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=3.3180\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega) \tag{30}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_n}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)\right]=-0.3260\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega) \tag{31}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_n}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)\right]=-2.4573\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) \tag{32}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_n}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)\right]=-0.3747\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega) \tag{33}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_n}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)\right]=1.8082\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega) \tag{34}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_n}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)\right]=-0.3156\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega) \tag{35}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_n}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=1.2766\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega) \tag{36}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Lambda^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)\right]=-2.0831\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega) \tag{37}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Lambda^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)\right]=-0.0006\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)\approx\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega) \tag{38}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Lambda^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)\right]=2.1823\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega) \tag{39}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Lambda^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)\right]=0.0586\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega) \tag{40}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Lambda^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=2.1823\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega) \tag{41}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^+}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)\right]=2.1417\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega) \tag{42}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^+}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=3.7338\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega) \tag{43}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)\right]=0.1168\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega) \tag{44}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^0}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=1.5732\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-X}(\omega) \tag{45}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^-}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)\right]=-2.1238\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega) \tag{46}$$

$$\frac{2}{\pi^2\alpha}\int\limits_{\omega_{\Sigma^-}}^{\infty}\frac{\mathrm{d}\omega}{\omega}\left[\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)-\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)\right]=-0.5316\,\mathrm{mb},\ \Rightarrow\ \sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)<\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega). \tag{47}$$

From these relations, one gets the following chain of inequalities

$$\begin{gathered}\sigma_{\mathrm{tot}}^{\gamma\Sigma^+\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma p\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Lambda^0\to X}(\omega)\approx\sigma_{\mathrm{tot}}^{\gamma\Sigma^0\to X}(\omega)>\\ \sigma_{\mathrm{tot}}^{\gamma\Xi^0\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma n\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Xi^-\to X}(\omega)>\sigma_{\mathrm{tot}}^{\gamma\Sigma^-\to X}(\omega)\end{gathered}$$

for total cross-sections of hadron photoproduction on ground state $(1/2)^+$ octet baryons to be valid in average for finite values of ω.

8. *Sum rule for photon target*

Let us consider the two-photon exchange electron-photon zero-angle scattering amplitude in the process

$$e(p,\lambda)+\gamma(k,\varepsilon)\to e(p,\lambda)+\gamma(k,\varepsilon), \tag{48}$$

in two-loop (α^3) approximation, as presented in Fig. 2, with $p^2=m_e^2$, $k^2=0$ and assuming that $s=2p\,k\gg m_e^2$.

Averaging over the initial electron and photon spin states (initial and final spin states are supposed to coincide) one can write down the amplitude of the process (48) in the following form

$$A^{e\gamma\to e\gamma}(s,t=0)=s\frac{\alpha}{4\pi^2}\int\frac{\mathrm{d}^2\mathbf{q}}{(\mathbf{q^2})^2}ds_1\sum_{\varepsilon}A_{\mu\nu\alpha\beta}^{\gamma\gamma\to\gamma\gamma}\frac{p^\mu p^\nu\varepsilon^\alpha\varepsilon^{*\beta}}{s^2}, \tag{49}$$

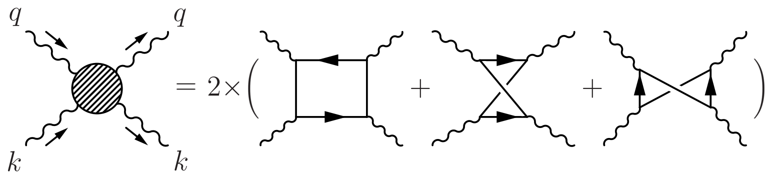

Fig. 2. Feynman diagram of $e\gamma \to e\gamma$ scattering with LBL mechanism to be realized by quark-loops.

where the light-cone projection of the light-by-light (LBL) scattering tensor takes the form

$$A^{\gamma\gamma\to\gamma\gamma}(s_1,\mathbf{q}) = A^{\gamma\gamma\to\gamma\gamma}_{\mu\nu\alpha\beta}\frac{p^\mu p^\nu \varepsilon^\alpha \varepsilon^{*\beta}}{s^2} = -\frac{8\alpha^2}{\pi^2}N_c Q_q^4 \int \mathrm{d}^4 q_- \left[\frac{S_1}{D_1} + \frac{S_2}{D_2} + \frac{S_3}{D_3}\right] \quad (50)$$

with

$$\frac{S_1}{D_1} = \frac{(1/4)Tr\hat{p}(\hat{q}_- + m_q)\hat{p}(\hat{q}_- - \hat{q} + m_q)\hat{\varepsilon}^*(\hat{q}_- - \hat{q} + \hat{k} + m_q)\hat{\varepsilon}(\hat{q}_- - \hat{q} + m_q)}{(q_-^2 - m_q^2)((q_- - q)^2 - m_q^2)^2((q_- - q + k)^2 - m_q^2)}, \quad (51)$$

$$\frac{S_2}{D_2} = \frac{(1/4)Tr\hat{p}(\hat{q}_- + m_q)\hat{p}(\hat{q}_- - \hat{q} + m_q)\hat{\varepsilon}^{(}(\hat{q}_- - \hat{q} - \hat{k} + m_q)\hat{\varepsilon}^*(\hat{q}_- - \hat{q} + m_q)}{(q_-^2 - m_q^2)((q_- - q)^2 - m_q^2)^2((q_- - q - k)^2 - m_q^2)}, \quad (52)$$

$$\frac{S_3}{D_3} = \frac{(1/4)Tr\hat{p}(\hat{q}_- + m_q)\hat{\varepsilon}(\hat{q}_- - \hat{k} + m_q)\hat{p}(\hat{q}_- + \hat{q} - \hat{k} + m_q)\hat{\varepsilon}^*(\hat{q}_- + \hat{q} + m_q)}{(q_-^2 - m_q^2)((q_- + q)^2 - m_q^2)((q_- + q + k)^2 - m_q^2)((q_- - k)^2 - m_q^2)}. \quad (53)$$

Here q_- means the quark four-momentum in the quark loop of the process $\gamma\gamma \to \gamma\gamma$, N_c is the number of colours in QCD, and Q_q is the charge of the quark q in electron charge units.

Now, taking the derivative of the relation (49) according to $d^2\mathbf{q}$ and investigating the analytic properties of the obtained expression in the s_1-plane, one gets the configuration as presented in Fig. 3, where also the path C of the integral expression

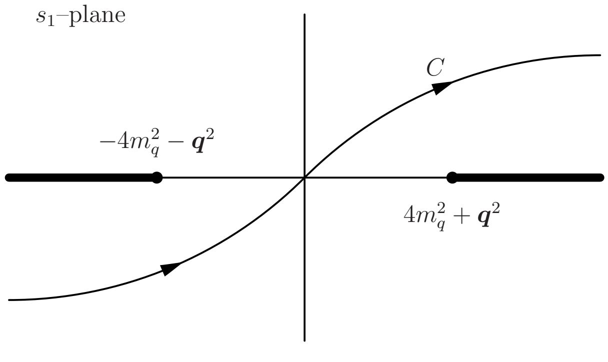

Fig. 3. The path C of an integration in (54).

$$I = \int_C \mathrm{d}s_1 \frac{\mathrm{d}A^{e\gamma\to e\gamma}(s_1,\mathbf{q})}{\mathrm{d}^2\mathbf{q}} \tag{54}$$

is drawn. When the integration contour is closed at the right side (on the s-channel cut) and at the left side (on the u-channel cut), one obtains the relation

$$\int_{-4m_q^2-\mathbf{q^2}}^{-\infty} \mathrm{d}s_1 \Delta_u \frac{\mathrm{d}A^{e\gamma\to e\gamma}(s_1,\mathbf{q})}{\mathrm{d}^2\mathbf{q}}|_{\mathrm{left}} = \int_{4m_q^2+\mathbf{q}^2}^{\infty} \mathrm{d}s_1 \Delta_s \frac{\mathrm{d}A^{e\gamma\to e\gamma}(s_1,\mathbf{q})}{\mathrm{d}^2\mathbf{q}}|_{\mathrm{right}}\,, \tag{55}$$

where the right s-channel discontinuity is related by Eq. (49), due to optical theorem in a differential form

$$\Delta_s \frac{\mathrm{d}A^{e\gamma\to e\gamma}(s,0)}{\mathrm{d}^2\mathbf{q}} = 2s \frac{\mathrm{d}\sigma^{e\gamma\to eq\bar{q}}}{\mathrm{d}^2\mathbf{q}}, \tag{56}$$

to the Q^2=$\mathbf{q}^2$=$-q^2$ dependent differential cross-section of $q\bar{q}$ pair creation by electron on photon, to be well known in the framework of QED [12] for l^+l^- pair creation

$$\begin{aligned} &\frac{4\alpha^3}{3(q^2)^2} f(\frac{\mathbf{q}^2}{m_q^2}) N_c Q_q^4 = \frac{\mathrm{d}\sigma^{e\gamma\to eq\bar{q}}}{\mathrm{d}\mathbf{q}^2}, \\ &f(\frac{\mathbf{q}^2}{m_q^2}) = (\mathbf{q}^2 - m_q^2)J + 1, \\ &J = \frac{4}{\sqrt{\mathbf{q}^2(\mathbf{q}^2+4m_q^2)}} \ln[\sqrt{\mathbf{q}^2/(4m_q^2)} + \sqrt{1+\mathbf{q}^2/(4m_q^2)}]. \end{aligned} \tag{57}$$

But the right-hand cut is related to two real quark production for $s_1 > 4m_q^2$, which is associated wit 2 jet production.

The left-hand cut contribution has the same form as in the QED case with constituent quark masses and as a result one obtains

$$\frac{4\alpha^3}{3(\mathbf{q}^2)^2} N_c \sum_q Q_q^4 f(\frac{\mathbf{q}^2}{m_q^2}) = \frac{\mathrm{d}\sigma^{e\gamma\to e2jets}}{\mathrm{d}\mathbf{q}^2}. \tag{58}$$

Finally, for the case of small $\mathbf{q}^2$, and applying the Weizsäcker-Williams relation, one comes to the sum rule for the photon target [6] as follows

$$\frac{14}{3} \sum_q \frac{Q_q^4}{m_q^2} = \frac{1}{\pi\alpha^2} \int_{4m_q^2}^{\infty} \frac{\mathrm{d}s_1}{s_1} \sigma_{\mathrm{tot}}^{\gamma\gamma\to 2jet}(s_1). \tag{59}$$

The quantity $\sigma_{\rm tot}^{\gamma\gamma\to 2\rm jets}(s_1)$ is assumed to decrease for large values of s_1. It corresponds to the events in $\gamma\gamma$ collisions with creation of two jets, which are not separated by rapidity gaps, and for which there is no experimental information until the present days.

The latter complicates a verification of the obtained sum rule for photon target.

9. *Conclusions*

Considering the very high energy peripheral electron-hadron scattering, with the production of a hadronic state X moving closely to the direction of initial hadron, then exploiting analytic properties of the forward retarded Compton scattering amplitude on the same hadron, for the case of small transferred momenta, new meson and baryon sum rules are derived. Evaluating the left-hand sides of the sum rules, the chains of inequalities for total cross-sections on hadron photoproduction on pseudoscalar mesons and $(1/2)^+$ octet baryons have been found.

Similarly, considering two-photon-exchange electron-photon zero-angle scattering process, and calculating explicitly the corresponding $\gamma\gamma \to \gamma\gamma$ subprocess in quark-loops approximation, the new sum rule for photon target are also derived.

Acknowledgements

The work was supported by the Slovak Grant Agency for Sciences VEGA, Grant No. 2/7116/27.

References

[1] V. De Alfaro, S. Fubini, G. Furlan and C. Rosetti, *Currents in Hadron Physics*, American Elsevier Publ. Company, Inc., New York (1973).

[2] K. Gottfried, Phys. Rev. Lett. **18** (1967) 1174.

[3] E. Kuraev, M. Sečansky and E. Tomasi-Gustafsson, Phys. Rev. D **73** (2006) 125016-1.

[4] S. Dubnička, A. Z. Dubničkova and E. A. Kuraev, Phys. Rev. D **75** (2007) 057901-1.

[5] S. Dubnička, A. Z. Dubničkova and E. A. Kuraev, Phys. Rev. D **74** (2006) 034023-1.

[6] E. Bartoš, S. Dubnička, A. Z. Dubničkova and E. A. Kuraev, Phys. Rev. D **76** (2007) 057901-1.

[7] E. Bartoš, S. Dubnička and E. A. Kuraev, Phys. Rev. D **70** (2004) 117901-1.

[8] V. N. Baier, V. S. Fadin, V. A. Khose and E. A. Kuraev, Phys. Rep. **78** (1981) 293.

[9] N. Cabibbo and L. A. Radicati, Phys. Let. **19** (1966) 697.

[10] Review of Particle Physics, J. Phys. G **33** (2006) 1.

[11] B. Kubis and U.-G. Meissner, Eur. Phys. J. C **18** (2001) 747.

[12] S. R. Kel'ner, Yad. Fiz. **5** (1967) 1092.

ZBROJNA PRAVILA ZA FOTOTVORBU HADRONA NA HADRONIMA I FOTONU

Izvodimo zbrojna pravila koja povezuju srednje-kvadratne polumjere naboja za pseudoskalarne mezone, Diracove srednje-kvadratne polumjere i anomalne magnetske momente za $(1/2)^+$ barione, te zbrojeve omjera četvrtih potencija kvarkovskih naboja s kvadratima masa kvarkova za slučaj fotona, s konvergentnim integralima udarnih presjeka hadronske fototvorbe na pseudoskalarnim mezonima, oktetnim barionima i preko udarnih presjeka procesa $\gamma\gamma \to 2\,\text{mlaza}$. Usput, izvodimo i odgovarajuće nizove nejednakosti za totalne udarne presjeke na mezonima i barionima, koji vrijede u prosjeku.