NONSINGULAR COSMOLOGICAL MODELS WITH A VARIABLE COSMOLOGICAL TERM Λ

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Exact solutions of the Einstein's field equations describing a spherically symmetric cosmological model without a big bang or any other kind of singularity recently obtained by Dadhich and Patel (2000) are revisited. The matter content of the model is a shear-free perfect fluid with isotropic pressure and a radial heat flux. Three different exact solutions are obtained both for perfect fluid and fluid with bulk viscosity. It turns out that the cosmological term $\Lambda(t)$ is a decreasing function of time, which is consistent with recent observations of type Ia supernovae.

1. Introduction

The problem of cosmological singularity is one of the most fundamental issues in modern theoretical cosmology. Due to the powerful singularity theorem [1, 2], it was widely believed that cosmological models must have initial singularity. However, in 1990 Senovilla [3] obtained the first singularity-free cosmological perfect-fluid (with a realistic equation of state $3p = \rho$) solution of the Einstein equation, and since then the possibility of constructing regular cosmologies was renewed. The interest for regular cosmologies had stifled for nearly 30 years due to the powerful singularity theorems, which seemed to preclude such spacetimes under very general requirements, such as chronology protecting, energy and generic conditions. The

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open way to regular cosmologies was found in violation of some technical premises of the theorems. The remarkable feature is the absence of an initial singularity, the curvature and matter invariants being regular and smooth everywhere. This corresponds to a cylindrically-symmetric spacetime filled with an isotropic radiation perfect fluid. For instance, it was shown by Chinea et al. [4] that the Senovilla spacetime did not possess a compact achronal set without edge and could not have closed trapped surfaces. However, the first results were not encouraging. The extension of the Senovilla solution to a family of spacetime left the set of regular models limited to a zero-measure subset surrounded by spacetime with Ricci and Weyl curvature singularities [5]. A thorough discussion of the model of such type can be found in Senovilla [6]. This family is shown to be included in a wider class of separable cosmological models, which comprises FLRW universe [7]. Other properties of these solutions, such as their inflationary behaviour, generalized Hubble law and the feasibility of constructing a realistic non-singular cosmological model are studied therein.

A large family of non-singular cosmological models and generalization thereof have been considered but they all are cylindrically symmetric [8] − [10]. For practical cosmology the spherical symmetry, however, is more appropriate. It is therefore pertinent to seek spherically symmetric nonsingular models. The first model of this kind was obtained by Dadhich [11] with an imperfect fluid with a heat flux. The model satisfied all energy conditions and had no singularity of any kind. Dadhich et al. [12] also obtained a non-singular model with null radiation flux. These models are both inhomogeneous and anisotropic and have a typical behaviour beginning with low density at $t \to -\infty$, contracting to high density at $t = 0$ and then again expanding to low density at $t \to \infty$. An interesting feature of the spacetime metric of these models is that it contains an arbitrary function of time which can be constrained to comply with the demand of non-singularity and energy conditions. Dadhich and Raychaudhuri [13] later showed how a particular choice of this function leads to a model of an ever existing spherically symmetric universe, oscillating between two regular states, which involves blue shifts as in the quasi steady state cosmological model of Hoyle, Burbige and Narlikar [14] and is filled with a non-adiabatic fluid with anisotropic pressure and radial heat flux. These observations led to the search of spherically symmetric singularity-free cosmological models with a perfect fluid source characterized by isotropic pressure. In this search, Tikekar [15] constructed two spherically symmetric singularity-free relativistic cosmological models, describing universes filled with non-adiabatic perfect fluid, accompanied by heat flow along radial direction. Recently, many researchers [16] − [20] have studied non-singular cosmological models in different context. From a purely theoretical point of view, the investigation of nonsingular cosmological models gives invaluable insight into the spacetime structure, the inherent nonlinear character of gravity and its interaction with matter fields. As a by-product, it also deepens our understanding of the singularity theorem, in particular the assumptions lying in their base [7].

Models with a dynamic cosmological term $\Lambda(t)$ are becoming popular as the cosmological-constant problem gets ameliorated in a natural way. There are signifi-

cant observational evidence for the detection of Einstein's cosmological constant, Λ or a component of material content of the universe, that varies slowly with time and space and so acts like Λ. New observations of type Ia supernovae (Garnavich et al. [21], Perlmutter et al. [22], Riess et al. [23], Schmidt et al. [24]), cosmic microwave background (CMB) anisotropies (e.g., Lineweaver [25]), galaxy surveys (e.g., Lahav and Bridle [26]), gravitational lensing (e.g., Chiba and Yoshii [27]), and the Lyα forest (e.g., Weinberg et al. [28]) argue for a nonzero cosmological "constant" with $\Omega_\Lambda (\equiv \Lambda/3H_0^2) \approx 0.6 - 0.7$. This quantity may *not be constant* as has been appreciated for 30 years (Bergmann [29], Wagoner [30], Linde [31], Kazanas [32]). Scalar fields (Dolgov [33], Abbott [34], Barr [35], Peeble and Ratra [36], Friemann et al. [37], Moffat [38], Starobinsky [39]), tensor fields (Hawking [40], Dolgov [41]), D-branes (Ellis, Mavromatos and Nanopoulos [42]), nonlocal effects (Banks [43], Linde [44]), wormholes (Coleman [45]), inflationary mechanisms (Brandenberger and Zhitnitsky [46], Peebles and Vilenkin [47]), and cosmological perturbations (Abramo, Brandenberger and Mukhanov [48]) have all been shown to give rise to an effective cosmological term that decays with time. Earlier researches on this topic, are contained in Zeldovich [49], Weinberg [50], Dolgov [51], Bertolami [52], Felten and Isaacman [53], Charlton and Turner [54], Sandaga [55], Carroll, Press and Turner [56]. Some of the recent discussions on the cosmological-constant "problem" and consequences on cosmology with a time-varying cosmological-constant have been discussed by Dolgov and Silk [57], Sahni and Starobinsky [58], Peebles [59], Padmanabhan [60], Carroll [61], Vishwakarma [62], and Pradhan et al. [63]. This motivates us to study the cosmological models with Λ varying with time.

Recently, Dadhich and Patel [64] obtained a shear-free nonsingular spherical model with heat flux. This model satisfies the weak and strong energy conditions and also has a physically acceptable fall-off behaviour in both r and t for physical and kinematic parameters. In this paper, motivated by the situation discussed above, we shall focus on the problem with varying cosmological constant in the presence of a perfect fluid and also in the presence of a bulk viscous fluid. We do this by extending the work of Dadhich and Patel [64] by including a varying cosmological constant. The remainder of this paper is organized as follows. In Sec. 2 we give a description of the cosmological models with their dynamical equations and solve them under the initial conditions inspired by Dadhich and Patel. We also investigate three different cosmological models for different values of the function $P(t)$ and discuss results for these regimes. Sec. 3 comprises bulk viscous universe. We present our discussion and conclusions in Sec. 4.

2. A perfect fluid universe revisited

In this section, we review the solutions obtained by Dadhich and Patel [64]. The metric of the model is given in the form

$$ds^2 = (r^2 + P)^{2n}dt^2 - (r^2 + P)^{2m} \left[dr^2 + r^2(d\theta^2 + \sin^2\theta \ d\phi^2) \right], \qquad (1)$$

where

$$2n = 2m \pm \sqrt{8m^2 + 8m + 1},$$

in particular,

$$2m = 1 - \sqrt{\frac{3}{2}} < 0, \quad 2n = \sqrt{\frac{3}{2}}. \tag{2}$$

Here $P = P(t)$ which can be chosen freely. The Einstein field equations for a perfect fluid with time-dependent cosmological constant and a radial heat flux read

$$R_{ik} - \frac{1}{2}Rg_{ik} + \Lambda g_{ik} = -\left[(\rho + p)u_i u_k - pg_{ik} + \frac{1}{2}(q_i u_k + q_k u_i)\right], \tag{3}$$

where we have set $8\pi G/c^2 = 1$, $u_i u^i = 1 = -q_i q^i$, $q_i u^i = 0$, ρ and p denote the fluid density and isotropic pressure, and q_i is the radial heat flux vector.
From Eqs. (1) and (3) we obtain

$$\rho = \frac{3m^2 \dot{P}^2}{(r^2 + P)^{2n+2}} - \frac{4m\{3P + (m+1)r^2\}}{(r^2 + P)^{2m+2}} + \Lambda, \tag{4}$$

$$p = -\frac{m}{(r^2 + P)^{2n+2}}\left[2(r^2 + P)\ddot{P} + (3m - 2n - 2)\dot{P}^2\right]$$

$$+ \frac{4}{(r^2 + P)^{2m+2}}\left[(m + n)P + n^2 r^2\right] - \Lambda, \tag{5}$$

$$q = \frac{4m(n+1)r\dot{P}}{(r^2 + P)^{n+2}}, \tag{6}$$

where $q_i = qg_i^1$. The expansion and acceleration are obtained as

$$\theta = \frac{3m\dot{P}}{(r^2 + P)^{n+1}}, \tag{7}$$

$$\dot{u}_r = -\frac{nr}{r^2 + P}. \tag{8}$$

We have the freedom of choosing the function $P(t)$ so as to give a non-singular behaviour to the above parameters. As a matter of fact, there are multiple choices (see, Dadhich and Patel [64]), for instant, $P(t) = a^2 + b^2 t^2$, $a^2 + e^{-bt^2}$, $a^2 + b^2 \cos \omega t$, $a^2 > b^2$. For all these choices, it is observed that all physical and kinematic parameters remain regular and finite for the entire range of variables. Note that the model admits an interesting oscillating behaviour in time, with oscillations

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between two finite regular states. Oscillating nonsingular models are quite novel and interesting in their own accord.

For a complete determinacy of the system, we assume an equation of state of the form

$$p = \gamma\rho, \quad 0 \le \gamma \le 1, \tag{9}$$

where γ is a constant.

2.1. Model 1

We set $P(t) = a^2 + b^2 t^2$, $a^2 > b^2$. In this case the matter density ρ, the fluid pressure p, the heat flux parameter q and kinematic parameter of expansion θ are found to be given by the following expressions

$$\rho = \frac{12m^2 b^4 t^2}{(r^2 + a^2 + b^2 t^2)^{2n+2}} - \frac{4m\left[3(a^2 + b^2 t^2) + (m+1)r^2\right]}{(r^2 + a^2 + b^2 t^2)^{2m+2}} + \Lambda, \tag{10}$$

$$p = -\frac{4mb^2\left[r^2 + a^2 + (3m - 2n - 1)b^2 t^2\right]}{(r^2 + a^2 + b^2 t^2)^{2n+2}} + \frac{4\left[(m+n)(a^2 + b^2 t^2) + n^2 r^2\right]}{(r^2 + a^2 + b^2 t^2)^{2m+2}} - \Lambda, \tag{11}$$

$$q = \frac{8m(n+1)rb^2 t}{(r^2 + a^2 + b^2 t^2)^{n+2}}, \tag{12}$$

$$\theta = \frac{6mb^2 t}{(r^2 + a^2 + b^2 t^2)^{n+1}}. \tag{13}$$

Equations (10) and (11), with the use of (9), reduce to

$$(1 + \gamma)\Lambda = -\frac{4mb^2\left[r^2 + a^2 + (3m - 2n - 1 + 3m\gamma)b^2 t^2\right]}{(r^2 + a^2 + b^2 t^2)^{2n+2}}$$

$$+\frac{4\left[(m + n + 3m\gamma)(a^2 + b^2 t^2) + \{n^2 + m(m+1)\gamma\}r^2\right]}{(r^2 + a^2 + b^2 t^2)^{2m+2}}. \tag{14}$$

From the above equations, it is evident that the matter density is always and everywhere positive, while positivity of pressure is ensured if $3m - 2n < 1$. The heat flux parameter $q > 0$ for $t < 0$, $q = 0$ for $t = 0$ and $q < 0$ for $t > 0$. Equation (13) implies that the model describes an expanding universe for $t < 0$ with $q > 0$ and a contracting universe for $t > 0$ for $q < 0$, the switching from contracting phase to phase of expansion occurring at $t = 0$.

From Eq. (14), we observe that the cosmological constant Λ is a decreasing function of time (see Fig. 1). We also observe that Λ approaches a small and positive value at late times which is supported by recent type Ia supernova observations [21] − [24].
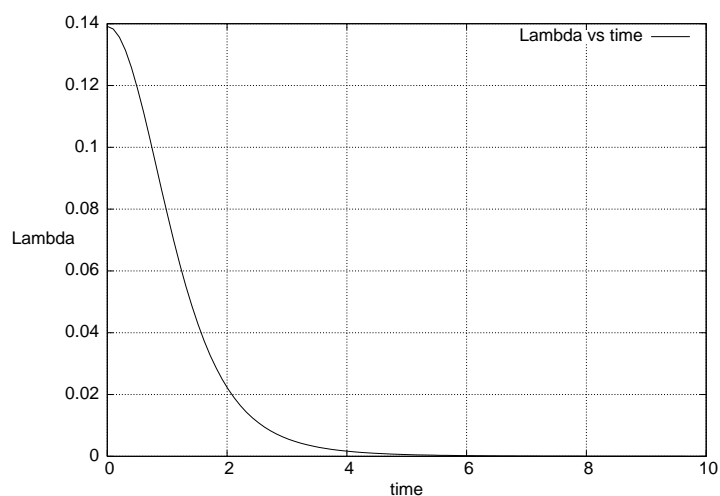
Fig. 1. *Variation of Λ with time for 2.1 Model 1. The values of parameters are: m = 1, n = $1 + \sqrt{17}/2$, $\gamma = 0.5$, a = 2, b = 1 and r = 1.*

2.2. Model 2

We set $P(t) = a^2 + e^{-bt^2}$, $a^2 > b^2$. In this case the matter density ρ, the fluid pressure p, the heat flux parameter q and kinematic parameter of expansion θ are found to be biven by the following expressions

$$\rho = \frac{12m^2b^2t^2e^{-2bt^2}}{(r^2 + a^2 + e^{-bt^2})^{2n+2}} - \frac{4m\left[3(a^2 + e^{-bt^2}) + (m+1)r^2\right]}{(r^2 + a^2 + e^{-bt^2})^{2m+2}} + \Lambda, \quad (15)$$

$$p = -\frac{4mbe^{-bt^2}\left[(r^2 + a^2)(2bt^2 - 1) - e^{-bt^2} + (3m - 2n)bt^2e^{-bt^2}\right]}{(r^2 + a^2 + e^{-bt^2})^{2n+2}}$$

$$+\frac{4\left[(m+n)(a^2 + e^{-bt^2}) + n^2r^2\right]}{(r^2 + a^2 + e^{-bt^2})^{2m+2}} - \Lambda, \quad (16)$$

$$q = -\frac{8mbr(n+1)te^{-bt^2}}{(r^2 + a^2 + e^{-bt^2})^{n+2}}, \quad (17)$$

$$\theta = -\frac{6mbte^{-bt^2}}{(r^2 + a^2 + e^{-bt^2})^{n+1}}. \quad (18)$$

By using Eq. (9) and eliminating $\rho(t)$ between (15) and (16), we obtain

$$(1+\gamma)\Lambda = -\frac{4mbe^{-bt^2}\left[\{3(1+\gamma)m - 2n\}bt^2e^{-bt^2} + (r^2 + a^2)(2bt^2 - 1) - e^{-bt^2}\right]}{(r^2 + a^2 + e^{-bt^2})^{2n+2}}$$

$$+\frac{4\left[\{(1+3\gamma)m + n\}(a^2 + e^{-bt^2}) + \{m(m+1)\gamma + n^2\}r^2\right]}{(r^2 + a^2 + e^{-bt^2})^{2m+2}}. \tag{19}$$

From the above equations, it is observed that the matter density is always and everywhere positive, while positivity of pressure is ensured if $3m - 2n > 0$. Thus it is also observed that the requirements of weak and strong energy conditions are fulfilled throughout the spacetime of this model. The dominant energy condition, which requires $\rho \geq p$, cannot, however, be satisfied, it is clearly violated for large r. Thus this model satisfies weak and strong but not the dominant energy condition.

From Eq. (19), it is observed that Λ first decreases, reaches a negative value then increases and becomes a constant small positive value (see Fig. 2). This could play the role of dark energy.
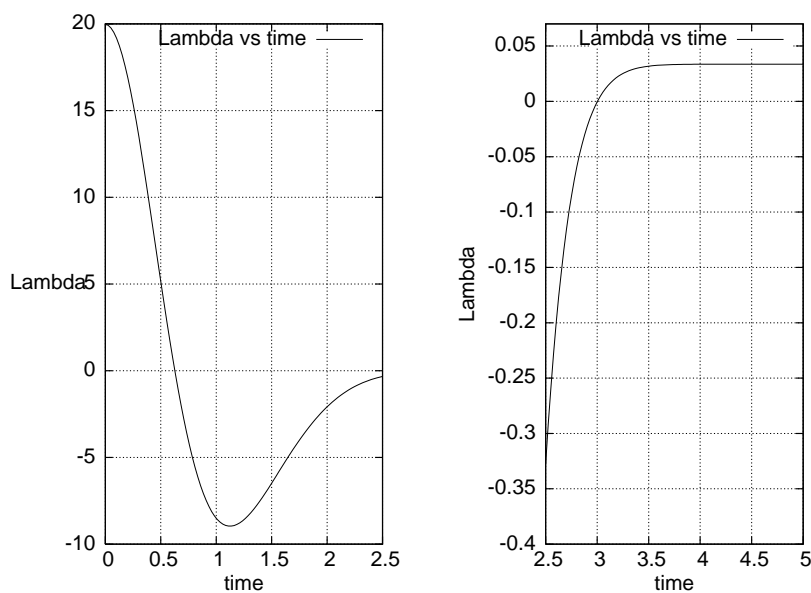

Fig. 2. Variation of Λ with time for 2.2 Model 2. The parameters are: m = 1, n = $1 - \sqrt{17}/2$, $\gamma = 0.5$, r = 1, a = 2 and b = 1.

2.3. Model 3

We set $P(t) = a^2 + b^2\cos\omega t$, $a^2 > b^2$. In this case the matter density ρ, the fluid pressure p, the heat flux parameter q and kinematic parameter of expansion

θ are given by the following expressions

$$\rho = \frac{3m^2b^4\omega^2\sin^2\omega t}{(r^2+a^2+b^2\cos\omega t)^{2n+2}} - \frac{4m[3(a^2+b^2\cos\omega t)+(m+1)r^2]}{(r^2+a^2+b^2\cos\omega t)^{2m+2}} + \Lambda, \quad (20)$$

$$p = \frac{mb^2\omega^2\left[2(r^2+a^2+b^2\cos\omega t)\cos\omega t - (3m-2n-2)\sin^2\omega t\right]}{(r^2+a^2+b^2\cos\omega t)^{2n+2}}$$

$$+\frac{4[(m+n)(a^2+b^2\cos\omega t)+n^2r^2]}{(r^2+a^2+b^2\cos\omega t)^{2m+2}} - \Lambda, \quad (21)$$

$$q = -\frac{4m(n+1)rb^2\omega\sin\omega t}{(r^2+a^2+b^2\cos\omega t)^{n+2}}, \quad (22)$$

$$\theta = -\frac{3b^2\omega\sin\omega t}{(r^2+a^2+b^2\cos\omega t)^{n+1}}. \quad (23)$$

Equations (20) and (21), with the use of (9, give

$$(1+\gamma)\Lambda = -\frac{mb^2\omega^2\left[\{3(1+b^2\gamma)m - 2(n+1)\}\sin^2\omega t - 2(r^2+a^2+b^2\cos\omega t)\cos\omega t\right]}{(r^2+a^2+b^2\cos\omega t)^{2n+2}}$$

$$+\frac{4\left[\{(1+3\gamma)m+n\}(a^2+b^2\cos\omega t)+\{m(m+1)\gamma+n^2\}r^2\right]}{(r^2+a^2+b^2\cos\omega t)^{2m+2}}. \quad (24)$$

From the above equations, it is observed that the matter density is always positive, whereas the pressure is non-negative if $3m - 2n < 0$. Thus it can be seen that the requirements of weak and strong energy conditions are fulfilled throughout the spacetime of this model but not the dominant energy condition. From Eq. (23), the expansion parameter indicates the universe of this model in the phase of contraction for $2\alpha\pi < \omega t < (2\alpha + 1)\pi$, where α takes on integer values only. During the phase of contraction $q < 0$ and during the expansion phase $q > 0$, while q vanishes when switching from contraction to expansion and from expansion to contraction occurs.

From Eq. (24), it is observed that the Λ is oscillating due to the properties of sinusoidal functions (see Fig. 3). It is also worth-noting that the average value (with respect to one period) of Λ is positive. Here we will have negative equation of state at late times required to support the current acceleration of universe.
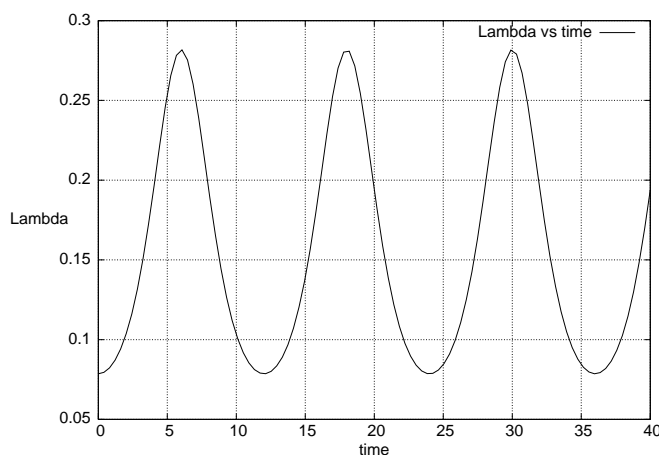
Fig. 3. *Variation of Λ with time for 2.3 Model 3. Here* m = 1, n = $1 + \sqrt{17}/2$, γ = 0.5, r = 1, a = 2, b = 1 *and* $\omega = \pi/6$.

3. Bulk viscous universe

The equations of bulk viscosity can be obtained from the general relativistic field equation when replace the effective pressure [49]

$$\bar{p} = p - \xi\theta, \tag{25}$$

where p is the pressure due to the present perfect fluid, ξ is the coefficient of bulk viscosity and θ is the expansion scalar. Thus, given $\xi(t)$, we can solve for cosmological parameters. In most of investigations involving bulk viscosity, it is assumed to be a simple power function of the energy density (Pavon [65], Maartens [66], Zimdahl [67])

$$\xi(t) = \xi_0\rho^k, \tag{26}$$

where ξ_0 and k are constants. If $k = 1$, Eq. (26) may correspond to a radiative fluid (Weinberg[50]). However, more realistic models (Santos[68]) are based on k lying in the range $0 \leq k \leq 1/2$.

Introducing (25) and (26) into (5), we obtain

$$p = \frac{3m\xi_0\rho^k\dot{P}}{(r^2 + P)^{n+1}} - \frac{m}{(r^2 + P)^{2n+2}}\left[2(r^2 + P)\ddot{P} + (3m - 2n - 2)\dot{P}^2\right]$$

$$+ \frac{4}{(r^2 + P)^{2m+2}}\left[(m + n)P + n^2r^2\right] - \Lambda. \tag{27}$$

3.1. Model 1

We set $P(t) = a^2 + b^2t^2$, $a^2 > b^2$. In this case we consider two following cases.

3.1.1. Case I : solution for $\xi = \xi_0$

When $k = 0$, Equation (26) reduces to $\xi = \xi_0$ (constant) and hence Eq. (27) with the help (9) and (10) reduces to the form

$$(1 + \gamma)\rho = \frac{6mb^2\xi_0 t}{(r^2 + a^2 + b^2t^2)^{n+1}} + \frac{4mb^2\left[(2n+1)b^2t^2 - (r^2 + a^2)\right]}{(r^2 + a^2 + b^2t^2)^{2n+2}}$$

$$-\frac{4\left[(2m - n)(a^2 + b^2t^2) + \{m(m+1) - n^2\}r^2\right]}{(r^2 + a^2 + b^2t^2)^{2m+2}}. \tag{28}$$

Eliminating $\rho(t)$ between Eqs. (10) and (28), we get

$$(1 + \gamma)\Lambda = \frac{6mb^2\xi_0 t}{(r^2 + a^2 + b^2t^2)^{n+1}} - \frac{4mb^2\left[r^2 + a^2 + \{3(1+\gamma)m - 2n - 1\}b^2t^2\right]}{(r^2 + a^2 + b^2t^2)^{2n+2}}$$

$$+\frac{4\left[\{(1+3\gamma)m + n\}(a^2 + b^2t^2) + \{n^2 + m(m+1)\gamma\}r^2\right]}{(r^2 + a^2 + b^2t^2)^{2m+2}}. \tag{29}$$

3.1.2. Case II : solution for $\xi = \xi_0\rho$

When $k = 1$, Eq. (26) reduces to $\xi = \xi_0\rho$ and hence Eq. (27), with the help of Eqs. (9) and (10), takes the form

$$\left[1 + \gamma - \frac{6\xi_0 mb^2 t}{(r^2 + a^2 + b^2t^2)^{n+1}}\right]\rho = \frac{4mb^2\{(2n+1)b^2t^2 - (r^2 + a^2)\}}{(r^2 + a^2 + b^2t^2)^{2n+2}}$$

$$+\frac{4\left[(n - 2m)(a^2 + b^2t^2) + r^2\{n^2 - m(m+1)\}\right]}{(r^2 + a^2 + b^2t^2)^{2m+2}}. \tag{30}$$

Eliminating $\rho(t)$ between Equations (30) and (10), we get

$$[(1 + \gamma)(r^2 + a^2 + b^2t^2)^{n+1} - 6\xi_0 mb^2 t]\Lambda =$$

$$24\xi_0 m^2 b^2 t \times \left[\frac{3mb^4t^2}{(r^2 + a^2 + b^2t^2)^{2n+2}} - \frac{3(a^2 + b^2t^2) + (m+1)r^2}{(r^2 + a^2 + b^2t^2)^{2m+2}}\right]$$

$$-\frac{4mb^2\left[r^2 + a^2 + \{3(1+\gamma)m - 2n - 1\}b^2t^2\right]}{(r^2 + a^2 + b^2t^2)^{n+1}} +$$

$$\frac{4\left[\{(1+3\gamma)m + n\}(a^2 + b^2t^2) + (n^2 - m^2 + 1)r^2\right]}{(r^2 + a^2 + b^2t^2)^{2m-n+1}}. \tag{31}$$

From Eqs. (29) and (31), we observe that the Λ is a decreasing function of time (see Fig. 4), and it approaches a small positive value which is similar to the previously discussed in Model 1 (Sec. 2.1).
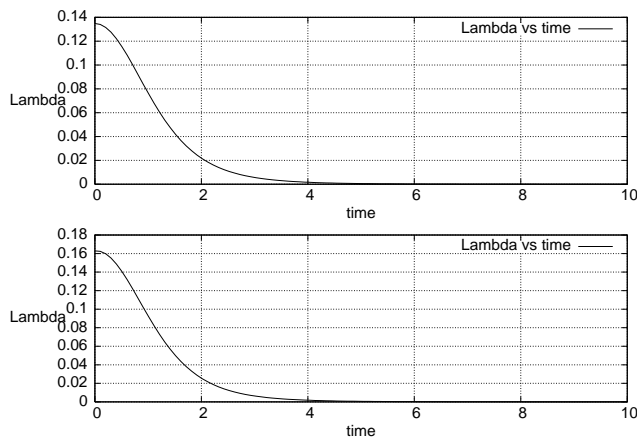

Fig. 4. *Variation of Λ with time for 3.1 Model 1, case I (lower panel) and II (upper panel). Here* $m = 1$, $n = 1 + \sqrt{17}/2$, $\gamma = 0.5$, $r = 1$, $a = 2$, $b = 1$ *and* $\xi_0 = 1$.

3.2. Model 2

We set $P(t) = a^2 + e^{-bt^2}$, $a^2 > b^2$. In this case we consider two following cases.

3.2.1. Case I : solution for $\xi = \xi_0$

When $k = 0$, Eq. (26) reduces to $\xi = \xi_0$ (constant) and hence Eq. (27) with the help of (9) and (15) reduces to the form

$$(1+\gamma)\rho = -\frac{6mb\xi_0 t e^{-bt^2}}{(r^2 + a^2 + e^{-bt^2})^{n+1}} +$$

$$\frac{4mbe^{-bt^2}\left[(2nbt^2+1)e^{-bt^2} - (r^2+a^2)(2bt^2-1)\right]}{(r^2+a^2+e^{-bt^2})^{2n+2}}$$

$$-\frac{4\left[(2m-n)(a^2+e^{-bt^2}) + \{m(m+1)-n^2\}r^2\right]}{(r^2+a^2+e^{-bt^2})^{2m+2}}. \tag{32}$$

Eliminating $\rho(t)$ between Eqs. (15) and (32), we get

$$(1+\gamma)\Lambda = -\frac{6mb\xi_0 t e^{-bt^2}}{(r^2+a^2+e^{-bt^2})^{n+1}}$$

$$-\frac{4mbe^{-bt^2}\left[\{3(1+\gamma)m-2n\}bt^2e^{-bt^2}+(r^2+a^2)(2bt^2-1)-e^{-bt^2}\right]}{(r^2+a^2+e^{-bt^2})^{2n+2}}$$

$$+\frac{4\left[\{(1+3\gamma)m+n\}(a^2+e^{-bt^2})+\{m(m+1)\gamma+n^2\}r^2\right]}{(r^2+a^2+e^{-bt^2})^{2m+2}}. \tag{33}$$

3.2.2. Case II : solution for $\xi = \xi_0\rho$

When $k = 1$, Eq. (26) reduces to $\xi = \xi_0\rho$ and hence Eq. (27) takes the form

$$\left[\gamma-\frac{6mb\xi_0e^{-bt^2}}{(r^2+a^2+e^{-bt^2})^{n+1}}\right]\rho =$$

$$-\frac{4mbe^{-bt^2}\left[(r^2+a^2)(2bt^2-1)-e^{-bt^2}+(3m-2n)bt^2e^{-bt^2}\right]}{(r^2+a^2+e^{-bt^2})^{2n+2}}$$

$$+\frac{4\left[(m+n)(a^2+e^{-bt^2})+n^2r^2\right]}{(r^2+a^2+e^{-bt^2})^{2m+2}}. \tag{34}$$

Eliminating $\rho(t)$ between Eqs. (34) and (15), we get

$$[(1+\gamma)(r^2+a^2+e^{-bt^2})^{n+1}-6mb\xi_0e^{-bt^2}]\Lambda =$$

$$24m^2b\xi_0e^{-bt^2}\left[\frac{3mb^2t^2e^{-2bt^2}}{(r^2+a^2+e^{-bt^2})^{2n+2}}-\frac{3(a^2+e^{-bt^2})+(m+1)r^2}{(r^2+a^2+e^{-bt^2})^{2m+2}}\right]$$

$$-\frac{4mbe^{-bt^2}\left[\{3(1+\gamma)m-2n\}bt^2e^{-bt^2}+(r^2+a^2)(2bt^2-1)-e^{-bt^2}\right]}{(r^2+a^2+e^{-bt^2})^{n+1}}$$

$$+\frac{4\left[\{(1+3\gamma)m+n\}(a^2+e^{-bt^2})+\{m(m+1)\gamma+n^2\}r^2\right]}{(r^2+a^2+e^{-bt^2})^{2m-n+1}}. \tag{35}$$

From Eq. (33), we observe that the value of Λ first increases slowly and suddenly reaches to peak, then it has sharp decrease to a negative value, again, it has a slow increment and finally becomes a small positive constant value (see Fig. 5 upper panel). From Eq. (35) we observe that Λ first decreases and then increases and finally approaches a small positive constant. This could play the role of dark energy.
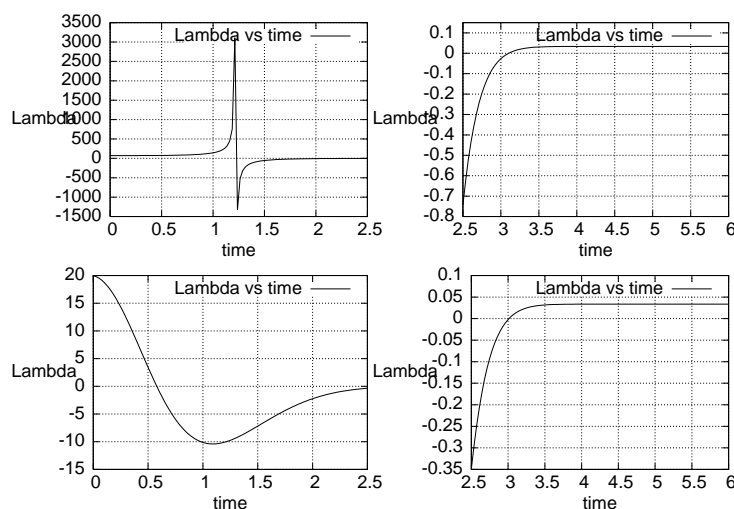
Fig. 5. Variation of Λ with time for 3.2 Model 2, cases I (lower panel) and II (upper panel). Here m = 1, n = $1 - \sqrt{17}/2$, γ = 0.5, r = 1, a = 2, b = 1 *and* ξ_0 = 1.

3.3. Model 3

We set $P(t) = a^2 + b^2 \cos \omega t$, $a^2 > b^2$. In this case we consider two following cases.

3.3.1. Case I : solution for $\xi = \xi_0$

When $k = 0$, Eq. (26) reduces to $\xi = \xi_0$(constant) and hence Eq. (27), with the help of (9) and (20), reduces to the form

$$(1 + \gamma)\rho = \frac{3\omega b^2 \xi_0 \sin \omega t}{(r^2 + a^2 + b^2 \cos \omega t)^{n+1}} +$$

$$\frac{mb^2\omega^2 \left[\{3m(b^2 - 1) + 2(n + 1)\} \sin^2 \omega t + 2(r^2 + a^2 + b^2 \cos \omega t) \cos \omega t\right]}{(r^2 + a^2 + b^2 \cos \omega t)^{2n+2}}$$

$$-\frac{4[(a^2 + b^2 \cos \omega t)(2m - n) - n^2 r^2]}{(r^2 + a^2 + b^2 \cos \omega t)^{2m+2}}. \tag{36}$$

Eliminating $\rho(t)$ between Eq. (20) and (36), we get

$$(1 + \gamma)\Lambda = \frac{3\omega b^2 \xi_0 \sin \omega t}{(r^2 + a^2 + b^2 \cos \omega t)^{n+1}}$$

$$-\frac{mb^2\omega^2 \left[\{3(1 + b^2\gamma)m - 2n\} \sin^2 \omega t - 2(r^2 + a^2 + b^2 \cos \omega t) \cos \omega t\right]}{(r^2 + a^2 + b^2 \cos \omega t)^{2n+2}}$$

$$+\frac{4\left[\{(1+3\gamma)m+n\}(a^2+b^2\cos\omega t)+\{m(m+1)\gamma+n^2\}r^2\right]}{(r^2+a^2+b^2\cos\omega t)^{2m+2}}. \tag{37}$$
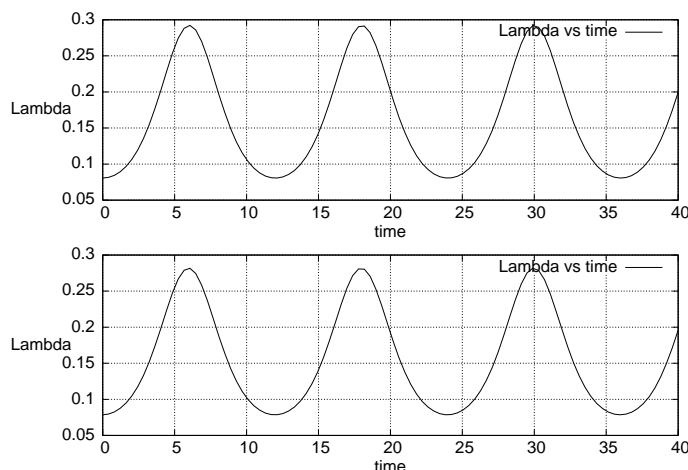

Fig. 6. Variation of Λ with time for 3.3 Model 3, case I (lower panel) and II (upper panel). Here $\text{m}=1$, $\text{n}=1+\sqrt{17}/2$, $\gamma=0.5$, $\text{r}=1$, $\text{a}=2$, $\text{b}=1$, $\xi_0=1$, $\omega=\pi/6$.

3.3.2. Case II : solution for $\xi=\xi_0\rho$

When $k=1$, Eq. (26) reduces to $\xi=\xi_0\rho$ and hence Eq. (27) takes the form

$$\left[\gamma-\frac{3\xi_0 b^2\omega\sin\omega t}{(r^2+a^2+b^2\cos\omega t)^{n+1}}\right]\rho=$$

$$\frac{mb^2\omega^2\left[2(r^2+a^2+b^2\cos\omega t)\cos\omega t-(3m-2n-2)\sin^2\omega t\right]}{(r^2+a^2+b^2\cos\omega t)^{2n+2}}$$

$$+\frac{4[(m+n)(a^2+b^2\cos\omega t)+n^2 r^2]}{(r^2+a^2+b^2\cos\omega t)^{2m+2}}. \tag{38}$$

Eliminating $\rho(t)$ between Eqs. (37) and (20), we obtain

$$\left[(1+\gamma)(r^2+a^2+b^2\cos\omega t)^{n+1}+3\xi_0 b^2\omega\sin\omega t\right]\Lambda=$$

$$+3\xi_0 b^2 m\omega\sin\omega t\left[\frac{3mb^4\omega^2\sin^2\omega t}{(r^2+a^2+b^2\cos\omega t)^{2n+2}}-\frac{4\{3(a^2+b^2\cos\omega t)+(m+1)r^2\}}{(r^2+a^2+b^2\cos\omega t)^{2m+2}}\right]$$

$$+\frac{mb^2\omega^2\left[2(r^2+a^2+b^2\cos\omega t)\cos\omega t-\{3m(1-\gamma b^2)-2(n+1)\}\sin^2\omega t\right]}{(r^2+a^2+b^2\cos\omega t)^{n+1}}$$

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$$+ \frac{4\left[\{(1+3\gamma)m+n\}(a^2+b^2\cos\omega t)+(m+n^2+1)r^2\right]}{(r^2+a^2+b^2\cos\omega t)^{2m-n+1}}. \tag{39}$$

From Eqs. (37) and (39), we observe that the Λ oscillates with time due to the properties of sinusoidal functions, present in these equations. The behaviour of cosmological term Λ may be observed from Fig. 6. The nature of these models are same as already discussed in Model 3 (Sec. 2.3).

4. Conclusions

We have obtained a new class of spherically-symmetric inhomogeneous cosmological models with a perfect fluid and also a bulk viscous fluid as the source of matter with a radial heat flux without a big bang or any other singularity. These are the shear-free nonsingular models. They are inhomogeneous and hence accelerating but not shearing. It is the heat flux that combines with pressure gradient to avoid singularity. From the point of view of realistic cosmology, these models share with the standard FRW model, spherical symmetry and the absence of shear.

The cosmological constant is a parameter describing the energy density of the vacuum (empty space), and a potentially important contribution to the dynamical history of the universe. The physical interpretation of the cosmological constant as vacuum energy is supported by the existence of the "zero point" energy predicted by quantum mechanics. In quantum mechanics, particle and antiparticle pairs are consistently being created out of the vacuum. Even though these particles exist for only a short amount of time before annihilating each other, they do give the vacuum a non-zero potential energy. In general relativity, all forms of energy should gravitate, including the energy of vacuum, hence the cosmological constant, too. A negative cosmological constant adds to the attractive gravity of matter, therefore universes with a negative cosmological constant are invariably doomed to re-collapse [69]. A positive cosmological constant resists the attractive gravity of matter due to its negative pressure. For most universes, the positive cosmological constant eventually dominates over the attraction of matter and drives the universe to expand exponentially [70].

The cosmological constants in all models given in Sections 2.1 and 3.1 are decreasing functions of time and they all approach a small and positive value at late times which are supported by the results from type Ia supernova observations recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [21], Perlmutter et al. [22], Riess et al. [23], Schmidt et al. [24]). Thus, with our approach, we obtain a physically relevant decay law for the cosmological term unlike other investigators where *adhoc* laws were used to arrive at mathematical expressions for the decaying vacuum energy. Our derived models provide a good agreement with the observational results. We have derived value for the cosmological constant Λ and attempted to formulate a physical interpretation for it.

This paper adds a novel family of shear-free models to Senovilla's first model

[3], a large family of cylindrical nonsingular models [5, 7, 8, 10] and a large family of spherical nonsingular models [11, 12, 64], avoiding cosmic singularity in the absence of shear.

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References

[1] R. Penrose, Phys. Rev. Lett. **14** (1965) 57.

[2] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-time*, Cambridge University Press, Cambridge (1973).

[3] J. M. M. Senovilla, Phys. Rev. Lett. **64** (1990) 2219.

[4] F. J. Chinea, L. Fernández-Jambrina and J. M. M. Senovilla, Phys. Rev. D **45** (1990) 481.

[5] E. Ruiz and J. M. M. Senovilla, Phys. Rev. D **45** (1992) 1995.

[6] J. M. M. Senovilla, *El Escorial Summer School on Gravitation and Relativity* (1992); Rotating Objects and Other Topics ed F. J. Chinea and L. M. González-Romero, Berlin, Springer (1993).

[7] J. M. M. Senovilla, Phys. Rev. D **53** (1996) 1799.

[8] N. Dadhich, L. K. Patel and R. Tikekar, Pramana - J. Phys. **44** (1995) 303.

[9] L. Fernández-Jambrina, Class. Quantum Grav. **14** (1997) 3407.

[10] J. M. M. Senovilla, Gen. Rel. Grav. **30** (1998) 701.

[11] N. Dadhich, J. Astrophys. Astr. **18** (1997) 343.

[12] N. Dadhich, L. K. Patel and R. Tikekar, Math Today **17** (1999) 17.

[13] N. Dadhich and A. K. Raychaudhuri, Mod. Phys. Lett. A **14** (1999) 2135.

[14] F. Hoyle, G. Burbidge and J. B. Narlikar, Not. R. Astron. Soc. **267** (1994) 1007.

[15] R. Tikekar, Pramana-J. Phys. **55** (2000) 623.

[16] J. M. Overduin, The Astrophys. J. **517** (1999) L1.

[17] K. A. Bronnikov, A. Dobosz and I.G. Dymnikova, Class. Quantum Grav. **20** (2003) 3797.

[18] L. Fernández-Jambrina and L. M. González-Romero, Int. J. Mod. Phys. D **7** (2004) 1; J. Math. Phys. **45** (2004) 2113.

[19] A. Pradhan, K. Srivastava and R. P. Singh, Fizika B (Zagreb) **15** (2006) 141.

[20] J. B. Pitts and W. C. Schieve, Found. Phys. **33** (2003) 1315, gr-qc/0406103.

[21] P. M. Garnavich et al. [Hi-z Supernova Team Collaboration], Astrophys. J. **493** (1998a) L53, astro-ph/9710123; Astrophys. J. **509** (1998b) 74, astro-ph/9806396.

[22] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. **483** (1997) 565, astro-ph/9608192; Nature **391** (1998) 51 astro-ph/9712212; Astrophys. J. **517** (1999) 565, astro-ph/9608192.

[23] A. G. Riess et al. [Hi-z Supernova Team Collaboration], Astron. J. **116** (1998) 1009 astro-ph/9805201.

[24] B. P. Schmidt et al. [Hi-z Supernova Team Collaboration], Astrophys. J. **507** (1998) 46 astro-ph/9805200.

[25] C. H. Lineweaver, astro-ph/9810334 (1998).

[26] O. Lahav and S.L. Bridle, astro-ph/9810169 (1998).

[27] M. Chiba and Y. Yoshii, astro-ph/9808321 (1998).

[28] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.

[29] P. G. Bergmann, Int. J. Theor. Phys. **1** (1968) 25.

[30] R. V. Wagoner, Phys. Rev. D **1** (1970) 3209.

[31] A. D. Linde, JETP Lett. **19** (1974) 183.

[32] D. Kazanas, ApJ **241** (1980) L59.

[33] A. D. Dolgov, in *The Very Early Universe*, eds. G. W. Gibbons, S. W. Hawking and S. T. C. Siklos, Cambridge University Press (1983) 449.

[34] L. F. Abbott, Phys. Lett. B **150** (1985) 427.

[35] S. M. Barr, Phys. Rev. D **36** (1987) 1691.

[36] P. J. E. Peebles and B. Ratra, ApJ **325** (1988) L17.

[37] J. A. Frieman, C. T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75** (1995) 2077.

[38] J. W. Moffat, Phys. Lett. B **357** (1995) 526.

[39] A. A. Starobinsky, LETP Lett. **68** (1998) 757, astro-ph/9810431.

[40] S. W. Hawking, Phys. Lett. B **134** (1984) 403.

[41] A. D. Dolgov, Phys. Rev. D **55** (1997) 5881.

[42] J. Ellis, N. E. Mavromatos and D. V. Nanopoulos, Gen. Rel. Grav. **32** (2000) 943, gr-qc/9810086.

[43] T. Banks, Nucl. Phys. B **309** (1988) 493.

[44] A. D. Linde, Phys. Lett. B **200** (1988) 272.

[45] S. Coleman, Nucl. Phys. B **310** (1988) 643.

[46] R. H. Brandenberger and A. R. Zhitnitsky, Phys. Rev. D **55** (1997) 4640.

[47] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D **59** (1999), 063505; astro-ph/9810509.

[48] L. R. W. Abramo, R. H. Brandenberger and V. F. Mukhanov, Phys. Rev. D **56** (1997) 4640.

[49] Ya. B. Zeldovich, Sov. Phys. - Uspekhi **11** (1968) 381.

[50] S. Weinberg, *Gravitation and Cosmology*, Wiley, New York (1972).

[51] A. D. Dolgov, M. V. Sazhin and Ya. B. Zeldovich, *Basics of Modern Cosmology*, Editions Frontiers (1990).

[52] O. Bertolami, Nuovo Cimento B **93** (1986) 36; Fortschr. Phys. **34** (1986) 829.

[53] J. E. Felten and R. Isaacman, Rev. Mod. Phys. **58** (1986) 689.

[54] J. C. Charlton and M. S. Turner, Astrophys. J. **313** (1987) 495.

[55] A. Sandage, Ann. Rev. Astron. Astrophys. **26** (1988) 561.

[56] S. M. Carroll, W. H. Press and E. L. Turner, Ann. Rev. Astron. Astrophys. **30** (1992) 499.

[57] A. D. Dolgov and J. Silk, Phys. Rev. D **47** (1993) 3144; **48** (1993) 2499.

[58] V. Sahni and A. Starobinsky, Int. J. Mod. Phys. D **9** (2000) 373, gr-qc/9904398.

[59] P. J. E. Peebles, Rev. Mod. Phys. **75** (2003) 559, astro-ph/0207347.

[60] T. Padmanabhan, Phys. Rep. **380** (2003) 235, hep-th/0212290.

[61] S. M. Carroll, Living Rev. Rel. **4** (2001) 1, astro-ph/00004075; astro-ph/0107571.

[62] R. G. Vishwakarma and Abdussattar, Phys. Rev. D **60** (1996) 063507.
R. G. Vishwakarma, Class. Quant. Grav. **17** (2000) 3833; **18** (2001) 1159; Gen. Rel. Grav. **33** (2001) 1973; Mon. Not. R. Astron. Soc. **331** (2002) 776 ; Class. Quant. Grav. **19** (2002) 4747.

[63] A. Pradhan, V. K. Yadav and N. N. Saste, Int. J. Mod. Phys. D **11** (2002) 857.
A. Pradhan, V. K. Yadav and I. Chakrabarty, Int. J. Mod. Phys. D **10** (2001) 339.
A. Pradhan and I. Aotemshi, Int. J. Mod. Phys. D **11** (2002) 1419.
A. Pradhan and H. R. Pandey, Int. J. Mod. Phys. D **12** (2003) 941 (gr-qc/0211002).
A. Pradhan and O. P. Pandey, Int. J. Mod. Phys. D **12** (2003) 1299.
A. Pradhan, S. K. Srivastav and K. R. Jotania, Czech. J. Phys. **54** (2004) 255.
A. Pradhan and S. K. Singh, Int. J. Mod. Phys. D **13** (2004) 503.
A. Pradhan and P. Pandey, Czech. J. Phys. **55** (2005) 749.
A. Pradhan, A. k. Yadav and L. Yadav, Czech. J. Phys. **55** (2005) 503.
A. Pradhan and P. Pandey, Astrophys. Space Sci. **301** (2006) 221.
A. Pradhan, G. P. Singh and S. Kotambkar, Fizika B (Zagreb) **15** (2006) 23.
C. P. Singh, S. Kumar and A. Pradhan, Class. Quant. Grav. **24** (2007) 455.
A. Pradhan, A. K. Singh and S. Otarod, Romanian J. Phys. **52** (2007) 445.

[64] N. Dadhich and L. K. Patel, Gravitation & Cosmology **6** (2000) 11.

[65] D. Pavon, J. Bafaluy and D. Jou, Class. Quant. Grav. **8** (1991) 357; in *Proc. Hanno Rund Conf. on Relativity and Thermodynamics* Ed. S. D. Maharaj, University of Natal, Durban (1996) 21.

[66] R. Maartens, Class. Quant. Grav. **12** (1995) 1455.

[67] W. Zimdahl, Phys, Rev. D **53** (1996) 5483.

[68] N. O. santos, R. S. Dias and A. Banerjee, J. Math. Phys. **26** (1985) 878.

[69] S. M. Carroll, W. H. Press and E. L. Turner, ARA&A **30** (1992) 499.

[70] C. S. Kochanek, ApJ **384** (1992) 1.

NESINGULARNI KOZMOLOŠKI MODELI S PROMJENLJIVIM KOZMOLOŠKIM ČLANOM Λ

Ponovno razmatramo egzaktna rješenja Einsteinovih jednadžbi polja za sferno simetrični kozmološki model bez velikog praska i drugih singulariteta, koji su nedavno razvili Dadhich i Patel (2000). Sadržaj tvari u modelu je perfektna tekućina bez posmika s izotropnim tlakom i radijalnim tokom topline. Izveli smo tri egzaktna rješenja za perfektnu tekućinu i tekućinu s volumnom viskoznošću. Dobivamo kozmološki član $\Lambda(t)$ koji opada s vremenom, što je u skladu s novim opažanjima supernova tipa Ia.