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BULK VISCOUS COSMOLOGICAL MODELS IN GENERAL RELATIVITY

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A new class of exact solutions of Einstein's field equations with bulk viscous fluid for an locally rotationally symmetric Bianchi type-I spacetime is obtained by using a variable deceleration parameter. We have described three cases, depending on the different forms of deceleration parameter, in which six models of the universe are obtained. The value of Hubble's constant H_0 is found to be less than unity for these models which are of the physical interest. Some physical and geometric properties of the models are also discussed.

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1. Introduction

The Bianchi cosmologies play an important role in theoretical cosmology and have been much studied since the 1960s. A Bianchi cosmology represents a spatially homogeneous universe, since by definition the spacetime admits a three-parameter group of isometries whose orbits are space-like hyper-surfaces. These models can be used to analyze aspects of the physical Universe which pertain to or which may be affected by anisotropy in the rate of expansion, for example, the cosmic microwave background radiation, nucleosynthesis in the early universe, and the question of the isotropization of the universe itself [1]. For simplification and description of the large scale behaviour of the actual universe, locally rotationally symmetric [henceforth

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referred as LRS] Bianchi type-I spacetime, have been widely studied [2]−[6]. When the Bianchi type-I spacetime expands equally in two spatial directions it is called locally rotationally symmetric spacetime. These kinds of models are interesting because Lidsey [7] showed that they are equivalent to a flat (FRW) universe with a self-interacting scalar field and a free massless scalar field, but produced no explicit example. Some explicit solutions were pointed out in Refs. [8] and [9].

The Einstein's field equations are a coupled system of highly non-linear differential equations and we seek physical solutions to the field equations for their applications in cosmology and astrophysics. In order to solve the field equations, we normally assume a form for the matter content or that spacetime admits killing vector symmetries [10]. Solutions to the field equations may also be generated by applying a law of variation for Hubble's parameter which was proposed by Berman [11]. In simple cases, the Hubble law yields a constant value of the deceleration parameter. It is worth observing that most of the well-known models of Einstein's theory and Brans-Dicke theory with curvature parameter $k = 0$, including inflationary models, are models with a constant deceleration parameter. In earlier literature, cosmological models with a constant deceleration parameter have been studied by Berman [11], Berman and Gomide [12], Johri and Desikan [13], Singh and Desikan [14], Maharaj and Naidoo [15], Pradhan et al. [16] and others. But redshift magnitude test has had a chequered history. During the 1960s and the 1970s, it was used to draw very categorical conclusions. The deceleration parameter q_0 was then claimed to lie between 0 and 1 and thus it was claimed that the Universe is decelerating. Recent observations [17, 18] of Type Ia Supernovae (SNe) allow to probe the expansion history of the Universe. The main conclusion of these observations is that the expansion of the Universe is accelerating. So we can consider the cosmological models with variable deceleration parameter. The readers are advised to see the papers by Vishwakarma and Narlikar [19] and Virey et al. [20] and references therein for a review on the determination of the deceleration parameter from the Supernovae data.

Most cosmological models assume that the matter in the Universe can be described by 'dust' (a pressure-less distribution) or at best a perfect fluid. The role of the bulk viscosity in the cosmic evolution, especially at its early stages, seems to be significant. The general criterion for bulk viscosity was given by Israel and Vardalas [21], Klimek [22] and Weinberg [23]. For example, the existence of the bulk viscosity is equivalent to a slow process of restoring equilibrium states (Landau and Lifshitz [24]). The presently observed high entropy per baryon in the Universe can be explained by involving some kind of dissipative mechanism (e. g., bulk viscosity). Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [25] for a review on cosmological models with bulk viscosity). The model studied by Murphy [26] possessed an interesting feature in that the big bang type of singularity of infinite spacetime curvature does not occur to be at finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors within the framework of general theory of relativity (Padmanabhan and Chitre [27], Pavon [28], Johri and Sudarshan [29], Maartens [30], Zimdahl [31], Santos et al. [32], Pradhan, Sarayakar and Beesham [33], Kalyani and Singh [34], Singh, Beesham and Mbokazi [35], Pradhan et al. [36]), Singh et al. [37], Bali and Pradhan [38]). This motivates the study cosmological bulk viscous fluid model.

Recently, Paul [39] has investigated LRS Bianchi type-I cosmological models with a variable deceleration parameter. In this paper, we propose to find LRS Bianchi type-I cosmological models in the presence of a bulk viscous fluid and we will generalize the solutions [39].

2. The metric and field equations

We consider the LRS Bianchi type-I metric in the form [5]

$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2 + dz^2)\,, \tag{1}$$

where A and B are functions of x and t. The stress energy-tensor in the presence of bulk stress has the form

$$T_{ij} = (\rho + \bar{p}) u_i u_j - \bar{p} g_{ij}\,, \tag{2}$$

where

$$\bar{p} = p - \xi u^i_{;i}\,. \tag{3}$$

Here ρ, p, $\bar{p}$ and ξ are the energy density, thermodynamical pressure, effective pressure and bulk viscous coefficient, respectively, and u_i is the four-velocity vector satisfying the relations

$$u_i u^i = 1\,. \tag{4}$$

The Einstein's field equations (in gravitational units $c = 1$, $G = 1$) read as

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi T_{ij}\,, \tag{5}$$

where R_{ij} is the Ricci tensor and $R = g^{ij} R_{ij}$ is the Ricci scalar. The Einstein's field equations (5) for the line element (1) has been set up as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{B'^2}{A^2 B^2} = -8\pi \bar{p}\,, \tag{6}$$

$$\dot{B}' - \frac{B' \dot{A}}{A} = 0\,, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{B''}{A^2 B} + \frac{A'B'}{A^3 B} = -8\pi\bar{p}\,, \tag{8}$$

$$\frac{2B''}{A^2 B} - \frac{2A'B'}{A^3 B} + \frac{B'^2}{A^2 B^2} - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = -8\pi\rho\,. \tag{9}$$

The energy conservation equation yields

$$\dot{\rho} + (\bar{p} + \rho)\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0\,, \tag{10}$$

where dots and primes indicate partial differentiation with respect to t and x, respectively.

3. Solution of the field equations

Equation (7), after integration, yields

$$A = \frac{B'}{\ell}\,, \tag{11}$$

where ℓ is an arbitrary function of x. Equations (6) and (8), with the use of Eq. (11), reduce to

$$\frac{B}{B'}\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\ddot{B}}{B}\right) + \frac{\dot{B}}{B'}\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{B'}{B}\right) + \frac{\ell^2}{B^2}\left(1 - \frac{B}{B'}\frac{\ell'}{\ell}\right) = 0\,. \tag{12}$$

Since A and B are explicit functions of x and t, so $\frac{B'}{B}$ is a function of x alone. Hence, after integrating Eq. (12) gives

$$B = \ell S(t)\,, \tag{13}$$

where S is a scale factor which is an arbitrary function of t. Thus from Eqs. (11) and (13), we have

$$A = \frac{\ell'}{\ell} S\,. \tag{14}$$

Now the metric (1) is reduced to the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - S^2\left[\mathrm{d}X^2 + e^{2X}(\mathrm{d}y^2 + \mathrm{d}z^2)\right], \tag{15}$$

where $X = \ln\,\ell$. The mass-density, effective pressure and Ricci scalar are obtained as

$$8\pi\rho = \frac{3}{S^2}\left[\dot{S}^2 - 1\right], \tag{16}$$

$$8\pi\bar{p} = \frac{1}{S^2}\left[1 - \dot{S}^2 - 2S\ddot{S}\right], \tag{17}$$

$$R = \frac{6}{S^2}\left[S\ddot{S} + \dot{S}^2 - 1\right]. \tag{18}$$

The function $S(t)$ remains undetermined. To obtain its explicit dependence on t, one may have to introduce additional assumption. To achieve this, we assume the deceleration parameter to be variable, i.e.

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{\dot{H} + H^2}{H^2}\right) = b\ (\text{variable}), \tag{19}$$

where $H = \frac{\dot{S}}{S}$ is the Hubble parameter. The above equation may be rewritten as

$$\frac{\ddot{S}}{S} + b\frac{\dot{S}^2}{S^2} = 0. \tag{20}$$

The general solution of Eq. (20) is given by

$$\int e^{\int (b/S)\mathrm{d}S}\mathrm{d}S = t + m, \tag{21}$$

where m is an integrating constant.

In order to solve the problem completely, we have to choose $\int(b/S)\mathrm{d}S$ in such a manner that Eq. (21) be integrable.

Of course, the choice of b, in (21) is quite arbitrary but, since we are looking for physically viable models of the Universe consistent with observations, we consider the following three cases:

4. *Solution for $b = -aS/\dot{S}^2$, where a is constant*

In this case, on integrating, Eq. (20) gives the exact solution

$$S = \frac{1}{2}at^2 + kt + d, \tag{22}$$

where k and d are constants of integration.

In this case, the mass-density, pressure and Ricci scalar are given by

$$8\pi\rho = \frac{3}{S^2}\left[(k + at)^2 - 1\right], \tag{23}$$

$$8\pi(p-\xi\theta) = -\frac{1}{S^2}\left[(k+at)^2 - 1 + 2a(\frac{1}{2}at^2 + kt + d)\right], \tag{24}$$

$$R = \frac{6}{S^2}\left[(k+at)^2 - 1 + a(\frac{1}{2}at^2 + kt + d)\right]. \tag{25}$$

Here ξ, in general, is a function of time. The expression for kinematical parameter expansion θ is given by

$$\theta = \frac{3(k+at)}{S}. \tag{26}$$

Thus, given $\xi(t)$, we can solve the equations. In most of investigations involving bulk viscosity it is assumed to be a simple power function of the energy density [28] −[30]

$$\xi(t) = \xi_0 \rho^n, \tag{27}$$

where ξ_0 and n are constants. For small density, n may even be equal to unity as used in Murphy's work for simplicity [7]. If $n = 1$, Eq. (27) may correspond to a radiative fluid [23]. However, more realistic models [32] are based on n lying in the regime $0 \leq n \leq \frac{1}{2}$.

On thermodynamical grounds, in conventional physics, ξ has to be positive; this being a consequence of the positive entropy change in irreversible processes. For simplicity and realistic models of physical importance, we consider the following two cases depending on ($n = 0, 1$):

4.1. Model I: solution for $\xi = \xi_0$

When $n = 0$, Eq. (27) reduces to $\xi = \xi_0$ = constant. Hence in this case Eq. (24), with the use of (26), leads to

$$8\pi p = \frac{24\pi\xi_0}{S}(k+at) - \frac{1}{S^2}\left[(k+at)^2 - 1 + 2a(\frac{1}{2}at^2 + kt + d)\right]. \tag{28}$$

4.2. Model II: solution for $\xi = \xi_0\rho$

When $n = 1$, Eq. (27) reduces to $\xi = \xi_0\rho$. Hence in this case Eq. (24), with the use of (26), reduces to

$$8\pi p = \frac{\{(k+at)^2 - 1\}}{S^3}\left[9\xi_0(k+at) - S\right] - \frac{2a(\frac{1}{2}at^2 + kt + d)}{S^2}. \tag{29}$$

Physical behaviour of the models:

The effect of the bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. We also observe here that Murphy's condition [7] about the absence of a big-bang-type singularity in the finite past in models with bulk viscous fluid, in general, is not true.

From Eqs. (23) and (25), it is observed that

$$\rho > 0 \quad \text{and} \quad R > 0 \quad \text{for} \quad t > \frac{1-k}{a}\,, \tag{30}$$

where $k < 1$. From Eqs. (25), (27) and (28), it is also observed that ρ, R and θ decrease as t increases.

We find that shear $\sigma = 0$ in the models. Hence $\sigma/\theta = 0$, which shows that the models are isotropic. From Hubble's parameter equation, $H = \dot{S}/S$, we have an epoch time t_0 given by

$$t_0 = \frac{1}{H_0} - \frac{k}{a} + \frac{\sqrt{a^2 + {H_0}^2(k^2 - 2ad)}}{aH_0}\,, \tag{31}$$

which gives that

$$at_0 + k = \frac{a}{H_0} - 1 + \frac{\sqrt{a^2 + {H_0}^2(k^2 - 2ad)}}{H_0} > 0\,. \tag{32}$$

From above equation we conclude that

$$\frac{a}{H_0} - 1 > 0\,, \tag{33}$$

which reduces to

$$H_0 < a\,. \tag{34}$$

From Eq. (31), we observe that $t_0 > 0$ for $H_0 < a/k$ and $k^2 > 2ad$, i.e.

$$H_0 < \frac{k}{2d}\,. \tag{35}$$

From Eq. (27), $R > 0$ implies that

$$(k + at)^2 > \sqrt{[1 - a(\frac{1}{2}at^2 + kt + d)]^2}. \tag{36}$$

It is evident from Eqs. (30) and (36) that $a < 1$ and hence from (34) we obtain

$$H_0 < 1\,. \tag{37}$$

The models, in general, represent expanding, non-shearing and isotropic universe. The models in the presence of bulk viscosity start expanding with a big bang at $t = 0$ when $d = 0$ and the expansion in the models decreases as time increases and the expansion stops at $t = \infty$ and $t = -k/a$.

5. Solution for $b = -atS/\dot{S}^2$, where a is constant

In this case, on integrating, Eq. (20) gives the exact solution

$$S = \frac{1}{6}at^3 + kt + d\,, \tag{38}$$

where k and d are constants of integration.

In this case, the mass-density, pressure and Ricci scalar are given by

$$8\pi\rho = \frac{3}{S^2}\left[\left(k + \frac{1}{2}at^2\right)^2 - 1\right], \tag{39}$$

$$8\pi(p - \xi\theta) = -\frac{1}{S^2}\left[\left(k + \frac{1}{2}at^2\right)^2 - 1 + 2at\left(\frac{1}{6}at^3 + kt + d\right)\right], \tag{40}$$

$$R = \frac{6}{S^2}\left[\left(k + \frac{1}{2}at^2\right)^2 - 1 + at\left(\frac{1}{6}at^3 + kt + d\right)\right]. \tag{41}$$

The expression for the kinematical parameter expansion θ is given by

$$\theta = \frac{3}{S}\left(k + \frac{1}{2}at^2\right), \tag{42}$$

where S is given by (41).

For simplicity and realistic models of physical importance, we consider the following two subcases:

5.1. Model I: solution for $\xi = \xi_0$

When $n = 0$, Eq. (27) reduces to $\xi = \xi_0 =$ constant. Hence in this case Eq. (40), with the use of (42), leads to

$$8\pi p = \frac{24\pi\xi_0}{S}\left(k + \frac{1}{2}at^2\right) - \frac{1}{S^2}\left[\left(k + \frac{1}{2}at^2\right)^2 - 1 + 2at\left(\frac{1}{6}at^3 + kt + d\right)\right]. \tag{43}$$

5.2. *Model II: solution for* $\xi = \xi_0 \rho$

When $n = 1$, Eq. (27) reduces to $\xi = \xi_0 \rho$. Hence in this case Eq. (40), with the use of (42), reduces to

$$8\pi p = \frac{\{(k + \frac{1}{2}at^2)^2 - 1\}}{S^3} \left[9\xi_0 (k + \frac{1}{2}at^2) - S\right] - \frac{2a(\frac{1}{6}at^3 + kt + d)}{S^2} . \tag{44}$$

Physical behaviour of the models:

From Eqs. (39) and (41), it is observed that $\rho > 0$ and $R > 0$ for $t > \sqrt{(2/a)(1-k)}$, where $k < 1$. From (42), we observe that θ decreases as t increases.

We find that shear $\sigma = 0$ in the models. Hence $\sigma/\theta = 0$ which shows that the models are isotropic. At any intermediate time $t = \sqrt{(2/a)(2-k)}$, $R > 0$ implies that

$$3 + \frac{4}{3}(2-k)(1+k) + d\sqrt{2a(2-k)} > 0 \, , \tag{45}$$

where $k < 2$. From Eq. (45), it is evident that

$$d\sqrt{2a(2-k)} > 0 \, . \tag{46}$$

From Hubble's parameter, $H = \dot{S}/S$, we obtain a cubic equation in t_0

$${t_0}^3 - \frac{3{t_0}^2}{H_0} + \frac{6kt_0}{a} - \frac{6}{aH_0}(k - dH_0) = 0 \, . \tag{47}$$

Solving Eq. (47), we obtain

$$t_0 = \frac{1}{H_0}, \quad a = 2k{H_0}^2 \;\; \text{and} \;\; d = \frac{a}{3{H_0}^3} \, , \tag{48}$$

where t_0 is an epoch time. Thus from Eq. (46)

$$4{H_0}^2 k(2-k) > 0 \, . \tag{49}$$

Hence Eq. (49), for $k < 2$, implies that

$$H_0 > 0 \, . \tag{50}$$

The models, in general, represent expanding, non-shearing and isotropic universe. The models in the presence of bulk viscosity start expanding with a big bang at $t = 0$ when $d = 0$ and the expansion in the models decreases as time increases and the expansion stops at $t = \infty$ and $t^2 = -2k/a$.

6. Solution for $b = -KS/\dot{S}^3$, where K is constant

In this case, on integrating, Eq. (20) gives the exact solution

$$S = \beta + \frac{(\alpha + 2Kt)^{3/2}}{3K}\,, \tag{51}$$

where α and β are constants of integration.

In this case, the mass-density, pressure and Ricci scalar are given by

$$8\pi\rho = \frac{3}{S^2}\left[(\alpha + 2Kt) - 1\right]\,, \tag{52}$$

$$8\pi(p - \xi\theta) = \frac{1}{S^2}\left[1 - (\alpha + 2Kt) - \frac{2K}{(\alpha + 2Kt)^{1/2}}\left\{\beta + \frac{(\alpha + 2Kt)^{\frac{3}{2}}}{3K}\right\}\right], \tag{53}$$

$$R = \frac{6}{S^2}\left[(\alpha + 2Kt) - 1 + \frac{K}{(\alpha + 2Kt)^{1/2}}\left\{\beta + \frac{(\alpha + 2Kt)^{\frac{3}{2}}}{3K}\right\}\right]. \tag{54}$$

The expansion θ is calculated as

$$\theta = \frac{3}{S}(\alpha + 2Kt)^{1/2}. \tag{55}$$

For simplicity and realistic models of physical importance, we consider the following two subcases:

6.1. Model I: solution for $\xi = \xi_0$

When $n = 0$, Eq. (27) reduces to $\xi = \xi_0 =$ constant. Hence in this case Eq. (53), with the use of (55), leads to

$$8\pi p = \frac{24\pi\xi_0}{S}(\alpha+2Kt)^{1/2} - \frac{1}{S^2}\left[\alpha+2Kt-1+\frac{2K}{(\alpha+2Kt)^{1/2}}\left\{\beta+\frac{(\alpha+2Kt)^{\frac{3}{2}}}{3K}\right\}\right]. \tag{56}$$

6.2. Model II: solution for $\xi = \xi_0\rho$

When $n = 1$, Eq. (27) reduces to $\xi = \xi_0\rho$. Hence in this case Eq. (53), with the use of (55), reduces to

$$8\pi p = \frac{(\alpha+2Kt-1)}{S^3}\left[9(\alpha+2Kt)^{1/2} - S\right] - \frac{2K}{S^2(\alpha+2Kt)^{1/2}}\left\{\beta+\frac{(\alpha+2Kt)^{\frac{3}{2}}}{3K}\right\}. \tag{57}$$

Physical behaviour of the models:

From Eq. (52), it is observed that $\rho > 0$ for $t > (1-\alpha)/2K$, where $\alpha < 1$. From (55), we observe that θ decreases as t increases.

We find that shear $\sigma = 0$ in the models. Hence $\sigma/\theta = 0$ which shows that the models are isotropic in nature.

From the Hubble's parameter, $H = \dot{S}/S$, we obtain

$$\chi_0{}^3 - 3K\left(\frac{\chi_0}{H_0} - \beta\right) = 0\,, \tag{58}$$

where $\chi_0 = \sqrt{\alpha + 2Kt_0}$ and t_0 is an epoch time. For the solution of Eq. (58) let us assume $\chi_0 = \beta$ and hence we have

$$\beta^2 = 3K\left(\frac{1}{H_0} - 1\right). \tag{59}$$

From Eq. (54), $R > 0$ implies that

$$4\chi^3 - 3\chi + 3\beta K > 0\,, \tag{60}$$

where $\chi = \sqrt{\alpha + 2Kt}$.

Solving Eq. (60) at any intermediate time, $t = (3-\alpha)/2K$, where $\alpha < 3$, we obtain

$$3 + K > 0\,. \tag{61}$$

From $\chi_0 = \sqrt{\alpha + 2Kt_0} = \beta$, we have

$$\alpha + 2K\,\frac{3-\alpha}{2K} = \beta^2,$$

which reduces to

$$\beta^2 = 3\,. \tag{62}$$

From Eqs. (59) and (62), we obtain

$$K = \frac{1}{(1/H_0) - 1}\,. \tag{63}$$

From Eqs. (61) and (63), we have

$$H_0 < 1.5\,. \tag{64}$$

From (63) in order that $K > 0$, $H_0 < 1$. Hence the value of Hubble's constant is less than unity.

7. *Conclusions*

In this paper we have described a new class of LRS Bianchi type-I cosmological models with a bulk viscous fluid as the source of matter by applying a variable deceleration parameter. Generally, the models are expanding, non-shearing and isotropic in nature. The models in the presence of bulk viscosity start expanding with a big bang at $t = 0$ when $d = 0$, the rate of expansion decreases as time increases and the expansion stops at $t = \infty$ and $t = -k/a$. The study of the results of the three deceleration parameter models of the universe showed that the Hubble's constant is less than 1.0. These mathematical results are of the physical interest. The scale factor is not obtained to be linearly related to the time as in the case of supernova cosmology model [40]. If $\xi = 0$ is set in the solutions obtained in this paper, we get the solutions obtained by Paul [39]. But in his paper [39], there are errors in the equations (15) and (16) which propagate throughout the paper and affect all results.

The coefficient of bulk viscosity is taken to be a power function of mass density. The effect of the bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. Murphy [26] has studied perfect fluid cosmological models with bulk viscosity and obtained that the big bang singularity may be avoided in the finite past. We also observe here that Murphy's condition [7] about the absence of a big bang type singularity in the finite past in models with bulk viscous fluid, in general, not true.

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References

[1] M. A. H. MacCallum, in *General Relativity: An Einstein Centenary Survey*, ed. S. W. Hawking and W. Israel, Cambridge University Press, Cambridge (1979).

[2] J. Hajj-Boutros and J. Sfeila, Int. J. Theor. Phys. **26** (1987) 98.

[3] Shri Ram, Int. J. Theor. Phys. **28** (1989) 917.

[4] A. Mazumder, Gen. Rel. Grav. **26** (1994) 307.

[5] A. Pradhan and A. K. Vishwakarma, SUJST **XII** Sec. B (2000) 42;
A. Pradhan, K. L. Tiwari and A. Beesham, Indian J. Pure Appl. Math. **32** (2001) 789;
I. Chakrabarty and A. Pradhan, Grav. & Cosm. **7** (2001) 55;
A. Pradhan and A. K. Vishwakarma, Int. J. Mod. Phys. D **8** (2002) 1195;
A. Pradhan and A. K. Vishwakarma, J. Geom. Phys. **49** (2004) 332.

[6] G. Mohanty, S. K. Sahu and P. K. Sahoo, Astrophys. Space Sci. **288** (2003) 523.

[7] J. E. Lidsey, Class. Quant. Grav. **9** (1992) 1239.

[8] J. M. Aguirregabiria, A. Feinstein and J. Ibanez, Phys. Rev. D **48** (1993) 4662.

[9] J. M. Aguirregabiria and L. P. Chimento, Class. Quant. Grav. **13** (1966) 3197.

[10] D. Kramer, H. Stephani, M. MacCallum and E. Hertt, *Exact Solutions of Einstein's Field Equations*, Cambridge University Press, Cambridge (1980).

[11] M. S. Berman, Nuovo Cimento B **74** (1983) 182.

[12] M. S. Berman and F. M. Gomide, Gen. Rel. Grav. **20** (1988) 191.

[13] V. B. Johri and K. Desikan, Pramana - J. Phys. **42** (1994) 473.

[14] G. P. Singh and K. Desikan, Pramana - J. Phys. **49** (1997) 205.

[15] S. D. Maharaj and R. Naidoo, Astrophys. Space Sci. **208** (1993) 261.

[16] A. Pradhan, V. K. Yadav and I. Chakrabarty, Int. J. Mod. Phys. D **10** (2001) 339. A. Pradhan and I. Aotemshi, Int. J. Mod. Phys. D **9** (2002) 1419.

[17] R. K. Knop et al., Astrophys. J. **598** (2003) 102.

[18] A. G. Riess et al., Astrophys. J. **607** (2004) 665.

[19] R. G. Vishwakarma and J. V. Narlikar, Int. J. Mod. Phys. D **14** (2005) 345.

[20] J. -M. Virey, P. Taxil, A. Tilquin, A. Ealet, C. Tao and D. Fouchez, Phys. Rev. D **72** (2005) 061302.

[21] W. Israel and J. N. Vardalas, Lett. Nouovo Cim. **4** (1970) 887.

[22] Z. Klimek, Post. Astron. **19** (1971) 165.

[23] S. Weinberg, Astrophys. J. **168** (1971) 175; *Graviatation and Cosmology*, Wiley, New York, p. 57 (1972).

[24] L. Landau and E. M. Lifshitz, *"Fluid Mechanics"*, Addison-Wesley, Mass., p. 304, (1962).

[25] Ø. Grøn, Astrophys. Space Sci. **173** (1990) 213.

[26] G. L. Murphy, Phys. Rev. D **8** (1973) 4231.

[27] T. Padmanabhan and S. M. Chitre, Phys. Lett. A **120** (1987) 433.

[28] D. Pavon, J. Bafaluy and D. Jou, Class. Quant. Grav. **8** (1991) 347; in *Proc. Hanno Rund Conf. on Relativity and Thermodynamics*, ed. S. D. Maharaj, University of Natal, Durban, p. 21 (1966).

[29] V. B. Johri and R. Sudarshan, Phys. Lett. A **132** (1988) 316.

[30] R. Maartens, Class. Quant. Grav. **12** (1995) 1455.

[31] W. Zimdahl, Phys. Rev. D **53** (1996) 5483.

[32] N. O. Santos, R. S. Dias and A. Banerjee, J. Math. Phys. **26** (1985) 878.

[33] A. Pradhan, R. V. Sarayakar and A. Beesham, Astr. Lett. Commun. **35** (1997) 283.

[34] D. Kalyani and G. P. Singh, *in New Direction in Relativity and Cosmology*, eds. V. de Sabbata and T. Singh, Hadronic Press, U. S. A., p. 41 (1997).

[35] T. Singh, A. Beesham and W. S. Mbokazi, Gen. Rel. Grav. **30** (1998) 537.

[36] A. Pradhan, V. K. Yadav and N. N. Saste, Int. J. Mod. Phys. D **11** (2002) 857.
A. Pradhan, S. K. Srivastav and K. R. Jotania, Czech. J. Phys. **54** (2004) 255.
A. Pradhan, L. Yadav and A. K. Yadav, Czech. J. Phys. **54** (2004) 487.
A. Pradhan and A. Rai, Astrophys. Space Sci. **291** (2004) 49.
A. Pradhan and P. Pandey, Czech. J. Phys. **55** (2005) 749.
A. Pradhan, S. K. Srivastav and M. K. Yadav, Astrophys. Space Sci. **298** (2005) 419.
A. Pradhan, L. Yadav and A. K. Yadav, Astrophys. Space Sci. **299** (2005) 31.
A. Pradhan, V. Rai and S. Otarod, Fizika B, **15** (2006) 57.
A. Pradhan, P. K. Singh, and K. R. Jotania, Czech. J. Phys. **56** (2006) 23.

[37] C. P. Singh, S. Kumar and A. Pradhan, Class. Quantum Grav. **24** (2007) 455.

[38] R. Bali and A. Pradhan, Chin. Phys. Lett. **24** (2007) 585.

[39] B. B. Paul, Pramana - J. Phys. **61** (2003) 1055.

[40] M. Sethi, A. Batra and D. Lohiya, Phys. Rev. D **60** (1999) 108301.

VOLUMNI VISKOZNI KOZMOLOŠKI MODELI U OPĆOJ RELATIVNOSTI

Izveli smo novu klasu egzaktnih rješenja Einsteinovih jednadžbi polja za volumnu viskoznu tekućinu i lokalno rotacijski simetričan prostor-vrijeme Bianchijevog tipa I, primjenjujući varijabilan parametar usporavanja. Opisali smo tri slučaja, ovisno o izrazu za parametar usporavanja, kojima smo postigli šest modela Svemira. U tim modelima nalazimo vrijednosti Hubbleove konstante H_0 manje od jedan, što je od interesa za fiziku. Raspravljamo neka fizička i geometrijska svojstva tih modela.

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