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LINEAR WAVES IN A SELF-GRAVITATING DUSTY PLASMA WITH DUST CHARGE FLUCTUATION

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In astrophysical environments, dynamics of large bodies such as planets, stars and satellites is solely governed by gravitational force, whereas electromagnetic force is the only force effective in controlling the dynamics of electrons, ions. It is very interesting to note that for the micron and submicron size dust grains these two forces become comparable, i.e. $Gm_{\rm d}^2/q_{\rm d}^2 \approx O(1)$, and the interplay between the gravitational and the electrostatic forces in the dynamics of such grains leads to various novel phenomena in the terrestrial and solar environment. Our motivation is to study linear waves excited in a dusty plasma due to self gravity. The constituents of our plasma system are electrons, ions, and micron- and submicron-sized dust grains. We consider this dusty plasma to be infinite, homogeneous with spatially uniform densities of the species. The ratio $Gm_{\rm d}^2/q_{\rm d}^2$ is very small for the electrons and ions, and so the effects of self gravity for these species may be neglected. Moreover, we have taken into account the ion-dynamics together with the dust-dynamics and charge fluctuation of the dust grains. Then we have derived a dispersion law associated with the analysis of the linear gravitational instability of the waves. In our analysis, we disregard the zero-order gravitational field and so Poisson's equation for the gravitational potential, ψ, has to be modified. The dispersion law has been analysed in detail and various cases are discussed.

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1. Introduction

In recent years, numerous workers have been carrying out investigations on various salient features of nonlinear waves in plasma contaminated by the dust grains encountered very often in space and laboratory plasmas. The dust grains are massive and charged. Their mass is of the order of $10^6 - 10^{10}$ times the proton mass and their charge is of the order of $10^3 - 10^7$ times the electronic charge. The presence of these dust particles in plasma (as impurity ions) can drastically modify the dispersion and the nonlinear properties, and at the same time, it introduces new time and space scales in the plasma behaviour leading to new waves and instabilities etc. So, a dusty plasma is a three component plasma containing electrons, ions and very massive charged grains of solid matter. Dusty plasma is usually found in the interstellar clouds, circumsteller clouds, interplanetary medium, cometary tails, planetary rings and the Earth's magnetosphere [1, 2]. A novel feature of dusty plasmas, when compared with usual electron-ion plasmas with different ion species or with electrons having different temperatures, is the charging of the grains which can fluctuate due to the collection of plasma (electron and ion) currents onto the grain surface.

In the absence of dust charge fluctuation, dusty plasma can support ultra-low frequency waves which propagate as normal modes. The first and one of the well investigated normal modes is the so called "dust-acoustic wave" (DAW) [3] which has been confirmed in recent laboratory experiments on dusty plasmas [4]. Another low-frequency mode supported by dusty plasma is "dust-ion-acoustic wave" (DIAW) [5]. On the other hand, when the grain charge fluctuation is selfconsistently included, the dust modes are found to be weakly damped [6, 7]. Furthermore, dense dusty plasmas support a new kind of ultra-low-frequency wave mode which may be called the "dust Coulomb wave" (DCW) [8, 9]. These waves are the normal modes of a dense dusty plasma arising solely due to grain charge fluctuations.

In astrophysical scenarios, the dynamics of the large bodies like planets, stars etc. is controlled by gravitational interaction while that of electrons and ions is governed by electromagnetic forces. It is now well established that for the dust particles these two forces are comparable. Because of the interplay between these two forces in the dynamics of the dust grains, many interesting phenomena take place in terrestrial and solar environments. Pandey et al. [10] have studied the Jeans instability of a dusty plasma considering dynamics of all plasma species but they have not taken into account the grain charge fluctuation. In the letter of Rao et al. [11], only Jeans instability due to the effect of self gravitation of the dust grains has been studied. They have considered only the dust dynamics assuming both the electron and the ion number densities to follow Boltzmann distribution, though they have included dust charge fluctuation. In our paper, an attempt has been made to study Jeans instability due to self gravity of the dust grains only, taking into account the dust charge fluctuation and ion dynamics, but assuming electrons to be Boltzmannian.

2. *Models and the dispersion relation*

In our plasma model, we consider a three-component dusty plasma containing electrons, ions and dust particles. We further include the self-gravitation because of the massive dust grains only. The number density of electrons is given by Boltzmann distribution

$$n_{\mathrm{e}} = n_{\mathrm{e}0} \exp\left\{ \frac{e\phi}{k_{\mathrm{B}} T_{\mathrm{e}}} \right\}, \tag{1}$$

where n_{e}, T_{e} and ϕ denote the number density, the temperature of electrons and the electrostatic plasma potential, respectively; k_{B} is the Boltzmann constant. Assuming the hydrodynamical description of the ion-fluid and the dust-fluid to be realistic, we can write the basic equations which govern the dynamics of the ions and the dust particles as (for one-dimensional wave propagation along the z-axis):

For the ions:

$$\begin{aligned} \frac{\partial n_{\mathrm{i}}}{\partial t} + \frac{\partial}{\partial z}(n_{\mathrm{i}} u_{\mathrm{i}}) &= 0\,, \\ \frac{\partial u_{\mathrm{i}}}{\partial t} + u_{\mathrm{i}} \frac{\partial u_{\mathrm{i}}}{\partial z} &= -\frac{e}{m_{\mathrm{i}}} \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial z} - \frac{k_{\mathrm{B}} T_{\mathrm{i}}}{m_{\mathrm{i}} n_{\mathrm{i}}} \frac{\partial n_{\mathrm{i}}}{\partial z}\,. \end{aligned} \tag{2}$$

For the dust grains:

$$\begin{aligned} \frac{\partial n_{\mathrm{d}}}{\partial t} + \frac{\partial}{\partial z}(n_{\mathrm{d}} u_{\mathrm{d}}) &= 0\,, \\ \frac{\partial v_{\mathrm{d}}}{\partial t} + v_{\mathrm{d}} \frac{\partial v_{\mathrm{d}}}{\partial z} &= -\frac{q_{\mathrm{d}}}{m_{\mathrm{d}}} \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial z}\,, \end{aligned} \tag{3}$$

where $T_{\mathrm{d}} \ll T_{\mathrm{i}} < T_{\mathrm{e}}$. ψ is the gravitational potential and q_{d} is the charge of the dust-grain which may be regarded as a dynamical variable having the equilibrium value $q_{\mathrm{d}0}$. The description will be closed if one considers the following equations which are the electrostatic and the gravitational Poisson's equation

$$\varepsilon_0 \frac{\partial^2 \phi}{\partial z^2} + n_{\mathrm{d}} q_{\mathrm{d}} + e n_{\mathrm{i}} - e n_{\mathrm{e}} = 0\,, \tag{4}$$

$$\frac{\partial^2 \psi}{\partial z^2} = 4\pi G m_{\mathrm{d}} n_{\mathrm{d}}\,. \tag{5}$$

Here n_{i}, n_{d} represent the number densities of the ions and the dust particles; m_{i}, m_{d} are the ion mass and the dust grain mass, respectively; u_{i}, u_{d} denote, respectively, the velocities of the ion-fluid and the dust-fluid along the direction of propagation of the wave and T_{i} is the ion-temperature. It has been found by Avinash et al. [12] that the inclusion of a non-static ion-response leads to a robust purely growing instability, the increment of which is much larger than that reported in Ref. [10]. Accordingly, the present instability is likely to play a very important role in understanding the phenomenon of condensation of charged dust grains in planetary rings as well as in galaxy formation. This is why we have incorporated the ion-dynamics.

The charge variation of the dust grains is given by

$$\frac{\partial q_\mathrm{d}}{\partial t} + u_\mathrm{d}\frac{\partial q_\mathrm{d}}{\partial z} = I_\mathrm{e} + I_\mathrm{i}\,, \tag{6}$$

where I_e and I_i are the electron-current and the ion-current [13] given by

$$\begin{aligned} I_\mathrm{e} &= -e\sqrt{8\pi}\,a^2 n_\mathrm{e}(\phi) v_\mathrm{te} \exp\left\{\frac{e\Psi}{k_\mathrm{B}T_\mathrm{e}}\right\}, \\ I_\mathrm{i} &= e\sqrt{8\pi}\,a^2 n_\mathrm{i}(\phi) v_\mathrm{ti}\left(1 - \frac{e\Psi}{k_\mathrm{B}T_\mathrm{i}}\right), \end{aligned} \tag{7}$$

where a denotes the radius of the dust grain, $\Psi = q_\mathrm{d}/(4\pi\varepsilon_0 a)$ is the grain surface potential relative to the ambient plasma potential and $q_\mathrm{d} = -Z_\mathrm{d}e$ where Z_d is the charge number on the surface of the dust grain. Other symbols occurring in Eq. (7) carry usual meanings.

In the equilibrium state, Eq. (6) becomes

$$I_\mathrm{e0} + I_\mathrm{i0} = 0\,. \tag{8}$$

This relation determines the equilibrium value of the dust charge, q_d0, which is given by

$$n_\mathrm{i0} v_\mathrm{ti}\left(1 - \frac{e\Psi_0}{k_\mathrm{B}T_\mathrm{i}}\right) = n_\mathrm{e0} v_\mathrm{te} \exp\left\{\frac{e\Psi_0}{k_\mathrm{B}T_\mathrm{e}}\right\}, \tag{9}$$

where $q_\mathrm{d0} = -eZ_\mathrm{d0}$ and $v_{\mathrm{t}\alpha} = \sqrt{k_\mathrm{B}T_\alpha/m_\alpha}$ (α = e for electron and i for ion) is the thermal velocity. To derive the linear dispersion relation, we have to linearise Eqs. (1) − (7), assuming the perturbations of various plasma variables given below to vary as $\exp\{\mathrm{i}(kz - \omega t)\}$,

$$\begin{aligned} &n_s = n_{s0} + n_{s1}\,, \quad u_s = u_{s1}\,, \quad u_\mathrm{d} = u_\mathrm{d0} + u_\mathrm{d1}\,, \\ &q_\mathrm{d} = q_\mathrm{d0} + q_\mathrm{d1}\,, \quad I_s = I_{s0} + I_{s1}\,, \quad \phi = \phi_1\,, \quad \psi = \psi_1\,, \end{aligned} \tag{10}$$

where s = e, i, for electron and ion, respectively. Only the dust grains have been assumed to have streaming velocities for obvious reasons.

After calculation, following usual technique, the linearised equations turn out to be

$$n_\mathrm{e1} = \frac{en_\mathrm{e0}}{k_\mathrm{B}T_\mathrm{e}}\,\phi_1\,, \tag{11}$$

$$-\omega n_\mathrm{i1} + kn_\mathrm{i0}u_\mathrm{i1} = 0\,, \tag{12}$$

$$-\omega n_\mathrm{i0}u_\mathrm{i1} + k\,\frac{k_\mathrm{B}T_i}{m_\mathrm{i}}\,n_\mathrm{i1} = -\frac{e}{m_\mathrm{i}}kn_\mathrm{i0}\phi_1 - kn_\mathrm{i0}\psi_1\,, \tag{13}$$

$$\begin{aligned}
n_{\rm d0}u_{\rm d1} &= \frac{\omega'}{k}n_{\rm d1}, & (14)\\
\omega' u_{\rm d1} &= \frac{kq_{d0}}{m_{\rm d}}\,\phi_1 + k\psi_1\,, & (15)\\
n_{\rm d0}q_{\rm d1} + q_{\rm d0}n_{\rm d1} + en_{\rm i1} - en_{\rm e1} &= k^2\varepsilon_0\phi_1\,, & (16)\\
4\pi G m_{\rm d} n_{\rm d1} &= -k^2\psi_1\,, & (17)\\
\big(\Omega_{\rm c} - {\rm i}\omega'\big)q_{\rm d1} &= I_0\left(\frac{n_{\rm i1}}{n_{\rm i0}} - \frac{n_{\rm e1}}{n_{\rm e0}}\right), & (18)
\end{aligned}$$

where

$$\begin{aligned}
\omega' &= \omega - ku_{\rm d0}\,,\\
\Omega_{\rm c} &= \frac{eI_0}{ak_{\rm B}T_{\rm e}}\left(1 + \frac{\sigma}{1 - e\Psi_0/(k_{\rm B}T_{\rm i})}\right),\\
\sigma &= \frac{T_{\rm e}}{T_{\rm i}}\,.
\end{aligned}$$

$\Omega_{\rm c}$ is the charging frequency and I_0 is the equilibrium current given by

$$I_0 = I_{\rm e0} = -I_{\rm i0} = e\sqrt{8\pi}\,a^2 n_{\rm i0}v_{\rm ti}\left(1 - \frac{e\Psi_0}{k_{\rm B}T_{\rm i}}\right), \tag{19}$$

where Ψ_0 is the equilibrium value of the grain surface potential. Again, the first order perturbed quantities like $n_{\rm i1}$, $n_{\rm d1}$, $q_{\rm d1}$ are expressed in terms of ϕ_1 as

$$\begin{aligned}
n_{\rm i1} &= n_{\rm io}\,\frac{\dfrac{\rm e}{\rm m_i} - \dfrac{q_{\rm d0}}{m_{\rm d}}\left(1 + \dfrac{\omega^2}{\omega_{\rm Jd}^2}\right)^{-1}}{\left(\dfrac{\omega}{k}\right)^2 - v_{\rm ti}^2}\,\phi_1\,, & (20)\\
n_{\rm d1} &= \frac{k^2 n_{\rm d0}q_{\rm d0}}{m_{\rm d}}\frac{\phi_1}{\omega'^2 + \omega_{\rm Jd}^2}\,, & (21)\\
q_{\rm d1} &= {\rm i}\,\frac{1}{\omega' + {\rm i}\Omega_{\rm c}}\,I_0\left(\frac{n_{\rm i1}}{n_{\rm i0}} - \frac{n_{\rm e1}}{n_{\rm e0}}\right), & (22)
\end{aligned}$$

where $\omega_{\rm Jd}$ is the Jeans frequency defined as $\omega_{\rm Jd}^2 = 4\pi G m_{\rm d} n_{\rm d0}$ and $n_{\rm i1}$, $n_{\rm e1}$ are given by Eqs. (20) and (11) respectively.

Using the relations (11), (20), (21) and (22) in the linearised Poisson's equation

(16), after algebraic manipulation one obtains

$$\left(\mathrm{i}\,\frac{1}{\omega'+\mathrm{i}\Omega_\mathrm{c}}\,\frac{n_\mathrm{d0}}{n_\mathrm{io}}\,I_0+e\right)n_\mathrm{i0}\,\frac{\dfrac{\mathrm{e}}{\mathrm{m_i}}-\dfrac{q_\mathrm{d0}}{m_\mathrm{d}}\,\dfrac{\omega_\mathrm{Jd}^2}{\omega'^2+\omega_\mathrm{Jd}^2}}{\dfrac{\omega}{k}^2-v_\mathrm{ti}^2}$$

$$-\left(\mathrm{i}\,\frac{1}{\omega'+\mathrm{i}\Omega_\mathrm{c}}\,I_0\,\frac{n_\mathrm{d0}}{n_\mathrm{e0}}+e\right)\frac{en_\mathrm{e0}}{k_\mathrm{B}T_\mathrm{e}}=k^2\varepsilon_0\left(1-\frac{\omega_\mathrm{pd}^2}{\omega'^2+\omega_\mathrm{Jd}^2}\right). \quad (23)$$

This is the dispersion relation which, on simplification, comes out to be a polynomial equation in ω of the form

$$A\omega^5+B\omega^4+C\omega^3+D\omega^2+E\omega+F=0\,, \quad (24)$$

where the complex coefficients A, B, ... are given in the Appendix. It is found that the coefficients are alternately real and imaginary in the absence of streaming of the dust particulates. In order to make a detailed analysis of (24) for studying various modes of wave propagation and instabilities, both growing and decaying, it is usual to express the wave frequency ω as $\omega=\omega_\mathrm{r}+\mathrm{i}\gamma$ where ω_r is the real part and γ is the growth rate (for positive value) or the decay rate (for negative value). If this is substituted in Eq. (24), then equating the real part and the imaginary part we obtain two simultaneous polynomial equations in ω_r and γ as

$$\begin{aligned}
A\omega_\mathrm{r}^5+B_\mathrm{r}\omega_\mathrm{r}^4+(-10A\gamma^2-4B_\mathrm{im}\gamma+C_r)\omega_\mathrm{r}^3+(-6B_\mathrm{r}\gamma^2&\\
-3C_\mathrm{im}\gamma+D_\mathrm{r})\omega_\mathrm{r}^2+(5A\gamma^4+4B_\mathrm{im}\gamma^3-3C_\mathrm{r}\gamma^2-2D_\mathrm{im}\gamma+E_\mathrm{r})\omega_\mathrm{r}&\\
+(B_\mathrm{r}\gamma^4+C_\mathrm{im}\gamma^3-D_\mathrm{r}\gamma^2-E_\mathrm{im}\gamma+F_\mathrm{r}) &= 0\,, \quad (25)\\
A\gamma^5+B_\mathrm{im}\gamma^4+(-10A\omega_\mathrm{r}^2-4B_\mathrm{r}\omega_\mathrm{r}-C_\mathrm{r})\gamma^3+(-6B_\mathrm{im}\omega_\mathrm{r}^2&\\
-3C_\mathrm{im}\omega_\mathrm{r}-D_\mathrm{im})\gamma^2+(5A\omega_\mathrm{r}^4+4B_\mathrm{r}\omega_\mathrm{r}^3+3C_\mathrm{r}\omega_\mathrm{r}^2+2D_\mathrm{r}\omega_\mathrm{r}+E_\mathrm{r})\gamma&\\
+(B_\mathrm{im}\omega_\mathrm{r}^4+C_\mathrm{im}\omega_\mathrm{r}^3+D_\mathrm{im}\omega_\mathrm{r}^2+E_\mathrm{im}\omega_\mathrm{r}+F_\mathrm{im}) &= 0\,, \quad (26)
\end{aligned}$$

where the coefficients B_r, C_r , ... and B_im, C_im, ... are defined in the Appendix.

3. Results and discussion

Equations (25) and (26) are two polynomial coupled equations in ω_r and γ of degree 5 (five). So it is not possible to have their analytical solutions. One can solve them numerically. But it has been found that it is troublesome to solve them numerically even using the effective software ‘MATHEMATICA’. Because of this technical difficulty, some terms occurring in Eqs. (25) and (26) have been dropped due to smallness. As a consequence, a cubic equation in γ has been obtained. For argon dusty plasma, it has been solved numerically with the typical parameter values:

$$n_{eo} = 94 \times 10^{13}\,\mathrm{m}^{-3}, \quad k_{\mathrm{B}}T_{\mathrm{e}} = 2.76 \times 10^{-19}\,\mathrm{J}\,, \quad m_d = 1.66 \times 10^{-17}\,\mathrm{kg}\,,$$
$$\Omega_c = 6.028 \times 10^5 \mathrm{sec}^{-1}, \quad I_0 = 0.643 \times 10^{-9}\,\mathrm{A}\,,$$
$$n_{\mathrm{d0}} = 5 \times 10^9\,\mathrm{m}^{-3}, \quad n_{\mathrm{i0}} = 10^{15}\,\mathrm{m}^{-3}, \quad Z_{\mathrm{d0}} = 1.2 \times 10^4, \quad \frac{m_{\mathrm{d}}}{m_{\mathrm{i}}} \approx 10^{20},$$
$$\omega_{\mathrm{pd}} \approx 1.77 \times 10^{-2}\,\mathrm{sec}^{-1}, \quad \omega_{\mathrm{Jd}} \approx 0.527 \times 10^{-2} \mathrm{sec}^{-1}.$$

Numerical analysis reveals that two roots are large negative numbers which are independent of k, while the third root is positive but very small ($\sim 10^{-21}$) in the absence of dust streaming. This positive root corresponds to the growing instability of the waves. It is very interesting to note that the growth rate comes out to be of the order of 10^{-6} if streaming of the dust particulates is considered. Apart from this, the growth rate, and thereby the growing instability, turns out to be markedly dependent on k. The mode of variation of the growth rate with k for a particular value of u_{d0}, the streaming velocity of the dust grains is displayed in Fig. 1. It shows that the growth rate increases in a nonlinear way with k. Moreover, it is

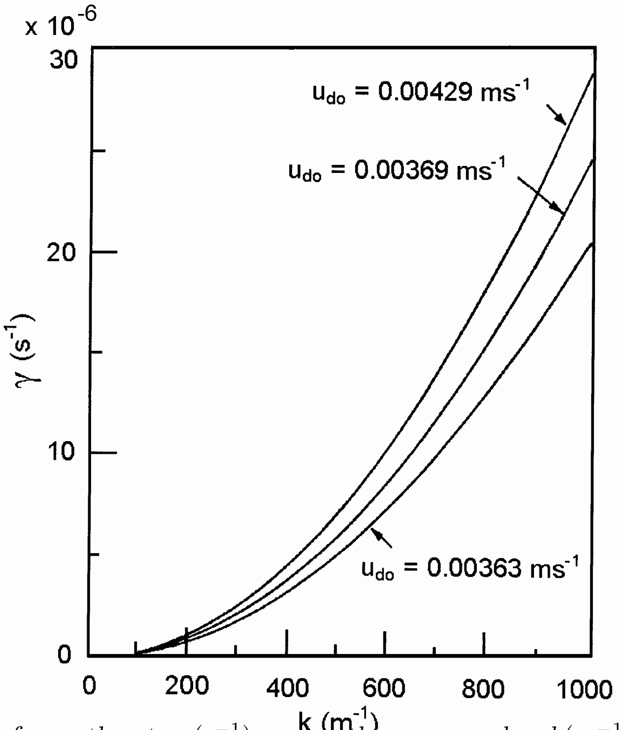

Fig. 1. Plot of growth rate $\gamma(\mathrm{s}^{-1})$ versus the wavenumber $k(\mathrm{m}^{-1})$ for the argon dusty plasma.

observed that the growth rate is greater when the streaming velocity of the dust grains is higher and vice versa for a fixed value of k. In this case $\omega_{\rm pd} > \omega_{\rm Jd}$. So, the dynamics of the charged particles is governed mostly by electrostatic forces.

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Appendix

$$A = \left(k^2\varepsilon_0 + \frac{e^2 n_{\rm e0}}{k_{\rm B}T_{\rm e}}\right)m_{\rm d}, \quad B = B_{\rm r} + {\rm i}B_{\rm im}, \quad C = C_{\rm r} + {\rm i}C_{\rm im},$$

$$D = D_{\rm r} + {\rm i}D_{\rm im}, \quad E = E_{\rm r} + {\rm i}E_{\rm im}, \quad F = F_{\rm r} + {\rm i}F_{\rm im}.$$

$$B_{\rm r} = -3ku_{\rm d0}A, \quad B_{\rm im} = A\Omega_{\rm c} + em_{\rm d}\frac{I_0 n_{\rm d0}}{k_{\rm B}T_{\rm e}},$$

$$C_{\rm r} = -e^2k^2n_{\rm i0}\frac{m_{\rm d}}{m_{\rm i}} + \left[\frac{e^2n_{\rm e0}}{k_{\rm B}T_{\rm e}}\big(3k^2u_{\rm d0}^2 + \omega_{\rm Jd}^2\big) + k^2\varepsilon_0\big(\omega_{\rm Jd}^2 + 3k^2u_{\rm d0}^2 - \omega_{\rm pd}^2\big)\right]m_{\rm d},$$

$$C_{\rm im} = 2ku_{\rm d0}m_{\rm d}\left[\frac{-e^2n_{\rm e0}}{k_{\rm B}T_{\rm e}}\left(\frac{I_0}{e}\frac{n_{\rm d0}}{n_{\rm e0}} + \Omega_{\rm c}\right) - k^2\varepsilon_0\Omega_{\rm c}\right],$$

$$D_{\rm r} = 3e^2k^3u_{\rm d0}n_{\rm i0}\frac{m_{\rm d}}{m_{\rm i}} - ku_{\rm d0}m_{\rm d}\frac{e^2n_{\rm e0}}{k_{\rm B}T_{\rm e}}\big(k^2u_{\rm d0}^2 + \omega_{\rm Jd}^2\big)$$

$$+k^3\varepsilon_0 m_{\rm d}u_{\rm d0}\big[\omega_{\rm pd}^2 - (k^2u_{\rm d0}^2 + \omega_{\rm Jd}^2)\big]$$

$$D_{\rm im} = -e^2k^2n_{\rm i0}\frac{m_{\rm d}}{m_{\rm i}}\left(\frac{I_0}{e}\frac{n_{\rm d0}}{n_{\rm i0}} + \Omega_{\rm c}\right) + \frac{e^2n_{\rm e0}}{k_{\rm B}T_{\rm e}}m_{\rm d}\big(k^2u_{\rm d0}^2 + \omega_{\rm Jd}^2\big)$$

$$\times\left(\frac{I_0}{e}\frac{n_{\rm d0}}{n_{\rm e0}} + \Omega_{\rm c}\right) - k^2\varepsilon_0 m_{\rm d}\Omega_{\rm c}\big[\omega_{\rm pd}^2 - (k^2u_{\rm d0}^2 + \omega_{\rm Jd}^2)\big],$$

$$E_{\rm r} = e^2k^2n_{\rm i0}\left[-\frac{m_{\rm d}}{m_{\rm i}}\big(\omega_{\rm Jd}^2 + 3k^2u_{\rm d0}^2\big) - Z_{\rm d0}\omega_{\rm Jd}^2\right],$$

$$E_{\rm im} = 2e^2k^3n_{\rm i0}u_{\rm d0}\frac{m_{\rm d}}{m_{\rm i}}\left(\Omega_{\rm c} + \frac{I_0}{e}\frac{n_{\rm d0}}{n_{\rm i0}}\right),$$

$$
\begin{aligned}
F_{\mathrm{r}} &= e^2 k^3 n_{\mathrm{i0}} u_{\mathrm{d0}} \left[\left(k^2 u_{\mathrm{d0}}^2 + \omega_{\mathrm{Jd}}^2\right) \frac{m_{\mathrm{d}}}{m_{\mathrm{i}}} + Z_{\mathrm{d0}} \omega_{\mathrm{Jd}}^2 \right], \\
F_{\mathrm{im}} &= -e^2 k^2 n_{\mathrm{i0}} \left(\frac{I_0}{e} \frac{n_{\mathrm{d0}}}{n_{\mathrm{i0}}} + \Omega_{\mathrm{c}} \right) \left[(k^2 u_{\mathrm{d0}}^2 + \omega_{\mathrm{Jd}}^2) \frac{m_{\mathrm{d}}}{m_i} + Z_{\mathrm{d0}} \omega_{\mathrm{Jd}}^2 \right].
\end{aligned}
$$

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LINEARNI VALOVI U PRAŠNJAVOJ PLAZMI POD DJELOVANJEM VLASTITE GRAVITACIJSKE SILE UZ IZMJENE NABOJA

U astrofizičkim uvjetima, za dinamiku velikih tijela, kao planeta i njihovih pratilaca, te zvijezda, važne su samo gravitacijske sile, dok elektromagnetske sile određuju dinamiku elektrona i iona. Međutim, vrlo je zanimljivo primijetiti da su te dvije sile usporedive, tj. $Gm_{\rm d}^2/q_{\rm d}^2 \approx O(1)$, u međudjelovanju čestica mikronske i podmikronske veličine, što vodi do novih pojava u omotaču Zemlje, Sunca i drugih nebeskih tijela. Zamisao ovog rada je proučavanje linearnih valova u prašnjavoj plazmi zbog vlastitih gravitacijskih sila. Sastav plazme su elektroni, ioni i čestice prašine mikronske i podmikronske veličine. Pretpostavljamo homogenu plazmu beskonačnih razmjera s jednolikom gustoćom tih čestica. Omjer $Gm_{\rm d}^2/q_{\rm d}^2$ je vrlo malen za elektrone i ione pa se gravitacijska sila tih čestica može zanemariti. Razmatramo zajedno dinamiku iona i čestica prašine, uzimajući u obzir promjene naboja zrnaca prašine. Izveli smo disperzijsku relaciju za analizu linearnih gravitacijskih nestabilnosti valova. U ovom razmatranju nismo uzeli u obzir gravitacijsko polje nultog reda, stoga se Poissonova jednadžba za gravitacijski potencijal, ψ, morala izmijeniti. Analizira se disperzijska relacija i raspravljaju različiti slučajevi.