Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
CODEN FIZBE7

ENTROPY OF COLORED QUARKS STATES AT FINITE TEMPERATURE

DAVID E. MILLER[a,b] and ABDEL-NASSER TAWFIK[a]

[a]*Fakultät für Physik, Universität Bielefeld, D-33501 Bielefeld, Germany*
E-mail addresses: om0@psu.edu, dmiller@physik.uni-bielefeld.de

[b]*Department of Physics, Pennsylvania State University, Hazleton, PA 18201, USA*
E-mail address: tawfik@physik.uni-bielefeld.de

Received 23 March 2006; Accepted 20 September 2006
Online 22 May 2007

The quantum entropy at finite temperatures is analyzed by using models for colored quarks making up the physical states of the hadrons. We explicitly work out some special models for the structure of the states of $SU(2)_c$ and $SU(3)_c$ related to the effects of temperature on the quantum entropy. We show that the entropy of the singlet states monotonically changes with the temperature. However, the structure of the octet states has a greater complexity which can be best characterized by two types, one of which is similar to that of the singlet states, while the other one reflects the existence of strong correlations between only two of the color states. For the sake of comparison, we work out the entropy for the *classical* Ising and the *quantum* XY spin chains. In the Ising model, the quantum entropy in the ground state does not directly enter into the partition function. It also does not depend on the number of spatial dimensions, but only on the number of quantum states making up the ground state. But the XY spin chain has a finite entropy at vanishing temperature. With the inclusion of the ground state, the results from the spin models are qualitatively similar to our models for the states of $SU(2)_c$ and $SU(3)_c$.

PACS numbers: 12.39.-x, 24.85.+p, 05.50.+q UDC 539.125, 536.75

Keywords: hadrons, colored quarks, quantum entropy, $SU(2)_c$ and $SU(3)_c$, singlet states, octet states, Ising and quantum XY spin chains

1. *Introduction*

In this paper we present a new method for evaluating the entropy of the colored quark states at finite temperature. Clearly, the entropy which we discuss here is the quantum entropy which is to a great extent different in nature from the *classical* (Boltzmann-Gibbs) entropy. From the classical point of view, the entropy characterizes an access to information between macroscopic and microscopic physical quantities using statistical criteria. It can be considered to be the number of possible microstates that the macrosystem can include. But the quantum entropy[1] can be calculated for very few degrees of freedom by using the density matrix and should not vanish at zero temperature. It reflects the uncertainties in the abundance of information about the quantum states in a system. The definition of quantum entropy dates back to the work of von Neumann [2] in the early thirties on the mathematical foundations of quantum theory. Another milestone has been set out by Stratonovich, when he introduced the reciprocative quantum entropy for two coupled systems. In the Lie algebra, the quantum fluctuation, and correspondingly the quantum entropy, are given by non-zero commutation relations. The most essential information we can retain from the physical systems is similar to measuring the eigenvalues of certain observables. The faultless measurement is to be actualized only if the degeneracy in the determined observable is entirely considered.

According to the Nernst's heat theorem, the entropy of a homogeneous system at zero temperature is expected to be zero. However, it has been proven that the mixing in ground states of subsystems together with the conservation of the total particle number with the temperature and the volume result in a finite entropy [3]. This is the so-called Gibbs paradox. On the other hand, we know that the effects of quantization on the fundamental laws of thermodynamics were generally well known to the founders of quantum theory [4]. Planck, who had successfully predicted the measured intensity distribution of different wavelengths by the postulation of a minimum energy amount for the light emitted from a dark cavity at a given temperature, had also realized that the massive particles with non-zero spin also possess a finite entropy related directly to that of the spin in the limit of low temperatures. This realization provided the chance for his students to resolve the Gibbs paradox. Furthermore, it also offered an exception to Nernst's heat theorem, which is usually stated under the name of the third law of thermodynamics that the entropy of a closed system in equilibrium must vanish in the low temperature limit. A clear general discussion of the entropy related to the Nernst's heat theorem was given by Schrödinger some years later in a lecture series [5]. In his selected example, Schrödinger took an N-particle system, in which each particle has two quantum states that contribute to the many particle ground state. Thus the ground state held $2N$ degenerate configurations in its structure, which must then provide for it an entropy of $N \ln 2$. This value is obviously independent of all thermodynamical quantities other than the number of particles N itself.

Furthermore, it is known that the more information present, the greater is the reduction of the entropy. In other words, the entropy is reduced by the amount of information distinguishing the different degenerate states of the system. The completely mixed state is that of minimal information. We mention here that this remark is completely consistent with the idea of confinement in the hadronic matter. If we were to consider a gas of N quarks or antiquarks in the same sense that Schrödinger [5] considered a gas of N degenerate two-level atoms, we should roughly expect an entropy of the form $N \ln 3$ for the $SU(3)_c$ quark structure in the color

[1] The authors of Ref. [1] distinguish between different entropies: the classical one they called the measurement entropy, and their relevant entropy refers to our quantum entropy.

singlet ground state of this pure composite system [6]. In this article, we shall attempt to elucidate the physical meaning of the quantum entropy for the colored quark states as well as investigate its behavior at finite temperature.

According to the third law of thermodynamics, it should be expected that the confined hadron bags – as a pure state – have zero entropy at zero temperature. However, these hadronic systems are treated as having subsystems composed of quantum elementary particles, which in their ground states can exhibit a finite value of the entropy. The quantum entropy of such subsystems reflects the degree of mixing and entanglement [7] inside the hadronic bag. In a recent article [6], it was shown that, in a quantum system with $SU(3)_c$ internal color symmetry, the ground state entropy arises from the mixing of the quantum color states of that system yielding a quantum entropy $\ln 3$. From this fact, we would expect that in a completely mixed ground state with N internal components, the entropy relates directly to the value $\ln N$. Nevertheless, one would expect some changes in this entropy in the presence of other states at finite temperatures. Such changes were mentioned in Ref. [6] in relation to the octet hadronic states as well as the changes in the thermodynamics from the quantum entropy at finite temperature [8].

Understanding the thermal behavior of quantum subsystems [9], which in our case are the individual colored quark states, should be very useful for various applications. First, it may well bring forth a method for a further explanation of the recent lattice results with the heavy quark-antiquark potentials [10 − 12] in the low-temperature limit. These lattice results for the entropy difference in a quark-antiquark color averaged state lead to the value of $-\ln 9$ at small distances [11, 12] for temperatures well below the deconfinement temperature. This simple numerical factor comes from the proper normalization of the color-averaged free energy. Our evaluation of the quantum entropy of one quark as part of the colorless singlet ground state [6] yields the value of $\ln 3$, so that for the colored quarks and antiquarks in the singlet ground state the quantum entropy is $2\ln 3$. This result is simply the opposite in sign to the normalization factor and lies that much above the usual entropy in the low-temperature limit. Furthermore, we believe that the investigation of the quantum subsystems at finite temperature might be useful for the understanding of the concept of confinement, which could exist at any given temperature. The possibility of cold dense quark matter deep in the interior of compact stellar objects provides another application for this work. Thus, we consider it possible that on account of the existence of a finite entropy inside the hadron bags, the confined quarks take on additional heating with increasing temperature so that the bag constant must be correspondingly modified [8]. Thereupon, the quark distributions inside the hadron bags give rise to an entanglement effect which is related to the quantum entropy.

In this work we shall further investigate the implications of the quantum entropy for the colored quarks related to the known properties of the standard model [13]. As previously mentioned [6], we shall not bring in other important properties of the quarks in the standard model like flavor, spin, isospin, chirality, electric charge and the spatial distribution of the quarks. Because we are not explicitly considering the quantum fields, such properties as gauge invariance do not enter into our considerations. From now on, we shall assume in this work that we are only looking at the color symmetry so that we shall leave out any name distinctions between the states. We shall further look into some specific properties of other quark structures. Thereafter, we shall investigate some specific properties of other states in relation to the ground state, which lead to some interesting results for the entropy. As a following investigation, we introduce and solve the quark quantum entropy some simple models of color mixing and unmixing at finite temperatures, and we check for each model for the limiting cases. Then we study the correlations of entangled spin systems. These spin systems are used to compare our results for measurement entropy of a system of colored quark states. Other degrees of freedom are not considered.

Before we introduce our models for the quantum entropy S at finite temperatures, one may ask why should S be significant. As we discussed above, there are many physical systems, for which one has to revise the third law of thermodynamics from its original form: the entropy remains finite even at absolute zero. When one ordinarily takes this situation into consideration, some of thermodynamical statements will necessarily become more complicated. Here we try to understand the reason for having a finite entropy at $T = 0$ in the quark matter in terms of the quantum correlations in the ground state. How the finite S would affect the thermodynamics will be discussed elsewhere. The effects of including S in the hadronic equation of a state at low temperatures we have studied in Ref. [8]. To our knowledge, the works of Elze [14] have been pioneering in addressing the question about the origin of the "entropy puzzle" in high-energy collisions. These articles established a theoretical framework for discussing how two hadrons scattering in the initial state undergo a hard interaction from a quantum mechanically pure initial state, which can result in a high-multiplicity event corresponding to a highly impure more or less thermal density matrix on the partonic level before hadronization. According to these articles [14], it is the entropy which clearly characterizes the quantum properties. Thereupon, the entropy production in the heavy-ion collisions is due to environmentally induced quantum decoherence in the observable subsystem. Therefore, we believe that there is no really comprehensible reason, at least theoretically, to consider a finite entropy in interpreting the particle multiplicity, while explicitly setting the entropy to zero in other comparable systems. We, like Elze, use the von Neumann prescription [2], which makes use of the eigenvalues of the reduced density matrices. We are able to give a first quantitative evaluation for the quarks' entropy within the hadrons [6, 8] in the singlet ground state. We have found that these ground states have the colors maximally mixed. In Ref. [15], we have evaluated qualitative models for the entropy in condensates of quark color superconductivity.

This paper is organized as follows: the next section is devoted to the formulation of the ground state, from which we develop models for the entropy of mixing of colored quark $SU(2)_c$ and $SU(3)_c$ states. Then we introduce our thermal models for the quantum entropy of these states. Some spin models are investigated in relation to the known exact solutions. The following section contains the discussion of our results. Finally, we end with the conclusion and outlook.

2. Formulation of the ground state entropy

In order to properly understand the ground state structure, we recall some common properties of spin and color quantum systems which relate to the ideas of superposition and entanglement [16]. The orthonormal basis usually taken [9] for spin $SU(2)$ can be written as $|0\rangle$ and $|1\rangle$ for the two basis states. For such two-state systems we have four combinations of $|ij\rangle$ which provide useful linear orthonormal combinations thereof. When we use the Pauli matrices σ^x, σ^y, σ^z together with the two dimensional identity matrix $\mathbb{1}_2$, we may easily write down the singlet and triplet structure for the structure of the states of $SU(2)$. The usual symmetric triplet and antisymmetric singlet states also provide a proper basis. After we have written down the density matrices ρ_t and ρ_s for each of the states, we find that after projecting out the second state, the single quark reduced density matrices are both in the form

$$\rho_{q,2} = \sum_i p_i \mathbb{1}_2 \,, \tag{1}$$

where p_i is the probability of i-th state. Hereupon, we can calculate the entropy S of the quantum states [2, 4], which makes direct use of the density matrix ρ

$$S = -\mathrm{Tr}(\rho \ln \rho) \,. \tag{2}$$

Even though this equation can be found in text books [2], we can still usefully apply it here. Thus the trace is taken over all quantum states Eq. (1). For quantum operators, the trace is independent of the representation. Therefore, the quantum states might well be used to write down either the quantum canonical or grand canonical partition function. The density matrix can be taken to be a mixing of subsystems within a closed system. Nevertheless, according to the Nernst's heat theorem, this enclosed system has zero entropy at zero temperature [3]. When, as is presently the case, the eigenvectors are known for ρ, we may directly write this form of the entropy in terms of the eigenvalues λ_i as follows

$$S = -\sum_i \lambda_i \ln \lambda_i \,. \tag{3}$$

It is obviously important to have positive eigenvalues. For the special case of a zero eigenvalue, we use the fact that $x \ln x$ vanishes in the small x limit. Then, for the density matrix ρ, we may interpret λ_i as the probability p_i of the i-th state. This meaning demands that $0 < p_i \leq 1$ Thus the orthonormality condition for the given states results in the condition ??. This is a very important condition for the entropy. Thus we can easily see that for $SU(2)$ the value of probabilities p_i is always 1/2, yielding the same total entropy for both the singlet and the triplet states

$$S_{q,2} = \ln 2 \,. \tag{4}$$

We get the same results if we apply the Schmidt decomposition on the pure state $|\Psi\rangle$, which consists of the basis states, $|0\rangle$ and $|1\rangle$. These states are used to construct sets of orthonormal states, $\{|0\rangle_i\}$ and $\{|1\rangle_i\}$, so that pure state can be written in the form

$$|\Psi\rangle = \sum_i \mathcal{N}_i \; |0\rangle_i \otimes |1\rangle_i \, . \tag{5}$$

$\mathcal{N}$ are real positive numbers, known as the Schmidt numbers, which satisfy

$$\sum_i \mathcal{N}_i^2 = 1 \, . \tag{7}$$

One important consequence of the last equation is that the reduced density matrices of just the basis states, $|0\rangle$ and $|1\rangle$, should have identical eigenvalues. After a substitution of $\mathcal{N}_i \equiv \lambda_i$ into Eq. (3), we find that $S_{q,2} = \ln 2$. Equation (5) can be generalized for decomposition into n subsystems. The pure state $|\psi\rangle$ can be expanded in a number of factorizable n-states. The number of coeficients is minimal so that Eq. (6) is still valid.

In the case of $SU(3)_c$, the state structures for the singlet and octet are very different. We recall that we have found for the single quark density matrix ρ_q the form [6]

$$\rho_{q,3} = \sum_i p_i \; \mathbb{1}_3 \, , \tag{7}$$

where $\mathbb{1}_3$ is the three-dimensional identity matrix.

We now apply the above definitions of the entropy to the $SU(3)_c$ quark states - as was done in Ref. [6]. It is clear that the original hadronic states are pure colorless states which posses zero entropy, as is expected by the third law of thermodynamics. For the meson, this is immediately obvious, since each colored quark state has the opposing colored antiquark state for the resulting colorless singlet state. The sum of all cycles determines the colorlessness of the baryon singlet state, thereby giving no entropy. However, the reduced density matrix for the individual quarks (antiquarks) ρ_q or $\rho_{\bar{q}}$ has a finite entropy. In this context, the reduced density matrix can be illustrated as a certain mixing inside the closed system [3]. Thus the whole system has zero entropy at zero temperature, the *sub*systems – as stated above – are expected to have a finite entropy at vanishing temperature.

For $SU(3)_c$, all eigenvalues λ_i in Eq. (3) have the same value 1/3. Thus we find for all quark (antiquark) in singlet states [6]

$$S_{q,3} = \ln 3 \, . \tag{8}$$

As a further exercise, we may compare this result with those for the quark octet states. The octet density matrices $\rho_{\mathrm{o},i}$ may be constructed from the eight Gell-Mann matrices $(\lambda)_i$ with $i = 1, 2 \ldots, 8$. The density matrix for each state is constructed by using the properties of $\Psi(\lambda)_i\Psi^*$. The first seven matrices all give the same value for the entropy, $\ln 2$, since all of these states are constructed only from the Pauli matrices. This result comes from the fact that these first seven octet states each involve only two of the three color states – that is, these octet states are not mixed states in all colors. Although the mixing of the two states is equal, it is not complete since the third color is absent. However, the eighth diagonal Gell-Mann matrix involves all three colors, but the mixing is unequal. It yields an entropy

$$S_{o,8} = \ln 3 - \frac{1}{3}\ln 2 \tag{9}$$

Thus we can clearly state that the entropy of any of the quark octet states is always smaller than of the quark color singlet state. This means that the colorless quark singlet state is the most probable individual state for the hadrons.

3. *Thermodynamical models for mixed colored quark states*

The structure of the color singlet hadronic ground state for $SU(3)_c$ was shown in Ref. [6] to have a complete uniform mixing of all colors of quarks and antiquarks for both the mesons and the baryons. We have seen above that this situation does not happen for the single quark (antiquark) entropy S_q or $S_{\bar{q}}$ in the presence of octet states, where the mixture is either partial or unequal. We now want to return to the single quark (antiquark) reduced density matrices $\rho_{q,2}$ or $\rho_{\bar{q},2}$, which we shall assume to be the same for the fundamental and antifundamental representations, since we are not considering the differences between flavors. In this section, we will extend these calculations of the structural entropy of the ground states for the $SU(2)_c$ and $SU(3)_c$ quarks in color singlet states to models for color mixing at finite temperature. In these models, the Boltzmann weighting for the finite-temperature states of the single colored quark states will contain the single-particle relativistic energies $\epsilon(p)$, which are given by $\sqrt{m^2+p^2}$ for the relativistic quark momentum p and mass m. Since the biggest effect at a given temperature T comes with the lowest value of $\epsilon(p)$, we may well assume that $\epsilon(p)$ is just determined by the lowest quark-mass threshold. Furthermore, $\epsilon(p)$ is also color independent.

3.1. *$SU(2)_c$ thermodynamical model*

We start our consideration of thermodynamics with a very simple model for quarks with an internal $SU(2)_c$ symmetry at a finite temperature T. We postulate that the single-quark reduced density matrix with two colors has the following form

$$\rho_{q,2}(T) = \frac{1}{2}\left[\left(1-\mathrm{e}^{-\epsilon(p)/T}\right)|0\rangle\langle 0| + \left(1+\mathrm{e}^{-\epsilon(p)/T}\right)|1\rangle\langle 1| = \frac{1}{2}\left[\mathbb{1}_2 - \sigma^z \mathrm{e}^{-\epsilon(p)/T}\right],\right. \tag{10}$$

which shifts the weighting of the eigenstates due to the temperature T. Thus we note that the total probability is still one, so that $\rho_{q,2}(T)$ will still have the correct probabilistic interpretation. Furthermore, we note that if we had taken all Pauli matrices with equal weighting in front of the Boltzmann factor, we would still be able to diagonalize the full $SU(2)$ reduced density matrix into this form. Then the eigenvalues from the temperature dependent reduced density matrix, Eq. (10), are found by using

$$\det\left[\langle i|\rho_q|j\rangle - \frac{\lambda}{2}\right] = \left[\frac{1}{2} - \frac{\mathrm{e}^{-\epsilon(p)/T}}{2\sqrt{3}} - \lambda\right]\left[\frac{1}{2} + \frac{\mathrm{e}^{-\epsilon(p)/T}}{2\sqrt{3}} - \lambda\right] - \frac{\mathrm{e}^{-2\epsilon(p)/T}}{6} = 0\,, \tag{11}$$

where the result is given by

$$\lambda_i(T) = \frac{1}{2}\left(1 - \sigma^z \mathrm{e}^{-\epsilon(p)/T}\right). \tag{12}$$

The values $i = \pm 1$ relate to the states $|0\rangle$ and $|1\rangle$, which are included in σ^z in Eq. (10). We remark at this point if the minus sign in front of the Pauli matrix were changed to a plus sign, the eigenvalues would be the same.

As it was above carried out for the ground state, we are now able to calculate the entropy $S_{q,2}(T)$ at finite temperature [2, 4] from the eigenvalues $\lambda_i(T)$ and under the assumption that the color at finite temperature have the same energy eigenstates, and we find

$$S_{q,2}(T) = -\sum_{i=\pm 1} \lambda_i(T) \ln \lambda_i(T) \tag{13}$$

$$= -\frac{1}{2}\left(1-\mathrm{e}^{-\epsilon(p)/T}\right) \ln\left[\frac{1}{2}\left(1-\mathrm{e}^{-\epsilon(p)/T}\right)\right] - \frac{1}{2}\left(1+\mathrm{e}^{-\epsilon(p)/T}\right) \ln\left[\frac{1}{2}\left(1+\mathrm{e}^{-\epsilon(p)/T}\right)\right]. \tag{14}$$

In the low-temperature limit, we can immediately find that the ground state quark entropy $S_{q,2}(T)$ has again, as in Eq. (4), the value of $\ln 2$, as in a completely equally mixed quark color state. However, in the high-temperature limit, the contribution to the entropy of the first state vanishes from deoccupation, while the second state becomes a pure state with a probability of one, which also contributes a vanishing value to the entropy. In Fig. 1 we look at some particular cases for $S_{q,2}(T)$, both at

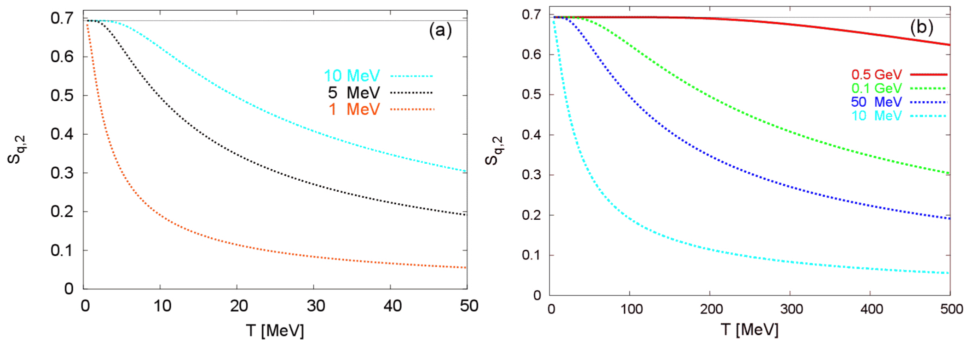

Fig. 1. Single quark entropy for various quark masses as a function of temperature T for $SU(2)$, Eq. (14). The left panel shows the results for the stated light quark masses. The right panel depicts the results for masses up to 500 MeV. *The dotted lines at the top represent the value of the ground state entropy at zero temperature in Eq. (4).*

lower and higher temperatures. In Fig. 1a we see how the single-quark entropy varies in a range of temperature up to 50 MeV with quark masses 1, 5 and 10 MeV. We notice that in all cases the entropies monotonically decrease in this region, meaning that the mixing of the states continually decreases. For the sake of comparison, in Fig. 1b we look at some larger quark masses in a much broader temperature range. We see the same tendency for the separation of the states. We note that with increasing quark masses, the range of temperature within which the entropy is entirely determined by the ground state value becomes wider and the entropy remains larger at higher temperatures. This reflects the importance of the ground state entropy for the massive quark systems. Thus, in this model for $SU(2)_c$ colored quark states, we find in the high-temperature limit a pure single colored quark state in the classical sense. Whereupon, we would expect a pure quark phase in which all correlations between the different colors have vanished – free quarks!

3.2. $SU(3)_c$ thermodynamical model

Now we construct a similar model for $SU(3)_c$ with the right quark mixing in the ground state, which also provides the proper probabilities for each of the states from the trace condition $\mathrm{Tr}\,\rho = \sum_i p_i = 1$ (Sect. 2). Again, we demand that the energy eigenstates remain the same for each of the color states $|i\rangle$. We think that the eighth (diagonal) Gell-Mann matrix λ_8 is not a very suitable choice as a weighting matrix for the thermal states, even though it maintains the trace condition for the probability. The eighth Gell-Mann matrix weights the third color twice as much as the other two taken individually. Therefore, we should look for another weighting matrix for the thermal states of $SU(3)$. Two possibilities, which we shall investigate, arise from the three complex roots of -1 and $+1$, respectively.

3.3. Unmixing model

This model is motivated by the previous results of $SU(2)_c$. The three complex roots of -1 are $\mathcal{R}_0 = -1$, $\mathcal{R}_1 = \exp(\mathrm{i}\pi/3)$ and $\mathcal{R}_2 = \exp(-\mathrm{i}\pi/3)$, and

$$\rho_{q,3} = \frac{1}{3}\left[\mathbb{1}_3 + \left(\mathcal{R}_0|0\rangle\langle 0| + \mathcal{R}_1|1\rangle\langle 1| + \mathcal{R}_2|2\rangle\langle 2|\right)\mathrm{e}^{-\epsilon(p)/T}\right] \tag{15}$$

Then we get three eigenvalues:

$$\begin{aligned}
\lambda_0 &= \frac{1}{3}\left(1 - \mathrm{e}^{-\epsilon(p)/T}\right), \\
\lambda_1 &= \frac{1}{3}\left(1 + \mathrm{e}^{\mathrm{i}\pi/3}\,\mathrm{e}^{-\epsilon(p)/T}\right), \\
\lambda_2 &= \frac{1}{3}\left(1 + \mathrm{e}^{-\mathrm{i}\pi/3}\,\mathrm{e}^{-\epsilon(p)/T}\right).
\end{aligned} \tag{16}$$

We may write the three roots as the weights w_i for the states $|i\rangle$ with the Boltzmann factor. Thus the reduced quark density matrix, Eq. (15), for $SU(3)_c$ reads

$$\rho_{q,3}(T) = \frac{1}{3}\big(\mathbb{1}_3 + |i\rangle w_i \mathrm{e}^{-\epsilon(p)/T}\langle i|\big)\,. \tag{17}$$

Clearly, in the low temperature limit we get back the completely mixed state with a probability 1/3 for each color state $|i\rangle$. We are able to calculate the entropy $S_{q,3}(T)$, Eq. (13), by carefully using the proper definitions for the complex logarithms

$$\begin{aligned} S_{q,3}(T) \quad = \quad & -\frac{1}{3}\big(1-\mathrm{e}^{-\epsilon(p)/T}\big)\,\ln\left[\frac{1}{3}\big(1-\mathrm{e}^{-\epsilon(p)/T}\big)\right] \\ & -\frac{1}{3}\big(1+\mathrm{e}^{\mathrm{i}\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\big)\,\ln\left[\frac{1}{3}\big(1+\mathrm{e}^{\mathrm{i}\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\big)\right] \\ & -\frac{1}{3}\big(1+\mathrm{e}^{-\mathrm{i}\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\big)\,\ln\left[\frac{1}{3}\big(1+\mathrm{e}^{-\mathrm{i}\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\big)\right] \end{aligned} \tag{18}$$

Using the properties of the logarithms of complex variable[2] we can write the real part as

$$z = \frac{1}{3}\big[1+\mathrm{e}^{-\epsilon(p)/T}+\mathrm{e}^{-2\epsilon(p)/T}\big]^{1/2}\,, \tag{19}$$

and the phase is given by

$$\theta = \arctan\left(\frac{\sqrt{3}\ \mathrm{e}^{-\epsilon(p)/T}}{2+\mathrm{e}^{-\epsilon(p)/T}}\right). \tag{20}$$

After a little algebra, we find that the single-quark quantum entropy at finite temperature for $SU(3)$ reads

$$S_{q,3}(T) = -\frac{1}{3}\big(1-\mathrm{e}^{-\epsilon(p)/T}\big)\,\ln\left[\frac{1}{3}\big(1-\mathrm{e}^{-\epsilon(p)/T}\big)\right] - 2z\big[\ln(z)\cos(\theta)-\theta\sin(\theta)\big]. \tag{21}$$

In the low-temperature limit, we have z just equal to $1/3$ and θ is exactly zero. Then the quark entropy is clearly again just $\ln 3$. However, in the limit of very high temperature, z becomes $1/\sqrt{3}$ and θ is just $\pi/6$. The contribution of the first term in Eq. (21) to $S_{q,3}(T)$ simply vanishes as was the case for $S_{q,2}(T)$ (review Fig. 1). However, the second term still remains at high temperatures leaving a limiting entropy of 0.8516. We see this effect in Fig. 2 where we have plotted $S_{q,3}(T)$ for various values of the quark masses. We note that the first term behaves qualitatively very similarly to the single quark for $SU(2)_c$ in Fig. 1. The entropy begins from $(\ln 3)/3$ and exponentially decreases with the temperature T. The second term has a remarkable dependence on T. For $T \to 0$, it has the value of $(2/3)\ln 3$, analogously to $SU(2)_c$. With increasing temperature, it rapidly goes to the asymptotic value $(\ln 3)/2+\pi\sqrt{3}/18$. Furthermore, we note that by decreasing the mass, it limits

[2] For the phase $\pi < \theta \leq \pi$, we define the complex variable as $\mathcal{Z} = \Re e Z \mathrm{e}^{\mathrm{i}\theta}$.

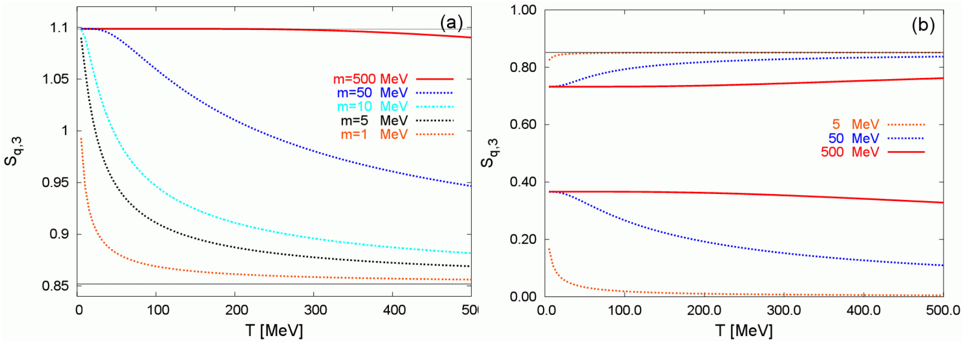

Fig. 2. The left panel shows the single-quark entropy change for various quark masses as a function of temperature T for $SU(3)$, Eq. (21). The right panel gives details on this behavior. We plot separately the two contributing terms of Eq. (21) for masses, 1, 50 and 500 MeV. *The curves in the bottom part of the figure represent only the first term.*

the range of temperature needed to reach this asymptotic region. Since the asymptotic value at high temperatures has remained more than three quarters of its ground-state value of $\ln 3$, there are still considerable correlations between two of the three color states. This observation points to an important fact about the structure of $SU(3)_c$ in these statistical models: the root structure forbids a complete cancellation of the real solutions which is needed to maintain the trace condition on the reduced density matrix.

Thus two states are always matched against one. Hence, for this model in the high-temperature limit, one color vanishes while the other two remain mixed and thereby correlated.

3.4. *Thermal mixing model*

The thermal effects cause an increase in the entropy when we take the three complex roots of $+1$, which are $\mathcal{R}_0 = +1$, $\mathcal{R}_1 = \exp \mathrm{i}2\pi/3$ and $\mathcal{R}_2 = \exp -\mathrm{i}2\pi/3$. The three eigenvalues are:

$$\lambda_0 = \frac{1}{3}\left(1 + \mathrm{e}^{-\epsilon(p)/T}\right),$$

$$\lambda_1 = \frac{1}{3}\left(1 + \mathrm{e}^{\mathrm{i}2\pi/3}\,\mathrm{e}^{-\epsilon(p)/T}\right), \tag{22}$$

$$\lambda_2 = \frac{1}{3}\left(1 + \mathrm{e}^{-\mathrm{i}2\pi/3}\,\mathrm{e}^{-\epsilon(p)/T}\right).$$

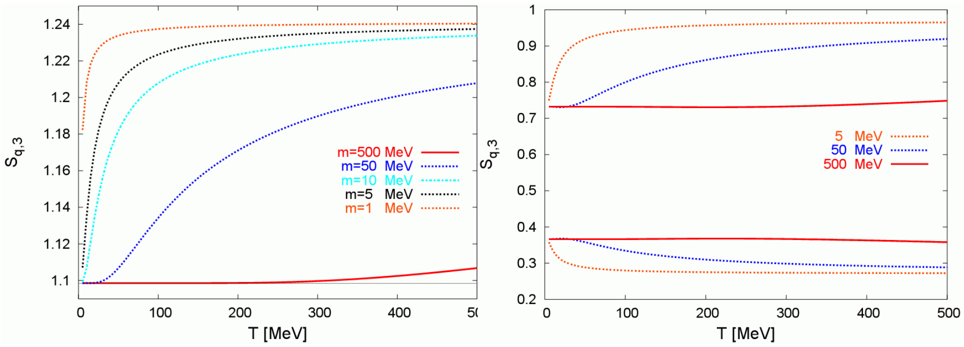

Fig. 3. The same as in Fig. 2 for the thermal mixing model.

The reduced quark density matrix for this solution given in Eq. (17) keeps the same form. Now the entropy takes the following form:

$$\begin{aligned} S_{q,3}(T) \quad &= \quad -\frac{1}{3}\left(1+\mathrm{e}^{-\epsilon(p)/T}\right)\, \ln\left[\frac{1}{3}\left(1+\mathrm{e}^{-\epsilon(p)/T}\right)\right] \\ & \quad -\frac{1}{3}\left(1+\mathrm{e}^{\mathrm{i}2\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\right)\, \ln\left[\frac{1}{3}\left(1+\mathrm{e}^{\mathrm{i}2\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\right)\right] \\ & \quad -\frac{1}{3}\left(1+\mathrm{e}^{-\mathrm{i}2\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\right)\, \ln\left[\frac{1}{3}\left(1+\mathrm{e}^{-\mathrm{i}2\pi/3}\ \mathrm{e}^{-\epsilon(p)/T}\right)\right] \end{aligned} \tag{23}$$

where

$$z = \frac{1}{3}\left[1-\mathrm{e}^{-\epsilon(p)/T}+\mathrm{e}^{-2\epsilon(p)/T}\right]^{1/2}, \tag{24}$$

and the phase is given by

$$\theta = \arctan\left(\frac{\sqrt{3}\ \mathrm{e}^{-\epsilon(p)/T}}{2-\mathrm{e}^{-\epsilon(p)/T}}\right). \tag{25}$$

After a little algebra, we find that the single-quark quantum entropy at finite temperature for $SU(3)$ reads

$$S_{q,3}(T) = -\frac{1}{3}\left(1+\mathrm{e}^{-\epsilon(p)/T}\right)\, \ln\left[\frac{1}{3}\left(1+\mathrm{e}^{-\epsilon(p)/T}\right)\right] - 2z\left[\ln(z)\cos(\theta)-\theta\sin(\theta)\right]. \tag{26}$$

Analytically, for large temperatures, z approaches $1/3$ and θ $\pi/3$. This means, the entropy approaches the asymptotic value $\ln 3-(2/3)\ln 2-\pi\sqrt{3}/9 \approx 1.239$.

The ground state favors the color singlet state with complete mixing. In these models, the high-temperature limit favors the octet states involving mostly two colors. This situation for $SU(3)_c$ can be contrasted with $SU(2)_c$ where the triplet and the singlet states have the same reduced density matrices except for the pure triplet states.

4. Spin models with strong correlations

Our objective in this section is to further investigate the structure of the entropy for some known spin models at finite temperature in which strong correlations exist. The general category of all spin models goes under the name of the Heisenberg model, which is central to the theory of magnetism [17]. In general, the Hamiltonian for these models involves the vector of spin matrices $\mathbf{s}(\mathbf{r}_i)$ for an electron at the position $\mathbf{r}_i$ coupled to another electron with the vector of spin matrices $\mathbf{s}(\mathbf{r}_j)$ at position $\mathbf{r}_j$

$$\mathcal{H} = -2\sum_{i<j} J_{ij}\ \mathbf{s}_i \cdot \mathbf{s}_j\ , \tag{27}$$

where J_{ij} is the exchange integral arising from the integration over the interaction containing the overlap of the spatial wavefunctions of the two electrons located at the two points $\mathbf{r}_i$ and $\mathbf{r}_j$. The relation of this exchange integral J_{ij} to the different spin directions $\mathbf{s}_i$ and $\mathbf{s}_j$ determines the local structure of the interaction.

4.1. Classical Ising spin chain

The simplest special case of the electron-electron interaction is the Ising nearest-neighbor chain interaction, for which only the spin matrices in the z-direction s_i^z ($\equiv \sigma^z$ Pauli spin matrix) and s_{i+1}^z are present. Then the interaction Hamiltonian in one dimension is simply

$$\mathcal{H}_{\text{Ising}} = -J\sum_i s_i^z s_{i+1}^z - h\sum_i s_i^z \tag{28}$$

where $J > 0$ is the simple nearest-neighbor constant coupling and h is the constant value of the external magnetic field. The interactions in the Ising model are completely classical since the spin operators s_i^z can be simply replaced by their diagonal values. The $SU(2)$ group structure ordinarily reduces to its centre $Z(2)$. Therefore, for the comparable anisotropic Heisenberg model, the quantum ground state has a maximum of two states. Thus each spin can equally well have one of the two directions: up and down. The one-dimensional entropy per spin at finite temperature for $h = 0$ is found to be

$$\frac{S(T)}{N} = \ln\left(1 + \mathrm{e}^{-2J/T}\right) + \frac{J}{T}\left[1 - \tanh\left(\frac{J}{T}\right)\right]. \tag{29}$$

We see these results in Fig. 4a, where we have purposely taken the energy scale for J to compare with $S_{q,2}(T)$ in Fig. 1a. It is clear that the entropy vanishes for low temperatures. However, in the high-temperature limit, $S \to \ln 2$. Actually, these limiting cases mean that the low temperature value of the Ising model does not properly reflect the ground-state structure of the full $SU(2)$ symmetry group. In $SU(2)$, there are two eigenstates at zero temperature, which lead to quantum

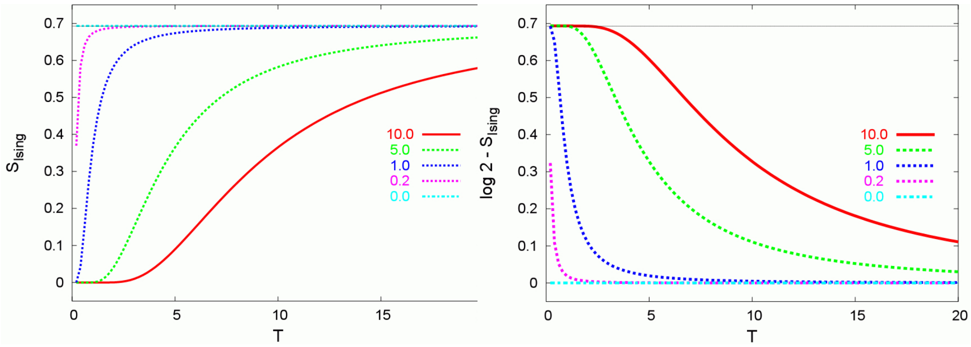

Fig. 4. The entropy for 1D Ising model without an external field calculated from $\mathcal{H}_{\rm HIsing}$ is shown in terms of the temperature T for different values of the exchange coupling $J > 0$. In the right panel, we graph the entropy difference from the $SU(2)$ ground-state entropy, $\ln 2$. This entropy difference (Eq. (32)) represents the ground-state entropy for the anisotropic Heisenberg chain at finite temperature.

entropy equals to $\ln 2$. Therefore, the results in Fig. 4a indicate the fact that the ground-state structure in the Ising model is not properly included in the partition function. It is clear that the deviation from the usual quantum $SU(2)$ structure is strongly dependent on the exchange coupling, $J > 0$. Thus for all temperatures in the limit of vanishing coupling, the entropy per spin is always equal to $\ln 2$.

When we make a comparison with our models given in Sect. 3.1, we are forced into the arbitrary inclusion of an additional part of the entropy related to the ground-state structure in the corresponding quantum model. Therefore, we recall the structure of the anisotropic Heisenberg model, for which we can write down a wavefunction for the strongly correlated spin matrices σ^z at $T = 0$. Thus the density matrix with $J > 0$ is written in terms of the two basis states $|0\rangle$ *and* $|1\rangle$. The anisotropic Heisenberg model in its ferromagnetic ground state with the two spin directions $|\uparrow\rangle$ and $|\downarrow\rangle$, respectively, has the density matrix given by

$$\rho = \sum |\Psi\rangle P_\Psi \langle\Psi| = \frac{1}{\sqrt{2}}\big(|0\rangle|0\rangle \pm |1\rangle|1\rangle\big)\frac{1}{\sqrt{2}}\big(\langle 0|\langle 0| \pm \langle 1|\langle 1|\big)\,, \tag{30}$$

where the $\pm$ contains the symmetrical and antisymmetrical combinations of the completely alligned two-spin states. P_Ψ is the probability of each state . By taking the partial trace Tr_p over Eq. (30), we get the reduced density matrix ρ^r by projecting out only the second components related to the neighboring spins, which yields

$$\rho^r = \mathrm{Tr}_p\, \rho = \frac{1}{2}\big(|0\rangle\langle 0| + |1\rangle\langle 0|\big)\,. \tag{31}$$

Obviously, the probability of each spin direction is such that the two eigenvalues from the reduced density matrix, Eq. (31), are identical with each (other?) given by $\lambda = 1/2$. We set this result into Eq. (3), which yields that the entropy of a single spin in this model at zero temperature is $S = \ln 2$. In the calculation of this value, we have assumed the small coupling limit. Thereby we considered only the spin *eigen*fluctuations in the ground state which are treated as quantum states.

As we have done in Sect. 3.1 (Eq. (10)), we subtract the temperature-dependent entropy from this value. This expression results in the temperature dependence of the quantum ground state entropy of the anisotropic Heisenberg model in the following form

$$S(T) \equiv \ln 2 - S_{\rm Ising}(T)\,. \tag{32}$$

These results are shown in Fig. 4b. At low temperatures, the entropy starts from the value ln 2. With increasing temperature, the ground state entropy of the model monotonically decreases. It reaches its asymptotic value of zero at high temperatures. Qualitatively, we got the same results for our models given in Sect. 3.1, which are graphically illustrated in Figs. 1 and 2. We clearly left arbitrary the units of the energy density in the Boltzmann term in Eq. (10) and of the exchange coupling in Eq. (29). Thus we are able to find a comparable quantum model by taking a special anisotropic case of the more general Heisenberg model.

4.2. *Quantum XY spin chain*

Another exactly solvable model directly related to the Heisenberg model is the XY model [18, 19]. In one spatial dimension, it is a highly-correlated quantum-mechanical system also in the ground state [20]. Furthermore, it is known [7, 20, 21] that the XY model in the ground state has a very complex structure containing an oscillating region for small values of h and γ, in addition to the expected ferromagnetic and paramagnetic phases for larger values of these parameters. Also, for a class of time-dependent magnetic fields, the magnetization has been shown to be nonergodic [20]. Recently, the correlations of the spins have been calculated using the reduced density matrix [7] in order to find the ground state entropy for a block of spins, yielding the expected logarithmic behavior. The Hamiltonian can be written in terms of the spin components in the x and y spin directions with an external field in the z-direction

$$\mathcal{H}_{XY} = \frac{1}{2}\sum_i \left[\big(1+\gamma\big) s_i^x s_{i+1}^x\big) + \big(1-\gamma\big) s_i^y s_{i+1}^y\big) - h \sum_i s_i^z \tag{33}$$

where γ is the spin anisotropy parameter in the x and y spin directions. The exact solution of this model has been found long ago [18, 19]. The spins are allowed to take an arbitrary angle, $k \in [0, 2\pi]$, with cyclic boundary conditions. By using the fermionic variables, $a_k^\dagger$, a_k, the Hamiltonian[3] may be written as

$$H = \sum_k \Lambda_k \left(a_k^\dagger a_k - \frac{1}{2}\right) \tag{34}$$

[3]The positive sign of Hamiltonian refers to attractive nearest-neighbor spin interactions. By rotating the chain along the spin z direction to every second spin, one can flip this sign [7].

$$\Lambda_k^2 \gamma^2 = (h-1)^2 + 4(h-1+\gamma^2)\sin^2\left(\frac{k}{2}\right) - 4(\gamma^2-1)\sin^4\left(\frac{k}{2}\right). \tag{35}$$

Then the entropy per spin site at finite temperature reads

$$\frac{S_\gamma(T)}{N} = \frac{1}{2\pi}\left[\int_0^{\pi/2} \mathrm{d}k\ \ln(1+\mathrm{e}^{-(\Lambda_k-\mu)/T}) + \frac{1}{T}\int_0^{\pi/2} \mathrm{d}k\ \frac{(\mu-\Lambda_k)\mathrm{e}^{\Lambda_k/T}}{\mathrm{e}^{\Lambda_k/T}+\mathrm{e}^{\mu/T}}\right]. \tag{36}$$

The entropy per spin as given in Eq. (36) represents the change of the mixing of the finite temperature state from the ground state due to the thermal effects. Since the quantum Ising model with $\gamma = 1$ has always the smallest value of S_{XY}, the effects of spin mixing disappear at the lower temperature. The value of the chemical potential μ is set equal to the Fermi energy calculated by integrating Λ_k over k, which is strictly valid only at zero temperature. From these plots we see

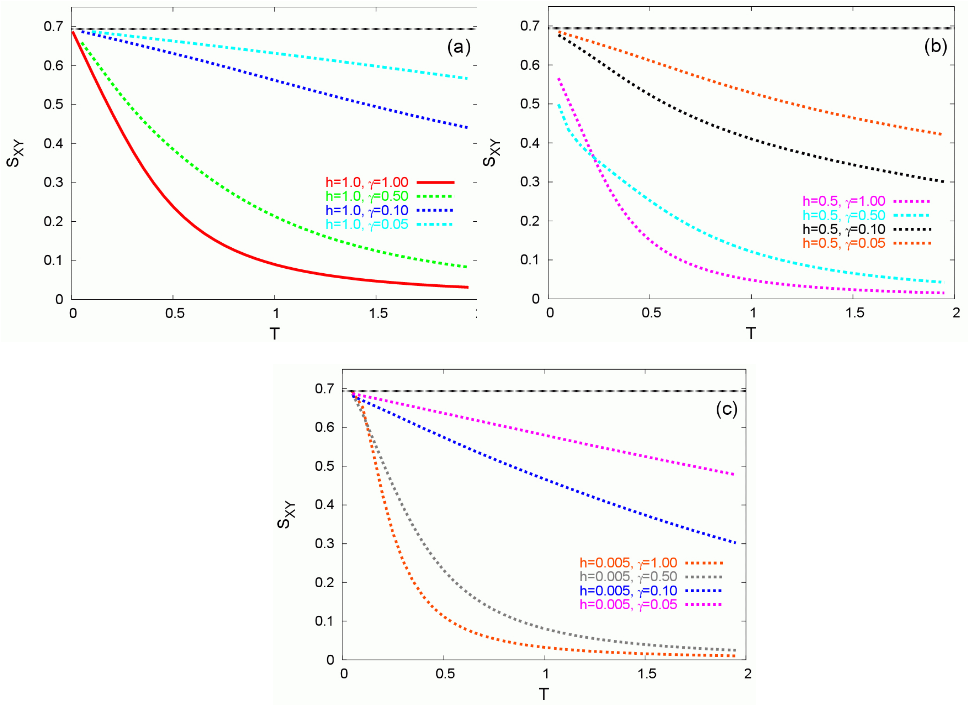

Fig. 5. Entropy per spin site is calculated for different temperatures in the XY-model. (a) Gives the results for constant $h = 1.0$, but different γ values. We note that with increasing T, the entropy S decreases exponentially, and the temperature range within which $S \to 0$ becomes broader with smaller γ values. Also, by deceasing γ, value the entropy increases. Then the system favors going toward its ground state value, $\ln 2$. Almost the same behavior can be seen in the plots (b) for $h = 0.5$ and (c) for $h = 0.005$.

that S_{XY} yields results analogous to our models for $SU(2)_c$ and $SU(3)_c$ discussed in Sect. 3, depending on the units of energy density, coupling, and the parameters h and γ. In Fig. 5, we plot the entropy per spin site against the temperature T for different values of h and γ. These results for the XY model, shown in Fig. 5, can be compared with those of the Ising model in Fig. 4 in so far as we include the dependence on the given parameters. The necessary conditions are that $\gamma = 1$ and $h \to 0$. We should notice the arbitrary units in both figures. In the limit $\gamma \to 0$, the value of the ground state entropy tends to stay constant at all temperatures. The same results are found in 1D Ising model when we consider the entropy difference in the limit $J \to 0$. Thus, we can conclude that the ground state entropy for the XY spin chain are qualitatively comparable with our models for the $SU(2)_c$ and $SU(3)_c$.

5. *Discussion*

In this work, we have calculated the entropy for the particular cases related to colored quark states in the hadron singlet and octet structures. From these calculations, we extended our considerations to finite temperatures. According to the third law of thermodynamics, the pure states, such as completely specified enclosed hadronic systems, possess a vanishing entropy at very low temperatures. Therefore, the hadron bags, as macroscopic isolated objects, are expected to have zero entropy at a vanishing temperature. However, if we further consider the *microscopic* hadron in terms of *subsystems* at low temperatures ranging up to the size of the quark mass, the quantum entropy is seen to have a finite value. Nevertheless, it is still a curious fact that the entropy of mixing is usually simply ignored or approximately taken to be zero even for the classical systems, even though it had been already recognized since the late nineteenth century [3]. Afterwards, it was confirmed by the laws of quantization of statistical states for the thermodynamics early in the last century [2, 4, 5]. Therefore, we think that there is no compelling reason to assume that the value of the ground-state entropy of a quantum or even classical *subsystems* is zero, just because one believes that the system to which they are assigned is enclosed and thereby must have a vanishing entropy. The *subsystems* have other degrees of mixing and thereby a finite entropy, even if the whole closed system is isolated and consequently has a vanishing entropy. Nevertheless, the ground-state entropy is important and, therefore, should be taken into consideration at low temperatures. As we have seen from our calculations, the upper region of the temperature of the importance for the ground-state entropy is generally characterized by the quark mass. For the case of the light quarks, when we are interested in such properties as the QCD phase transition or the structure of quark matter at very high temperatures, the ground state entropy of the colored quarks has no longer any significance. On the other hand, in the interior of compact stellar objects, cold dense quark matter is highly probable. In this case, the ground-state entropy would play an important role, especially, in the understanding of properties of superconductivity in cold dense quark matter [22]. The quantum entropy could be very important for the phase transition from nuclear matter to quark matter in the hybrid stars, which may determine the stability and structure of these compact stars [23, 24].

The quark and antiquark states mathematically build up the Schmidt decompositions of meson state, where the Schmidt numbers simply represent the normalization of their wavefunctions. Furthermore, from the quantum teleportation, we know that the colored quarks and antiquarks can be considered as mutual purifications for each other. Each singlet state is equally weighted in the decomposition of meson states, from which each state possesses an equal probability. On the other hand, each singlet-quark color state has the same degree of mixing. Therefore, each quark or antiquark has a finite entropy even though the meson state, as a pure isolated state, must have zero entropy. In particular for the baryon states, we must use the doubly-reduced density matrix, for which each single-quark state appears twice in the six contributing terms. The totally-reduced density matrix gives the effect of the spatial mixing of each of the single subsystems, here the quarks or the antiquarks. Then the resulting quantum entropy at zero temperature gives the maximal quantum entropy for the completely mixed states.

As we have seen above, the octet states are much more complex than the singlet states. This is true because the octets have many more and quite differently structured states. After we had looked at the Pauli matrices for $SU(2)_c$, we realized that the eighth Gell-Mann matrix, λ_8, includes other states with a different weighting. Thus for the octet generally, no single state can be considered to be a purification of the other states. At finite temperature, the octet structure becomes even more complex. The reason for this is that with increasing temperature, the correlations between the states become stronger. At very high temperatures, the correlated states reach their asymptotic value. Thus at high temperatures, the states of the Pauli type become more probable. Furthermore, the pure singlet ground state becomes no longer possible. Thus, the unoccupied states become more available at high temperatures.

We have postulated simple models for the thermal dependence of the ground state entropy. We saw that the singlet state is a completely mixed state with the maximum value of the entropy given by $\ln 3$ at vanishing temperature. We have used a Boltzmann-like factor for the thermal dependence of the entropy. With increasing temperature, the change of the ground state entropy still continues. For $SU(2)_c$ at high temperatures, the mixing and consequently the entropy vanish entirely. We have noted that by lowering the values of the quark masses, the range of temperatures limits the asymptotic region. However, for $SU(3)_c$, we see that the asymptotic value at high temperatures for the unmixing model has retained more than three quarters of its ground state value of $\ln 3$. Therefore, there are still considerable correlations between two of the three color states. This observation comes from the complex structure of $SU(3)_c$. Obviously, in this case, the root structure forbids a complete cancellation of the real solutions which is needed to maintain the trace condition on the reduced density matrix. Thus two states are always matched against one, which forms the subsystem. Hence, in the high-temperature limit, one color state vanishes, while the other two remain mixed and thereby correlated. We can conclude that ground state favors the color singlet state with complete mixing.

$SU(3)_c$ in the high-temperature limit favors the octet states involving in most cases only two colors. This situation for $SU(3)_c$ can be contrasted with $SU(2)_c$, where the triplet and the singlet states have the same reduced density matrices, except for the pure triplet states. However, such phenomena do not happen in the thermal mixing model. In this case, the thermal effects lead to further mixing of all color states. This fact is clear from Fig. 3, where the entropy steadily increases from the ground state value $\ln 3$ to a larger asymptotic value.

In order to further study a model for the ground state structure in relation to the $SU(2)_c$ entropy, we utilized some known *classical* and *quantum* spin chains, which also have strong correlations at finite temperature. We investigated the structure of the Hamiltonians at finite temperature. The simplest and most widely used *classical* spin model is the one-dimensional Ising model. Another solvable model which is related to the anisotropical one-dimensional Heisenberg model is the XY spin chain. It is a highly-correlated quantum-mechanical system in the ground state. One of the most valuable results we have gotten from our investigation here is that the classical spin model is not able to successfully describe the ground state. The resulting entropy, calculated directly from the partition function, is simply zero at zero temperature. Thus we plot the entropy differences of the finite temperature states to the ground state. If we additionally include thermal terms similar to those in our models for $SU(2)_c$ and $SU(3)_c$, the entropy starts from zero and monotonically increases up to the asymptotic value of $\ln 2$. Also, we have found that this asymptotic limit does not depend on the spatial dimension. Particularly with regard to the XY model, it has a nontrivial structure, depending on the parameters of the spin asymmetry and external field, which has been quite recently investigated through the correlation functions in the ground state [7].

6. Conclusions and outlook

In this work we have compared the quantum definitions as contrasted to the classical concepts of entropy in relation to the temperature. We have noticed that, generally, the quantum definition is important in the low-temperature limit, while the classical concepts usually relate to higher temperatures. First, we have discussed the ground-state entropy, which is strictly a quantum definition and does not itself appear in classical physics. We have evaluated and contrasted the symmetry structure for both the $SU(2)_c$ and $SU(3)_c$ color groups. For these symmetries, we have used simple thermodynamical models involving the color ground state entropy to show how the quantum mixing entropy disappears for the entropy differences with increasing temperature. From this result, we have also discussed how this disappearance can give rise to new pure states in the high-temperature limit.

One motivation for this work has been to study the correlation between the quarks and antiquarks. In recent QCD lattice simulations, one can find indications of ground-state behavior at short distances and low temperatures [10, 11, 12]. These results could very well relate to the entropy we described above, which is currently under investigation in relation to certain models [25]. As a next step, we want to look further into the thermodynamical properties of these correlations in relation to the present study [26]. There are many applications of the finite entropy of colored quarks at zero and very low temperatures much below the temperature of QCD phase transition from hadron to quark-gluon plasma [27, 28]. This endeavor could help demonstrate the usefulness of the quantum entropy in the description of the thermal properties of the strong interaction at very low temperatures. In this sense, we have already considered the effects of the finite value of the quantum entropy at low temperature on the pressure inside the hadron bag [8]. Furthermore, in forthcoming works we want to include the effects of the gluons and the chiral symmetry in relation to the quantum entropy. In relation to the correlations, we have studied the effects of the quantum entropy on the condensates of quark pairs with strong correlations and at very low temperatures and very high quark chemical potentials [15, 29].

Thus we have seen in several different models how the usual thermodynamical entropy, gotten from the evaluation of the partition function, acts as a means of disorganizing the ground state. It undoes the entanglement in some cases completely and in others only partially. Therefore, it has the effect of lowering the correlations between the quarks and antiquarks seen in the ground state.

Acknowledgements

The authors would like to thank Frithjof Karsch, Krzysztof Redlich and Helmut Satz for the very helpful discussions. D.E.M. is very grateful to the Pennsylvania State University Hazleton for the sabbatical leave of absence and to the Fakultät für Physik der Universität Bielefeld.

References

[1] R. Balian, M. Vénéroni and N. Balazs, Europhys. Lett. **1** (1986) 1.

[2] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, Springer Verlag, Berlin (1932).

[3] M. Planck, *Vorlesungen über Thermodynamik*, Walter De Gruyter and Co., Berlin (1964).

[4] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics*, Vol. 3, *Quantum Mechanics* and Vol. 5, *Statistical Physics*, Pergamon Press, Oxford (1987).

[5] E. Schrödinger,*Statistical Thermodynamics*, Cambridge University Press, Cambridge (1946).

[6] D. E. Miller, *Entropy for $SU(3)_c$ Quark States*, arXiv:hep-ph/0306302, Eur. Phys. J. C **34** (2004) 435.

[7] G. Vidal, J. I. Latorre, R. Rico, and A. Kitaev, Phys. Rev. Lett. **90** (2003) 227902; J. I. Latorre, R. Rico and G. Vidal, Quant. Inf. Comput. **4** (2004) 48; arXiv:quant-ph/0304098.

[8] D. E. Miller and A. Tawfik, *The Effects of Colored Quark Entropy on the Bag Pressure*, arXiv:hep-ph/0309139, J. Phys. G **30** (2004) 371.

[9] R. P. Feynman, *Statistical Physics*, eds. R. Kikuchi, H. A. Feiveson and J. Shahan, Benjamin Inc., Reading, Massachusetts (1972).

[10] F. Zantow, O. Kaczmarek, F. Karsch and P. Petreczky, *Workshop on Strong and Electroweak Matter (SEWM 2002)*, Heidelberg, Germany, 2-5 Oct 2002.

[11] O. Kaczmarek, S. Ejiri, F. Karsch, E. Laermann and F. Zantow, *Heavy Quark Free Energies and the Renormalized Polyakov Loop in full QCD*, Prog. Theor. Phys. Suppl. **153** (2004) 287; hep-lat/0312015.

[12] F. Zantow, Doctoral Thesis, University of Bielefeld, 2004.

[13] J. F. Donoghue, E. Golovich and B. R. Holstein, *Dynamics of the Standard Model*, Cambridge University Press, Cambridge (1992).

[14] H. Th. Elze, Nucl. Phys. B **436** (1995) 213; Nucl. Phys. Proc.Suppl. BC **39** (1995) 169; Phys. Lett. B **369** (1996) 295.

[15] D. E. Miller and A. Tawfik, *Entanglement on Condensates Involving Strong Interactions*, arXiv:hep-ph/0312368.

[16] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, Cambridge University Press, Cambridge (2000).

[17] K. Yosida, *Theory of Magnetism*, Springer-Verlag, Berlin (1996).

[18] E. Lieb, T. Schultz and D. Mattis, Ann. Phys. **16** (1961) 407.

[19] S. Katsura, Phys. Rev. **127** (1962) 1508.

[20] E. Barouch, B. M. McCoy and M. Dresden, Phys. Rev. A **2** (1970) 1075; E. Barouch and B. M. McCoy, Phys. Rev. A **3** (1971) 786.

[21] M. Henkel, *Conformal Invariance and Critical Phenomena*, Springer- Verlag, Berlin (1999).

[22] D. H. Rischke and R. D. Pisarski, Color Superconductivity in Cold, Dense Quark Matter, arXiv:nucl-th/0004016.

[23] D. Blaschke, H. Grigorian, D. N. Aguilera, S. Yasui and H. Toki, *Effects of Quark Matter and Color Superconductivity in Compact Stars*, AIP Conf. Proc. **660** (2003) 209; arXiv:hepph/0301087.

[24] M. Buballa, F. Neumann, M. Oertel and I. Shovkovy, *Quark Mass Effects on the Stability of Hybrid Stars*, Phys. Lett. B **595** (2004) 36; arXiv:hep-ph/0311078.

[25] F. Karsch, D. Miller and A. Tawfik, *Finite Temperature Entropy from Static Quark-Antiquark Potentials*, in preparation.

[26] D. E. Miller and A. Tawfik, in preparation

[27] F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. C **29** (2003) 549.

[28] F. Karsch, K. Redlich and A. Tawfik, Phys. Lett. B **571** (2003) 67.

[29] D. E. Miller and A. Tawfik, *Entropy for Color Superconductivity in Quark Matter*, Acta Phys. Pol. B **35** (2004) 2165; arXiv:hep-ph/0405175.

ENTROPIJA STANJA OBOJENIH KVARKOVA NA KONAČNOJ TEMPERATURI

Analiziramo kvantnu entropiju na konačnim temperaturama proučavajući modele obojenih kvarkova koji grade fizička stanja hadrona. Podrobno razlažemo posebne modele za strukturu stanja $SU(2)_c$ i $SU(3)_c$ koji tumače učinke temperature na kvantnu entropiju. Pokazujemo da se entropija singletnih stanja monotono mijenja s temperaturom. Međutim, struktura tripletnih stanja znatno je zamršenija, što se najbolje može prikazati dvama opisima, jednom koji je sličan opisu singletnih stanja, i drugom koji odražava pojavu snažnih korelacija samo među dvama stanjima boje. Radi usporedbe, računamo entropiju za *klasični* Isingov model i *kvantnih* XY spinskih lanaca. U Isingovom modelu, kvantna entropija osnovnog stanja ne ulazi izravno u particijsku funkciju. Ona također ne ovisi o broju prostornim dimenzija, već samo o broju kvantnih stanja koja tvore osnovno stanje. Ali XY spinski lanac zadržava konačnu entropiju ako temperatura iščezava. Ako se uključi osnovno stanje, ishodi spinskih modela slični su našim modelima za $SU(2)_c$ i $SU(3)_c$.