

## FAMILY STRUCTURE OF LEPTONS AND THEIR CURRENTS OF AN AXIAL-VECTOR NATURE

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All neutrinos have a non-zero mass and regardless of whether a neutrino is of the Dirac or Majorana type, it can possess both anapole and electric dipole moments. Between the corresponding form factors there appears a connection, for example, for neutrino scattering on spinless nuclei. We discuss a theory in which a mass consists of vector and axial-vector components responsible for separation of leptonic current into the vector and axial-vector parts of the same charge or dipole moment. Such a model can explain the absence of truly neutral neutrino vector interactions and the availability of an axial-vector structure of a Majorana mass. Thereby, it relates the two neutrinos of different nature. We derive an equation which relates the masses to a ratio of the anapole and electric dipole form factors of any charged lepton and its neutrino as a consequence of their unification in families of doublets and singlets. This testifies in favour of the existence of the left (right) dileptons and paradileptons of the axial-vector currents. Each of them answers to conservation of an axial-vector charge and any lepton flavour. Therefore, an axial-vector mass, anapole and electric dipole moment of the neutrino become proportional, respectively, to an axial-vector mass, anapole and electric dipole moment of a particle of the same family.

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### 1. Introduction

The nature has been created so that to any type of charged lepton corresponds a kind of neutrino [1]. Such pairs can constitute the leptonic families of the left-handed  $SU(2)_L$ -doublets as well as of the right-handed  $SU(2)_R$ -singlets. This gives the right to define their family structure in the united form [2]:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, (\nu_e, e^-)_R, \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, (\nu_\mu, \mu^-)_R, \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, (\nu_\tau, \tau^-)_R, \dots, \quad (1)$$

$$\begin{pmatrix} \bar{\nu}_e \\ e^+ \end{pmatrix}_R, (\bar{\nu}_e, e^+)_L, \begin{pmatrix} \bar{\nu}_\mu \\ \mu^+ \end{pmatrix}_R, (\bar{\nu}_\mu, \mu^+)_L, \begin{pmatrix} \bar{\nu}_\tau \\ \tau^+ \end{pmatrix}_R, (\bar{\nu}_\tau, \tau^+)_L, \dots \quad (2)$$

Each family distinguishes from others by an individual flavour [3, 4]. There exist, therefore, the three ( $l = e, \mu, \tau$ ) lepton flavours:

$$L_l = \begin{cases} +1 & \text{for } l_L^-, l_R^-, \nu_{lL}, \nu_{lR}, \\ -1 & \text{for } l_R^+, l_L^+, \bar{\nu}_{lR}, \bar{\nu}_{lL}, \\ 0 & \text{for remaining particles.} \end{cases} \quad (3)$$

Conservation of all types of lepton flavours

$$L_l = \text{const}, \quad (4)$$

or full lepton number

$$L_e + L_\mu + L_\tau = \text{const} \quad (5)$$

is practically not excluded [5]. Its legality follows from the fact that both (4) and (5) become possible owing to a formation of the united dileptons

$$(l_L^-, \bar{\nu}_{lR}), (l_R^-, \bar{\nu}_{lL}), \quad (6)$$

$$(l_R^+, \nu_{lL}), (l_L^+, \nu_{lR}) \quad (7)$$

and paradileptons

$$\{(l_L^-, \bar{\nu}_{lR}), (l_R^+, \nu_{lL})\}, \{(l_R^-, \bar{\nu}_{lL}), (l_L^+, \nu_{lR})\} \quad (8)$$

of the vector nature [2]. For example, in the  $\beta$ -decays

$$n \rightarrow p^\pm e^\mp \bar{\nu}_e(\nu_e), \quad p^\pm \rightarrow n e^\pm \nu_e(\bar{\nu}_e), \quad (9)$$

$$\mu^\mp \rightarrow e^\mp \bar{\nu}_e(\nu_e) \nu_\mu(\bar{\nu}_\mu), \quad \tau^\mp \rightarrow e^\mp \bar{\nu}_e(\nu_e) \nu_\tau(\bar{\nu}_\tau), \quad (10)$$

as well as in other phenomena with vector currents. Such systems can also explain the conservation of summed charge.

According to the theory of unification of fermions [2], the mass  $m_{\nu_l}$ , charge  $e_{\nu_l}$  and vector moment  $\mu_{\nu_l}$  of the neutrino are proportional, respectively, to the mass  $m_l$ , charge  $e_l$  and vector moment  $\mu_l$  of a particle of the same leptonic family

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_e : m_\mu : m_\tau, \quad (11)$$

$$e_{\nu_e} : e_{\nu_\mu} : e_{\nu_\tau} = e_e : e_\mu : e_\tau, \quad (12)$$

$$\mu_{\nu_e} : \mu_{\nu_\mu} : \mu_{\nu_\tau} = \mu_e : \mu_\mu : \mu_\tau. \quad (13)$$

However, it is known [6, 7] that the interaction of leptons with virtual photon may be described by the vertex operator

$$\Gamma_\mu(p, p') = \Gamma_\mu^V(p, p') + \Gamma_\mu^A(p, p'), \quad (14)$$

including both the vector ( $V$ ) and axial-vector ( $A$ ) parts:

$$\Gamma_\mu^V(p, p') = \bar{u}(p', s')[\gamma_\mu f_{1l}(q^2) - i\sigma_{\mu\lambda} q_\lambda f_{2l}(q^2)]u(p, s), \quad (15)$$

$$\Gamma_\mu^A(p, p') = \bar{u}(p', s')\gamma_5[\gamma_\mu g_{1l}(q^2) - i\sigma_{\mu\lambda} q_\lambda g_{2l}(q^2)]u(p, s). \quad (16)$$

Here  $\sigma_{\mu\lambda} = [\gamma_\mu, \gamma_\lambda]/2$ ,  $q = p - p'$ ,  $p(s)$  and  $p'(s')$  denote the four-momentum (helicity) of the neutrino before and after the interaction. The functions  $f_{1l}(q^2)$  and  $f_{2l}(q^2)$  at  $q^2 = 0$  give the full electric charge and vector dipole moment of this particle [8]

$$e_l = f_{1l}(0), \quad \mu_l = f_{2l}(0), \quad (17)$$

on which there exist earlier and comparatively new laboratory and astrophysical restrictions [9, 10]. The value of  $e_l$  for the lepton (antilepton) and its neutrino (antineutrino) has the negative (positive) sign.

The values of  $g_{1l}(0)$  and  $g_{2l}(0)$  define the static size of the neutrino anapole [11] and electric dipole moments:

$$a_l = \frac{1}{m_l} \left( \frac{g_{1l}(0)}{f_{1l}(0)} \right)^2 f_{2l}(0), \quad d_l = g_{2l}(0). \quad (18)$$

Of them  $a_l$  can also be measured [12], and for  $d_l$  some experimental and cosmological limits are known [9, 10].

In the framework of the recent presentations about the nature of these currents,  $g_{1l}(q^2)$  must be CP-symmetrical, but P-antisymmetrical [11]. The function  $g_{2l}(q^2)$  is C-even, but CP-odd [13]. The existence of  $g_{1l}(q^2)$  is incompatible with the gauge invariance. In other words, the terms  $g_{il}(q^2)$  can exist only in the case when the mirror symmetry is violated in the absence of gauge invariance. This possibility is realized if a particle has a self inertial mass [14].

A massive Dirac ( $l = \nu_D$ ) neutrino can, therefore, possess each of the discussed currents [14, 15, 16]. Their structure at  $e = |e|$  for the light ( $\nu_D = \nu_e$ ) neutrino in one-loop approximation has the form

$$f_{1\nu_D}(0) = \frac{3eG_F m_{\nu_D}^2}{4\pi^2\sqrt{2}}, \quad f_{2\nu_D}(0) = \frac{3eG_F m_{\nu_D}}{8\pi^2\sqrt{2}}, \quad (19)$$

$$g_{1\nu_D}(0) = \frac{3eG_F m_{\nu_D}^2}{4\pi^2\sqrt{2}}, \quad g_{2\nu_D}(0) = \frac{3eG_F m_{\nu_D}}{4\pi^2\sqrt{2}}. \quad (20)$$

From the general considerations, it follows that  $f_{il}(q^2)$  and  $g_{il}(q^2)$  describe the neutrino vector and axial-vector form factors. If this is so, the standard  $SU(2)_L \otimes U(1)$  theory [17] states that the functions  $f_{il}(q^2)$  and  $g_{il}(q^2)$  are proportional to  $g_{V_i}$  and  $g_{A_i}$ , namely, to the coupling constants of the neutrino vector and axial-vector currents.

From such a point of view, each of (19) and (20) takes place only in the framework of the  $(V - A)$  model, in which

$$f_{il}(q^2) \rightarrow \frac{1}{g_{V_i}} f_{il}(q^2) = f_{il}(q^2), \quad (21)$$

$$g_{il}(q^2) \rightarrow \frac{1}{g_{A_i}} g_{il}(q^2) = g_{il}(q^2), \quad (22)$$

where and further  $g_{V_i} = g_{A_i} = 1$ . Therefore, the form factors  $f_{1l}(0)$  and  $g_{1l}(0)$  must be identical [18] parameters of the  $(V - A)$  theory of a massive neutrino [19, 20]. The account of the latter leads us to the conclusion that

$$g_{1l}(0) - f_{1l}(0) = 0. \quad (23)$$

Insofar as a neutrino [21] of the Majorana type ( $l = \nu_M$ ) is concerned, we start from the fact [22] that it has no vector interaction ( $g_{V_{\nu_M}} = 0$ ), and its axial-vector interaction is stronger ( $g_{A_{\nu_M}} = 2g_{A_{\nu_D}}$ ) than of a Dirac fermion. At the same time, with the aid of (22), it is not difficult to see [14, 23] that truly neutral neutrino ( $\nu_M = \nu_1$ ) anapole and electric dipole moments are, according to (20), equal to

$$g_{1\nu_M}(0) = \frac{3eG_F m_{\nu_M}^2}{2\pi^2\sqrt{2}}, \quad g_{2\nu_M}(0) = \frac{3eG_F m_{\nu_M}}{2\pi^2\sqrt{2}}. \quad (24)$$

Such a conclusion one can make by investigating the processes with nuclei in which appear the most diverse relations between the properties of Dirac and Majorana neutrinos. Their nature, as we shall see below, gives the possibility to define the structure of masses of neutrinos of both types.

It is also relevant to include in the discussion the united dependence of the axial-vector form factors of leptons of the same family. This allows to elucidate the ideas of each of the laws of conservation of summed charge, lepton flavours and full lepton number.

The above questions will be illuminated in this work by studying the behavior of light neutrinos of a different nature and of electrons in the elastic axial-vector scattering on a spinless nucleus as a consequence of the availability of rest mass, anapole and electric dipole moments of elementary particles and their longitudinal polarization.

## 2. Relation of axial-vector currents of neutrino and electron

The amplitude of polarized massive Dirac and Majorana fermions scattering by nuclei in the limit of one-photon exchange may be written as

$$M_{fi}^E = \frac{4\pi\alpha}{q^2} \bar{u}(p', s') \gamma_5 [\gamma_\mu g_{1l}(q^2) - i\sigma_{\mu\lambda} q_\lambda g_{2l}(q^2)] u(p, s) \langle f | J_\mu^\gamma(q) | i \rangle. \quad (25)$$

Here  $l = e = e_{L,R}$ ,  $\nu = \nu_D = \nu_e = \nu_{eL,R}$  or  $\nu_M = \nu_1 = \nu_{1L,R}$ , and  $J_\mu^\gamma$  denotes the Coulomb current of a nucleus [24].

According to (25), the interaction cross section of longitudinal neutrinos and electrons with the field of a spinless nucleus has the following structure

$$\frac{d\sigma_E^{A_l}(\theta_l, s, s')}{d\Omega} = \frac{1}{2} \sigma_o^l \{ (1 + ss') g_{1l}^2 + 4m_l^2 \eta_l^{-2} (1 - ss') g_{2l}^2 \operatorname{tg}^2 \frac{\theta_l}{2} \} F_E^2(q^2), \quad (26)$$

in which

$$\sigma_o^l = \frac{\alpha^2 \cos^2(\theta_l/2)}{4E_l^2 (1 - \eta_l^2) \sin^4(\theta_l/2)}, \quad \eta_l = \frac{m_l}{E_l},$$

$$E_l = \sqrt{p^2 + m_l^2}, \quad F_E(q^2) = ZF_c(q^2),$$

$$q^2 = -4E_l^2 (1 - \eta_l^2) \sin^2 \frac{\theta_l}{2}.$$

Here  $\theta_l$  is the axial-vector scattering angle,  $E_l$  is the particle energy,  $F_c(q^2)$  is the nuclear charge form factor ( $F_c(0) = 1$ ). The index  $A_l$  indicates the absence of the neutrino vector currents.

The terms  $(1 + ss')$  and  $(1 - ss')$  describe the axial-vector Coulomb interactions of the left ( $s = -1$ ) - and right ( $s = +1$ )-handed neutrinos leading to the scattering with ( $s' = -s$ ) or without ( $s' = s$ ) flip of their spin. Taking this into account, we can write (26) as

$$d\sigma_E^{A_l}(\theta_l, s) = d\sigma_E^{A_l}(\theta_l, g_{1l}, s) + d\sigma_E^{A_l}(\theta_l, g_{2l}, s), \quad (27)$$

$$\frac{d\sigma_E^{A_l}(\theta_l, g_{1l}, s)}{d\Omega} = \frac{d\sigma_E^{A_l}(\theta_l, g_{1l}, s' = s)}{d\Omega} = \sigma_o^l g_{1l}^2 F_E^2(q^2), \quad (28)$$

$$\frac{d\sigma_E^{A_l}(\theta_l, g_{2l}, s)}{d\Omega} = \frac{d\sigma_E^{A_l}(\theta_l, g_{2l}, s' = -s)}{d\Omega} = 4m_l^2 \eta_l^{-2} \sigma_o^l g_{2l}^2 F_E^2(q^2) \operatorname{tg}^2 \frac{\theta_l}{2}. \quad (29)$$

Averaging over  $s$  and summing over  $s'$ , one can reduce the cross section (26) to the following form

$$d\sigma_E^{A_i}(\theta_l) = d\sigma_E^{A_i}(\theta_l, g_{1l}) + d\sigma_E^{A_i}(\theta_l, g_{2l}), \quad (30)$$

$$\frac{d\sigma_E^{A_i}(\theta, g_{1l})}{d\Omega} = \sigma_o^l g_{1l}^2 F_E^2(q^2), \quad (31)$$

$$\frac{d\sigma_E^{A_i}(\theta, g_{2l})}{d\Omega} = 4m_l^2 \eta_l^{-2} \sigma_o^l g_{2l}^2 F_E^2(q^2) \operatorname{tg}^2 \frac{\theta_l}{2}. \quad (32)$$

Thus, (27) and (30) would seem allow the conclusion that either incoming neutrinos are strictly longitudinally polarized, or they possess no polarization. But we can say that this is not quite so. The point is that the spin structure of their interaction with the field of emission can not be defined regardless of the medium properties where it originates. In other words, among the incoming and scattered neutrinos one can find both longitudinally polarized and unpolarized particles, each of which has suffered a strong change in his spin nature. Under such circumstances, a flux of outgoing neutrinos is a partially ordered set of the scattered fermions.

So, we must recognize that (26) constitutes the naturally united set of the axial-vector cross sections:

$$d\sigma_E^{A_i} = \{d\sigma_E^{A_i}(\theta_l, g_{1l}, s), \quad d\sigma_E^{A_i}(\theta_l, g_{2l}, s), \quad d\sigma_E^{A_i}(\theta_l, g_{1l}), \quad d\sigma_E^{A_i}(\theta_l, g_{2l})\}. \quad (33)$$

Of course, this class with a partial order is a partially ordered set [25] if between some pairs of its elements  $d\sigma_E^{A_i}(\theta_l, g_{1l}, s)$  and  $d\sigma_E^{A_i}(\theta_l, g_{2l}, s)$  there exists a relation

$$d\sigma_E^{A_i}(\theta_l, g_{1l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{2l}, s) \quad (34)$$

such that

- 1)  $d\sigma_E^{A_i}(\theta_l, g_{1l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{1l}, s)$  (reflexivity),
- 2)  $d\sigma_E^{A_i}(\theta_l, g_{1l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{2l}, s)$  and  $d\sigma_E^{A_i}(\theta_l, g_{2l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{1l}, s)$  imply  $d\sigma_E^{A_i}(\theta_l, g_{2l}, s) = d\sigma_E^{A_i}(\theta_l, g_{1l}, s)$  (antisymmetricality),
- 3)  $d\sigma_E^{A_i}(\theta_l, g_{1l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{2l}, s)$  and  $d\sigma_E^{A_i}(\theta_l, g_{2l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{1l}, s)$  say  $d\sigma_E^{A_i}(\theta_l, g_{1l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{1l}, s)$  (transitivity).

From our earlier developments [14, 23], we find that any massive neutrino, regardless whether it is a Dirac or a Majorana particle, has simultaneously both anapole and electric dipole. A non-zero value of one of elements of class (33) will, therefore, indicate to the existence of each of them. This becomes possible owing to the structural dependence of all elements of the set (33) which establishes the relation (34) so that it satisfies one more condition:  $d\sigma_E^{A_i}(\theta_l, g_{1l}, s) \leq d\sigma_E^{A_i}(\theta_l, g_{2l}, s)$

implies  $d\sigma_E^{A_l}(\theta_l, g_{2l}, s) \leq d\sigma_E^{A_l}(\theta_l, g_{1l}, s)$  (symmetry). Thus, if (34) is both symmetric and antisymmetric, it is more natural to expect that it appears in (33) as a relation of equality.

It should also be observed that the cross sections of the axial-vector interactions of longitudinal and unpolarized leptons are not different

$$\frac{d\sigma_E^{A_l}(\theta_l, s)}{d\sigma_E^{A_l}(\theta_l)} = 1. \quad (35)$$

At first sight, the latter gives the possibility to compare separately the contribution of the currents  $g_{1l}(q^2)$  or  $g_{2l}(q^2)$  to the scattering cross sections of longitudinal polarized and unpolarized particles. On the other hand, such a comparison of (27) and (30) shows that

$$\frac{d\sigma_E^{A_l}(\theta_l, g_{il}, s)}{d\sigma_E^{A_l}(\theta_l, g_{il})} = 1, \quad (36)$$

and consequently, the class (33) having a partial order is of those partially ordered sets, in which the transitivity of relation (34) leads to an equality implied from its symmetry and antisymmetry. At the same time, the set (33) can possess each of these properties even at any permutation of elements. This is of course intimately connected with the character of their structure depending on nature of the discussed types of interactions.

They show that between the contributions of  $g_{1l}(q^2)$  and  $g_{2l}(q^2)$  to the cross sections both in (27) and in (30) the same identity takes place, for which the interratios of possible pairs of elements of class (33) establish four most diverse structural equations.

For elucidation of their ideas, it is desirable to apply to the two of them:

$$\frac{d\sigma_E^{A_l}(\theta_l, g_{2l}, s)}{d\sigma_E^{A_l}(\theta_l, g_{1l}, s)} = 1, \quad \frac{d\sigma_E^{A_l}(\theta_l, g_{2l})}{d\sigma_E^{A_l}(\theta_l, g_{1l})} = 1, \quad (37)$$

or to the two remaining equalities.

Such a choice is based logically on the fact that the anapole  $g_{1l}(q^2)$  and electric dipole  $g_{2l}(q^2)$  correspond to the two forms of the same regularity of an axial-vector nature of the same charged lepton which unites all elements of the set (33) in a unified whole.

Inserting the explicit values of  $d\sigma_E^{A_l}$  in any of Eqs. (37), we find that

$$4m_l^2 \frac{g_{2l}^2(q^2)}{g_{1l}^2(q^2)} \frac{\text{tg}^2(\theta_l/2)}{\eta_l^2} = 1. \quad (38)$$

If choose a particle energy  $E_l \gg m_l$ , at which  $\eta_l \rightarrow 0$ , then for the case  $q^2 \rightarrow 0$  when  $\theta_l \rightarrow 0$ , the limit is

$$\lim_{\eta_l \rightarrow 0, \theta_l \rightarrow 0} \frac{\text{tg}^2(\theta_l/2)}{\eta_l^2} = \frac{1}{4},$$

because of which the solution (38) takes the form [23]

$$m_l \frac{g_{2l}(0)}{g_{1l}(0)} = \pm 1. \quad (39)$$

Together with (38), that implies that  $g_{1l}(q^2)$  and  $g_{2l}(q^2)$  are the functions which have the different values and dimensionality for the same value of the square of the four-dimensional momentum transfer. According to (37), this reflects the unified nature of their interaction with the field of emission. As a consequence, the scattered flux with a partial order consists of a partially ordered set of outgoing fermions. They constitute the class (33) so that the relation (34) in it be reflexive, symmetric, antisymmetric and transitive.

It should be mentioned, however, that the vertex  $\Gamma_\mu^A$  is defined, for example, in many articles as [26]

$$\Gamma_\mu^A(p, p') = \bar{u}(p', s') \gamma_5 [\gamma_\mu q^2 G_{1l}(q^2) - i \sigma_{\mu\lambda} q_\lambda G_{2l}(q^2)] u(p, s). \quad (40)$$

Here  $G_{1l}(0)$  gives the dimensional anapole:  $a_l = G_{1l}(0)$ . In other words, the difference in (16) and (40) is that

$$g_{1l}(q^2) = q^2 G_{1l}(q^2) \quad \text{and} \quad g_{2l}(q^2) = G_{2l}(q^2). \quad (41)$$

With these conditions, (31), (32) and (36) replace (37) by

$$\frac{G_{2l}^2(q^2)}{4m_l^2 G_{1l}^2(q^2)} \frac{\eta_l^2 \operatorname{tg}^2(\theta_l/2)}{(1 - \eta_l^2)^2 \sin^4(\theta_l/2)} = 1. \quad (42)$$

Taking into account that

$$\lim_{\eta_l \rightarrow 0, \theta_l \rightarrow 0} \frac{\eta_l^2 \operatorname{tg}^2(\theta_l/2)}{(1 - \eta_l^2)^2 \sin^4(\theta_l/2)} = 1,$$

it is not difficult to get from (42) the following relation

$$\frac{G_{2l}(0)}{2m_l G_{1l}(0)} = \pm 1. \quad (43)$$

Comparing (39) and (43), we find [14] that

$$G_{1l}(0) = \frac{g_{1l}(0)}{2m_l^2}. \quad (44)$$

In both definitions (18) and (44), as expected from simple reasoning [11], the anapole can not change its own value, so that there exists a relation among the parameters

$$\frac{1}{m_l} \left( \frac{g_{1l}(0)}{f_{1l}(0)} \right)^2 f_{2l}(0) = G_{1l}(0). \quad (45)$$



The equations (23), (44) and (45) are reduced to an equality [23]

$$g_{1l}(0) - 2m_l f_{2l}(0) = 0. \quad (46)$$

Its insertion in (39) allows to conclude that

$$g_{2l}(0) - 2f_{2l}(0) = 0. \quad (47)$$

Such a behavior of the neutrino vector and axial-vector moments, together with (23), testifies also about the availability of a latent structure of lepton-photon vertex depending on the particle mass.

### 3. Relation of massive neutrinos of different nature

We see that in the case of truly neutral neutrinos ( $l = \nu_M$ ), it is convenient to replace the individual dependence between the anapole and electric dipole form factors (39) by

$$g_{1\nu_M}(0) - m_{\nu_M} g_{2\nu_M}(0) = 0. \quad (48)$$

It reflects the availability of a unified structure of Dirac and Majorana neutrino axial-vector currents. It is important to elucidate whether there exists a connection of massive neutrinos of both types. The elastic scattering processes

$$\nu(\nu_D, \nu_M) + A(Z) \xrightarrow{\gamma} \nu'(\nu'_D, \nu'_M) + A(Z), \quad (49)$$

$$\bar{\nu}(\bar{\nu}_D, \bar{\nu}_M) + A(Z) \xrightarrow{\gamma} \bar{\nu}'(\bar{\nu}'_D, \bar{\nu}'_M) + A(Z) \quad (50)$$

are particularly interesting, because the incoming fluxes include the Dirac and Majorana neutrinos.

Of course, this presentation is not a standard one. The number of particles and the structural phenomena originating in the processes (49) or (50) coincide, as follows from considerations of symmetry. Such a conformity takes place regardless whether the neutrino is a Dirac or a Majorana fermion. Then it is possible, for example, to study the united processes (49) and (50) in the presence of only a Dirac neutrino anapole and electric dipole moment of a Majorana particle.

To investigate further, we make the following replacements in Eq. (26):

$$g_{1l}(q^2) \rightarrow \frac{1}{g_{A\nu_D}} g_{1\nu_D}(q^2),$$

$$g_{2l}(q^2) \rightarrow \frac{1}{2g_{A\nu_D}} g_{2\nu_M}(q^2),$$

$$m_l \rightarrow m_{\nu_D} \rightarrow m_{\nu_M} \rightarrow m_\nu.$$

Thus, only a part is obtained of a general picture of elastic scattering in which it is additionally assumed that

$$2g_{1\nu_D}(0) - m_\nu g_{2\nu_M}(0) = 0. \quad (51)$$

The equations (48) and (51), together with the first of Eqs. (20) at  $m_\nu = m_{\nu_D} = m_{\nu_M}$  lead us once again to Eqs. (24). The indications are in favour of a certain latent regularity of the nature of mass.

#### 4. *United dependence of the axial-vector form factors of the neutrino and electron*

We now remark that as in (34), any relation

$$d\sigma_E^{A_l}(\theta_l, g_{1l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{2l}, s) \quad (52)$$

defined between the elements  $d\sigma_E^{A_l}(\theta_l, g_{1l}, s)$  and  $d\sigma_E^{A_l}(\theta_l, g_{2l}, s)$  of the set (33) is a relation of equivalence [25] only in the case when it satisfies the conditions

- 1)  $d\sigma_E^{A_l}(\theta_l, g_{1l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{1l}, s)$  (reflexivity),
- 2)  $d\sigma_E^{A_l}(\theta_l, g_{1l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{2l}, s)$  says  $d\sigma_E^{A_l}(\theta_l, g_{2l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{1l}, s)$  (symmetricality),
- 3)  $d\sigma_E^{A_l}(\theta_l, g_{1l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{2l}, s)$  and  $d\sigma_E^{A_l}(\theta_l, g_{2l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{1l}, s)$  imply  $d\sigma_E^{A_l}(\theta_l, g_{1l}, s) \sim d\sigma_E^{A_l}(\theta_l, g_{1l}, s)$  (transitivity).

Such an equivalence relation separates the class (33) to the possible subsets so that they have no general elements. The set (33) may be symbolically presented as

$$d\sigma_E^{A_l} = \{d\sigma_E^{A_l}(\theta_l, s), d\sigma_E^{A_l}(\theta_l)\}. \quad (53)$$

Its subclasses reflect just the fact that each of Eqs. (27) and (30) constitutes a kind of a united set of cross sections:

$$d\sigma_E^{A_l}(\theta_l, s) = \{d\sigma_E^{A_l}(\theta_l, g_{1l}, s), d\sigma_E^{A_l}(\theta_l, g_{2l}, s)\}, \quad (54)$$

$$d\sigma_E^{A_l}(\theta_l) = \{d\sigma_E^{A_l}(\theta_l, g_{1l}), d\sigma_E^{A_l}(\theta_l, g_{2l})\}. \quad (55)$$

These classes are, according to (35), equal. Similar equality can exist only in the case when the scattering corresponds either to a vector ( $V_l$ ) or to an axial-vector ( $A_l$ ) component of leptonic current.

Here it is relevant to note that any neutrino possesses simultaneously only one of the currents,  $V_l$  or  $A_l$ . In this situation, the interference between the two interactions of a different nature may serve as a certain indication to the appearance of the unified system of the two massive neutrinos of the most diverse currents [27].

It is not excluded, however, that regardless of the values of scattering cross sections of polarized and unpolarized particles in the field of a nucleus, their interratio for any charged lepton and its neutrino has the same value. In our case, from (35), we are led to the following relation:

$$\frac{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, s)}{d\sigma_E^{A\nu_l}(\theta_{\nu_l})} = \frac{d\sigma_E^{A_l}(\theta_l, s)}{d\sigma_E^{A_l}(\theta_l)}. \quad (56)$$

In conformity with ideas of Eqs. (36),

$$\frac{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, g_{i\nu_l}, s)}{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, g_{i\nu_l})} = \frac{d\sigma_E^{A_l}(\theta_l, g_{il}, s)}{d\sigma_E^{A_l}(\theta_l, g_{il})}, \quad (57)$$

expressing the circumstance that the sets (54) and (55) both for  $\nu_l$  and for  $l$  consist of the same elements.

Thus, on the basis of (38), we can relate with confidence the masses to a ratio of the axial-vector currents of each charged lepton and its neutrino if reflexivity, symmetry and transitivity of an equivalence relation (52) hold regardless of the particle type, owing to which the interratio of any pair of elements from subclasses (54) and (55) for  $\nu_l$  and  $l$  coincides. That gives the right to establish four more most diverse identities.

To show their features, one can use two of them:

$$\frac{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, g_{2\nu_l}, s)}{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, g_{1\nu_l}, s)} = \frac{d\sigma_E^{A_l}(\theta_l, g_{2l}, s)}{d\sigma_E^{A_l}(\theta_l, g_{1l}, s)}, \quad (58)$$

$$\frac{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, g_{2\nu_l})}{d\sigma_E^{A\nu_l}(\theta_{\nu_l}, g_{1\nu_l})} = \frac{d\sigma_E^{A_l}(\theta_l, g_{2l})}{d\sigma_E^{A_l}(\theta_l, g_{1l})}, \quad (59)$$

or the two remaining relationships.

Because of (31), (32) and (36), any of (58) and (59) allows to derive the same equation:

$$m_{\nu_l} \frac{g_{2\nu_l}(0)}{g_{1\nu_l}(0)} = \pm m_l \frac{g_{2l}(0)}{g_{1l}(0)}. \quad (60)$$

Another possibility is that by inserting (41) in (28), (29), (31) and (32), one can also find from (58) and (59) that

$$\frac{G_{2\nu_l}^2(q^2)}{4m_{\nu_l}^2 G_{1\nu_l}^2(q^2)} \frac{\eta_{\nu_l}^2 \operatorname{tg}^2(\theta_l/2)}{(1 - \eta_{\nu_l}^2)^2 \sin^4(\theta_l/2)} = \frac{g_{2l}^2(q^2)}{4m_l^2 g_{1l}^2(q^2)} \frac{\eta_l^2 \operatorname{tg}^2(\theta_l/2)}{(1 - \eta_l^2)^2 \sin^4(\theta_l/2)}. \quad (61)$$

Disclosure of the uncertainties that define (43), leads us from (61) to the following result

$$\frac{G_{2\nu_l}(0)}{2m_{\nu_l}G_{1\nu_l}(0)} = \pm \frac{G_{2l}(0)}{2m_lG_{1l}(0)}. \quad (62)$$

Its comparison with (60) at  $G_{2l}(0) = g_{2l}(0)$  establishes (44) and thereby confirms the fact that the existence both of an individual and of the united connections of the axial-vector cross sections of charged lepton and its neutrino scattering is, by itself, not excluded.

## 5. Conclusion

Our study of the behavior of massive neutrinos of Dirac and Majorana types in a nucleus Coulomb field shows clearly that between the properties of these particles there exist well defined relations. We have established an individual [23] and the united relations of the anapole and electric dipole form factors of each charged lepton and its neutrino. Such regularities, however, encounter many problems which reflect the characteristic features of the latent nature of the inertial mass.

At the same time, it is clear that (23), (44), (46) and (47) define the anapole and electric dipole moments of any neutrino or charged lepton:

$$a_l = \frac{g_{1l}(0)}{2m_l^2} = \frac{e_l}{2m_l^2}, \quad d_l = g_{2l}(0) = \pm \frac{e_l}{m_l}. \quad (63)$$

These values, together with (19), (20) and (24) require the elucidation of the ideas of each of the existing types of charges and masses.

From the point of view of mass-charge duality, any of the electric ( $E$ ), weak ( $W$ ) and strong ( $S$ ) charges testifies in favour of the existence of a kind of inertial mass [28]. The neutrino mass and charge are naturally united in rest mass  $m_l^U$  and charge  $e_l^U$  equal to all the mass and charge:

$$m_l = m_l^U = m_l^E + m_l^W + m_l^S + \dots, \quad (64)$$

$$e_l = e_l^U = e_l^E + e_l^W + e_l^S + \dots. \quad (65)$$

In the framework of the standard electroweak theory, a Majorana ( $l = \nu_M$ ) neutrino has no electric mass ( $m_{\nu_M}^E = 0$ ) nor a Coulomb charge ( $e_{\nu_M}^E = 0$ ). Usually, it is accepted that the axial-vector terms of leptonic current (14) appear owing to the weak interaction [26]. Therefore, it seems that form factors  $g_{i\nu_M}(0)$  arise at the expense of a truly neutral neutrino weak mass.

On the other hand, as is known, a massive Dirac neutrino interaction with a weak field of emission may be expressed by a neutral current [17] consisting of vector and axial-vector parts. According to the mass-charge duality [28], this implies that

not only the weak charge  $e_l^W$  of a particle but also its weak mass  $m_l^W$  includes both vector and axial-vector components.

A given circumstance seems to indicate that the electromagnetic form factors  $f_{i\nu_e}(0)$  and  $g_{i\nu_e}(0)$  appear, respectively, due to the vector and axial-vector parts of the neutrino weak mass.

Furthermore, if it turns out that any dipole moment can exist only in the presence of a kind of charge [8], from the point of view of each lepton-photon vertex (15) or (16), it should be expected that the functions  $f_{il}(q^2)$  and  $g_{il}(q^2)$  correspond in nature to the vector and axial-vector components of the same well known Dirac ( $i = 1$ ) or Pauli ( $i = 2$ ) interaction.

It is clear from the above considerations that  $f_{1l}(q^2)$  and  $g_{1l}(q^2)$  characterize the vector and axial-vector parts of the electric charge, and  $f_{2l}(q^2)$  and  $g_{2l}(q^2)$  describe their dipole moments. They explain the availability of a vector [2] as well as of an axial-vector component of the Coulomb mass.

Thus, it follows that each part ( $K = E, W, S, \dots$ ) of the neutrino mass contains the vector and axial-vector components. We can, therefore, separate any of the united rest mass (64) and charge (65) into the vector and the axial-vector parts:

$$m_l = m_{V_l}^K + m_{A_l}^K, \quad (66)$$

$$e_l = e_{V_l}^K + e_{A_l}^K. \quad (67)$$

With regard to the question about truly neutral neutrinos, their mass is strictly of an axial-vector type. Therefore, a Majorana neutrino has not a vector nature.

So, we have learned that (51) relates an axial-vector part of Coulomb mass to form factors of a Dirac particle anapole and a Majorana neutrino electric dipole moment. Therefore, from the point of view of the suggested theory of mass, each of the above established relations (23), (46) and (47) between the vector and axial-vector currents of the two neutrinos of the same leptonic families must be interpreted as an indication to the existence of a kind of the unified system of the two left (right)-handed neutrinos of a different nature from the same purely neutrino families [14]. They are of course the united parafermions

$$(\nu_D^L, \bar{\nu}_M^R), \quad (\nu_D^R, \bar{\nu}_M^L), \quad (68)$$

$$(\bar{\nu}_D^R, \nu_M^L), \quad (\bar{\nu}_D^L, \nu_M^R) \quad (69)$$

which appear in the presence of the field of emission both of fermions with  $V_l$  currents and of fermions of  $A_l$  currents. Their scattering on nuclei is described also by the interference contribution  $f_{il}(q^2)g_{il}(q^2)$  of the interaction vector  $f_{il}(q^2)$  and axial-vector  $g_{il}(q^2)$  parts [27, 29].

This convinces us here that the appearance of any self interference term  $g_{il}^2(q^2)$  in the cross section (26) can be explained by the availability in all families of doublets or singlets of a hard axial-vector connection between the two left (right)-handed

particles of a definite type. It has the crucial value for steadiness of individual dileptons of an axial-vector nature

$$(l_L^-, l_R^+), \quad (l_R^-, l_L^+), \quad (70)$$

$$(\nu_D^L, \bar{\nu}_D^R), \quad (\nu_D^R, \bar{\nu}_D^L), \quad (71)$$

$$(\nu_M^L, \bar{\nu}_M^R), \quad (\nu_M^R, \bar{\nu}_M^L). \quad (72)$$

One of sharply expressed features of the discussed types of interactions is the equality of each of interratios of the possible pairs of elements of subsets (54) and (55) in the processes with charged lepton and its neutrino. This corresponds in nature to the definite flavour. In other words, any of (58) and (59) is valid only for particles of the same type of lepton.

As a consequence, the structural dependence (60) is in favour of coexistence of each charged lepton and its neutrino. They can, therefore, constitute the naturally united families of the left-handed  $SU(2)_L$ -doublets as well as of the right-handed  $SU(2)_R$ -singlets [2].

To express the idea more clearly, it is desirable to present (60) in the form

$$m_{\nu_l} g_{1l}(0) g_{2\nu_l}(0) - m_l g_{1\nu_l}(0) g_{2l}(0) = 0. \quad (73)$$

Turning to the cross section (27), we remark that the anapole  $g_{1l}$  does not change the direction of the particle spin, while the electric dipole moment  $g_{2l}$  is responsible for its flip [29]. The latter confirms the fact that the same lepton cannot have simultaneously both left - and right-handed helicities. Therefore, each interference term in (73) implies the existence of a kind of connected system of the two types of left (right)-handed leptons of the axial-vector currents from the same families of doublets or singlets.

It is seen that the equations (60) and (73) relate the two united dileptons of an axial-vector nature. Such systems can appear because of conservation of lepton flavours, for example, in any of  $\beta$ -decays (9) and (10) as well as in the scattering on spin-zero nuclei [27], if among incoming particles not only leptons but also their neutrinos are present.

We have already mentioned that formation of the left (right) dileptons and paradileptons in the processes with leptonic currents  $f_{il}(q^2)$  is responsible for conservation of vector part of the electric charge.

According to these results, each of earlier experiments [6, 9, 30] about conservation of summed electric charge and any type of lepton numbers in reactions (9), (10) and

$$\gamma e^- \rightarrow e^- \nu_e \bar{\nu}_e, \quad e^- e^+ \rightarrow \nu_e \bar{\nu}_e, \quad (74)$$

$$\nu_e e^- \rightarrow \nu_e e^-, \quad \bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^- \quad (75)$$

may serve as the source of facts confirming the existence of dileptons and paradileptons of a different nature.

By following the structure of the united relation (60), we have

$$\frac{m_{\nu_l}}{m_l} = \frac{g_{1\nu_l}(0)}{g_{1l}(0)} \frac{g_{2l}(0)}{g_{2\nu_l}(0)}. \quad (76)$$

The latter together with the ideas of full lepton number conservation law predicts the size of the neutrino axial-vector mass

$$m_{\nu_e} : m_{\nu_\mu} : m_{\nu_\tau} = m_e : m_\mu : m_\tau. \quad (77)$$

One can also find from (60) with the help of (18) and (44) that

$$a_{\nu_e} : a_{\nu_\mu} : a_{\nu_\tau} = a_e : a_\mu : a_\tau, \quad (78)$$

$$d_{\nu_e} : d_{\nu_\mu} : d_{\nu_\tau} = d_e : d_\mu : d_\tau. \quad (79)$$

Thus, unlike the earlier presentations on the families of leptons [32, 32], the discussed theory of unification of fermions [2] leads us to a correspondence principle that the axial-vector mass, anapole and electric dipole moment of the neutrino are proportional, respectively, to the axial-vector mass, anapole and electric dipole moment of a particle of the same family.

Finally, the experimental observation of the above noted regularities in the nature of fermions appears to be possible by measuring the effects of the nuclear charge field [2, 14]. Of course, to make such a subtle measurement one should create devices with sufficiently high sensitivity.

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## STRUKTURA LEPTONSKIH OBITELJI I NJIHOVE STRUJE AKSIJALNO-VEKTORSKE NARAVI

Svaki neutrino ima masu različitu od nula i može imati i anapolni i električni dipolni moment neovisno o tome je li Diracovog ili Majoraninog tipa. Među njihovim faktorima oblika javlja se poveznica, npr. u raspršenju neutrina na jezgrama bez spina. Razmatramo teoriju u kojoj se masa sastoji od vektorske i aksijalno-vektorske sastavnice zbog kojih se leptonska struja razdvaja na vektorski i aksijalno-vektorski dio za jednak naboj ili dipolni moment. Taj model može objasniti odsustvo stvarno neutralnih vektorskih međudjelovanja neutrina i mogućnost aksijalno-vektorske strukture Majoranine mase. Tako se povezuju dva neutrina različite naravi. Izvodimo jednadžbu koja povezuje mase u omjerima anapolnih i električnih dipolnih faktora oblika za bilo koji nabijen lepton i njegov neutrino na osnovi njihovog skupljanja u dubletne i singletne obitelji. To ukazuje na postojanje lijevih (desnih) dileptona i paradileptona aksijalno-vektorskih struja. Svaki od njih zadovoljava sačuvanje aksijalno-vektorskog naboja za svaki leptonski okus. Stoga su aksijalno-vektorska masa, anapolni i električni dipolni moment razmjerni aksijalno-vektorskoj masi, anapolnom i električnom dipolnom momentu čestice jedne obitelji.