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MULTIFRACTALITY AND SLOW PARTICLE PRODUCTION IN ^{24}Mg–AgBr INTERACTION AT 4.5 AGeV ENERGY

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We investigated the multifractality of target fragments of ^{24}Mg–AgBr interaction at low energy (4.5 AGeV) in emission angle phase space, using a new method as proposed by Takagi. The analysis involves the step of measuring the generalised dimension D_q, which in turn deducts the multifractal behaviour of target fragments. We also determine the multifractal specific heat.

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1. Introduction

To characterize mathematically at-all-scales intrinsically-irregular systems, the concept of fractality is needed. The word fractal, coined by B. Mandelbrot [1], means to create irregular fragments. A fractal structure has the property that if one magnifies a small portion of it, the same complexity is shown as for the entire system. This feature establishes the absence of regularity in the system. According to Mandelbrot, there is a fractal face to the geometry of nature. The power-law behaviour of normalised scaled factorial moments also suggests the existence of fractal properties in multiparticle production [2 – 4]. The nonuniform fractality, i.e. the multifractality, has been the focal point of a number of theoretical investigations [5, 6] on dynamics of multiparticle production. The earlier G-moment method to study the multifractality is faulty in this respect that experimental data sets do not show exact linearity in a log–log plot of moment against bin size, as expected from the mathematical formula. This problem may arise due to the nonvalidity of the assumed mathematical limit (number of points tends to infinity) for the real experimental data where number of particles in each event is always finite. To remove this difficulty, Takagi proposed a method [7] which reveals that the fractal properties are associated with multiparticle production process at high energies. This method has been properly applied to study the fractal characteristics in electron–positron annihilation [8, 9] and UA5 data on proton-antiproton collisions [10]. It is well known that pions produced in relativistic interactions are very informative about the dynamics of particle production process. But to get a deep insight, it is essential to study target fragments, which can be a good source of information. On the other hand, Bershadskii [11] reported that the constant specific heat (CSH) approximation, widely used in the ordinary thermodynamics, is applicable to the multifractal data. Moreover, the constant specific heats, calculated from a few sets of data corresponding to different types of interactions, are approximately the same and, therefore, may turn out to be a universal characteristic of the multiparticle production process at high energies. This creates a new path to carry out study on constant specific heat approximation in multiparticle production in high-energy reactions. This report represents a work on the fractal nature of particles evaporated from a target in ^{24}Mg–AgBr interactions at 4.5 AGeV using Takagi's procedure, and also to study multifractal specific heat in emission-angle phase space.

It is interesting to note that this analysis reveals also multifractal behaviour of target fragments.

2. *Experimental details*

In order to probe the fractality in target fragmentation, we precisely study the interactions initiated by ^{24}Mg in AgBr at 4.5 AGeV. The data set used in the present analysis was obtained by irradiating stacks of G5 nuclear emulsion plates by a ^{24}Mg beam of incident energy 4.5 AGeV at JINR Dubna. The scanning of the plates was carried out with the help of a high resolution Leitz Metalloplan microscope provided with an online computer system. The details of scanning and measurement are given in our previous works [12].

For this analysis, we have considered only the slow-target fragments known as black particles. According to the emulsion terminology, black tracks are identified by ionisation lying between $1.4\,I_0 < I < 10\,I_0$, where I_0 is the minimum ionisation of a singly charged particle. These black tracks have the range < 3 mm.

The spatial angle of emission (θ) of each black track with respect to the beam axis is calculated by noting the space coordinates of the interaction center (x_0, y_0, z_0), one point on the incident beam track and one point on the respective secondary track, and applying simple coordinate geometry. The average multiplicity of produced black particles of the sample is 9.6 ± 0.47.

3. *Method of study*

We consider the target fragmentation process at some incident energy. In such a process, the particle distribution is considered in a phase space x. A single event contains n particles. The multiplicity n changes from event to event according to the distribution $P_n(x)$. The selected phase space interval of length x is divided into M bins of equal size, the width of each bin being $\delta x = x/M$. Then the multiplicity distribution for a single bin is denoted as $P_n(\delta x)$ for $n = 0, 1, 2, 3 \ldots$, where we assume that the inclusive particle distribution $\mathrm{d}n/\mathrm{d}x$ is constant, and $P_n(\delta x)$ is independent of the location of the bin. n hadrons contained in a single event are distributed in the interval $x = x_{\max} - x_{\min}$. The multiplicity n changes from event to event according to the distribution $P_n(x)$. If the number of independent events is Ω, then the particles produced in those events are distributed in ΩM bins of size δx. Let N be the total number of target-associated slow particles produced in these Ω events and n_{aj} the multiplicity of black particles in the jth bin of the ath event.

The theory of multifractals [13] has been motivated to consider the normalized density P_{aj} defined by

$$P_{aj} = n_{aj}/N\,. \tag{1}$$

This is of course also true when $N \to \infty$. Then one has to consider the Takagi's moment of order q as

$$T_q(\delta x) = \ln \sum_{a=1}^{\Omega} \sum_{j=1}^{M} P_{aj}^q \quad \text{for} \quad q > 0\,, \tag{2}$$

which behaves like a linear function of the logarithm of the resolution $R(\delta x)$,

$$T_q(\delta x) = A_q + B_q \ \ln R(\delta x)\,,$$

where A_q and B_q are constants independent of δx. If such a behaviour is observed for a considerable range of $R(\delta x)$, a generalised dimension may be determined as

$$D_q = B_q/(q-1)\,. \tag{3}$$

Now, evaluating the double sum of P_{aj}^q, Takagi showed [7] that for a sufficiently large Ω,

$$\ln\langle n^q\rangle = A_q + (B_q + 1) \ \ln R(\delta x)\,.$$

While analyzing real data [8−10], it was observed [14] that a plot of $\ln\langle n^q\rangle$ against δx saturates for large x regions. This deviation may be due to the non-flat behaviour of $\mathrm{d}n/\mathrm{d}x$ in the large x region. Takagi suggested that $\langle n\rangle$ would be a better choice for the resolution $R(\delta x)$ because $\mathrm{d}n/\mathrm{d}\langle x\rangle$ is flat by definition [13, 16]. Choosing $R(\delta x) = \langle n\rangle$, one has

$$\ln\langle n^q\rangle = A_q + (B_q + 1) \ \ln\langle n\rangle \qquad (4)\,,$$

a simple linear relation between $\ln\langle n^q\rangle$ and $\ln\langle n\rangle$. The generalized dimension D_q can be obtained from the slope values using the relation (3).

The case with $q = 1$ is obtained by taking an appropriate limit [13]. D_1 can be obtained from the relation

$$\langle n \ln n \rangle / \langle n \rangle = C_1 + D_1 \ln \langle n \rangle \,, \tag{5}$$

where C_1 is a constant. So far the methodology is developed for non-overlapping x bins, but it can be applicable for overlapping bins [15].

It is well known that in classical thermodynamics, the constant specific heat approximation (CSH) is widely applicable in many important cases, e.g. the specific heat of gases and solids is constant, independent of temperature over a greater or smaller temperature interval [16]. This approximation is also applicable to multifractal data of target fragmentation process, which has already been shown by Bershadskii [11].

One can get the generalised dimension D_q from CSH approximation as

$$D_q = (a - c) + c \; \ln q/(q - 1) \,. \tag{6}$$

Here c is the specific heat, a is some other constant and q can be treated as the inverse of temperature, $q = 1/T$ [16].

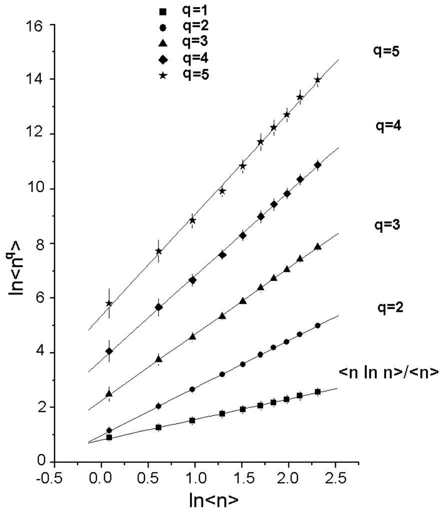
If one plots D_q versus $\ln q/(q-1)$, the slope of the graph will give the value of specific heat. In this paper, we investigate the multifractal nature of ^{24}Mg-AgBr interaction at 4.5 AGeV and from the knowledge of D_q, we calculate the specific heat. It is observed that the generalised dimension D_q decreases with the increase of order number q. This reflects the multifractal geometry in the case of particles evaporated from the target.

4. Analysis and discussion

To analyze our data, it is considered that a single event contains n black particles distributed in a particular interval in emission-angle phase space. This interval is divided into overlapping bins whose size is varied in steps of 0.1.The central value is 0. In Fig. 1, $\ln\langle n^q \rangle$ is plotted against $\ln\langle n \rangle$ for $q = 1, 2, 3$ and 4. A linear behaviour is obtained in all of the above log-log plot. The generalized dimensions D_2, D_3, D_4, D_5 are obtained from the slopes of such linear plots by using Eq. (3). D_1 is obtained from Eq. (5). In Fig. 2, D_q is plotted against q. This plot shows that D_q decreases gradually with the increase of q, which suggests multifractality in the case of target-evaporated particles. We also plot D_q against $\ln q/(q-1)$ (Fig. 3), which is also a linear plot. From the slope of its linearity, we get the specific heat. The generalized dimensions are given in Table 1. The calculated specific heat for these interaction data is

$$0.19 \pm 0.017 \,.$$

This analysis shows that even at very low energy (4.5 AGeV) the target fragments from the ^{24}Mg–AgBr interaction show multifractal behaviour.



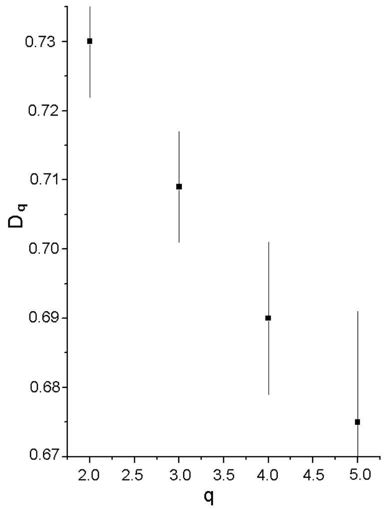


Fig. 1 (left). Variation of $\ln\langle nq\rangle$ *with* $\ln\langle n\rangle$.

Fig. 2. Variation of D_q *with* q.

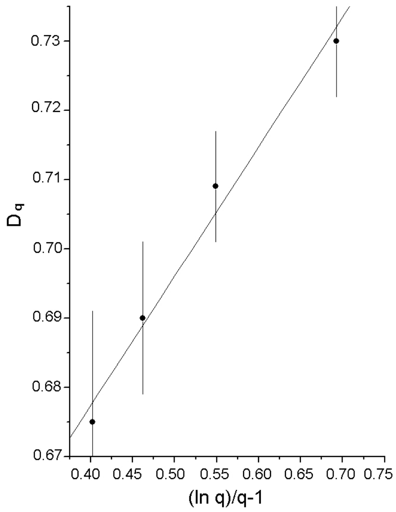


Fig. 3. Variation of D_q *with* $\ln q/(q-1)$.

TABLE 1. Values of D_q *for different* q *values of* ^{24}Mg–AgBr *interaction.*

Data	D_1	D_2	D_3	D_4	D_5
^{24}Mg - AgBr at 4.5 AGeV	0.750	0.730	0.709	0.690	0.675

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MNOGOFRAKTALNOST I TVORBA SPORIH ČESTICA U SUDARIMA ^{24}Mg–AgBr NA ENERGIJI 4.5 *A*GeV

Istražujemo mnogofraktalnost izlaznih sporih čestica u sudarima ^{24}Mg–AgBr na niskoj energiji (4.5 *A*GeV) u faznom prostoru kuta emisije, primjenom nove metode koju je predložio Takagi. U analizi se određuje poopćena dimenzija D_q iz koje se izvode mnogofraktalna svojstva izlaznih čestica. Također izvodimo mnogofraktalnu specifičnu toplinu.