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GENERATION OF BIANCHI TYPE V BULK VISCOUS COSMOLOGICAL MODELS WITH TIME-DEPENDENT Λ-TERM

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Bianchi type V bulk viscous fluid cosmological models are investigated with dynamic cosmological term $\Lambda(t)$. Using a generation technique (Camci et al., 2001), it is shown that the Einstein's field equations are solvable for any arbitrary cosmic scale function. Solutions for particular forms of cosmic scale functions are also obtained. The cosmological constant is found to be a decreasing function of time, which is supported by results from recent type Ia supernovae observations. Some physical and geometrical aspects of the models are also discussed.

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1. Introduction

The study of Bianchi type V cosmological models creates increasing interest as these models contain special isotropic cases and permit arbitrarily small anisotropy levels at some instant of cosmic time. This property makes them suitable as models of our universe. The homogeneous and isotropic Friedmann-Robertson-Walker (FRW) [1, 2] cosmological models, which are used to describe standard cosmological models, are particular cases of the Bianchi type I, V and IX universes, according to

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whether the constant curvature of the physical three-space, $t =$ constant, is zero, negative or positive. These models will be interesting to construct cosmological models of the types which are of class one. Present cosmology is based on the FRW model which is completely homogeneous and isotropic. This is in agreement with observational data about the large-scale structure of the universe. However, although homogeneous but anisotropic models are more restricted than the inhomogeneous models, they explain a number of observed phenomena quite satisfactorily. This stimulates the research for obtaining exact anisotropic solutions of the Einstein's field equations (EFEs) as cosmologically accepted physical models for the universe (at least in the early stages). Roy and Prasad [3] have investigated Bianchi type V universes which are locally rotationally symmetric and are of embedding class one, filled with perfect fluid, with heat conduction and radiation. Bianchi type V cosmological models have been studied by other researchers (Farnsworth [4], Maartens and Nel [5], Wainwright et al. [6], Collins [7], Meena and Bali [8] and Pradhan et al. [9, 10]) in different contexts.

Models with a dynamic cosmological term $\Lambda(t)$ are becoming popular as they solve the cosmological-constant problem in a natural way. There is a significant observational evidence for the detection of Einstein's cosmological constant, Λ, or a component of material content of the universe, that varies slowly with time and space and so acts like Λ. Recent cosmological observations by High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [11], Perlmutter et al. [12], Riess et al. [13], Schmidt et al. [14]) strongly favour a significant and positive Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitudes and red-shift of type Ia supernova suggest that our universe may be an accelerating one with a large fraction of the cosmological density in the form of cosmological Λ-term. Earlier researches on this topic, are contained in Lodovico [15], Weinberg [16], Dolgov [17 – 19], Bertolami [20], Ratra and Peebles [21], Carrol, Press and Turner [22]. Some of the recent discussions on the cosmological-constant "problem" and consequence on cosmology with a time-varying cosmological-constant have been discussed by Tsagas and Maartens [23], Sahni and Starobinsky [24], Peeble [25], Padmanabhan [26], Vishwakarma [27] and Pradhan et al. [28]. This motivates us to study the cosmological models with Λ varying with time.

The distribution of matter can be satisfactorily described by a perfect fluid due to the large-scale distribution of galaxies in our universe. However, observed physical phenomena, such as the large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggest an analysis of dissipative effects in cosmology. Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. Misner [29] has studied the effect of viscosity on the evolution of cosmological models. The role of viscosity in cosmology has been investigated by Weinberg [30]. Nightingale [31]. Heller and Klimek [32] have obtained viscous universes without initial singularity. The model studied by Murphy [33] possesses an interesting feature in which the big-bang type singularity of infinite space-time curvature does not occur at a finite past. However, the relationship assumed by Murphy between the viscosity coefficient and the matter density is not acceptable at large density. Thus, we should consider the presence of material distribution other than a perfect fluid to obtain realistic cosmological models (see Grøn [34] for a review on cosmological models with bulk viscosity). The effect of bulk viscosity on the cosmological evolution has been investigated by a number of authors in the framework of general theory of relativity. This motivates us to study the cosmological bulk viscous fluid model.

In recent years, several authors (Hajj-Boutros [35], Hajj-Boutros and Sfeila [36], Ram [37], Mazumder [38] and Pradhan and Kumar [39]) have investigated the solutions of EFEs for homogeneous but anisotropic models by using different generation techniques. Bianchi spaces I-IX are useful tools in constructing models of spatially homogeneous cosmologies (Ellis and MacCallum [40], Ryan and Shepley [41]). From these models, homogeneous Bianchi type V universes are the natural generalization of the open FRW model which eventually isotropize. Recently, Camci et al. [42] derived a new technique for generating exact solutions of EFEs with perfect fluid for Bianchi type V space-time. Very recently, Pradhan et al. [43] have obtained Bianchi type V perfect fluid cosmological models with time dependent Λ-term.

In this paper, in what follows, we will discuss Bianchi type V cosmological models obtained by augmenting the energy–momentum tensor of a bulk viscous fluid by a term that represents the cosmological constant varying with time, and later generalize the solutions of Refs. [37, 42, 43]. This paper is organized as follows: The field equations and the generation technique are presented in Section 2. We relate three of the metric variables by solving the off-diagonal component of EFEs, and find a second integral which is used to relate the remaining two metric variables. In Section 3, for the particular form of each metric variables, some solutions are presented separately and solutions of Camci et al. [42], Ram [37] and Pradhan et al. [43] are shown to be particular cases of these solutions. Kinematical and dynamical properties of all solutions are also studied in this section. In Section 4, we give the concluding remarks.

2. *Field equations and generation technique*

In this section, we review the solutions obtained by Pradhan et al. [43]. The usual energy-momentum tensor is modified by addition of the term

$$T_{ij}^{(\mathrm{vac})} = -\Lambda(t) g_{ij} \,, \tag{1}$$

where $\Lambda(t)$ is the cosmological term and g_{ij} is the metric tensor. Thus the new stress energy-momentum tensor in the presence of bulk stress is given by

$$T_{ij} = (\bar{p} + \rho) u_i u_j - \bar{p} g_{ij} - \Lambda(t) g_{ij} \,, \tag{2}$$

where

$$\bar{p} = p + \xi u^i_{;i}\,. \tag{3}$$

Here, ρ, p, $\bar{p}$, ξ and u are, respectively, the energy density, isotropic pressure, effective pressure, bulk viscous coefficient and the fluid four-velocity vector of distribution such that $u^i u_i = 1$. In general, ξ is a function of time.

We consider the space-time metric of the spatially homogeneous Bianchi type V of the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2(t)\mathrm{d}x^2 - e^{2\alpha x}\left[B^2(t)\mathrm{d}y^2 + C^2(t)\mathrm{d}z^2\right], \tag{4}$$

where α is a constant. For the energy momentum tensor (2) and Bianchi type V space-time (4), Einstein's field equations

$$R_{ij} - \frac{1}{2}Rg_{ij} = -8\pi T_{ij} \tag{5}$$

yield the following five independent equations

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} - \frac{\alpha^2}{A^2} = -8\pi(\bar{p} + \Lambda)\,, \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4 C_4}{AC} - \frac{\alpha^2}{A^2} = -8\pi(\bar{p} + \Lambda)\,, \tag{7}$$

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} - \frac{\alpha^2}{A^2} = -8\pi(\bar{p} + \Lambda)\,, \tag{8}$$

$$\frac{A_4 B_4}{AB} + \frac{A_4 C_4}{AC} + \frac{B_4 C_4}{BC} - \frac{3\alpha^2}{A^2} = 8\pi(\rho - \Lambda)\,, \tag{9}$$

$$\frac{2A_4}{A} - \frac{B_4}{B} - \frac{C_4}{C} = 0\,. \tag{10}$$

Here and in what follows the suffix 4 by the symbols A, B, C and ρ denote differentiation with respect to t. The Bianchi identity ($T^{ij}_{;j} = 0$) takes the form

$$\rho_4 + (\rho + p)\theta = 0\,. \tag{11}$$

It is worth noting here that our approach suffers from a lack of Lagrangian approach. There is no known way to present a consistent Lagrangian model satisfying the necessary conditions discussed in this paper.

The physical quantities expansion scalar θ and shear scalar σ^2 have the following expressions:

$$\theta = u^i_{;i} = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\,, \tag{12}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left[\theta^2 - \frac{A_4 B_4}{AB} - \frac{A_4 C_4}{AC} - \frac{B_4 C_4}{BC}\right]. \tag{13}$$

Integrating Eq. (10) and absorbing the integration constant into B or C, we obtain

$$A^2 = BC\,, \tag{14}$$

without any loss of generality. Thus, elimination of $\bar{p}$ from Eqs. (6) - (8) gives the condition of isotropy of pressures

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 = 2\frac{C_{44}}{C} + \left(\frac{C_4}{C}\right)^2, \tag{15}$$

which on integration yields

$$\frac{B_4}{B} - \frac{C_4}{C} = \frac{k}{(BC)^{3/2}}\,, \tag{16}$$

where k is a constant of integration. Hence for the metric function B or C from the above first order differential Eq. (16), some scale transformations permit us to obtain new metric function B or C.

Firstly, under the scale transformation $\mathrm{d}t = B^{1/2}\mathrm{d}\tau$, Eq. (16) takes the form

$$CB_\tau - BC_\tau = kC^{-1/2}\,, \tag{17}$$

where subscript represents derivative with respect to τ. Considering Eq. (17) as a linear differential equation for B, where C is an arbitrary function, we obtain

$$(i)\ B = k_1 C + kC\int\frac{\mathrm{d}\tau}{C^{5/2}}\,, \tag{18}$$

where k_1 is an integrating constant. Similarly, using the transformations $\mathrm{d}t = B^{3/2}\mathrm{d}\tilde{\tau}$, $\mathrm{d}t = C^{1/2}\mathrm{d}T$ and $\mathrm{d}t = C^{3/2}\mathrm{d}\tilde{T}$ in Eq. (16), after some algebra we obtain, respectively,

$$(ii)\ B(\tilde{\tau}; k_2, k) = k_2 C\ \exp\left(k\int\frac{\mathrm{d}\tilde{\tau}}{C^{3/2}}\right), \tag{19}$$

$$(iii)\ C(T; k_3, k) = k_3 B - kB\int\frac{\mathrm{d}T}{B^{5/2}}\,, \tag{20}$$

and

$$(iv)\ C(\tilde{T}; k_4, k) = k_4 B\ \exp\left(k\int\frac{\mathrm{d}\tilde{T}}{B^{3/2}}\right), \tag{21}$$

where k_2, k_3 and k_4 are constants of integration. Thus choosing any given function B or C in cases (i), (ii), (iii) and (iv), one can obtain B or C and hence A from (14).

3. Generation of new solutions

We consider the following four cases:

3.1. Case (I): Let $C = \tau^n$ (n is a real number satisfying $n \neq \frac{2}{3}$)

In this case, Eq. (18) gives

$$B = k_1 \tau^n + \frac{2k}{2-5n} \tau^{1-3n/2}, \tag{22}$$

and then from (14), we obtain

$$A^2 = k_1 \tau^{2n} + \frac{2k}{2-5n} \tau^{1-n/2}. \tag{23}$$

Hence the metric (4) reduces to the new form

$$\mathrm{d}s^2 = \left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)[\mathrm{d}\tau^2 - \tau^n \mathrm{d}x^2] - e^{2\alpha x}\left[\left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)^2 \mathrm{d}y^2 + \tau^{2n}\mathrm{d}z^2\right], \tag{24}$$

where

$$\ell = \frac{k}{2-5n} \quad \text{and} \quad \ell_1 = 1 - \frac{3n}{2}.$$

For this derived model, Eq. (24), the effective pressure, energy density and cosmological constant are given by

$$8\pi(\bar{p} + \Lambda) = \left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)^{-3}\left[-2k_1^2 n(n-1)\tau^{2n-2} - k_1 \ell n(10-13n)\tau^{-(\ell_1+2n)}\right.$$
$$\left. - \frac{\ell^2(4+4n-11n^2)}{2}\tau^{-3n}\right] + \alpha^2\tau^{-n}\left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)^{-1}, \tag{25}$$

$$8\pi(\rho - \Lambda) = \left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)^{-3}\left[3k_1^2 n^2\tau^{2n-2} + 3k_1\ell n(2-n)\tau^{-(\ell_1+2n)}\right.$$
$$\left. + \frac{\ell^2(4+4n-11n^2)}{2}\tau^{-3n}\right] - 3\alpha^2\tau^{-n}\left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)^{-1}. \tag{26}$$

We assume for the specification of ξ that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{27}$$

where γ ($0 \leq \gamma \leq 1$) is a constant.

Thus, given $\xi(t)$, we can solve the system for the physical quantities. In most of investigations involving bulk viscosity, it is assumed to be a simple power function of the energy density (Pavon et al. [44]; Maartens [45]; Zimdahl [46]; Santos et al. [47]),

$$\xi(t) = \xi_0 \rho^w, \tag{28}$$

where ξ_0 and w are real constants. For small density, w may even be equal to unity as used in Murphy's work [48] for simplicity. If $w = 1$, Eq. (28) may correspond to a radiative fluid (Weinberg [16]). Near the big bang, $0 \leq w \leq \frac{1}{2}$ is a more appropriate assumption (Belinskii and Khalatnikov [49]) to obtain realistic models.

For simplicity and realistic models of physical importance, we consider the following two cases ($w = 0, 1$).

3.1.1. Model I: Solution for $w = 0$

When $w = 0$, Eq. (28) reduces to $\xi = \xi_0 =$ constant. Hence in this case, with the use of (26), (27) and (28), Eq. (25) leads to

$$8\pi(1+\gamma)\rho = {D_1}^{-3}\Big[n(n+2)k_1^2\tau^{2(n-1)} + 2(5n-2)k_1\ell n\tau^{-(\ell_1+2n)}\Big] +$$

$$ {D_1}^{-1}\Big[-2\alpha^2\tau^{-n} + 24\pi\xi_0\left(k_1 n\tau^{n-1} + \frac{1}{2}\ell(2-n)\tau^{-\frac{3n}{2}}\right){D_1}^{-\frac{1}{2}}\Big], \tag{29}$$

where

$$D_1 = k_1\tau^n + 2\ell\tau^{\ell_1}.$$

Eliminating $\rho(t)$ from Eqs. (26) and (29), we obtain

$$8\pi(1+\gamma)\Lambda = {D_1}^{-3}\Big[(1-2n-3n\gamma)nk_1^2\tau^{2(n-1)} - \{(10-13n)+3(2-n)\gamma\}k_1\ell n\tau^{-(\ell_1+2n)}$$

$$-\frac{1}{2}(4+4n-11n^2)(1+\gamma)\ell^2\tau^{-3n}\Big] +$$

$$(1+3\gamma)\alpha^2\tau^{-n}{D_1}^{-1} + 24\pi\xi_0\left[nk_1\tau^{n-1} + \frac{1}{2}\ell(2-n)\tau^{-\frac{3n}{2}}\right]{D_1}^{-\frac{3}{2}}. \tag{30}$$

3.1.2. Model II: Solution for $w = 1$

When $w = 1$, Eq. (28) reduces to $\xi = \xi_0\rho$. Hence in this case, with the use of (26), (27) and (28), Eq. (25) leads to

$$8\pi\rho = \frac{\Big[n(n+2)k_1^2\tau^{2(n-1)} + 2n(5n-2)k_1\ell\tau^{-(\ell_1+2n)} - 2\alpha^2\tau^{-n}{D_1}^2\Big]}{{D_1}^3\Big[(1+\gamma) - 3\xi_0\{nk_1\tau^{n-1} + \frac{1}{2}(2-n)\ell\tau^{-\frac{3n}{2}}\}{D_1}^{-\frac{3}{2}}\Big]}. \tag{31}$$

Eliminating $\rho(t)$ between (26) and (31), we obtain

$$8\pi\Lambda = \frac{\left[n(n+2)k_1^2\tau^{2(n-1)} + 2n(5n-2)k_1\ell\tau^{-(\ell_1+2n)} - 2\alpha^2\tau^{-n}{D_1}^2\right]}{{D_1}^3\left[(1+\gamma) - 3\xi_0\{nk_1\tau^{n-1} + \frac{1}{2}(2-n)\ell\tau^{-\frac{3n}{2}}\}{D_1}^{-\frac{3}{2}}\right]}$$

$$-\frac{1}{{D_1}^3}\left[3n^2k_1^2\tau^{2(n-1)} + 3n(2-n)k_1\ell\tau^{-(\ell_1+2n)} + \frac{1}{2}(4+4n-11n^2)\ell^2\tau^{-3n} - 3\alpha^2\tau^{-n}{D_1}^2\right]. \tag{32}$$

From Eqs. (29) and (30), we observe that at the time of early universe, the the energy density $\rho(t)$ and cosmological constant $(\Lambda(t))$ decrease when time increases (see Fig. 1). We also observe that the value of Λ is small and positive at late times, which is supported by recent type Ia supernovae observations [11−14]. In Model II, from Eqs. (31) and (32), we find that for a large range of parameters the energy density decreases with time very sharply and becomes negative (even if it is positive initially) and then remains negative throughout the evolution. The cosmological constant shows singular behaviour closer to the origin. Then it shows an eratic behaviour and at later stage it remains a negative constant value. It seems that Model II may not be a physical model of the universe.

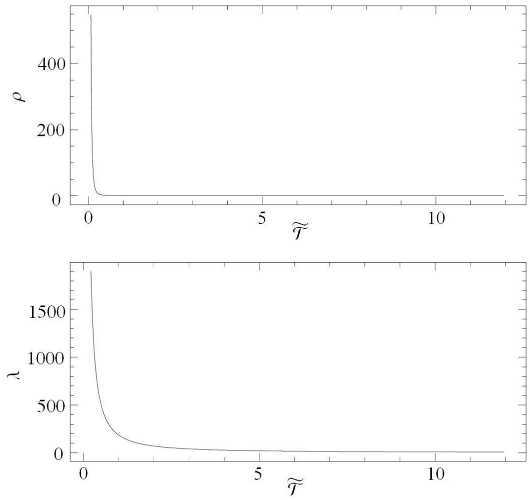

Fig. 1. (Top) Plot of $\rho \to \tau$ and (bottom) $\Lambda \to \tau$ for parameters $n = 1/4$, $k_1 = 1$, $\gamma = 0.5$, $\alpha = 1$, $\xi_0 = 1$ and rest of the constants are set to 1.

The metric (24) is a four-parameter family of solutions to EFEs with a bulk viscous fluid. Using the scale transformation $dt = B^{\frac{1}{2}}d\tau$ in Eqs. (12) and (13) for this case, one obtains the scalar expansion θ and the shear σ

$$\theta = 3\left[k_1 n\tau^{n-1} + \frac{\ell(2-n)}{2}\tau^{-3n/2}\right]\left(k_1\tau^n + 2\ell\tau^{\ell_1}\right)^{-3/2}, \tag{33}$$

$$\sigma = \frac{1}{2} k \tau^{-3n/2} \left(k_1 \tau^n + 2\ell \tau^{\ell_1} \right)^{-3/2}. \tag{34}$$

Eqs. (33) and (34) lead to

$$\frac{\sigma}{\theta} = \frac{k}{6} \left[k_1 n \tau^{n-\ell_1} + \frac{\ell(2-n)}{2} \right]^{-1}. \tag{35}$$

Now, we consider four subcases for the parameters Λ, n, k and k_1.

In the subcase $\Lambda = 0$ and $\xi_0 = 0$, the metric (24) with expressions p, ρ, θ and σ for this model are the same as those of solution (18) of Camci et al. [42]. If we set $\xi_0 = 0$, the metric (24) gives the solution obtained by Pradhan et al. [43].

In the subcase $\Lambda = 0$, $n = 0$, after a suitable inverse time transformation, we find that

$$\mathrm{d}s^2 = \mathrm{d}t^2 - K_1 (t + t_0)^{2/3} \mathrm{d}x^2 - e^{2\alpha x} \left[K_1 (t + t_0)^{4/3} \mathrm{d}y^2 + \mathrm{d}z^2 \right], \tag{36}$$

where t_0 is a constant of integration and $K_1 = (3k/2)^{2/3}$. The expressions p, ρ, θ and σ for this model are not given here, since it is observed that the physical properties of this one are same as that of the solution (24) of Ram [37].

In the subcase $\Lambda = 0$, $k = 0$, after inverse time transformation and rescaling, the metric (24) reduces to

$$\mathrm{d}s^2 = \mathrm{d}t^2 - K_2 (t + t_1)^{4n/(n+2)} \left[\mathrm{d}x^2 + e^{2\alpha x} (\mathrm{d}y^2 + \mathrm{d}z^2) \right], \tag{37}$$

where t_1 is a constant of integration and $K_2 = \big((n+2)/2\big)^{4n/(n+2)}$. For this solution, when $n = 1$ and $\alpha = 0$, we obtain Einstein and de Sitter [50] dust-filled universe. For $K_2 = 1$, $t_1 = 0$ and $n = 2m/(2-m)$, where m is a parameter in Ram's paper [37], the solution (37) reduces to the metric (14) of Ram [37]. In latter case, if also $\alpha = 0$, then we get the Minkowski space-time.

Now, in subcase $\Lambda = 0$ and $k_1 = 0$, after some algebra, the metric (24) takes the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - 2\ell K_3 (t + t_2)^{2/3} \left[\mathrm{d}x^2 + e^{2\alpha x} \left(a t^{m_1} \mathrm{d}y^2 + a^{-1} t^{-m_1} \mathrm{d}z^2 \right) \right], \tag{38}$$

where t_2 is a constant, $a t^{m_1} = 2\ell K_3^{(2-5n)/(2-n)} (t + t_2)^{(2(2-5n))/(3(2-n)}$ and $K_3 = \left[\big(3(2-n)\big) / \big(4\sqrt{2\ell}\big) \right]^{2/3}$. For $t_2 = 0$, $k = \frac{2}{3}$ and $n = 0$ from (38), we obtain that the solution (24) of Ram [37].

Some physical aspects of model

The model (24) has the barrel singularity at $\tau = \tau_0$ given by

$$\tau_0 = \left[\frac{k_1 (5n-2)}{2k} \right]^{2/((2-5n))},$$

which corresponds to $t = 0$. For $n \neq 2/5$ from (24), it is observed that at the singularity state $\tau = \tau_0$, p, ρ, Λ, θ and σ are infinitely large. At $t \to \infty$, which corresponds to $\tau \to \infty$ for $n < 2/5$ and $k > 0$, or $\tau \to 0$ for $n > 2/5$ and $k < 0$, p, ρ, Λ, θ and σ vanish. Therefore, for $n \neq 2/5$, the solution (24) represents an anisotropic universe exploding from $\tau = \tau_0$, i.e. $t = 0$, which expands for $0 < t < \infty$. We also find that the ratio σ/θ tends to a finite limit as $t \to \infty$, which means that the shear scalar does not tend to zero faster than the expansion. Hence, the model does not approach isotropy for large values of t.

In the subcase $\Lambda = 0$, $k = 0$, the ratio (35) tends to zero, then the model approaches isotropy, i.e. shear scalar σ goes to zero. For the model (37), p and ρ tend to zero as $t \to \infty$; the model would give an essentially empty universe at large time. The dominant energy condition given by Hawking and Ellis [51] requires that

$$\rho + p \geq 0, \quad \rho + 3p \geq 0\,. \tag{39}$$

Thus, we find for the model (37) that $n(2-n) \geq 0$. Hence for the values $0 \leq n \leq 2$, the universe (37) satisfies the strong energy condition, i.e. $\rho + 3p \geq 0$. Also this model is sheer-free and expanding.

In the subcase $\Lambda = 0$, $k_1 = 0$, for $n \neq 2/5$, 2, it is observed from relations (29) - (34) that p, ρ, θ and σ are infinitely large at the singularity state $t = -t_2$. When $t \to \infty$, these quantities vanish. We also find that the ratio σ/θ is a constant. This shows that the cosmological model (38) does not approach isotropy for large value of t. In this model, the dominant energy conditions (39) are then verified for $6 - 5n - 25n^2 \geq 0$. Since $n \neq 2/5$, the model (38) satisfies the strong energy condition for $-3/5 \leq n \leq 2/5$.

In each of the subcases, all obtained solutions (36), (37) and (38) satisfy the Bianchi identity given in Eq. (10).

3.2. Case (II): Let $C = \tilde{\tau}^{\,n}$ (n is a real number satisfying $n \neq \dfrac{2}{3}$)

In this case Eq. (19) gives

$$B = k_2 \tilde{\tau}^n \exp\left(M \tilde{\tau}^{\ell_1}\right), \tag{40}$$

and from (14), we obtain

$$A^2 = k_2 \tilde{\tau}^{2n} \exp\left(M \tilde{\tau}^{\ell_1}\right), \tag{41}$$

where $M = k/\ell_1$. Hence the metric (4) reduces to the form

$$\mathrm{d}s^2 = \tilde{\tau}^{4(1-\ell_1)/3} \left[\tilde{\tau}^{2(1-\ell_1)/3} e^{3M\tilde{\tau}^{\ell_1}} \mathrm{d}\tilde{\tau}^2 - e^{M\tilde{\tau}^{\ell_1}} \mathrm{d}x^2 \right.$$
$$\left. - e^{2\alpha x} \left(e^{2M\tilde{\tau}^{\ell_1}} \mathrm{d}y^2 + \mathrm{d}z^2 \right) \right], \tag{42}$$

where the constant k_2 is taken, without any loss of generality, equal to 1. This metric is a three-parameter family of solutions to EFEs with a perfect fluid.

For the above model, the distribution of matter and nonzero kinematical parameters are obtained as

$$8\pi(\bar{p}+\Lambda) = 2n\tilde{\tau}^{2(\ell_1-2)} + 3nk\tilde{\tau}^{3\ell_1-4}$$

$$+\frac{k^2}{2}\tilde{\tau}^{4(\ell_1-1)} + \alpha^2\tilde{\tau}^{4(\ell_1-1)/3}e^{-3M\tilde{\tau}^{\ell_1}}, \tag{43}$$

$$8\pi(\rho-\Lambda) = 3n^2\tilde{\tau}^{2(\ell_1-2)} + 3nk\tilde{\tau}^{3\ell_1-4}$$

$$+\frac{k^2}{2}\tilde{\tau}^{4(\ell_1-1)} - 3\alpha^2\tilde{\tau}^{4(\ell_1-1)/3}e^{-3M\tilde{\tau}^{\ell_1}}. \tag{44}$$

For simplicity and realistic models of physical importance, we consider the following two cases ($w = 0, 1$).

3.2.1. Model I: Solution for $w = 0$

When $w = 0$, Eq. (28) reduces to $\xi = \xi_0 =$ constant. Hence in this case, Eq. (43), with the use of (44), (27) and (28), leads to

$$8\pi(1+\gamma)\rho = n(2+3n)\tilde{\tau}^{2(\ell_1-2)} + 6nk\tilde{\tau}^{(3\ell_1-4)} + k^2\tilde{\tau}^{4(\ell_1-1)} - 2D_3 + 24\pi\xi_0 D_2, \tag{45}$$

where

$$D_2 = n\tilde{\tau}^{(\ell_1-2)} + \frac{1}{2}k\tilde{\tau}^{2(\ell_1-1)},$$

$$D_3 = \alpha^2\tilde{\tau}^{\frac{4}{3}(\ell_1-1)}e^{-3M\tilde{\tau}^{\ell_1}}.$$

Eliminating $\rho(t)$ from Eqs. (44) and (45), we obtain

$$8\pi(1+\gamma)\Lambda = n(2-3n\gamma)\tilde{\tau}^{2(\ell_1-2)} + 3nk(1-\gamma)\tilde{\tau}^{(3\ell_1-4)}$$

$$+\frac{1}{2}k^2(1-\gamma)\tilde{\tau}^{4(\ell_1-1)} + (1-3\gamma)D_3 + 24\pi\xi_0 D_2\,. \tag{46}$$

3.2.2. Model II: Solution for $w = 1$

When $w = 1$, Eq. (28) reduces to $\xi = \xi_0 \rho$. Hence in this case, Eq. (43), with the use of (44), (27) and (28), leads to

$$8\pi\rho = \frac{n(2+3n)\tilde{\tau}^{2(\ell_1-2)} + 6nk\tilde{\tau}^{(3\ell_1-4)} + k^2\tilde{\tau}^{4(\ell_1-1)} - 2D_3}{(1+\gamma) - 3\xi_0 D_2} \,. \tag{47}$$

Eliminating $\rho(t)$ from Eqs. (44) and (47), we obtain

$$8\pi\Lambda = \frac{n(2+3n)\tilde{\tau}^{2(\ell_1-2)} + 6nk\tilde{\tau}^{(3\ell_1-4)} + k^2\tilde{\tau}^{4(\ell_1-1)} - 2D_3}{(1+\gamma) - 3\xi_0 D_2}$$

$$-3n^(2)\tilde{\tau}^{2(\ell_1-2)} - 3nk\tilde{\tau}^{(3\ell_1-4)} - \frac{1}{2}k^2\tilde{\tau}^{4(\ell_1-1)} + 3D_3 \,. \tag{48}$$

In Model I, from Eqs. (45) and (46), we observe that the energy density $\rho(t)$ and cosmological constant ($\Lambda(t)$) decrease when time increases (see Fig. 2). Here we find the energy density always positive. We also observe that the value of Λ is small and positive at late times which is supported by recent type Ia supernovae observations [11 − 14]. From Eqs. (47) and (48), we observe that Model II has a similar behaviour as Model I, so it is not reproduced here.

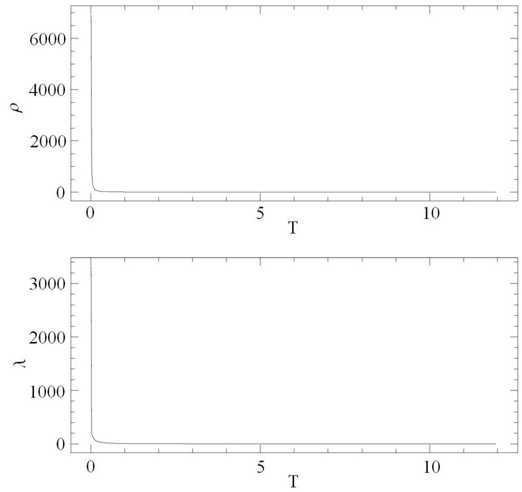

Fig. 2. (Top) Plot of $\rho \to \tilde{\tau}$ and (bottom) $\Lambda \to \tilde{\tau}$ for parameters $n = 0.45$, $K = 2.0$, $k_1 = 2$, $\gamma = 0.5$, $\alpha = 1$, $\xi_0 = 1$, $M = 1$ and rest of the constants are set to 1.

The scalar of expansion θ and the shear σ are obtained as

$$\theta = 3\left[n\tilde{\tau}^{\ell_1-2} + \frac{k}{2}\tilde{\tau}^{2(\ell_1-1)}\right], \tag{49}$$

$$\sigma = \frac{k}{2}\tilde{\tau}^{2(\ell_1-1)}e^{-3M\tilde{\tau}^{\ell_1}}, \tag{50}$$

From Eqs. (49) and (50), we have

$$\frac{\sigma}{\theta} = \frac{k}{6\,(n\tilde{\tau}^{-\ell_1}+k/2)}\,. \tag{51}$$

In the subcase $\Lambda = 0$ and $\xi_0 = 0$, the metric (42) with expressions p, ρ, θ and σ for this model are same as that of the solution (27) of Camci et al. [42].

In the subcase $\Lambda = 0$, $\xi_0 = 0$, $\ell_1 = 1$ (i.e. $n = 0$), we find a similar solution to (36), and hence this subclass is omitted. For $k = 0$, the ratio (51) is zero and hence there is no anisotropy.

After a suitable coordinate transformation, the metric (42) can be written as

$$\mathrm{d}s^2 = \mathrm{d}t^2 - K_4(t+t_3)^{2\ell_1}\left[\mathrm{d}x^2 + e^{2\alpha x}(\mathrm{d}y^2+\mathrm{d}z^2)\right]\,, \tag{52}$$

where t_3 is a constant and $K_4 = \left[2/(2-3M_1)\right]^{2M_1}$, $M_1 = 2n/(2+3n) \neq \frac{2}{3}$, where M_1 is a new parameter. When $M_1 = 0$ and $\ell_1 = 0$, from (52), we get the solution (12) of Ram [37].

Some physical aspects of model

The models have a singularity at $\tilde{\tau} \to -\infty$ for $\ell_1 > 0$ or $\tilde{\tau} \to 0$ for $\ell_1 < 0$, which corresponds to $t \to 0$. It is a point-type singularity for $\ell_1 > 0$, whereas it is a cigar or a barrel singularity when $\ell_1 < 0$. At $t \to \infty$, which correspond to $\tilde{\tau} \to \infty$ for $\ell_1 > 0$ or $\tilde{\tau} \to 0$ for $\ell_1 < 0$, from Eqs. (45) - (50), we obtain that for $\ell_1 > 0$, $p, \rho \to 0$, and $\sigma, \theta \to 0$ $(k > 0)$, - ∞ $(k < 0)$; for $\ell_1 < 0$, similar to the above ones. Then, clearly, for a realistic universe, it must be that as $\tilde{\tau} \to -\infty$, n and k are positive and ℓ_1 is an odd positive number; as $\tilde{\tau} \to 0$, k is positive, and ℓ_1 an even negative number. Also, since $\lim_{\tilde{\tau}\to\infty} \frac{\sigma}{\theta} \neq 0$, these models do not approach isotropy for large values of $\tilde{\tau}$.

In the subcase $k = 0$ for the metric (52), the effective pressure, density and cosmological constant are given by

$$8\pi(\bar{p}+\Lambda) = \frac{\ell_1(2-3\ell_1)}{(t+t_3)^2} + \frac{\alpha^2}{K_4(t+t_3)^{2\ell_1}}\,, \tag{53}$$

$$8\pi(\rho-\Lambda) = \frac{3\ell_1}{(t+t_3)^2} - \frac{3\alpha^2}{K_4(t+t_3)^{2\ell_1}}\,. \tag{54}$$

When $\Lambda = 0$ and $\xi_0 = 0$, the pressure and energy density are the same as that given in Eq. (44) of paper Camci et al. [42]. In this case, the weak and strong energy conditions (39) for this solution are identically satisfied when $\ell_1(1-\ell_1) \geq 0$, i.e. $0 \leq \ell_1 \leq 1$. This model is shear-free and expanding with $\theta = 3\ell_1/(t+t_3)$.

3.3. Case (III): Let $B = T^n$ (n is a real number).

In this case, Eq. (20) gives

$$C = k_3 T^n - 2\ell T^{\ell_1}\,, \tag{55}$$

and then from (14), we obtain

$$A^2 = k_3 T^{2n} - 2\ell T^{\ell_1 + n}\,. \tag{56}$$

Hence the metric (4) takes the new form

$$\mathrm{d}s^2 = \left(k_3 T^n - 2\ell T^{\ell_1}\right)[\mathrm{d}t^2 - T^n \mathrm{d}x^2] -$$

$$e^{2\alpha x}\left[T^{2n}\mathrm{d}y^2 + \left(k_3 T^n - 2\ell T^{\ell_1}\right)^2 \mathrm{d}z^2\right]. \tag{57}$$

For the four-parameter family solution (57), the physical and kinematical quantities are given by

$$8\pi(\bar{p} + \Lambda) = \left[-\frac{\ell^2}{2}(11n^2 - 4n - 4)T^{-3n} + \ell k_3 n(13n - 10)T^{\ell_1 + n} -\right.$$

$$\left. 2k_3^2 n(n-1)T^{2n-2}\right]\left(k_3 T^n - 2\ell T^{\ell_1}\right)^{-3} + \alpha^2 T^{-n}\left(k_3 T^n - 2\ell T^{\ell_1}\right)^{-1}, \tag{58}$$

$$8\pi(\rho - \Lambda) = \left[-\frac{\ell^2(11n^2 - 4n - 14)}{2}T^{-3n} - 3\ell k_3 n(2 - n)T^{\ell_1 + n} +\right.$$

$$\left. 3k_3^2 n^2 T^{2n-2}\right]\left(k_3 T^n - 2\ell T^{\ell_1}\right)^{-3} - 3\alpha^2 T^{-n}\left(k_3 T^n - 2\ell T^{\ell_1}\right)^{-1}. \tag{59}$$

3.3.1. **Model I: Solution for** $w = 0$

When $w = 0$, Eq. (28) reduces to $\xi = \xi_0 =$ constant. Hence in this case, Eq. (58), with the use of (59), (27) and (28), leads to

$$8\pi(1+\gamma)\rho = \left[n(n+2){k_3}^2 T^{2(n-1)} + 16n(n-1)k_3\ell T^{(\ell_1+n)} - (11n^2 - 4n - 9)\ell^2 T^{-3n}\right]{D_4}^{-3}$$

$$-2\alpha^2 T^{-n}{D_4}^{-1} + 24\pi\xi_0 D_5 {D_4}^{-3/2}, \tag{60}$$

where

$$D_4 = k_3 T^n - 2\ell T^{\ell_1},$$

$$D_5 = k_3 n T^{n-1} + \frac{1}{2}(n-2)\ell T^{-3n/2}.$$

Eliminating $\rho(t)$ from Eqs. (59) and (60), we obtain

$$8\pi(1+\gamma)\Lambda = \Big[\{-\frac{1}{2}n\ell^2(1-\gamma)(11n-4) - \ell^2(7\gamma-2)\}T^{-3n}$$

$$+n\ell k_3\{13n-10+3\gamma(2-n)\}T^{\ell_1+n} - nk_3{}^2(2n-2+3n\gamma)T^{2(n-1)}\Big]D_4{}^{-3}$$

$$+(1+3\gamma)\alpha^2 T^{-n}D_4{}^{-n} + 24\pi\xi_0 D_5 D_4{}^{-3/2}. \tag{61}$$

3.3.2. Model II: Solution for $w = 1$

When $w = 1$, Eq. (28) reduces to $\xi = \xi_0\rho$. Hence in this case, Eq. (58), with the use of (59), (27) and (28), leads to

$$8\pi\rho = \frac{1}{[(1+\gamma) - 3\xi_0 D_5 D_4{}^{-\frac{3}{2}}]}\Big[-\ell^2(11n^2-4n-9)T^{-3n} + 16n(n-1)\ell k_3 T^{\ell_1+n}$$

$$+n(n+2)k_3{}^2T^{2(n-1)}\Big]D_4{}^{-3} - \alpha^2T^{-n}D_3{}^{-1}. \tag{62}$$

Eliminating $\rho(t)$ from Eqs. (59) and (62), we obtain

$$8\pi\Lambda = \frac{1}{[(1+\gamma) - 3\xi_0 D_5 D_4{}^{-\frac{3}{2}}]}\Big[\{-\ell^2(11n^2-4n-9)T^{-3n} + 16n(n-1)\ell k_3 T^{\ell_1+n}$$

$$+n(n+2)k_3{}^2T^{2(n-1)}\}D_4{}^{-3} - \alpha^2T^{-n}D_3{}^{-1}\Big] -$$

$$\Big[\{-\frac{1}{2}(11n^2-4n-14)\ell^2T^{-3n} - 3n(2-n)\ell k_3T^{\ell_1+n} + 3n^2k_3{}^2T^{2(n-1)}\}D_4{}^{-3} + 3\alpha^2T^nD_4{}^{-1}\Big]. \tag{63}$$

In Model I, from Eqs. (60) and (61), we observe that the energy density $\rho(t)$ and cosmological constant $(\Lambda(t))$ are decreasing functions of time (see Fig. 3). The energy density is always positive. We also observe that the value of Λ is small and positive at late times which is supported by recent type Ia supernovae observations [11–14]. From Eqs. (62) and (63), we observe that Model II has the similar behaviour as Model I, so it is not reproduced here.

The scale of expansion and the shear are obtained as

$$\theta = 3\left[\frac{\ell(n-2)}{2}T^{-3n/2} + k_3 n T^{n-1}\right]\left(k_3T^n - 2\ell T^{\ell_1}\right)^{-3/2}, \tag{64}$$

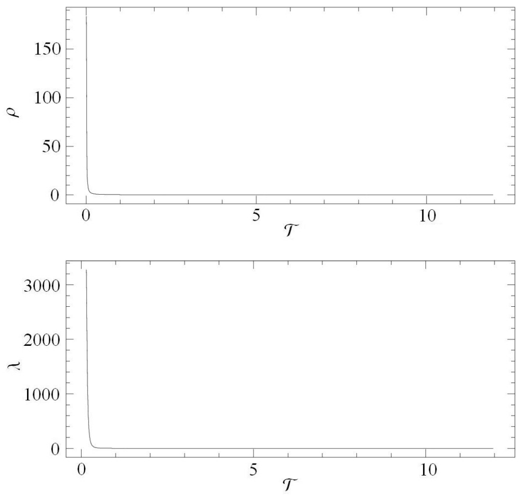

Fig. 3. (Top) Plot of $\rho \to T$ *and (bottom)* $\Lambda \to T$ *for parameters* $n = 0.45$, $K = 2.0$, $k_3 = 1$, $\gamma = 0.5$, $\alpha = 1$, $\xi_0 = 1$ *and rest of the constants are set to* 1.

$$\sigma = \frac{kT^{-3n/2}}{2}\left(k_3 T^n - 2\ell T^{\ell_1}\right)^{-3/2}. \tag{65}$$

From (64) and (65), we get

$$\frac{\sigma}{\theta} = \frac{k}{6}\left[k_3 n T^{-\ell_1+n} + \frac{\ell(n-2)}{2}\right]^{-1}. \tag{66}$$

In the subcase $\Lambda = 0$ and $\xi = 0$, the metric (57) with expressions p, ρ, θ and σ for this model are same as that of solution (34) of Camci et al. [42]. If we set $\xi_0 = 0$, this metric (57) represents the solution obtained by Pradhan et al. [43].

In the subcase $\Lambda = 0$, $\xi = 0$, $n = 0$, after an inverse transformation, the metric (57) reduces to the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - K_5(t+t_4)^{2/3}\mathrm{d}x^2 - e^{2\alpha x}\left[\mathrm{d}y^2 + K_5^2(t+t_4)^{4/3}\mathrm{d}z^2\right], \tag{67}$$

where t_4 is an integrating constant. This model is different from the model (36) by a change of scale.

In the subcase $\Lambda = 0$, $k = 0$, the same model as (37) is obtained.

Further, in the subcase $\Lambda = 0$, $\xi = 0$ $k_3 = 0$, we see that the metric (57) takes the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - 2\ell K_6(t+t_5)^{2/3}\left[\mathrm{d}x^2 + e^{2\alpha x}\left(bt^{m_2}\mathrm{d}y^2 + b^{-1}t^{-m_2}\mathrm{d}z^2\right)\right], \tag{68}$$

where t_5 is a constant, $bt^{m_2} = 2\ell K_6^{(2-5n)/(2-n)}(t + t_5)^{[2(2-5n)]/[3(2-n)]}$ and $K_6 = \Big[\big(3(2-n)\big)/\big(4\sqrt{2\ell}\,\big)\Big]^{2/3}$. This metric is only different from (37) by a change of sign. Also, in each of the subcases, the physical and kinematical properties of obtained metric are same as that of Case(I). Therefore, we do not consider them here.

3.4. *Case (IV): Let $B = \tilde{\tau}^n$, where n is any real number.*

In this case, Eq. (21) gives

$$C = k_4 \tilde{\tau}^n \exp\left(\frac{k}{\ell_1}\tilde{\tau}^{\ell_1}\right), \tag{69}$$

and then from (14), we obtain

$$A^2 = k_4 \tilde{\tau}^{2n} \exp\left(\frac{k}{\ell_1}\tilde{\tau}^{\ell_1}\right). \tag{70}$$

Hence the metric (4) reduces to

$$\mathrm{d}s^2 = \tilde{\tau}^{2n} \exp\left(\frac{k}{\ell_1}\tilde{\tau}^{\ell_1}\right)\left[\tilde{\tau}^n \exp\left(\frac{2k}{\ell_1}\tilde{\tau}^{\ell_1}\right) - \mathrm{d}x^2\right]$$

$$-e^{2\alpha x}\left[\mathrm{d}y^2 + \exp\left(\frac{2k}{\ell_1}\tilde{\tau}^{\ell_1}\right) - \mathrm{d}z^2\right], \tag{71}$$

where, without any loss of generality, the constant k_4 is taken equal to 1. Expressions for physical and kinematical parameters for the model (71) are not given here, but it is observed that the properties of the metric (71) are same as that of the solution (42), i.e. the Case (II).

4. *Concluding remarks*

In this paper we have described new exact solutions of EFEs for Bianchi type V spacetime with a bulk viscous fluid as the source of matter and cosmological term Λ varying with time. Using a generation technique introduced by Camci et al. [42], it is shown that the Einstein's field equations are solvable for any arbitrary cosmic scale function. Starting from particular cosmic functions, new classes of spatially homogeneous and anisotropic cosmological models have been investigated for which the fluids are acceleration- and rotation-free, but they do have expansion and shear. For $\alpha = 0$ in the metric (4), we obtained metrics as LRS Bianchi type I model (Hajj-Boutros [35], Hajj-Boutros and Sfeila [36], Ram [37], Mazumder [38], Pradhan and Kumar [39]) and Pradhan et al. [43]. It is also seen that the solutions obtained by Camci et al. [42], Ram [37], Pradhan and Kumar [39] and Bianchi type V models studied by Pradhan et al. [43] are particular cases (except one) of our solutions.

The cosmological constants in all models given in Section 3 are decreasing functions of time (except one, Model II of Case (I)), and they all approach a small positive value at late times, what is supported by the results from the supernova observations recently obtained by the High-z Supernova Team and Supernova Cosmological Project (Garnavich et al. [11], Perlmutter et al. [12], Riess et al. [13], Schmidt et al.[14]).

The features of these new solutions are that from our wide range of choices of parameters for Model I of Cases (I), (II), and (III), it is apparent from figures that energy density $\rho(t)$ and dynamic cosmological term $\Lambda(t)$ are decreasing function of time, remain positive and small at later stage during evolution. These two quantities remain finite and do not become zero at later stage. Hence this seem to be physically viable to explore physical mechanism for further detail study depending on relevance of the physical problem. These studies show that the bulk viscous effect is apparent on $\Lambda(t)$. In many astrophysical situations the viscosity calculated using statistical consideration do not give satisfactory viscous number. Hence the specific thing about bulk viscosity will depend on the details of the viscous nature of the matter. We have explored Model II of Cases (I), (II) and (III). So we feel that it is not necessary to display behaviour of $\rho(t)$ and $\Lambda(t)$. We have studied these models and no anomaly has been observed.

Model II of Case (I) is explored for a variety of parameters. We find that in many cases, energy density decreases sharply and becomes negative very fast and at a later stage also remains negative, but there is a slight increase at later stage and remains constant (negative constant), which we feel as an undesirable feature. So it is not discussed in the paper. Also, in many cases it is initially oscillatory. In this case, $\Lambda(t)$ shows very peculiar behaviour. Only in a few cases, Λ is initially negative, then becomes positive but in all cases during initial evolution Λ has singularities when t is close to zero and a few at later finite times. The cause of this behaviour is not very apparent from the model. Otherwise, for a wide range of parameters, Λ shows eratic behaviour. In most of the cases, singular and eratic behaviour of $\Lambda(t)$ is persistent and the cause is not known. So, we do not discuss it further. This may require a detailed separate study.

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IZVOD KOZMOLOŠKIH MODELA V BIANCHIJEVE VRSTE S VOLUMNIM TRENJEM I VREMENSKI-OVISNIM ČLANOM Λ

Istražujemo kozmološke modele V-e Bianchijeve vrste s volumnim trenjem i dinamičnim kozmološkim članom $\Lambda(t)$. Primjenom metode izvođenja (Camci et al., 2001) pokazujemo da se Einsteinove jednadžbe polja mogu riješiti za proizvoljnu funkciju kozmičke mjere. Postigli smo rješenja za posebne funkcije kozmičkih mjera. Nalazimo da je kozmološka konstanta opadajuća funkcija vremena, što je u skladu s nedavnim opažanjima supernova Ia. Raspravljamo također neka fizička svojstva modela.