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PLANE SYMMETRIC INHOMOGENEOUS BULK VISCOUS DOMAIN WALL IN LYRA GEOMETRY

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Some bulk viscous general solutions are found for domain walls in Lyra geometry in the plane symmetric inhomogeneous spacetime. Expressions for the energy density and pressure of domain walls are derived in both cases of uniform and time varying displacement field β. The viscosity coefficient of bulk viscous fluid is assumed to be a power function of mass density. Some physical consequences of the models are also given. Finally, the geodesic equations and acceleration of the test particle are discussed.

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1. Introduction

Topological structures could be produced at phase transitions in the Universe as it cooled [1] − [5]. Phase transitions can also give birth to solitonlike structures such as monopoles, strings and domain walls [6]. Within the context of general relativity, domain walls are immediately recognizable as especially unusual and interesting sources of gravity. Domain walls form when discrete symmetry is spontaneously broken [7]. In the simplest models, symmetry breaking is accomplished by a real scalar field ϕ whose vacuum manifold is disconnected. For example, suppose that the scalar potential for ϕ is $U(\phi) = \lambda(\phi^2 - \mu^2)^2$. The vacuum manifold

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for ϕ then consists of two points [$\phi = \mu$ and $\phi = -\mu$]. After symmetry breaking, different regions of the Universe can settle into different parts of the vacuum with domain walls forming the boundaries between these regions. As was pointed out by Zel'dovich *et al.* [6], the stress-energy of domain walls is composed of surface energy density and strong tension in two spatial directions, with the magnitude of the tension equal to that of the surface energy density. This is interesting because there are several indications that tension acts as a repulsive source of gravity in general relativity, whereas pressure is attractive. We note, however that this analysis neglects the effects of gravity [8]. Locally, the stress energy for a wall of arbitrary shape is similar to that of a plane-symmetric wall having both surface energy density and surface tension. Closed-surface domain walls collapse due to their surface tension. However, the details of the collapse for a wall with arbitrary shape and finite thickness are largely unknown.

The spacetime of cosmological domain walls has now been a subject of interest for more than a decade since the work of Vilenkin [9] and Ipser and Sikivie [10] who use Israel's thin wall formalism [11] to compute the gravitational field of an infinitesimally thin planar domain wall. After the original work [9, 10] for thin walls, attempts focused on trying to find a perturbative expansion in the wall thickness [8, 12]. With the proposition by Hill, Schramn and Fry [13] of a late phase transition with thick domain walls, there were some efforts in finding exact thick-wall solution [14, 15]. Recently, Bonjour *et al.* [16] considered gravitating thick domain wall solutions with planar and reflection symmetry in the Goldstone model. Bonjour *et al.* [17] also investigated the spacetime of a thick gravitational domain wall for a general potential $V(\phi)$. Jensen and Soleng [18] have studied anisotropic domain walls where the solution has naked singularities and the generic solution is unstable to Hawking decay.

The investigation of relativistic cosmological models usually has the energy momentum tensor of matter generated by a perfect fluid. To consider more realistic models, one must take into account the viscosity mechanisms, which have already attracted the attention of many researchers. Most of the studies in cosmology involve a perfect fluid. Large entropy per baryon and the remarkable degree of isotropy of the cosmic microwave background radiation, suggests that we should analyze dissipative effects in cosmology. Further, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the recombination era [19], decay of massive superstring modes into massless modes [20], gravitational string production [21, 22] and particle creation effect in the grand unification era. It is known that the introduction of bulk viscosity can avoid the big bang singularity. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models (see Grøn [23] for a review on cosmological models with bulk viscosity). A uniform cosmological model filled with fluid which possesses pressure and second (bulk) viscosity was developed by Murphy [24]. The solutions that he found exhibit an interesting feature that the big bang type singularity appears in the infinite past.

Einstein (1917) geometrized gravitation. Weyl, in 1918, was inspired by it and he was the the first to unify gravitation and electromagnetism in a single spacetime geometry. He showed how one can introduce a vector field in the Riemannian spacetime with an intrinsic geometrical significance. But this theory was not accepted as it was based on non-integrability of length transfer. Lyra [25] introduced a gauge function, i.e., a displacement vector in Riemannian spacetime which removes the non-integrability condition of a vector under parallel transport. In this way Riemannian geometry was given a new modification by him and the modified geometry was named as Lyra's geometry.

Sen [26] and Sen and Dunn [27] have proposed a new scalar-tensor theory of gravitation and constructed the field equations analogous to the Einstein's field equations, based on Lyra's geometry which in normal gauge may be written in the form

$$R_{ij} - \frac{1}{2}g_{ij}R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4}g_{ij}\phi_k\phi^k = -8\pi G T_{ij}\,, \tag{1}$$

where ϕ_i is the displacement vector and other symbols have their usual meanings.

Halford [28] pointed out that the constant vector displacement field ϕ_i in Lyra's geometry plays the role of the cosmological constant Λ in the normal general relativistic treatment. It was shown by Halford [29] that the scalar-tensor treatment based on Lyra's geometry predicts the same effects, within observational limits as the Einstein's theory. Several investigators [30] – [43] have studied cosmological models based on Lyra geometry in different contexts. Soleng [31] has pointed out that the cosmologies based on Lyra's manifold with constant gauge vector ϕ will either include a creation field and be equal to Hoyle's creation field cosmology [44] – [46] or contain a special vacuum field which together with the gauge vector term may be considered as a cosmological term. In the latter case, the solutions are equal to the general relativistic cosmologies with a cosmological term.

The Universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous. But during the early stages of evolution, it is unlikely that it could have had such a smoothed out picture. Hence, we consider plane symmetry which provides an opportunity for the study of inhomogeneity. Recently Pradhan *et al.* [47] have studied plane symmetric domain wall in the presence of a perfect fluid.

Motivated by the situations discussed above, we shall focus in this paper upon the problem of establishing a formalism for studying the general solutions for domain wall in Lyra geometry in the plane symmetric inhomogeneous spacetime metric in the presence of bulk viscous fluid. Expressions for the energy density and pressure of domain walls are obtained in both cases of uniform and time varying displacement field β. This article is organized as follows: The metric and the basic equations are presented in Section 2. In Section 3, we deal with the solution of the field equations. The Subsection 3.1 contains the solution of the uniform displacement field ($\beta = \beta_0$, constant). This section also contains two different models and also the physical consequences of these models. The Subsection 3.2 deals with the solution with time-varying displacement field ($\beta = \beta_0 t^\alpha$). This subsection also contains two different models and their physical consequences are discussed. The geodesic equations and accelerations of the test particle are discussed in Section 4. Finally, in Section 5 concluding remarks are given.

2. The metric and basic equations

Thick domain walls are characterized by the energy-momentum tensor of a viscous field which has the form

$$T_{ik} = \rho(g_{ik} + w_i w_k) + \bar{p} w_i w_k, \quad w_i w^i = -1\,, \tag{2}$$

where

$$\bar{p} = p - \xi w^i_{;i}\,. \tag{3}$$

Here ρ, p, $\bar{p}$, and ξ are the energy density, the pressure in the direction normal to the plane of the wall, the effective pressure and the bulk viscous coefficient, respectively, and w_i is a unit space-like vector in the same direction.

The displacement vector ϕ_i in Eq. (1) is given by

$$\phi_i = (0, 0, 0, \beta)\,, \tag{4}$$

where β may be considered constant as well as a function of the time coordinate, like the cosmological constant in Einstein's theory of gravitation.

The energy momentum tensor T_{ij} in comoving coordinates for thick domain walls takes the form

$$T^0_0 = T^2_2 = T^3_3 = \rho, \quad T^1_1 = -\bar{p}, \quad T^0_1 = 0\,. \tag{5}$$

We consider the most general plane-symmetric spacetime metric suggested by Taub [48]

$$\mathrm{d}s^2 = e^A(\mathrm{d}t^2 - \mathrm{d}z^2) - e^B(\mathrm{d}x^2 + \mathrm{d}y^2)\,, \tag{6}$$

where A and B are functions of t and z.

Using Eq. (5), the field Eqs. (1) for the metric (6) reduce to

$$\frac{e^{-A}}{4}(-4B'' - 3B'^2 + 2A'B') + \frac{e^{-A}}{4}(\dot{B}^2 + 2\dot{B}\dot{A}) - \frac{3}{4}e^{-A}\beta^2 = 8\pi\rho\,, \tag{7}$$

$$\frac{e^{-A}}{4}(-B'^2 - 2B'A') + \frac{e^{-A}}{4}(-4\ddot{B} + 3\dot{B}^2 - 2\dot{A}\dot{B}) + \frac{3}{4}e^{-A}\beta^2 = -8\pi\bar{p}\,, \tag{8}$$

$$\frac{e^{-A}}{4}[-2(A'' + B'') - B'^2] + \frac{e^{-A}}{4}[2(\ddot{A} + \ddot{B}) + \dot{B}^2] + \frac{3}{4}e^{-A}\beta^2 = 8\pi\rho\,, \tag{9}$$

$$-\dot{B}' + \dot{B}(A' - B') + \dot{A}B' = 0\,. \tag{10}$$

In order to solve the above set of field equations, we assume the separable form of the metric coefficients as follows

$$A = A_1(z) + A_2(t), \quad B = B_1(z) + B_2(t) \,. \tag{11}$$

From Eqs. (10) and (11), we obtain

$$\frac{A_1'}{B_1'} = \frac{(\dot{B}_2 - \dot{A}_2)}{B_2} = m \,, \tag{12}$$

where m is considered as the separation constant.

Equation (12) yields the solution

$$A_1 = mB_1 \,, \tag{13}$$

$$A_2 = (1 - m)B_2 \,. \tag{14}$$

Again, subtracting Eq. (9) from Eq. (7) and using Eq. (11), we obtain

$$A_1'' - B_1'' - B_1'^2 + A_1'B_1' = \ddot{A}_2 + \ddot{B}_2 - \dot{A}_2\dot{B}_2 + 3\beta^2 = k \,, \tag{15}$$

where k is another separation constant.

With the help of Eqs (13) and (14), Eq. (15) may be written as

$$(m - 1)[B_1'' + B_1'^2] = k \,, \tag{16}$$

$$(2 - m)\ddot{B}_2 + (m - 1)\dot{B}_2^2 = k - 3\beta^2. \tag{17}$$

3. *Solutions of the field equations*

In this section we shall obtain exact solutions for thick domain walls in some cases.

Using the substitution $u = e^{B_1}$ and $a = \dfrac{k}{1-m}$, Eq. (16) takes the form

$$u'' + au = 0 \,, \tag{18}$$

which has the solution

$$e^{B_1} = u = c_1 \sin(z\sqrt{a}) + c_2 \cos(z\sqrt{a}) \,, \quad \text{when } a > 0 \,, \tag{19}$$

where c_1 and c_2 are integrating constants. Equation (19) represent the general solution of the differential Eq. (18) when $a > 0$. It may be noted that Rahaman *et al.* [37] have obtained a particular solution for the case $a < 0$ in the presence of the perfect fluid. Recently Pradhan *et al.* [47] have investigated a general solution in the presence of the perfect fluid.

Equation (17) may be written as

$$\ddot{B}_2 - \frac{(1-m)}{(2-m)}\dot{B}_2^2 + \frac{3}{(2-m)}\beta^2 = \frac{k}{2-m}\,. \tag{20}$$

Now we shall consider uniform and time varying displacement field β separately.

3.1. *Case I: Uniform displacement field ($\beta = \beta_0$, constant)*

By the use of the transformation $v = e^{-[(1-m)/(2-m)]B_2}$, Eq. (19) reduces to

$$\ddot{v} + bv = 0\,, \tag{21}$$

where

$$b = \frac{(1-m)(k-3\beta_0^2)}{(2-m)^2}\,.$$

Again, it can be easily seen that Eq. (21) has the solution

$$e^{-[(1-m)/(2-m)]B_2} = v = \bar{c_1}\sin(t\sqrt{b}) + \bar{c_2}\cos(t\sqrt{b}) \quad \text{when } b > 0\,, \tag{22}$$

where $\bar{c_1}$ and $\bar{c_2}$ are integrating constants. Hence the metric coefficients have the explicit forms when $a > 0$ and $b > 0$,

$$e^A = [c_1\sin(z\sqrt{a}) + c_2\cos(z\sqrt{a})]^m \times [\bar{c_1}\sin(t\sqrt{b}) + \bar{c_2}\cos(t\sqrt{b})]^{(m-2)}\,, \tag{23}$$

$$e^B = [c_1\sin(z\sqrt{a}) + c_2\cos(z\sqrt{a})] \times [\bar{c_1}\sin(t\sqrt{b}) + \bar{c_2}\cos(t\sqrt{b})]^{-(m-2)/(1-m)}. \tag{24}$$

With the help of Eqs. (23) and (24), the energy density and pressure can be obtained from Eqs. (7) and (8) as given by

$$32\pi\rho = e^{-A}\left[4a + a\left(\frac{Z_1}{Z_2}\right)^2(1+m) + \frac{(3-m)(2-m)^2}{(1-m)^2}b\left(\frac{T_2}{T_1}\right)^2 - 3\beta_0^2\right], \tag{25}$$

$$32\pi(p-\xi\theta) =$$

$$e^{-A}\left[a(1{+}m)\left(\frac{Z_1}{Z_2}\right)^2 + \frac{4b(2{-}m)}{(1{-}m)} + \frac{b(2{-}m)(2m^2{-}7m{+}2)}{(1{-}m)^2} \times \left(\frac{T_2}{T_1}\right)^2 - 3\beta_0^2\right], \tag{26}$$

where

$$Z_1 = c_1 - c_2\tan(z\sqrt{a})\,,$$
$$Z_2 = c_2 + c_1\tan(z\sqrt{a})\,,$$
$$T_1 = \bar{c}_2 + \bar{c}_1\tan(t\sqrt{b})\,,$$
$$T_2 = \bar{c}_1 + \bar{c}_2\tan(t\sqrt{b})\,.$$

Here ξ, in general, is a function of time. The expression for kinematical parameter expansion θ is given by

$$\theta = \frac{e^{-A/2}}{(m-1)} \left(\frac{T_3}{T_1}\right), \tag{27}$$

where $T_3 = \bar{c}_1 - \bar{c}_2 \tan(t\sqrt{b})$. Thus, given $\xi(t)$, we can solve Eq. (26). In most investigations involving bulk viscosity, it is assumed to be a simple power function of the energy density [49]–[52]

$$\xi(t) = \xi_0 \rho^n, \tag{28}$$

where ξ_0 and n are constants. For a small density, n may even be equal to unity as used in Murphy's work for simplicity [24]. If $n = 1$, Eq. (28) may correspond to a radiative fluid [53]. Near the big bang, $0 \leq n \leq \frac{1}{2}$ is a more appropriate assumption [54] to obtain realistic models.

For simplicity and realistic models of physical importance, we consider the following two cases ($n = 0, 1$).

3.1.1. **Model I: solution for $\xi = \xi_0$**

When $n = 0$, Eq. (28) reduces to $\xi = \xi_0 =$ constant. Hence in this case Eq. (26), with the use of (27), leads to

$$32\pi p = \frac{32\pi\xi_0 e^{-A/2}}{(m-1)} \left(\frac{T_3}{T_1}\right) + e^{-A} \left[a(1+m)\left(\frac{Z_1}{Z_2}\right)^2 + \frac{4b(2-m)}{(1-m)} + \frac{b(2-m)(2m^2-7m+2)}{(1-m)^2}\left(\frac{T_2}{T_1}\right)^2 - 3\beta_0\right]. \tag{29}$$

3.1.2. **Model II: solution for $\xi = \xi_0\rho$**

When $n = 1$, Eq. (28) reduces to $\xi = \xi_0\rho$ and hence Eq. (26), with the use of (27), leads to

$$32\pi p = \mathrm{e}^{-A}\left[a(1+m)(1+T_4) + 4aT_4 + \frac{4b(2-m)}{(1-m)} + \frac{b(2-m)}{(1-m)^2}\left(\frac{T_2}{T_1}\right)^2 \times \left\{(2-m)(3-m)T_4 + 2m^2 - 7m + 2\right\} - 3(T_4+1){\beta_0}^2\right], \tag{30}$$

where $T_4 = \dfrac{32\pi\xi_0 e^{-a/2}}{(m-1)} \left(\dfrac{T_3}{T_1}\right)$.

From the above results of both models, it is evident that at any instant the domain wall density ρ and pressure p in the perpendicular direction decrease on both sides of the wall away from the symmetry plane, and both vanish as $z \longrightarrow \pm\infty$. The space times in both cases are reflection symmetrical with respect to the wall. All these properties are very much expected for a domain wall. It can be also seen that the viscosity, as well as the displacement field β, exhibit essential influence on the character of the solutions.

3.2. Case II: Time-varying displacement field ($\beta = \beta_0 t^\alpha$)

Using the aforesaid power law relation between the time coordinate and the displacement field, Eq. (19) may be written as

$$\ddot{w} - \left[\frac{3(1-m)\beta_0^2}{4(2-m)^2}\, t^{2\alpha} - \frac{k(1-m)}{(2-m)^2}\right] w = 0\,, \tag{31}$$

where

$$w = e^{-(1-m)/(2-m)}\, B_2\,. \tag{32}$$

Now, it is difficult to find a general solution of Eq. (31). Hence we consider a particular case of physical interest. It is believed that β^2 appears to play the role of a variable cosmological term $\Lambda(t)$ in Einstein's equation. Considering $\alpha = -1$ and $\beta = \beta_0/t$, Eq. (31) reduces to

$$t^2\ddot{w} + \left[\frac{k(1-m)}{(2-m)^2} t^2 - \frac{3}{4}\frac{(1-m)}{(2-m)^2}\beta_0^2\right] w = 0\,. \tag{33}$$

Eq. (33) yields the general solution

$$w t^{r+1} = (t^3 D)^r \left[\frac{c_1 e^{ht} + c_2 e^{-ht}}{t^{2r-1}}\right], \tag{34}$$

where

$$D \equiv \frac{\mathrm{d}}{\mathrm{d}t}\,,$$
$$r = \frac{1}{2}[\{1 + \frac{3(1-m)}{(2-m)^2}\beta_0^2\}^{\frac{1}{2}} - 1]\,,$$
$$h^2 = \frac{k(1-m)}{(2-m)^2}\,.$$

For $r = 1$, $\beta_0^2 = \dfrac{8(2-m)^2}{3(1-m)}$, Eq. (34) suggests

$$w = \left(h - \frac{1}{t}\right) c_3 e^{ht} - \left(h + \frac{1}{t}\right) c_4 e^{-ht}, \tag{35}$$

where c_3 and c_4 are integrating constants.

Hence the metric coefficients have the explicit forms when $a > 0$ as

$$e^A = \left[c_1 \sin(z\sqrt{a}) + c_2 \cos(z\sqrt{a})\right]^m \times \left[\left(h - \frac{1}{t}\right) c_3 e^{ht} - \left(h + \frac{1}{t}\right) c_4 e^{-ht}\right]^{(m-1)}, \quad (36)$$

$$e^B = \left[c_1 \sin(z\sqrt{a}) + c_2 \cos(z\sqrt{a})\right] \times \left[\left(h - \frac{1}{t}\right) c_3 e^{ht} - \left(h + \frac{1}{t}\right) c_4 e^{-ht}\right]^{-(2-m)/(1-m)}. \quad (37)$$

With the help of Eqs. (36) and (37), the energy density and pressure can be obtained from Eqs. (7) and (8)

$$32\pi\rho = e^{-A}\left[4a + a(1+m)\left(\frac{Z_1}{Z_2}\right)^2 + \frac{(3-m)(2-m)^2}{(1-m)^2}\left(\frac{c_3 h^2 t}{T_6} - \frac{1}{t}\right)^2 - \frac{3\beta_0^2}{t^2}\right], \quad (38)$$

$$32\pi(p - \xi\theta) = e^{-A}\left[a(1+m)\left(\frac{Z_1}{Z_2}\right)^2 - \frac{(1+2m)(2-m)^2}{(1-m)^2}\left(\frac{c_3 h^2 t}{T_6} - \frac{1}{t}\right)^2 - \frac{4(2-m)}{(1-m)}\left\{\frac{1}{t^2} - \frac{4c_4 h^3 t e^{-2ht}}{T_6} + h^2\left(\frac{T_5}{T_6}\right)^2\right\} - \frac{3\beta_0^2}{t^2}\right], \quad (39)$$

where

$$T_5 = c_3 + c_4(1 + 2ht)e^{-2ht},$$
$$T_6 = c_3(ht - 1) - c_4(1 + ht)e^{-2ht}.$$

The expression for the kinematical parameter expansion θ is given by

$$\theta = \frac{(hT_7 + T_8)(m^2 - 4m + 5)}{2(m-1)} e^{-A/2}, \quad (40)$$

where

$$T_7 = (ht - 1)c_3 + (ht + 1)c_4 e^{-2ht},$$
$$T_8 = c_3 + c_4 e^{-2ht}.$$

In this case we again consider the following two cases ($n = 0, 1$).

3.2.1. Model I: solution for $\xi = \xi_0$

When $n = 0$, Eq. (28) reduces to $\xi = \xi_0 = constant$. Hence in this case Eq. (39), with the use of (40), leads to

$$32\pi p = \frac{32\pi\xi_0(hT_7 + T_8)(m^2 - 4m + 5)}{2(m-1)T_6} e^{-A/2} + e^{-A}\left[a(1+m)\left(\frac{Z_1}{Z_2}\right)^2 - \frac{(1+2m)(2-m)^2}{(1-m)^2}\left(\frac{c_3 h^2 t}{T_6} - \frac{1}{t}\right)^2 - \frac{4(2-m)}{(1-m)} \times \left\{\frac{1}{t^2} - \frac{4c_4 h^3 t e^{-2ht}}{T_6} + h^2\left(\frac{T_5}{T_6}\right)^2\right\} - \frac{3\beta_0^2}{t^2}\right]. \quad (41)$$

3.2.2. Model II: solution for $\xi = \xi_0 \rho$

When $n = 1$, Eq. (28) reduces to $\xi = \xi_0\rho$. Hence in this case, Eq. (39) with the use of (40), leads to

$$32\pi p = e^{-A}\left[4a + a(1+m)\left(\frac{Z_1}{Z_2}\right)^2 + \frac{(3-m)(2-m)^2}{(1-m)^2}\left(\frac{c_3h^2t}{T_6} - \frac{1}{t}\right)^2 - \frac{3\beta_0^2}{t^2}\right]T_9$$

$$+e^{-A}\left[a(1+m)\left(\frac{Z_1}{Z_2}\right)^2 - \frac{(1+2m)(2-m)^2}{(1-m)^2}\left(\frac{c_3h^2t}{T_6} - \frac{1}{t}\right)^2 - \frac{4(2-m)}{(1-m)}\times\right.$$

$$\left.\left\{\frac{1}{t^2} - \frac{4c_4h^3te^{-2ht}}{T_6} + h^2\left(\frac{T_5}{T_6}\right)^2\right\} - \frac{3\beta_0^2}{t^2}\right], \tag{42}$$

where

$$T_9 = \frac{16\pi\xi_0(hT_7 + T_8)(m^2 - 4m + 5)}{(m-1)T_6}e^{-A/2}.$$

From the above results, it is evident in both cases that at any instant the domain wall density ρ and pressure p in the perpendicular direction decrease on both sides of the wall away from the symmetry plane and both vanish as $z \longrightarrow \pm\infty$. The space times in both cases are reflection symmetrical with respect to the wall. All these properties are very much expected for a domain wall. It can be also seen that the viscosity, as well as the displacement field β exhibit essential influence on the character of the solutions.

4. *Study of geodesics*

The trajectory of the test particle $x^i\{t(\lambda), x(\lambda), y(\lambda), z(\lambda)\}$ in the gravitational field of the domain wall can be determined by integrating the geodesic equations

$$\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\lambda^2} + \Gamma^\mu_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\beta}{\mathrm{d}\lambda} = 0\,, \tag{43}$$

for the metric (6). It has been already mentioned in Ref. [37], the acceleration of the test particle in the direction perpendicular to the domain wall (i.e. in the z-direction) may be expressed as

$$\ddot{z} = \frac{e^{B-A}}{2}\frac{\partial B}{\partial z}(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}\frac{\partial A}{\partial z}(\dot{t}^2 + \dot{z}^2) - \frac{\partial A}{\partial z}\dot{t}\dot{z}\,. \tag{44}$$

By simple but lengthy calculation, one can get an expression for the acceleration which may be positive, negative, or zero, depending on suitable choice of the constants. This implies that the gravitational field of the domain wall may be repulsive, or attractive in nature, or without a gravitational effect.

5. *Conclusions*

The present study deals with plane-symmetric domain wall within the framework of Lyra geometry, in the presence of bulk viscous fluid. The essential difference between the cosmological theories based on Lyra geometry and Riemannian geometry lies in the fact that the constant vector displacement field β arises naturally from the concept of gauge in Lyra geometry, whereas the cosmological constant Λ was introduced in *ad hoc* fashion in the usual treatment. Currently the study of domain walls and cosmological constant have gained renewed interest due to their application in structure formation in the Universe. Recently Rahaman *et al.* [37] presented a cosmological model for domain wall in Lyra geometry under a specific condition by taking displacement fields β as constant. The cosmological models based on varying displacement vector field β have widely been considered in the literature in different contexts [32] – [36]. Motivated by these studies, it is worthwhile to consider domain walls with a time varying β in Lyra geometry. In this paper both cases viz., constant and time-varying displacement field β, are discussed in the context of domain walls within the framework of Lyra geometry.

The study on domain walls in this article successfully describes various features of the Universe. A network of domain walls would accelerate the expansion of the Universe, but it would also exert a repulsive force expected to help the formation of large-scale structures. An interesting result that emerged in this work is that the pressure perpendicular to the wall is non-zero.

The effect of bulk viscosity is to produce a change in perfect fluid and hence exhibit essential influence on the character of the solution. We observe here that Murphy's conclusion [24] about the absence of a big bang type singularity in the infinite past in models with bulk viscous fluid, in general, is not true. The results obtained in [20] also show that, it is, in general, not valid, since for some cases big bang singularity occurs in finite past.

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RAVNINSKI SIMETRIČAN NEHOMOGEN VOLUMNO VISKOZAN DOMENSKI ZID U LYRINOJ GEOMETRIJI

Našli smo neka opća rješenja za domenske zidove u Lyrinoj geometriji za volumno viskozan nehomogen prostor-vrijeme i ravninsku simetriju. Izveli smo izraze za gustoću energije i tlak domenskih zidova za stalno i za vremenski promjenljivo posmačno polje β. Pretpostavljamo da je koeficijent viskoznosti volumne viskozne tekućine dan s potencijom gustoće mase. Opisujemo neke izvode modela. Na kraju, raspravljamo geodetske jednadžbe i ubrzanje ispitne čestice.