Printed ISSN 1330–0016
Online ISSN 1333–9133
CD ISSN 1333–8390
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# COSMOLOGICAL MODELS WITH A VARIABLE $\Lambda$ TERM IN HIGHER DIMENSIONAL SPACETIME

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Received 10 June 2005; Revised manuscript received 26 December 2005
Accepted 11 October 2006 Online 10 November 2006

We present cosmological models with variable cosmological term ($\Lambda$) in the context of higher-dimensional spacetime. It has been observed that in these models particle horizon exists and the cosmological term is decaying with time. Further, it is shown that the new models solve entropy problem and generate several models obtained in four-dimensional spacetime.

PACS numbers: 98.80.-k, 98.80.Es UDC 524.83

Keywords: cosmology, cosmological constant, higher dimensional spacetime

## 1. *Introduction*

There have been several attempts to unify gravity with other fundamental forces in nature. In the last two decades more efforts have been made to study the theories in which dimensions of spacetime are greater than the (3+1) of the observable Universe. Chodos and Detweiler [1] have suggested that present four-dimensional structure of the Universe might have been preceded by a higher-dimensional structure which at a later stage becomes effectively four-dimensional due to dimensional reduction process in which 4D spacetime expands while the extra dimensions either contract to the unobserved Planckian length scale or remain constant. It has been suggested by Marciano [2] that the experimental detection of time variations of fundamental constants should be a strong evidence for the existence of extra dimensions. Further, developments in super string theory and super gravity

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theory have generated great interest in particle physics and cosmology to study higher-dimensional theory [3]. The string theory in which the matter fields are confined to a 3-dimensional "brane-world" embeded in $1+3+d$ dimensions, while the gravitational field can propagate in $d$ extra dimensions, has inspired researchers and created great interest in higher-dimensional gravity theories. The brane-world model, presented by Randall and Sundrum [4], has attracted much attention as an alternative to the standard four-dimensional cosmology because standard four-dimensional gravity is recovered on the brane in the low-energy limit [5]. Due to the possibility of the large extra-dimensions in the brane-world models, the cosmological models of the early Universe must be studied with careful consideration of the effect of the bulk geometry [6]. Recently, Sasaki [7] has presented review on recent progress in the brane-world cosmology. Very recently, Alam and Sahni [8] have compared the predictions of brane-world models to two recently released Supernova data sets: the Gold data [9] and the data from the Supernova Legacy Survey (SNLS)[10]. They have shown that brane-world models satisfy both sets of SNe data. In recent years, several authors (Refs. [11 − 13] and references therein) have studied multidimensional inhomogeneous cosmological models in a different context.

The cosmological term ($\Lambda$) was originally introduced in cosmology by Einstein in 1917 to obtain a static solution of his field equations. Subsequently, Friedmann presented expanding solutions to the Einstein equations and cosmological term was rejected by Einstein in view of Hubble's discovery in favour of expanding Universe. However, cosmological term has attracted researchers from time to time and its cosmological consequences have been studied [14 − 16]. Linde [17] has suggested that cosmological 'constant' is a function of temperature and related to the spontaneous symmetry breaking process, thus it should be a function of time in a spatially homogeneous expanding Universe as temperature varies with time. The simplest phenomenological approach to solve this problem is to allow the effective cosmological term to vary with time, which enables it to relax to its present value($\Lambda_0 \leq 10^{-56}$ cm$^{-2}$) in an expanding Universe. In the context of the quantum field theory a cosmological term corresponds to the energy density of the vacuum. The inflationary cosmology indicates a large value of the cosmological term at the early epoch which might have accelerated the expansion of the Universe whereas current observations suggest much smaller value of the cosmological term at later epoch and hence it is speculated that the Universe inflates extremely rapidly by the decaying cosmological 'constant' in the period between the baryogenesis and primordial nucleosynthesis. Presently cosmological 'constant' is considered as one of the most important problems in cosmology as it resolves many outstanding problems in a natural way. Many aspects of $\Lambda$ related to the age problem, classification of models, classical tests, observational constraints, structure formation and gravitational lenses were discussed by several authors (Refs. [18 − 23] and references therein). In the spirit of quantum cosmology, Chen and Wu [18] argued that as $\Lambda$ has dimension of inverse length squared, one can express

$$\Lambda \propto \frac{1}{l_{\mathrm{Pl}}^2}\left[\frac{l_{\mathrm{Pl}}}{R}\right]^n, \qquad (\hbar = c = 1)\,, \tag{1}$$

where $l_{\rm Pl}$ is the Planck length. Here, the term $\hbar$ does not appear because general relativity is a classical theory. It is obvious to put $n = 2$ in the above expression. The relation $\Lambda \propto R^{-2}$ does not fall in conflict with the high degree of isotropy of the cosmic background radiation. This *ansatz* was generalized by Carvalho *et al.* [19] by including a term proportional to the Hubble parameter. They have suggested a decaying $\Lambda$ of the form

$$\Lambda = (n+1)[\alpha R^{-2} + \beta H^2]\,, \tag{2}$$

where $n = 2$, and $\alpha$ and $\beta$ are adjustable dimensionless parameters. The additional term $\beta H^2$ can modify some features of cosmological models related to the age and low energy density problems. For the sake of generality, Waga [20] has presented following generalized law for $\Lambda$

$$\Lambda = \alpha R^{-2} + \beta H^2 + \lambda\,. \tag{3}$$

Here $\lambda$ is a dimensionless constant. Viana and Liddle [24] have considered the scalar field perturbation in cosmological models which they suggested as an alternative to cosmological 'constant' models. Recent observations (Perlmutter *et al.* [25] and Riess *et al.* ([9, 26]), strongly support a significant and positive value of $\Lambda$. Their findings arise from the study of more than 50 type Ia supernovae with red-shifts in the range $0.10 \le z \le 0.83$ and these suggest Friedmann models with negative pressure matter such as a cosmological term, domain walls or cosmic strings (Vilenkin [27], Garnavich *et al.* [28]). Sahni [29] reviewed the observational evidences for a small $\Lambda$ at the present epoch and suggests that when the high redshift observations of type Ia supernovae are combined with CMB observations, that strongly supports a flat Universe with $\Omega_m + \Omega_\Lambda \simeq 1$. By modifying the Chen and Wu '*ansatz*', Vishwakarma *et al.* [30] have investigated some cosmological models with $\Lambda$ which fit to the angular size red-shift relation data very well and demand cosmic expansion with a positive decreasing $\Lambda$. Diaz-Rivera and Pimentel [31] have made a detailed study of cosmological models with decaying $\Lambda$ in scalar tensor theories. Some of the recent discussions on the cosmological constant "problem" and consequences on cosmology with a time-varying cosmological constant are contained in Bertolami [32], Ratra and Peebles [33], Dolgov *et al.* [34], Sahni and Starobinsky [35], Padmanbhan [36] and Pradhan *et al.* [37]. Motivated by the aforesaid research works, in the present paper, higher-dimensional cosmological models have been studied with $\Lambda$ of the form given in Eq. (2). The cosmological consequences of $\Lambda$ in the context of higher dimensions are discussed.

## 2. *Field equations*

We consider higher dimensional Friedmann-Robertson-Walker type spacetime metric [38]

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}x_n^2)\right], \tag{4}$$

where

$$\mathrm{d}x_n^2 = \mathrm{d}\theta_1^2 + \sin^2\theta_1 \mathrm{d}\theta_2^2 + ....... + \sin^2\theta_1 \sin^2\theta_2 ......... \sin^2\theta_{n-1}\mathrm{d}\theta_n^2 \,,$$

$R(t)$ is the scale factor, $k = 0, \pm 1$ the curvature parameter and $D = n + 2$ the total number of dimensions.

Einstein field equations with cosmological 'constant' for perfect fluid distribution are given by

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi G[(\rho + p)u_i u_j - p g_{ij}] - \Lambda g_{ij} \,, \tag{5}$$

where $u_i$ stands for $(n + 2)$ fluid velocity in comoving co-ordinates, $\rho$ and $p$ are the energy density and perfect fluid pressure, respectively. The Einstein's field equations (5) for the spacetime metric (4) yield two independent equations

$$\frac{n(n+1)}{2} \left[ \left( \frac{\dot{R}}{R} \right)^2 + \frac{k}{R^2} \right] = 8\pi G\rho + \Lambda \,, \tag{6}$$

$$\frac{n(n+1)}{2} \frac{\ddot{R}}{R} = -4\pi G \left[ (n-1)\rho + (n+1)p - \frac{\Lambda}{4\pi G} \right] . \tag{7}$$

Equations (6) and (7) yield the energy conservation equation

$$\dot{\rho} + (n+1)\Big(\rho + p\Big)\frac{\dot{R}}{R} = -\frac{\dot{\Lambda}}{8\pi G} \,. \tag{8}$$

Introducing the energy density, vacuum, curvature, Hubble and deceleration parameters

$$\Omega_h = \frac{16\pi G\rho}{n(n+1)H^2} \,, \qquad \Omega_{h\Lambda} = \frac{2\Lambda}{n(n+1)H^2} \,,$$

$$\Omega_k = \frac{k}{R^2 H^2}, \qquad H = \frac{\dot{R}}{R} \,, \qquad q = -\frac{\ddot{R}}{R H^2} \,,$$

Equations (6) and (7) may be written as

$$1 + \Omega_k = \Omega_h + \Omega_{h\Lambda} \,, \tag{9}$$

$$q = \frac{\Omega_h}{2} \left[ n - 1 + (n+1)\frac{p}{\rho} \right] - \Omega_{h\Lambda} \,, \tag{10}$$

where the subscript $h$ stands for higher dimensional values of these quantities.

Eliminating $\Omega_{h\Lambda}$ from Eqs. (9) and (10), and using the equation of state $p = (\gamma - 1)\rho$, we obtain

$$\frac{2}{(n+1)\gamma}\Big[1 + q + \Omega_k\Big] = \Omega_h\,. \tag{11}$$

Since $\Omega_h \geq 0$, Eq. (11) indicates $1 + q + \Omega_k \geq 0$. If we neglect the curvature parameter, one can see that the deceleration parameter $q$ always satisfies the condition $q \geq -1$. Assuming $\Omega_h = 0$, we can get the de Sitter model. Considering $\gamma = 1$ for matter-dominated present epoch ($t = t_0, n = 2$), we recover the result of Waga [20]

$$\frac{2}{3}\Big[1 + q_0 + \Omega_{k0}\Big] = \Omega_0\,. \tag{12}$$

The subscript “0” stands for the present value of the parameter.

From Eq. (12), it may be seen that $\Omega_0 < \frac{2}{3}$ requires $q_0 < 0$. The dynamical estimate suggest $\Omega_0 = 0.2 \pm 0.1$ and observations do not rule-out the negative value of $q_0$ [19].

It has been argued [39] that an accelerating scale factor $R(t)$ and entropy production are essential conditions for a dynamical solution of the horizon and flatness problems. To solve the entropy problem, cosmological models of the Universe with cosmological ‘constant’ have already been presented in the literature (see Ozer and Taha [40], Salim and Waga [41]).

The usual entropy law in general relativity may be written as

$$T\mathrm{d}S \equiv \mathrm{d}(\rho V) + p\mathrm{d}V = 0\,, \tag{13}$$

while, in the present case, with the help of Eq. (8), it is modified to

$$T\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{1}{8\pi G}\delta R^{n+1}\frac{\mathrm{d}\Lambda}{\mathrm{d}t}\,, \tag{14}$$

where $V = \delta R^{n+1}$ [21]. Equation (14) gives a mathematical relation between entropy production and cosmological term $\Lambda$.

A combination of Eqs. (6) and (7) with (2) produces the second-order differential equation

$$R\ddot{R} + A\dot{R}^2 - B = 0\,, \tag{15}$$

where

$$A = \frac{\gamma(n+1)}{2n}\Big[n - 2\beta\Big] - 1\,, \tag{16}$$

$$B = k - \frac{\gamma(n+1)}{2n}\Big[nk - 2\alpha\Big]. \tag{17}$$

The first integral of Eq. (15) gives

$$\dot{R}^2 = CR^{-2A} + \frac{B}{A} \qquad (A \neq 0)\,, \tag{18}$$

and

$$\dot{R}^2 = C + 2B\ \log\ R \qquad (A = 0)\,. \tag{19}$$

Here $C$ is an integration constant.

## 3. *Cosmological models*

In this section, different cases are considered to discuss behaviour of cosmological models.

### 3.1. *Case I* ($\beta = \frac{n}{2} - \frac{2n}{(n+1)\gamma}$)

With the assumption $\beta = \dfrac{n}{2} - \dfrac{2n}{(n+1)\gamma}$, Eq. (16) suggests $A = 1$ and hence Eq. (18) reduces to

$$R\dot{R} = (C + BR^2)^{1/2}, \tag{20}$$

which on integration yields

$$R(t) = [B(t-t_0)^2 + 2(t-t_0)H_0R_0^2 + R_0^2]^{1/2}. \tag{21}$$

Equation (21) gives the non-singular expanding model of the Universe. The model shows that there is a possibility of the Universe passing through a minimum at $t = t_0 - H_0R_0^2/B$, when $B > R_0^2H_0^2$. This shows that solutions are non-singular only for some range of parameters, whereas they are singular when $B = \dfrac{(2t_0H_0 - 1)R_0^2}{t_0^2}$. Using Eq. (21), we obtain the expressions for $\Lambda$ and $\rho$

$$\Lambda = \frac{(n+1)}{R^4}\left[\alpha R^2 + \beta\{B(t-t_0) + H_0R_0^2\}^2\right], \tag{22}$$

$$\rho = \frac{(n+1)}{16\pi GR^4}\left[\{n-2\beta\}\{B(t-t_0) + H_0R_0^2\}^2 + \{nk - 2\alpha\}R^2\right]. \tag{23}$$

From Eq. (23), we may see that for $\beta \leq \dfrac{n}{2}$ and $\alpha \leq \dfrac{nk}{2}$, the energy density is always positive. The cosmological term ($\Lambda$) and energy density ($\rho$) are decreasing with evolution of the Universe. It can be easily observed from Eq. (22) that the cosmological constant is a decreasing function of time and approaches a small positive value as time progresses (i.e the present epoch).

With the help of result (22), Eq. (14) produces

$$T\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{n+1}{4\pi G}\delta\{B(t-t_0)+H_0R_0^2\}R^{n-3}\left[\alpha - B\beta + \frac{2\beta\{B(t-t_0)+H_0R_0^2\}^2}{R^2}\right]. \tag{24}$$

This model suggests that entropy problem can be solved for all $\alpha \geq B\beta$. When $\gamma = 4/(n+1)$, $\beta$ vanishes and $\Lambda \propto R^{-2}$ which is the case in Ref. [18] for $n = 2$.

According to the Gibbs' integrability condition, one cannot independently specify an equation of state for the pressure and temperature ([42, 43]). The equation of state for temperature may be considered as $T \propto e^{\int \mathrm{d}p/(\rho(p)+p)}$ which with the help of equation $p = (\gamma - 1)\rho$ gives

$$T = T_0\rho^{(\gamma-1)/\gamma}\,. \tag{25}$$

From Eqs. (23) and (25), it is clear that temperature of the Universe is decreasing with expansion of the Universe.

### 3.2. *Case II* ($\alpha = 0$)

Considering $\alpha = 0$, for flat model ($k = 0$), Eqs. (17) and (18) yield

$$R(t) = R_0\left[\frac{\gamma(n+1)}{2n}\{n-2\beta\}H_0(t-t_0)+1\right]^{2n/\gamma(n+1)(n-2\beta)}. \tag{26}$$

This represents a non-singular model of the Universe. Further, using Eq. (26), the expressions for $\Lambda$ and $\rho$ in terms of the time co-ordinate $t$ may be obtained as

$$\Lambda = (n+1)\beta H_0^2\left(\frac{R_0}{R}\right)^{\gamma(n+1)(n-2\beta)/n}, \tag{27}$$

$$16\pi G\rho = (n+1)\,[n-2\beta]\,H_0^2\left(\frac{R_0}{R}\right)^{\gamma(n+1)(n-2\beta)/n}. \tag{28}$$

The condition $\beta \leq n/2$ ensures the positivity of the energy density. From Eq. (27), it is observed that the cosmological constant is a decreasing function of time and it approaches a small positive value as time progresses (i.e the present epoch).

Again, we can obtain the singular model of the Universe by integrating Eq. (18) and assuming the initial condition $R(0) = 0$. In this case we get

$$R(t) = R_0\left[\frac{\gamma(n+1)}{2n}\{n-2\beta\}H_0t\right]^{2n/\gamma(n+1)[n-2\beta]}. \tag{29}$$

Further, assuming $t = t_0$, $n = 2$ and $\gamma = 1$ for the present epoch (matter dominated Universe), one may get

$$H_0 t_0 = \frac{2}{3(1-\beta)} \,. \tag{30}$$

This is similar to the result obtained by Freese *et al.* [44].

From Eqs. (14) and (27), we obtain

$$T\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{(n+1)^2\beta\gamma}{8\pi G}\delta H_0^3\left[1-\frac{2\beta}{n}\right]\left(\frac{R_o}{R}\right)^{3\gamma(n+1)[n-2\beta]/2n} R^{(n+1)}. \tag{31}$$

Again, Eq. (31) solves the entropy problem for $\beta \le n/2$.

### 3.3. *Case III : Radiation dominated phase*

The radiation dominated case is relevant during early stages of the Universe where the extra dimensions might have a significant role to play as all dimensions including extra dimensions can be treated on the same footing and refer to the instant before the Universe passes through compactification transition. Using $\gamma = \frac{4}{3}$ for radiation, from Eqs. (16) – (18) and (2) follows

$$\dot{R}^2 = C_1 R^{[2n(1-2n)+8(n+1)\beta]/3n} + \left[\frac{4(n+1)\alpha - n(2n-1)k}{n(2n-1)-4(n+1)\beta}\right], \tag{32}$$

$$\Lambda = (n+1)\beta C_1 R^{4(n+1)(2\beta-n)/3n} + \left[\frac{n(n+1)(2n-1)[\alpha - k\beta]}{n(2n-1)-4(n+1)\beta}\right] R^{-2}\,. \tag{33}$$

From Eq. (33), it is observed that the cosmological constant is a decreasing function of time and approaches a small positive value as time progresses (i.e the present epoch).

Again, by use of Eqs. (32) – (33), one may obtain energy density from Eq. (6) as

$$\frac{16\pi G\rho_r}{n(n+1)} = C_1\left[1-\frac{2\beta}{n}\right]R^{4(n+1)(2\beta-n)/3n} + \left[\frac{6(\alpha-k\beta)}{n(2n-1)-4(n+1)\beta}\right]R^{-2}, \tag{34}$$

$$\frac{16\pi G\rho_v}{n(n+1)} = \beta C_1 R^{4(n+1)(2\beta-n)/3n} + \left[\frac{n(2n-1)[\alpha-k\beta]}{n(2n-1)-4(n+1)\beta}\right]R^{-2}. \tag{35}$$

It has been assumed that the Universe couples only with the dominant component. With $n = 2$, our models reduces to those obtained by Carvalho *et al.* [19]. Again, if we take $n = 2$, $\beta = 0$ and $C_1$, $\alpha$ always positive, the cosmological scenario of [18] can be obtained.

As it has been mentioned in the Refs. [18] and [19], the constant $\alpha$ plays the role of the curvature parameter $k$. We consider

$$\alpha = \frac{n(2n-1)}{4(n+1)} k \,, \tag{36}$$

then Eq. (32) yields

$$C_1 = {H_0}^2 {R_0}^{4(n+1)(n-2\beta)/3n} \,,$$

$$R = R_0 \left[ \frac{2(n+1)(n-2\beta)}{3n} H_0 t \right]^{3n/2(n+1)(n-2\beta)} . \tag{37}$$

Equation (37) suggests that the age of the Universe can be calculated from

$$t_0 = \frac{3n}{2(n+1)(n-2\beta)} {H_0}^{-1} \tag{38}$$

Now, if we consider the flat model ($k=0$, i.e. $\alpha = 0$), then Eqs. (33) − (35) take the form

$$\Lambda = (n+1)\beta {H_0}^2 \left( \frac{R_0}{R} \right)^{[4(n+1)(n-2\beta)]/3n} \,, \tag{39}$$

$$\frac{16\pi G \rho_r}{n(n+1)} = \left( 1 - \frac{2\beta}{n} \right) {H_0}^2 \left( \frac{R_0}{R} \right)^{[4(n+1)(n-2\beta)]/3n} \,, \tag{40}$$

$$\frac{16\pi G \rho_v}{n(n+1)} = \beta {H_0}^2 \left( \frac{R_0}{R} \right)^{[4(n+1)(n-2\beta)]/3n} . \tag{41}$$

Further, Eqs. (40) and (41) suggest the relation $\rho_r = \left( \frac{1}{\beta} - \frac{2}{n} \right) \rho_v$.

At this stage one may find a relation connecting the scale factor $R(t)$ and temperature of radiation as

$$T_{\rm rad} = T_0 \left( \frac{R_0}{R} \right)^{[(n+1)(n-2\beta)]/3n} . \tag{42}$$

Further, by use of Eq. (37), the relation (42) suggests that temperature is decreasing with evolution of the Universe.

### 3.4. Case IV : Matter dominated phase

Now we consider matter-dominated phase to examine cosmological consequences of dimensionality during cosmic evolution.

$$\dot{R}^2 = C_2 R^{[-n(n-1)+2(n+1)\beta]/n} + \left[\frac{2(n+1)\alpha - n(n-1)k}{n(n-1) - 2(n+1)\beta}\right], \tag{43}$$

$$\Lambda = (n+1)\beta C_2 R^{(n+1)(2\beta-n)/n} + \left[\frac{n(n+1)(n-1)(\alpha-\beta k)}{n(n-1)-2(n+1)\beta}\right] R^{-2}, \tag{44}$$

$$\frac{16\pi G\rho_m}{n(n+1)} = C_2\left[1 - \frac{2\beta}{n}\right] R^{(n+1)(2\beta-n)/n} + \left[\frac{4(\alpha-\beta k)}{n(n-1)-2(n+1)\beta}\right] R^{-2}, \tag{45}$$

$$\frac{16\pi G\rho_v}{n(n+1)} = \beta C_2 R^{(n+1)(2\beta-n)/n} + \left[\frac{n(n-1)(\alpha-\beta k)}{n(n-1)-2(n+1)\beta}\right] R^{-2}. \tag{46}$$

If the constant $\alpha$ is related to the curvature parameter $k$ as

$$\alpha = \frac{n(n-1)k}{2(n+1)}, \tag{47}$$

then Eq. (43) on integration gives

$$C_2 = {H_0}^2 {R_0}^{(n+1)(n-2\beta)/n},$$

$$R = R_0\left[\frac{(n+1)(n-2\beta)}{2n} H_0 t\right]^{2n/(n+1)(n-2\beta)}. \tag{48}$$

Equation (48) suggests that the time interval $t_0$ elapsed in quasi-FRW stage is

$$t_0 = \frac{2n}{(n+1)(n-2\beta)} {H_0}^{-1} \tag{49}$$

Again, considering the flat model($k = 0$ i.e. $\alpha = 0$), Eqs. (44)-(46) take the form

$$\Lambda = (n+1)\beta {H_0}^2 \left(\frac{R_0}{R}\right)^{(n+1)(n-2\beta)/n}, \tag{50}$$

$$\frac{16\pi G\rho_m}{(n+1)} = \left(\frac{n-2\beta}{n}\right) {H_0}^2 \left(\frac{R_0}{R}\right)^{(n+1)(n-2\beta)/n}, \tag{51}$$

$$\frac{16\pi G\rho_v}{(n+1)} = \beta {H_0}^2 \left(\frac{R_0}{R}\right)^{(n+1)(n-2\beta)/n}. \tag{52}$$

In this case also, we can easily find the relation $\rho_m = \left(\frac{1}{\beta} - \frac{2}{n}\right)\rho_v$. From Eqs. (48) and (50), we observe that the cosmological constant is a decreasing function of time and approaches a small positive value as time progresses (i.e the present epoch).

## 4. *Conclusions*

In this work, we have presented cosmological models with varying $\Lambda$ in higher dimensions which yield several four-dimensional results when n = 2. The analytic expressions of cosmological variables indicate a clear dependence of dynamical properties on dimensionality of the spacetime. The horizon distance, $d_{\mathrm{H}}(t)$ at time $t$ is the proper distance travelled by light emitted at $t = t_{\mathrm{e}}$:

$$\mathrm{d_H}(t) = R(t) \lim_{t_{\mathrm{e}} \to 0} \left[ \int_{t_{\mathrm{e}}}^{t} \frac{\mathrm{d}t'}{R(t')} \right]. \tag{53}$$

If the horizon distance is finite throughout, then we can say that the causal communication between two observers exists. It can be easily seen that horizon distance is finite in all models. The boundary of horizon is smaller during higher dimensions. The derived models solve the entropy problem. In both cases, the radiation-dominated phase and the matter-dominated phase, the ratio of the energy density and cosmological constant depends on the constant $\beta$ and dimensionality of the spacetime. This may present a significant problem to structure formation or brane world cosmology.

The values of the cosmological constant in all models are found to be decreasing functions of time and they all approach small and positive values which is supported by the results from recent Ia supernovae observations (Perlmutter *et al.* [25], Riess *et al.* ([9],[26]), Garnavich *et al.* [28]).

*Acknowledgements*

GPS and AP would like to thank Inter-University Center for Astronomy and Astrophysics, Pune, India for providing facilities under Associateship Programme where part of this work was carried out. SK is thankful to Principal of G. H. R. College of Engineering for continuous encouragement and VNIT for providing necessary facilities. Authors are also thankful to the referee for his constructive suggestions.

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## KOZMOLOŠKI MODELI S PROMJENLJIVIM ČLANOM $\Lambda$ U VIŠEDIMENZIJSKOM PROSTORU-VREMENU

Predstavljamo kozmološke modele s promjenljivim kozmološkim članom ($\Lambda$) u okviru višedimenzijskog prostora-vremena. Pokazuje se da u tim modelima postoji čestični horizont i da kozmološki član opada s vremenom. Nadalje, pokazujemo kako novi modeli rješavaju problem entropije i polazišta su više modela u četiridimenzijskom prostoru-vremenu.