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GRAVITATIONAL FIELS OF A NON-STATIONARY GLOBAL MONOPOLE IN HIGHER DIMENSION

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In this paper, we study the gravitational field of a non-stationary global monopole in higher dimension by using field-theoretic energy-momentum tensors for monopole configuration. The solutions are obtained using the functional separability of the metric coefficients. We have shown that the monopole exerts attractive gravitational effects on test particles.

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1. *Introduction*

It is well known that the usual 4-dimensional space-time is not sufficient to explain the unification of the gravitational force with other forces in nature. So the higher-dimensional theory may be useful to study this problem. It was first suggested by Kaluza and by Klein [1] to unify gravity and electromagnetic field by studying a five-dimensional model. In cosmology, the higher-dimensional theory is successful in studies of the very early stages of evolution of the Universe. Solutions of Einstein's field equations are believed to be of physical relevance possibly at the extremely early times before the Universe underwent the compactification transitions [2]. It is argued that the early Universe had undergone a number of phase transitions as it cooled down from the hot initial phase. One of immediate consequences of these phase transitions is the formation of defects. Monopoles are point-like topological objects that may have arisen during the phase transitions in the early Universe [3]. Both for particle physicists and cosmologists, global monopoles are important objects which have been predicted to exist in grand unified theory. Using a suitable scalar field, it can be shown that spontaneous symmetry breaking can give rise to such objects which are nothing but topological knots in the vacuum expectation value of the scalar field, and most of their energy is concentrated in a small region near the monopole core. From the topological point of view, they are formed in the vacuum manifold M when M contains surfaces which can not be continuously shrunk to a point, i.e., when $\pi_2(M) \neq I$. Such monopoles have Goldstone fields with energy density decreasing with the distance as inverse square law. They are also found to have some interesting features in the sense that a monopole exerts no gravitational force on its surrounding non-relativistic matter, but the space around them has a deficit solid angle [3].

Barriola and Vilenkin (BV) [4] first showed the existence of such a monopole solution resulting from the breaking of the global $SO(3)$ symmetry of a triplet scalar field in a Schwarzschild background. Their work is related to a static model of the monopole space-time. Banerji et al. [5] have extended the work of BV to higher dimensions. In a recent work, Bronnikov and Meierovich [6] studied gravitational properties of global monopole in the $(D = d + 2)$-dimensional space-time. Recently, Chakraborty [7] described a non-stationary monopole solution by considering a field-theoretic energy-momentum tensor. In the present paper, we extend the work of Chakraborty to a higher-dimensional model.

2. *The basic equations*

The most general five-dimensional non-static metric ansatz describing a monopole is given by

$$\mathrm{d}s^2 = -A\mathrm{d}t^2 + B\mathrm{d}r^2 + C\mathrm{d}\Omega^2 + E\mathrm{d}\Psi^2 \,. \tag{1}$$

A, B, C and E are functions of r and t, and Ψ is the fifth coordinate. We closely follow the formalism of Chakraborty [7] and take the Lagrangian that gives rise to monopoles as

$$L = \frac{1}{2}\partial_\mu \Phi^a \partial^\mu \Phi^a - \frac{1}{4}\lambda(\Phi^a\Phi^a - \eta^2)^2 \,, \tag{2}$$

where Φ^a is the triplet scalar field $a = 1, 2, 3$ and η is the energy scale of symmetry breaking. For a non-static monopole, we do not write the explicit form of the field configuration of Φ^a, but take it as implicit form. The energy-momentum tensor for the above Lagrangian is given by [7]

$$T_\mu^\gamma = \nabla_\mu \Phi^a \cdot \nabla^\gamma \Phi^a - L\delta_\mu^\gamma \,. \tag{3}$$

So the explicit expressions of the Einstein equations ($G_\mu^\gamma = T_\mu^\gamma$) for the metric

(1) are

$$\frac{1}{2B}\left[-2\frac{C^{11}}{C}-\frac{E^{11}}{E}+\frac{(C^1)^2}{2C^2}+\frac{(E^1)^2}{2E^2}+\frac{B^1C^1}{BC}+\frac{B^1E^1}{2BE}-\frac{E^1C^1}{EC}\right] \tag{4}$$

$$+\frac{1}{2A}\left[\frac{(C^*)^2}{2C^2}+\frac{B^*C^*}{BC}+\frac{B^*E^*}{2BE}+\frac{E^*C^*}{EC}\right]+\frac{1}{C}$$

$$=\frac{1}{2}\left[-\frac{1}{A}(\Phi^{a*})^2-\frac{1}{B}(\Phi^{a1})^2+\frac{1}{2}\lambda(\Phi^a\Phi^a-\eta^2)^2\right],$$

$$\frac{1}{2B}\left[-\frac{(C^1)^2}{2C^2}-\frac{A^1C^1}{AC}-\frac{A^1E^1}{2AE}-\frac{E^1C^1}{EC}\right] \tag{5}$$

$$+\frac{1}{2A}\left[2\frac{C^{**}}{C}+\frac{E^{**}}{E}-\frac{(C^*)^2}{2C^2}-\frac{(E^*)^2}{2E^2}-\frac{A^*C^*}{AC}-\frac{A^*E^*}{2AE}+\frac{E^*C^*}{EC}\right]+\frac{1}{C}$$

$$=\frac{1}{2}\left[\frac{1}{A}(\Phi^{a*})^2+\frac{1}{B}(\Phi^{a1})^2+\frac{1}{2}\lambda(\Phi^a\Phi^a-\eta^2)^2\right],$$

$$\frac{1}{2B}\left[-\frac{C^{11}}{C}-\frac{E^{11}}{E}-\frac{A^{11}}{A}+\frac{(A^1)^2}{2A^2}+\frac{(C^1)^2}{2C^2}+\frac{(E^1)^2}{2E^2}\right. \tag{6}$$

$$\left.+\frac{B^1C^1}{2BC}+\frac{B^1A^1}{2BA}+\frac{B^1E^1}{2BE}-\frac{E^1C^1}{2EC}-\frac{E^1A^1}{2EA}-\frac{A^1C^1}{2AC}\right]$$

$$\frac{1}{2A}\left[\frac{C^{**}}{C}+\frac{E^{**}}{E}+\frac{B^{**}}{B}-\frac{(C^*)^2}{2C^2}-\frac{(B^*)^2}{2B^2}-\frac{(E^*)^2}{2E^2}\right.$$

$$\left.-\frac{A^*C^*}{2AC}-\frac{A^*E^*}{2AE}-\frac{A^*B^*}{2AB}+\frac{E^*C^*}{2EC}+\frac{B^*C^*}{2BC}+\frac{E^*B^*}{2EB}\right]$$

$$=\frac{1}{2}\left[\frac{1}{A}(\Phi^{a*})^2+\frac{1}{B}(\Phi^{a1})^2+\frac{1}{2}\lambda(\Phi^a\Phi^a-\eta^2)^2\right],$$

$$\frac{1}{2B}\left[-2\frac{C^{11}}{C}-\frac{A^{11}}{A}+\frac{(C^1)^2}{2C^2}-\frac{(A^1)^2}{2A^2}+\frac{B^1C^1}{BC}+\frac{B^1A^1}{2BA}-\frac{A^1C^1}{AC}\right] \tag{7}$$

$$+\frac{1}{2A}\left[2\frac{C^{**}}{C}+\frac{B^{**}}{B}-\frac{(C^*)^2}{2C^2}-\frac{(B^*)^2}{2B^2}-\frac{A^*C^*}{2AC}-\frac{A^*B^*}{2AB}+-\frac{B^*C^*}{2BC}\right]+\frac{1}{C}$$

$$=\frac{1}{2}\left[\frac{1}{A}(\Phi^{a*})^2+\frac{1}{B}(\Phi^{a1})^2+\frac{1}{2}\lambda(\Phi^a\Phi^a-\eta^2)^2\right],$$

$$\frac{C^{*1}}{C}+\frac{E^{*1}}{2E}-\frac{C^*C^1}{2C^2}-\frac{E^*E^1}{2E^2}-\frac{A^1C^*}{2AC}-\frac{A^1E^*}{4AE}-\frac{B^*C^1}{2BC}-\frac{E^1B^*}{4EB}=\Phi^{a*}\cdot\Phi^{a1}, \tag{8}$$

The field equation for the scalar triplet Φ^a is

$$\frac{1}{A}\left[\frac{\Phi^{a**}}{\Phi^a}+\frac{\Phi^{a*}}{\Phi^a}\left(\frac{A^*}{A}-\frac{B^*}{B}-2\frac{C^*}{C}-\frac{E^*}{E}\right)\right] \tag{9}$$

$$+\frac{1}{B}\left[\frac{\Phi^{a11}}{\Phi^a}+\frac{\Phi^{a1}}{\Phi^a}\left(\frac{A^1}{A}-\frac{B^1}{B}+2\frac{C^1}{C}+\frac{E^1}{E}\right)\right]+\lambda(\Phi^a\Phi^a-\eta^2),$$

['$*$' and '^{1}' are differentiations w.r.t. t and r, respectively].

3. *Solutions to the field equations*

As the field equations are complicated to solve, we shall assume the separable form of the metric coefficients as follows

$$A=A_1(r)A_2(t),\quad B=B_1(r)B_2(t),\quad C=C_1(r)C_2(t),\quad E=E_1(r)E_2(t). \tag{10}$$

Without any loss of generality, one can assume

$$A_2(t)=B_1(r)=1. \tag{11}$$

because $A_2(t)$ or $B_1(r)$ different from unity results in a scaling of time or radial coordinates. Also, we have taken the scalar field triplet in the separable form as

$$\Phi^a(r,t)=\Phi^a_1(r)+\Phi^a_2(t), \tag{12}$$

Using the above separable forms and Eq. (11), we get from Eq. (8)

$$\frac{C_2^*}{2C_2}\left[\frac{C_1^1}{C_1}-\frac{A_1^1}{A_1}\right]+\frac{E_2^*}{4E_2}\left[\frac{E_1^1}{E_1}-\frac{A_1^1}{A_1}\right]-\frac{B_2^*}{4B_2}\left[\frac{2C_1^1}{C_1}+\frac{E_1^1}{E_1}\right]=\Phi_2^{a*}\cdot\Phi_1^{a1}. \tag{13}$$

This leads to a possible choice

$$\frac{C_1^1}{C_1} - \frac{A_1^1}{A_1} = 0\,, \tag{14}$$

so that

$$\frac{E_2^*}{4E_2}\left[\frac{E_1^1}{E_1} - \frac{A_1^1}{A_1}\right] - \frac{B_2^*}{4B_2}\left[\frac{2C_1^1}{C_1} + \frac{E_1^1}{E_1}\right] = \Phi_2^{a*}\cdot\Phi_1^{a1}\,, \tag{15}$$

After adding Eqs. (4) and (5) and subtraction of twice Eq. (7), we get

$$\frac{1}{2B}\left[2\frac{C^{11}}{C} - \frac{E^{11}}{E} + 2\frac{A^{11}}{A} - \frac{(C^1)^2}{2C^2} + \frac{(E^1)^2}{2E^2}\right. \tag{16}$$

$$\left. + \frac{(A^1)^2}{A^2} + \frac{B^1E^1}{2BE} - \frac{A^1E^1}{2AE} - 2\frac{E^1C^1}{EC} - 2(\Phi^{*1})^2\right]$$

$$= -\frac{1}{2A}\left[\frac{E^{**}}{E} - 2\frac{B^{**}}{B} + \frac{(C^*)^2}{C^2} + \frac{(B^*)^2}{B^2} - \frac{(E^*)^2}{2E^2}\right.$$

$$\left. - \frac{A^*E^*}{2AE} + \frac{A^*B^*}{AB} + 2\frac{E^*C^*}{EC} + \frac{E^*B^*}{2EB} + 2(\Phi_1^{a*})^2\right].$$

We shall now solve these equations with the following relations among the metric coefficients

$$A_1 = aE_1^n \quad \text{and} \quad B_2 = bE_2^m\,, \tag{17}$$

where a, b, m and n are arbitrary constants. From Eq. (15) and using Eq.(17), we get

$$\Phi_2^{a*} = \frac{c}{p}\frac{E_2^*}{E_2} \quad \text{and} \quad \Phi_1^{a1} = p\frac{E_1^1}{E_1}\,, \tag{18}$$

where p is the separation constant and $c = \frac{1}{4}[1 - n - m(1 + 2n)]$. Now eliminating Φ_1^{a1} and using the separable forms and Eq. (17), we get from Eq. (16)

$$\frac{E_1^{11}}{E_1} - d\frac{E_1^{12}}{E_1^2} = eE_1^{-n}\,, \tag{19}$$

where $d = \{2p^2 - 4n^2 + (13/2)n - 1/2\}/(4n - 1)$, $e = q/[a(4n - 1)]$ and q is a separation constant. The integral form of E_1 is

$$\int\left[D_1E_1^{2d} + \frac{2e}{n + 2 - 2d}E_1^{2-n}\right]^{-1/2}\,, \tag{20}$$

where D_1 and r_0 are integration constants. For different choices of the constants, the solutions for E_1 are

$$\text{Case I} : q = 0, \quad E_1 \propto (r - r_0)^{1/(1-d)} , \tag{21}$$

$$\text{Case II} : D_1 = 0, \quad E_1 \propto (r - r_0)^{2/n} . \tag{22}$$

Then from Eq.(19) we get

$$\text{Case III} : n = \frac{1}{4}, \quad \text{from Eq. (19) we get } E_1 \propto (r - r_0)^8 \text{ provided } p^2 < 7/16 , \tag{23}$$

$$\text{CaseIV} : n = 1, d = 1 : \; E_1 = \frac{1}{2D_1 \exp(\sqrt{D_1} r))} (\exp(\sqrt{D_1} r) + 2e)^2 , \tag{24}$$

$$\text{CaseV} : n = 2, d = 1 : \; E_1 = \sqrt{\frac{e}{(d-2)D_1}} \cosh(\sqrt{D_1}(r - r_0)) , \tag{25}$$

Proceeding in a similar way, the integral form of E_2 is

$$\int \left[D_2 E_2^{2g} + \frac{2f}{m + 2 - 2g} E_2^{2-m} \right]^{-1/2} \mathrm{d}E_2 = \pm (t - t_0) , \tag{26}$$

where D_2 and t_0 are integration constants, and $f = q/[b(2m-1)]$ and $g = [2/p^2 - m^2 + v^2 + 2v + (5/2)m - 1/2]/(2m-1)$. We introduce the assumption

$$C_2 = E_2^v , \tag{27}$$

where v is an arbitrary constant. The solution set for the time part E_2 is as follows:

$$\text{CaseI} : q = 0, \quad E_2 \propto (t - t_0)^{1/(1-g)} , \tag{28}$$

$$\text{CaseII} : D_2 = 0, \quad E_2 \propto (t - t_0)^{2/m} , \tag{29}$$

$$\text{CaseIII} : m = \frac{1}{2}, \quad E_2 \propto (t - t_0)^4 , \tag{30}$$

$$\text{CaseIV} : m = 1, g = 1 : \quad E_2 = \frac{1}{2D_2 \exp(\sqrt{D_2} t))} (\exp(\sqrt{D_2} t) + 2f)^2 , \tag{31}$$

$$\text{CaseV} : m = 2, g = 1 : \quad E_2 = \sqrt{\frac{f}{(g-2)D_2}} \cosh(\sqrt{D_2}(t - t_0)) . \tag{32}$$

From Eq.(18), we get

$$\Phi_1^a(r) = p \ln E_1 + \Phi_{01}^a \quad \text{and} \quad \Phi_2^a(t) = (c/p) \ln E_2 + \Phi_{02}^a , \tag{33}$$

where Φ_{01}^a and Φ_{02}^a are integration constants.

4. *Concluding remarks*

In this paper, we study the gravitational field of non-stationary monopole in a higher dimension by considering a field-theoretic energy-momentum tensor. It is important to note that our higher-dimensional non-stationary metric is not conformally flat and hence it represents a monopole [7]. The solutions we have obtained in the present paper are not the most general. However, the presented solutions are perhaps the only exact analytical solutions obtained so far. The expression for our metric (1) is

$$\mathrm{d}s^2 = -E_1^n \mathrm{d}t^2 + E_2^m \mathrm{d}r^2 + E_1^n E_2^v \mathrm{d}\Omega_2^2 + E_1 E_2 \mathrm{d}\Psi_2^2 \,.$$

If we define

$$T = \int E_2^{-m/2} \mathrm{d}t \quad \text{and} \quad R = \int E_1^{-n/2} \mathrm{d}r \,,$$

then the above metric can be written as

$$\mathrm{d}s^2 = E_1^n E_2^m [-\mathrm{d}T^2 + \mathrm{d}R^2 + E_2^{v-m} \mathrm{d}\Omega^2 + E_1^{1-n} E_2^{1-m} \mathrm{d}\Psi^2 \,.$$

The metric describes a solid-angle deficiency, which depends both on radial and time coordinates. It may be interesting to recast the whole formalism in an effective four-dimensional background. One can note that the diagonal metric we obtained for a five-dimensional space-time, corresponding to a spherically symmetric monopole, appears as an effective four-dimensional metric given by

$$\mathrm{d}s^2 = -C_1 \mathrm{d}t^2 + C_2^{m/v} \mathrm{d}t^2 + C_1 C_2 \mathrm{d}\Omega_2^2 \,.$$

If we choose $v = 1$, then our solution is exactly the same as was obtained by Chakraborty [7], after choosing suitably the arbitrary constants. Another aspect of the monopole is the effect on test particle by its gravitational field. Let us consider an observer with the four-velocity given by

$$V_i = \sqrt{C_1} \delta_i^t \,.$$

Then we obtain the acceleration vector A_i as

$$A^i = V^i_{;k} V^k = (C_1^1 / C_1) C_1^{-2} \delta_r^i \,.$$

For the above solutions (23) – (25), one can see that A_r is positive. But the solutions (21) and (22) give A_r a positive value with some restrictions. Hence, the monopole exhibits an attractive nature to the observer. Non-separable solutions of the field equation give more insight into the problem and this will be the aim of our future study.

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GRAVITACIJSKO POLJE MIRNOG GLOBALNOG MONOPOLA U VIŠOJ DIMENZIJI

U ovom radu proučavamo gravitacijsko polje mirnog globalnog monopola kojega oblikujemo u više-dimenzijskom prostoru primjenom tenzora energije-impulsa teorije polja. Rješenja postižemo rabeći funkcijsku razdvojivost metričkih koeficijenata. Pokazujemo da monopol proizvodi privlačan gravitacijski učinak.