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ON THE MØLLER ENERGY-MOMENTUM COMPLEX OF THE MELVIN MAGNETIC UNIVERSE

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We use the Møller energy-momentum complex to calculate the energy of the Melvin magnetic universe. The energy distribution depends on the magnetic field. In the expression for the energy, the first term represents twice the classical value of energy and the other terms are due to the relativistic correction.

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1. Introduction

The subject of energy-momentum localization in general relativity continues to be an open one because there is no given yet a generally accepted expression for the energy-momentum density. Related on the method that used the energy-momentum complexes we can say that there are various energy-momentum complexes including those of Einstein [1], Landau and Lifshitz [2], Papapetrou [3], Bergmann [4], Weinberg [5] and Møller [6]. Also, there are doubts that these prescriptions could give acceptable results for a given space-time. The problem is that with different energy-momentum complexes we can obtain different expressions for the energy associated with a given space-time. This is because most of the energy-momentum complexes are restricted to the use of particular coordinates. But the results obtained by several authors [7]-[10] demonstrated that the energy-momentum complexes are good tools for evaluating the energy and momentum in general relativity. Even they are coordinate dependent, various energy-momentum complexes give the same energy distribution for a given space-time.

Aguirregabiria, Chamorro and Virbhadra [7] obtained that the energy-momentum complexes of Einstein [1], Landau and Lifshitz [2], Papapetrou [3], and Weinberg [5] give the same result for the energy distribution for any Kerr–Schild metric. Also, recently, Virbhadra investigated [9] if these definitions lead to the same result for the most general nonstatic spherically symmetric metric and found they disagree. Only the energy-momentum complex of Einstein gives the same expression for the energy when the calculations are performed in the Kerr–Schild Cartesian and Schwarzschild Cartesian coordinates.

On the other hand, the Møller energy-momentum complex [6] allows to compute the energy in any coordinate system. Also, many results recently obtained [11]-[12] support the conclusion given by Lessner [13] in his recent paper that the Møller definition is a powerful concept of energy and momentum in general relativity. The viewpoint of Lessner [13] is that "The energy-momentum four-vector can only transform according to special relativity only if it is transformed to a reference system with an everywhere constant velocity. This cannot be achieved by a global Lorentz transformation". He pointed out that the problem lies with the interpretation of the result from a special relativistic point of view instead of a general relativistic one. In conclusion, the Møller prescription can be used with success to evaluate the energy distribution of a given space-time.

In this paper we calculate the energy distribution of the Melvin magnetic universe [14]-[15] in the Møller prescription. Through the paper we use geometrized units ($G = 1, c = 1$) and follow the convention that Latin indices run from 0 to 3.

2. Energy in the Møller prescription

The Melvin magnetic universe [14, 15] is described by the electrovac solution of the Einstein–Maxwell equations and consists of a system of parallel magnetic lines of forces in equilibrium under their mutual gravitational attraction. The Einstein–Maxwell equations are

$$R_i^{\ k} - \frac{1}{2}\, g_i^{\ k}\, R = 8\,\pi\, T_i^{\ k}, \tag{1}$$

$$\frac{1}{\sqrt{-g}}\left(\sqrt{-g}\, F^{ik}\right)_{,k} = 4\,\pi\, J^i, \tag{2}$$

$$F_{ij,k} + F_{jk,i} + F_{ki,j} = 0. \tag{3}$$

The energy-momentum tensor of the electromagnetic field is given by

$$T_i^{\ k} = \frac{1}{4\,\pi}\left[-F_{im}\, F^{km} + \frac{1}{4}\, g_i^{\ k}\, F_{mn}\, F^{mn}\right]. \tag{4}$$

The electrovac solution corresponds to $J^i = 0$ and is given by the metric

$$\mathrm{d}s^2 = L^2\,(\mathrm{d}t^2 - \mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2) - L^{-2}\, r^2\, \sin^2\theta\, \mathrm{d}\varphi^2, \tag{5}$$

where

$$L = 1 + \frac{1}{4} B_0^2 r^2 \sin^2\theta. \tag{6}$$

The Cartan components of the magnetic field are

$$\begin{aligned} H_r &= L^{-2}\, B_0 \cos\theta, \\ H_\theta &= -L^{-2}\, B_0 \sin\theta. \end{aligned} \tag{7}$$

B_0 is the magnetic field parameter and is a constant in the solution given by (5) and (6).

The energy-momentum tensor has the non-vanishing components

$$\begin{aligned} T_1{}^1 &= -T_2{}^2 = \frac{B_0^2 (1 - 2\sin^2\theta)}{8\pi L^4}, \\ T_0{}^0 &= -T_3{}^3 = \frac{B_0^2}{8\pi L^4}, \\ T_2{}^1 &= -T_1{}^2 = \frac{2 B_0^2 \sin\theta \cos\theta}{8\pi L^4}. \end{aligned} \tag{8}$$

The Møller energy-momentum complex $M_i{}^k$ [6] is given by

$$M_i{}^k = \frac{1}{8\pi} \chi_i{}^{kl}{}_{,l}\ , \tag{9}$$

where the quantity $\chi_i{}^{kl}$ is the Møller superpotential and satisfies the antisymmetric property

$$\chi_i{}^{kl} = -\chi_i{}^{lk} = \sqrt{-g} \left(\frac{\partial g_{in}}{\partial x^m} - \frac{\partial g_{im}}{\partial x^n} \right) g^{km} g^{nl}. \tag{10}$$

Also, $M_i{}^k$ satisfies the local conservations laws

$$\frac{\partial M_i{}^k}{\partial x^k} = 0. \tag{11}$$

$M_0{}^0$ is the energy density and $M_\alpha{}^0$ are the momentum density components.

The energy is given by

$$E = \iiint M_0{}^0 \mathrm{d}x^1 \mathrm{d}x^2 \mathrm{d}x^3 = \frac{1}{8\pi} \iiint \frac{\partial \chi_0{}^{0l}}{\partial x^l} \,\mathrm{d}x^1\, \mathrm{d}x^2\, \mathrm{d}x^3. \tag{12}$$

For the Melvin magnetic universe, we obtain that the non-zero χ_i^{kl} components are given by

$$\chi_0^{01} = \frac{B_0^2\, r^3\, \sin^3\theta}{(1 + 1/4\, B_0^2\, r^2\, \sin^2\theta)},$$
$$\chi_0^{02} = \frac{B_0^2\, r^2\, \cos\theta\, \sin^2\theta}{(1 + 1/4\, B_0^2\, r^2\, \sin^2\theta)}. \tag{13}$$

After some calculations, applying the Gauss theorem and plugging (13) into (12), we obtain the energy distribution

$$E(r) = \frac{1}{3}\, B_0^2\, r^3 - \frac{1}{15}\, B_0^4\, r^5 + \frac{1}{70}\, B_0^6\, r^7. \tag{14}$$

Putting G and c at their places, we get

$$E(r) = \frac{1}{3}\, B_0^2\, r^3 - \frac{1}{15}\, \frac{G}{c^4}\, B_0^4\, r^5 + \frac{1}{70}\, \frac{G^2}{c^8}\, B_0^6\, r^7. \tag{15}$$

The first term represents twice of the classical value of energy [16] obtained in the Landau and Lifshitz and Papapetrou prescriptions. The other terms are due to the relativistic corrections.

For the energy distribution given by (14), we have the graphic representation in Fig. 1, where E is plotted against r on the x-axis and B_0 on the y-axis.

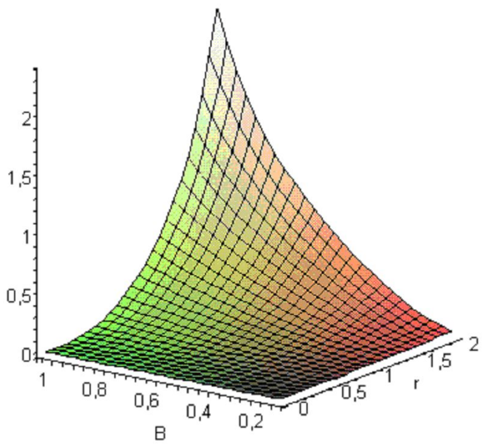

Fig. 1. Dependence of the energy (ordinate, according to Eq. (14)) and strength of magnetic field (B) on distance.

3. Discussion

The subject of the localization of energy in general relativity continues to be an open one ever since Einstein gave his important result of the special theory of relativity that mass is equivalent to energy. Misner et al. [17] sustained that to look for a local energy-momentum means that one is looking for the right answer to the wrong question. Also, they concluded that the energy is localizable only for spherical systems. On the other hand, Cooperstock and Sarracino [18] demonstrated that if the energy is localizable in spherical systems then it is also localizable in any space-time. Many results recently obtained sustain the viewpoint of Bondi [19] who's opinion is that "a nonlocalizable form of energy is not admissible in general relativity, because any form of energy contributes to gravitation and so its location can in principle be found". Also, Chang, Nester and Chen [20] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum. They concluded that there exists a direct relationship between energy-momentum complexes and quasilocal expressions because every energy-momentum complex is associated with a legitimate Hamiltonian boundary term. Very important is the Cooperstock hypothesis [21] which states that the energy and momentum are confined to the regions of non-vanishing energy-momentum tensor of the matter and all non-gravitational fields.

In this paper, we use the Møller prescription to compute the energy distribution of the Melvin magnetic universe. We obtain that the energy distribution given in Ref. 14 depends on the magnetic field. The first term amounts about twice the classical value of energy [16] obtained in the Landau and Lifshitz and Papapetrou prescriptions. The other terms are due to the relativistic corrections. The result is different from that obtained by Xulu [16] using the energy-momentum complexes of Landau and Lifshitz and those of Papapetrou. Let us stress that the Møller energy-momentum complex does not require to carry out calculations in any particular coordinates.

References

[1] C. Møller, Ann. Phys. (NY) **4** (1958) 347; A. Trautman, in *Gravitation: an Introduction to Current Research*, ed. L. Witten, Wiley, New York (1962) p. 169; R. P. Wallner, Acta Physica Austriaca **52** (1980) 121.

[2] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, Pergamon Press (1987) p. 280.

[3] A. Papapetrou, Proc. Roy. Irish. Acad. **A52** (1948) 11.

[4] P. G. Bergmann and R. Thompson, Phys. Rev. **89** (1953) 400.

[5] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of General Theory of Relativity*, John Wiley and Sons, Inc., New York (1972) p. 165.

[6] C. Møller, Ann. Phys. (NY) **4** (1958) 347.

[7] J. M. Aguirregabiria, A. Chamorro and K. S. Virbhadra, Gen. Rel. Grav. **25** (1996) 1393.

[8] K. S. Virbhadra, Phys. Rev. D **41** (1990) 1086; S. Virbhadra, Phys. Rev. D **42** (1990) 2919; N. Rosen and K. S. Virbhadra, Gen. Rel. Grav. **25** (1993) 429; K. S. Virbhadra and J. C. Parikh, Phys. Lett. B **331** (1994) 302; K. S. Virbhadra, Pramana - J. Phys. **45** (1995) 215.

[9] K. S. Virbhadra, Phys. Rev. D **60** (1999) 104041.

[10] A. Chamorro and K. S. Virbhadra, Pramana - J. Phys. **45** (1995) 181; A. Chamorro and K. S. Virbhadra, Int. J. Mod. Phys. D **5** (1997) 251; S. S. Xulu, Int. J. Theor. Phys. **37** (1998) 1773; S. S. Xulu, Int. J. Mod. Phys. D **7** (1998) 773; E. C. Vagenas, Int. J. Mod. Phys. **A18** (2003) 5949; I. Radinschi, Acta Physica Slovaca **49(5)** (1999) 789; I. Radinschi, Mod. Phys. Lett. A **15**, Nos. 11&12 (2000) 803; T. Bringley, Mod. Phys. Lett. A **17** (2002) 157; M. Súkeník and J. Sima, gr-qc/0101026; M. Sharif, Int. J. Mod. Phys. D **13** (2004) 1019; M. Sharif, Int. J. Mod. Phys. A **17** 1175 (2002); M. P. Dabrowski and J. Garecki, Phys. Rev. D **70** (2004) 043511; R. Gad, Astrophys. Space. Sci. **293** (2004) 453.

[11] S. S. Xulu, Mod. Phys. Lett. A **15** (2000) 1511; PhD Thesis, hep-th/0308070; S. S. Xulu, Astrophys. Space. Sci. **23** (2003) 283; I-Ching Yang, Ching-Tzung Yeh, Rue-Ron Hsu and Chin-Rong Lee, Int. J. Mod. Phys. D **6** (1997) 349; I-Ching Yang, Wei-Fui Lin and Rue-Ron Hsu, Chin. J. Phys. **37** (1999) 113; E. C. Vagenas, Int. J. Mod. Phys. A **18** (2003) 5781; E. C. Vagenas, Mod. Phys. Lett. A **19** (2004) 213; M. Sharif and Tasmin Fatima, Int. J. Mod. Phys. A **20** (2005) 4309; gr-qc/0410004; R. Gad, Astrophys. Space. Sci. **295** (2005) 459; R. Gad, Astrophys. Space. Sci. **293** (2004) 453; T. Grammenos, Mod. Phys. Lett. A **20** (2005) 1741; gr-qc/0412120.

[12] I. Radinschi, Mod. Phys. Lett. A **15 (35)** (2000) 2171; I. Radinschi, Fizika B (Zagreb) B **9(4)** (2000) 203; I. Radinschi, APH N. S., Heavy Ion Physics **12** (2000) 47; I. Radinschi, Acta Physica Slovaca **50(4)** (2000) 609; I. Radinschi, Chin. J. Phys. **39(5)** (2001) 1; I. Radinschi, Mod. Phys. Lett. A **16(10)** (2001) 673; I-Ching Yang and I. Radinschi, Mod. Phys. Lett. A **17** (2002) 1159; I-Ching Yang and I. Radinschi, gr-qc/0202075; I. Radinschi, gr-qc/0302047; I-Ching Yang and I. Radinschi, gr-qc/0309130; I-Ching Yang and I. Radinschi, Chin. J. Phys. **41** (2003) 326; I-Ching Yang and I. Radinschi, Chin. J. Phys. **42(1)** (2004) 40.

[13] G. Lessner, Gen. Rel. Grav. **28** (1996) 527.

[14] M.A. Melvin, Phys. Lett. **8** (1964) 65.

[15] F.J. Ernst, J. Math. Phys. **15** (1974) 1409.

[16] S. S. Xulu, Int. J. Mod. Phys. A **15** (2000) 4849.

[17] C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation*, W. H. Freeman and Co., NY (1973) 603.

[18] F. I. Cooperstock and R. S. Sarracino, J. Phys. A: Math. Gen. **11** (1978) 877.

[19] H. Bondi, Proc. Roy. Soc. London A **427** (1990) 249.

[20] Chia-Chen Chang, J. M. Nester and Chiang-Mei Chen, Phys. Rev. Lett. **83** (1999) 1897.

[21] F. I. Cooperstock, Mod. Phys. Lett. A **14** (1999) 1531.

MØLLEROV KOMPLEKS ENERGIJE-IMPULSA U MELVINOVOM MAGNETSKOM SVEMIRU

Primijenili smo Møllerov kompleks energije-impulsa u računu energije Melvinovog magnetskog svemira. Raspodjela energije ovisi o magnetskom polju. Prvi član u izrazu za energiju dvaput je veći od klasičnoga, a drugi su članovi relativističke popravke.