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$t \to bWh^0$ AND $t \to bWA^0$ DECAYS AND POSSIBLE CP VIOLATING EFFECTS

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We study the charged $t \to bWh^0$ and $t \to bWA^0$ decays in the framework of the general two Higgs doublet model, so called model III and beyond. Here, we take the Yukawa couplings complex and introduce a new complex parameter due to the physics beyond the model III, to switch on the CP violating effects. We predict the branching ratios as $BR(t \to bWh^0) \sim 10^{-6}$ and $BR(t \to bWA^0) \sim 10^{-8}$. Furthermore, we find a CP asymmetry, of the order of 10^{-2}, for both decays.

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1. Introduction

Because of its large mass, the top quark has many decay channels and this opens a new window to test the standard model (SM) and to get some clues about the new physics beyond SM. In the literature, there are various studies in SM and beyond it [1−13]. The rare flavor-changing transitions $t \to cg\,(\gamma, Z)$ (g =gluon) have been studied in Refs. [4, 6], $t \to cH^0$ in Refs. [2, 6−9] and $t \to cl_1 l_2$, where l_1, l_2 are different lepton flavors, in Ref. [10]. The SM prediction for the branching ratio (BR) of the process $t \to cg\,(\gamma, Z)$ is 4×10^{-11} (5×10^{-13}, 1.3×10^{-13}) [2], and $t \to cH^0$ is of the order of the magnitude of $10^{-14} - 10^{-13}$ in SM [7]. These are not measurable quantities even at the highest luminosity accelerators. Possible new physics effects are the candidates for the enhancement of *BR*s of the above processes. The decays $t \to cH^0$ and $t \to cl_1 l_2$ have been analyzed in Refs. [9] and [10], respectively, in the framework of the general two Higgs doublet model (model III). In these studies, it has been observed that there could be a strong enhancement in *BR*, almost seven orders of magnitude, compared to the one in the SM, for the decay $t \to cH^0$; a measurable *BR* of the order of the magnitude of $10^{-8} - 10^{-7}$ is expected for the decay $t \to cl_1l_2$. In Ref. [11], $t \to cV(VV)$, $(V = W, g, \gamma, Z)$ decays have been analysed in the topcolor-assisted technicolor theory.

The charged $t \to b$ transitions exist in the SM model and have been studied in the literature extensively. The top decay $t \to bW$ has been analysed (see Ref. [12] and references therein) in the two Higgs doublet model and $t \to bWZ$ decay has been studied in Ref. [13].

The present work is devoted to the analysis of the charged $t \to b\,W\,h^0$ and $t \to b\,W\,A^0$ decays in the framework of the general two Higgs doublet model (model III). This decay occurs at the tree level with the extended Higgs sector since the scalar bosons h^0 and A^0 exist in the new sector. We study *BR*s of the above decays and obtain values of the order of magnitude of 10^{-6} and 10^{-8}, respectively. Furthermore, we search for the possible CP (C=charge conjugation, P=parity) violating effects. To study a nonzero CP asymmetry (A_{CP}), we take the Yukawa coupling for $b\,h^0\,(A^0)\,b$ transition complex and introduce a new complex parameter, where its complexity comes from some type of radiative corrections, due to the model beyond the model III (see Sect. 2). We obtain A_{CP} of the order of magnitude of 10^{-2} and observe that these physical quantities can give valuable information about physics beyond the SM, and allow the determination of free parameters introduced in these models.

The paper is organized as follows: in Sect. 2, we present the *BR* and A_{CP} of the decay $t \to b\,W\,h^0(A^0)$ in the framework of the model III. Sect. 3 is devoted to the discussion and our conclusions.

2. $t \to bWh^0$ *and* $t \to bWA^0$ *decays with possible CP violating effects*

If one respects the current mass values of $h^0(A^0)$, namely $m_{h^0} \sim 85\,\mathrm{GeV}$ ($m_{A^0} \sim 90\,\mathrm{GeV}$), the charged $t \to bWh^0(A^0)$ is kinematically possible and does not exist in the SM model. With the minimal extension of the Higgs sector the CP odd new Higgs scalar A^0 arises and the $t \to bWA^0$ decay at the tree level is permitted. In this model, $t \to bWh^0$ decay is possible at the tree level, where h^0 is the new CP even Higgs scalar and, in general, it mixes with the SM one, H^0. In this section, we study these top decays in the general two Higgs doublet model, so called model III and the possible CP violation.

The $t \to bWh^0(A^0)$ decay is created by the charged $t \to bW$ process and the neutral $t \to t^* h^0(A^0)$ or $b^* \to bh^0(A^0)$ ($t^*(b^*)$ = virtual t (b) quarks) processes, which are controlled by the Yukawa interaction

$$\mathcal{L}_Y = \eta^U_{ij} \bar{Q}_{iL} \tilde{\phi}_1 U_{jR} + \eta^D_{ij} \bar{Q}_{iL} \phi_1 D_{jR} + \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + h.c. \;, \quad (1)$$

where L and R denote chiral projections $L(R) = 1/2(1 \mp \gamma_5)$, ϕ_i for $i = 1, 2$, are the two scalar doublets, $\bar{Q}_{iL}$ are left handed quark doublets, $U_{jR}(D_{jR})$ are right handed up (down) quark singlets, with family indices i, j. The Yukawa matrices $\eta^{U,D}_{ij}$ and $\xi^{U,D}_{ij}$ have in general complex entries. By considering the gauge and CP invariant Higgs potential which spontaneously breaks $SU(2) \times U(1)$ down to $U(1)$ as

$$\begin{aligned} V(\phi_1, \phi_2) &= c_1(\phi_1^+\phi_1 - v^2/2)^2 + c_2(\phi_2^+\phi_2)^2 \\ &+ c_3[(\phi_1^+\phi_1 - v^2/2) + \phi_2^+\phi_2]^2 + c_4[(\phi_1^+\phi_1)(\phi_2^+\phi_2) - (\phi_1^+\phi_2)(\phi_2^+\phi_1)] \\ &+ c_5[Re(\phi_1^+\phi_2)]^2 + c_6[Im(\phi_1^+\phi_2)]^2 + c_7 \; . \end{aligned} \tag{2}$$

with real parameters $c_i, (i = 1, 2, ...7)$, and choosing the parametrization for ϕ_1 and ϕ_2 as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\left(\begin{array}{c} 0 \\ v + H^0 \end{array}\right) + \left(\begin{array}{c} \sqrt{2}\chi^+ \\ i\chi^0 \end{array}\right)\right] ; \phi_2 = \frac{1}{\sqrt{2}}\left(\begin{array}{c} \sqrt{2}H^+ \\ H_1 + iH_2 \end{array}\right) . \tag{3}$$

with the vacuum expectation values,

$$< \phi_1 >= \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ v \end{array}\right) ; < \phi_2 >= 0 \; , \tag{4}$$

the H_1 and H_2 become the mass eigenstates h^0 and A^0, respectively, since no mixing occurs between two CP-even neutral bosons H^0 and h^0, at the tree level. This scenario permits one to collect SM particles in the first doublet and new particles in the second one. Furthermore the flavor changing (FC) interaction can be obtained as

$$\mathcal{L}_{Y,FC} = \xi^{U\,\dagger}_{ij} \bar{Q}_{iL} \tilde{\phi}_2 U_{jR} + \xi^D_{ij} \bar{Q}_{iL} \phi_2 D_{jR} + h.c. \; , \tag{5}$$

with the couplings $\xi^{U,D}$ for the FC charged interactions

$$\begin{aligned} \xi^U_{ch} &= \xi^U_N \, V_{CKM} \; , \\ \xi^D_{ch} &= V_{CKM} \, \xi^D_N \; , \end{aligned} \tag{6}$$

where $\xi^{U,D}_N$ is defined by the expression

$$\xi^{U(D)}_N = (V^{U(D)}_{R(L)})^{-1} \xi^{U,(D)} V^{U(D)}_{L(R)} \; , \tag{7}$$

V_{CKM} in Eq. (6) is the Cabibbo-Kobayashi-Maskawa matrix and $V^{U,D}_{R,L}$ are the rotation matrices acting on the up and down type quarks, with left and right chirality, respectively. Notice that the index "N" in $\xi^{U,D}_{\rm N}$ denotes the word "neutral".

Using the relevant diagrams for the $t \to bWh^0(A^0)$ decay, which are given in Fig. 1, and taking into account only the real Yukawa couplings $\xi^D_{N,bb}$, $\xi^U_{N,tt}$, the

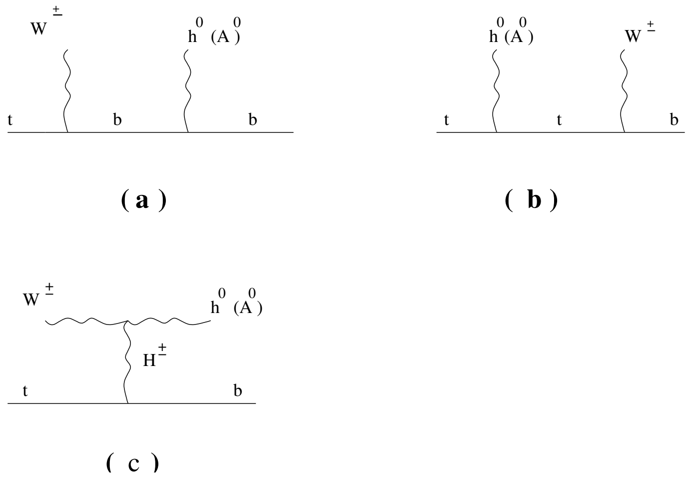

Fig. 1. The diagrams which contribute to the decay $t \to bWh^0(A^0)$.

matrix element square $|M|^2_{h^0}$ ($|M|^2_{A^0}$) reads

$$|M|^2_{h^0\,(A^0)}(p1, p_b, k, q) = \xi^D_{N,bb}\, \xi^U_{N,tt}\, f_1(h^0\,(A^0)) + (\xi^D_{N,bb})^2\; f_2(h^0\,(A^0)) + (\xi^U_{N,tt})^2\, f_3(h^0\,(A^0)) \tag{8}$$

where

$$\begin{aligned} f_1(h^0) &= 16\,|V_{tb}|^2\, m_b\, m_t \Bigg(m_W^2 \Big(s_2^2(h^0) - s_1^2(h^0) + 2\, s_1(h^0)\, s_2(h^0) \Big) x_{h^0} \\ &+ 2\, s_1(h^0) \Big((s_1(h^0) - s_2(h^0))\, k.(p_1 - p_b) + 2\, s_1(h^0)\, p_1.p_b \Big) \\ &+ \frac{1}{m_W^2} \Big(- (s_2^2(h^0) + 2\, s_1^2(h^0) + 2\, s_1(h^0)\, s_2(h^0))\, (k.q)^2 + 2\, s_1(h^0)\, (2\, s_1(h^0) \\ &+ s_2(h^0))\, k.q\, q.(p_1 - p_b) + 8\, s_1^2(h^0)\, p_1.q\, p_b.q \Big) \Bigg)\,, \\ f_2(h^0) &= 8\,|V_{tb}|^2 \Bigg(- m_W^2 \Big(s_2(h^0) + s_1(h^0) \Big)^2 x_{h^0}\, p_1.p_b \\ &+ \frac{1}{m_W^2}\, k.q \Big((s_2(h^0) + 2\, s_1(h^0)) \\ &\times\, (s_2(h^0)\, k.q\, p_1.p_b + 2\, s_1(h^0)\, k.p_b\, q.p_1) - 2\, s_1(h^0)\, s_2(h^0)\, k.p_1\, p_b.q \Big) \\ &+ 2\, s_1^2(h^0)\, (k.p_1\, k.p_b - x_{h^0}\, q.p_1\, q.p_b) \Bigg)\,, \end{aligned}$$

$$
\begin{aligned}
f_3(h^0) &= 8\,|V_{tb}|^2 \Bigg(- m_W^2 \Big(4\, s_1(h^0)\,(s_1(h^0) - s_2(h^0))\, x_t\, k.p_b \\
&+ \Big((s_1(h^0) + s_2(h^0))^2\, x_{h^0} - 4\, s_1^2(h^0)\, x_t \Big)\, p_1.p_b \Big) \\
&+ \frac{1}{m_W^2}\, k.q\, \Big(s_2(h^0)\,(s_2(h^0) + 2\, s_1(h^0))\, k.q\, p_1.p_b - 2\, s_1(h^0)\, s_2(h^0)\, q.p_1\, k.p_b \\
&+ 2\, s_1(h^0)(2\, s_1(h^0) + s_2(h^0))\, k.p_1\, q.p_b \Big) \\
&+ 2\, s_1(h^0)\, \Big(s_1(h^0)\, k.p_1\, k.p_b \\
&- \Big(2\,(2\, s_1(h^0) + s_2(h^0))\, x_t\, k.q + s_1(h^0)\,(x_{h^0} - 4\, x_t)\, q.p_1 \Big)\, q.p_b \Big) \Bigg) , \\
f_1(A^0) &= 16\,|V_{tb}|^2\, m_b\, m_t \Bigg(m_W^2 \Big(s_2^2(A^0) + s_1^2(A^0) \Big)\, x_{A^0} \\
&+ \frac{1}{m_W^2} \Big(2\, s_1^2(A^0) - s_2^2(A^0) \Big)\, (k.q)^2 \Bigg) , \\
f_2(A^0) &= 8\,|V_{tb}|^2 \Bigg(- m_W^2 \Big(s_2(A^0) + s_1(A^0) \Big)^2 x_{A^0}\, p_1.p_b \\
&+ \frac{1}{m_W^2}\, k.q\, \Big((s_2(A^0) + 2\, s_1(A^0))\, (s_2(A^0)\, k.q\, p_1.p_b + 2\, s_1(A^0)\, k.p_b\, p_1.q) \\
&- 2\, s_1(A^0)\, s_2(A^0)\, k.p_1\, p_b.q \Big) + 2\, s_1^2(A^0)\, (k.p_1\, k.p_b - x_{A^0}\, q.p_1\, q.p_b) \Bigg) , \\
f_3(A^0) &= 8\,|V_{tb}|^2 \Bigg(- m_W^2 \Big(s_1(h^0) - s_2(h^0) \Big)^2 x_{A^0}\, p_1.p_b + \frac{1}{m_W^2}\, k.q\, \Big(s_2(A^0)\, (s_2(h^0) \\
&- 2\, s_1(h^0))\, k.q\, p_1.p_b + 2\, s_1(A^0)\, (2\, s_1(A^0) - s_2(A^0))\, k.p_1\, p_b.q) \\
&+ 2\, s_1(A^0)\, s_2(A^0)\, k.p_b\, p_1.q \Big) + 2\, s_1^2(A^0)\, \Big(k.p_1\, k.p_b - x_{A^0}\, p_1.q\, p_b.q \Big) \Bigg) . \qquad (9)
\end{aligned}
$$

Here the functions $s_{1(2,3)}(h^0(A^0))$ are

$$
\begin{aligned}
s_1(h^0) &= -\frac{g_W}{4\, m_W^2\, (1 + x_t - 2\, \frac{p_1.q}{m_W^2})} , \\
s_2(h^0) &= \frac{g_W}{2\, m_W^2\, (1 + x_{h^0} - y_t - 2\, \frac{k.q}{m_W^2})} ,
\end{aligned}
$$

$$s_{1(2)}(A^0) \;\; = \;\; (-)\, s_{1(2)}(h^0 \to A^0)\,, \tag{10}$$

with weak coupling constant g_W, $x_{h^0(A^0)} = m^2_{h^0(A^0)}/m^2_W$, $x_t = m_t^2/m_W^2$ and $y_t = m^2_{H^\pm}/m^2_W$ and p_1, p_b, q and k are four momentum of t quark, b quark, W boson and Higgs scalar $h^0(A^0)$, respectively.

Finally, using the well known expression defined in the t quark rest frame

$$\begin{aligned} d\Gamma_{h^0\,(A^0)} \;\; &= \;\; \frac{(2\pi)^4}{12\, m_t}\, \delta^{(4)}(p_1 - p_b - k - q)\, \frac{\mathrm{d}^3 p_b}{(2\pi)^3\, 2\, E_b}\, \frac{\mathrm{d}^3 q}{(2\pi)^3\, 2\, E_W}\, \frac{\mathrm{d}^3 k}{(2\pi)^3\, 2\, E_{h^0(A^0)}} \\ &\times \;\; |M|^2_{h^0\,(A^0)}(p_1, p_b, k, q) \end{aligned} \tag{11}$$

and the total decay width $\Gamma_T \sim \Gamma(t \to bW)$ as $\Gamma_T = 1.55\,\mathrm{GeV}$, we get the BR for the decay $t \to bWh^0(A^0)$.

Now, we would like to study a possible CP violating effects, which can give comprehensive information about the free parameters of the model used. For the process under consideration, the CP violation can be obtained by choosing the complex Yukawa couplings in general, namely, taking the parametrizations

$$\begin{aligned} \xi^U_{N,tt} &= |\xi^U_{N,tt}|\, e^{\mathrm{i}\theta_{tt}}\,, \\ \xi^D_{N,bb} &= |\xi^D_{N,bb}|\, e^{\mathrm{i}\theta_{bb}}\,, \end{aligned} \tag{12}$$

where $e^{\mathrm{i}\theta_{tt}}\,(e^{\mathrm{i}\theta_{bb}})$ is the CP violating phase appearing in the Yukawa coupling $\xi^U_{N,tt}\,(\xi^U_{N,bb})$ driving the $t - t\,(b - b)$ transition. However, this choice is not enough to get non-zero A_{CP}

$$A_{CP} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \tag{13}$$

where $\bar{\Gamma}$ is the decay width for the CP conjugate process. This forces one to go beyond the model III and try to obtain a new complex quantity so that its complexity does not come from the Yukawa couplings but from some radiative corrections. Under the light of this discussion, we introduce an additional complex correction χ to $b \to b$ transition, which may come from the new model beyond the model III as

$$\begin{gathered} (\xi^D_{N,bb} + \xi^{D*}_{N,bb}) \;\; + \;\; (\xi^D_{N,bb} - \xi^{D*}_{N,bb})\gamma_5 + \chi \\ \text{and} \\ (\xi^{D*}_{N,bb} - \xi^D_{N,bb}) \;\; - \;\; (\xi^D_{N,bb} + \xi^{D*}_{N,bb})\gamma_5 + \chi\gamma_5\,, \end{gathered}$$

Here we take the magnitude of χ at most $|\chi| \sim 10^{-2}$, which is more than one order smaller compared to the vertex due to model III. In this case, we take the correction to the $t \to t$ transition small since the strength of $t \to t$ transition is weaker compared to the strength of the $b \to b$ transition, with respect to our choice (see the section Discussion)

At this stage, we introduce a model beyond the model III as follows: The multi Higgs doublet model which contains more than two Higgs doublets in the Higgs sector can be a candidate. The choice of three Higgs doublets brings new Yukawa couplings which are responsible for the interactions between new Higgs particles and the fermions. The Yukawa Lagrangian in the three Higgs doublet model (3HDM) reads

$$\begin{aligned} \mathcal{L}_Y &= \eta^U_{ij}\bar{Q}_{iL}\tilde{\phi_1}U_{jR} + \eta^D_{ij}\bar{Q}_{iL}\phi_1 D_{jR} + \xi^{U\,\dagger}_{ij}\bar{Q}_{iL}\tilde{\phi_2}U_{jR} + \xi^D_{ij}\bar{Q}_{iL}\phi_2 D_{jR} \\ &+ \rho^U_{ij}\bar{Q}_{iL}\tilde{\phi_3}U_{jR} + \rho^D_{ij}\bar{Q}_{iL}\phi_3 D_{jR} + h.c. \ , \end{aligned} \tag{14}$$

where $\rho^{U(D)}_{ij}$ is the new coupling and ϕ_3 can be chosen as

$$\phi_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}F^+ \\ H_3 + iH_4 \end{pmatrix} \ , \tag{15}$$

with vanishing vacuum expectation value. The fields F^+ and $H_3\,(H_4)$ represent the new charged and CP even (odd) Higgs particles, respectively. Notice that the other Yukawa couplings and Higgs particles in Eq. (14) are the ones existing in the model III. Now, we choose the additional Yukawa couplings $\rho^{U(D)}_{ij}$ real and take into account the radiative corrections to the $b \to b$ transition which comes from the contributions of third Higgs doublet for the decay under consideration. Here the complexity of the parameter should come from the radiative corrections but not from the new Yukawa couplings. We can take this complex contribution as a source for the additional part χ. Since the number of free parameters, namely masses of new Higgs particles $m_{F^\pm}$, m_{H_3}, m_{H_4} and the new Yukawa couplings $\rho^{U(D)}_{ij}$, increases, there arises a difficulty to restrict them. However, the overall uncertainity coming from these free parameters lies in the contribution of χ and it can be overcome by the possible future measurement of the CP violation for our process.

Finally, by using the definition

$$A^{h^0(A^0)}_{CP}(E_W, E_b) = \frac{\dfrac{\mathrm{d}^2\Gamma(t \to bWh^0(A^0))}{\mathrm{d}E_b\mathrm{d}E_W} - \dfrac{\mathrm{d}^2\Gamma(\bar{t} \to \bar{b}\bar{W}h^0(A^0)}{\mathrm{d}E_b\mathrm{d}E_W}}{\dfrac{\mathrm{d}^2\Gamma(t \to bWh^0(A^0))}{\mathrm{d}E_b\mathrm{d}E_W} + \dfrac{\mathrm{d}^2\Gamma(\bar{t} \to \bar{b}\bar{W}h^0(A^0)}{\mathrm{d}E_b\mathrm{d}E_W}} \tag{16}$$

we obtain the differential $A_{CP}(E_W, E_b)$ for the process $t \to bWh^0(A^0)$ as

$$A^{h^0(A^0)}_{CP}(E_W, E_b) \quad = \quad |\bar{\xi}^D_{N,bb}|\,|\chi|\,\sin\theta_{bb}\,\sin\theta_\chi\frac{\Phi^{h^0(A^0)}}{D^{h^0(A^0)}} \ , \tag{17}$$

where

$$
\begin{aligned}
\Phi^{h^0} &= 4\, m_t\, s_1(h^0)\, |V_{tb}|^2 \Bigg(4\Big(2\, E_W^2\, m_t\, s_1(h^0)\, (x_t - 2) - E_b^2\, (2\, E_W + m_t)\, s_2(h^0) \\
&\times (1 + x_t) + E_b\, E_W \Big(E_W (s_2(h^0)(1 + 3\, x_{h^0} - 3\, x_t) + 4\, s_1(h^0)(1 + 2\, x_{h^0})) \\
&+ m_t\, (s_2(h^0) + 2\, s_1(h^0)\, (2\, x_{h^0} + x_t))\Big)\Big) + m_W^2 \Big(m_t (s_2(h^0)\, (-1 + (x_{h^0} - x_t)^2) \\
&+ 4\, s_1(h^0)\, (1 + x_t + x_{h^0}) + 2\, E_b\, (-2\, s_1(h^0)\, (1 + 2\, x_{h^0} + x_t) \\
&+ s_2(h^0)\, (-1 + x_{h^0} - x_t)\, (2\, x_t - 1)) - 2\, E_W\, (s_2(h^0)\, (x_{h^0} - x_t)\, (1 - x_t + x_{h^0}) \\
&+ s_1(h^0)\, (4 + 2\, x_{h^0}\, (2 + 2\, x_{h^0} - x_t) + 2\, x_t\, (3 - x_t)))\Big) \\
&+ \frac{8}{m_W^2} \Big(E_b\, E_W^2\, (E_b (2\, E_W + m_t)\, s_2(h^0) - 4\, E_W\, m_t\, s_1(h^0))\Big)\Bigg), \\
\Phi^{A^0} &= 4\, m_t\, s_1(A^0)\, |V_{tb}|^2 \Bigg(4\Big(2\, E_W^2\, m_t\, s_1(A^0)\, (x_t - 2) - E_b^2\, (2\, E_W + m_t)\, s_2(A^0) \\
&\times (1 + x_t) + E_b\, E_W \Big(E_W (s_2(A^0)(1 + 3\, x_{A^0} - 3\, x_t) + 4\, s_1(A^0)(1 + 2\, x_{A^0})) \\
&+ m_t\, (s_2(A^0) + 2\, s_1(A^0)\, (2\, x_{A^0} + x_t))\Big)\Big) \\
&+ m_W^2 \Big(- m_t (s_2(A^0)\, (-1 + (x_{A^0} - x_t)^2) + 4\, s_1 A^0\, (1 + x_t + x_{A^0}) \\
&+ 2\, E_b\, (2\, s_1(A^0)\, (1 + 2\, x_{A^0} + x_t) + s_2(A^0)\, (-1 + x_t + 2\, x_t^2 + x_{A^0} (1 - 2\, x_t)) \\
&+ 2\, E_W\, \Big(s_2(A^0)\, (x_{A^0} - x_t)\, (1 - x_t + x_{A^0}) + s_1(A^0)\, (4 + 4\, x_{A^0}\, (1 + x_{A^0} - 2\, x_t) \\
&+ 2\, x_t\, (3 - x_t))\Big)\Big) \\
&+ \frac{8}{m_W^2} \Big(E_b\, E_W^2\, (E_b (2\, E_W + m_t)\, s_2(A^0) - 4\, E_W\, m_t\, s_1(A^0))\Big)\Bigg), \qquad (18)
\end{aligned}
$$

with $\chi = e^{\mathrm{i}\,\theta_\chi}\, |\chi|$, $\bar{\xi}^D_{N,bb} = e^{\mathrm{i}\,\theta_{bb}}\, |\bar{\xi}^D_{N,bb}|$. Notice that we do not present the functions D^{h^0} and D^{A^0} since their explicit expressions are long. Here the functions $s_1(h^0(A^0))$ and $s_2(h^0(A^0))$ are given in Eq.(10).

3. Discussion

This section is devoted to the analysis of the *BR* and A_{CP} of the decays $t \to bWh^0$ and $t \to bWA^0$ in the framework of the model III and beyond. In our numerical analysis we use the form of the coupling $\bar{\xi}^{U(D)}_{N,ij}$, which is defined as $\xi^{U(D)}_{N,ij} = \sqrt{\frac{1}{\sqrt{2}} 4\, G_F}\, \bar{\xi}^{U(D)}_{N,ij}$.

Since the model III contains a large number of free parameters, such as Yukawa couplings, $\bar{\xi}^{U(D)}_{N,ij}$, the masses of new Higgs bosons, $H^{\pm}$, h^0 and A^0, we try to restrict them by using experimental measurements. In our calculations, we neglect all Yukawa couplings except $\bar{\xi}^{U}_{N,tt}$ and $\bar{\xi}^{D}_{N,bb}$, due to their their light flavor contents. In addition to this, we neglect the off-diagonal coupling $\bar{\xi}^{U}_{N,tc}$, since it is small compared to $\bar{\xi}^{U}_{N,tt}$ (see Ref. [14]). One of the most important experimental measurements for the prediction of the constraint region for the couplings $\bar{\xi}^{U}_{N,tt}$ and $\bar{\xi}^{D}_{N,bb}$ is the the CLEO measurement [15]

$$BR(B \to X_s \gamma) = (3.15 \pm 0.35 \pm 0.32) \times 10^{-4} \ . \tag{19}$$

and our procedure is to restrict the Wilson coefficient $C_7^{\rm eff}$ which is the effective coefficient of the operator $O_7 = (e/16\pi^2) \bar{s}_\alpha \sigma_{\mu\nu} (m_b R + m_s L) b_\alpha \mathcal{F}^{\mu\nu}$ (see Ref. [14] and references therein), in the region $0.257 \leq |C_7^{\rm eff}| \leq 0.439$, where the upper and lower limits were calculated using Eq. (19) and all possible uncertainities in the calculation of $C_7^{\rm eff}$ [14]. In the calculation of A_{CP}, $\bar{\xi}^{D}_{N,bb}$ ($\bar{\xi}^{U}_{N,tt}$) they are taken complex (real), and a new small complex parameter χ is introduced, for the physics beyond the model III. In the following, we choose $|r_{tb}| = \left| \frac{\bar{\xi}^{U}_{N,tt}}{\bar{\xi}^{D}_{N,bb}} \right| < 1$. Notice that in the figures, BR and A_{CP} are restricted to the region between the solid (dashed) lines for $C_7^{\rm eff} > 0$ ($C_7^{\rm eff} < 0$). There are two possible solutions for $C_7^{\rm eff}$ due to the cases where $|r_{tb}| < 1$ and $r_{tb} > 1$. In the case of complex Yukawa couplings, only the solutions obeying $|r_{tb}| < 1$ exist.

In Fig. 2, we plot $BR(t \to bWh^0)$ with respect to $\bar{\xi}^{D}_{N,bb}/m_b$ for $m_{H^\pm} = 400\,$GeV, $m_{h^0} = 85\,$GeV. As shown in this figure, BR is of the order of magnitude of 10^{-6} and it increases with the increasing value of the $\bar{\xi}^{D}_{N,bb}/m_b$. Its magnitude (the restriction region) is larger (broader) for $C_7^{\rm eff} > 0$ compared to the one for $C_7^{\rm eff} < 0$.

Figure 3 is devoted to the same dependence of the $BR(t \to bWA^0)$ for $m_{H^\pm} = 400\,$GeV, $m_{A^0} = 90\,$GeV. For this process, the BR is of the order of magnitude of 10^{-8}, almost 2 orders of magnitude smaller than $BR(t \to bWh^0)$. It increases with increasing value of $\bar{\xi}^{D}_{N,bb}/m_b$, and its magnitude (the restriction region) is larger (broader) for $C_7^{\rm eff} > 0$ compared to the one for $C_7^{\rm eff} < 0$. Furthermore, the restriction region is sensitive to the parameter $\bar{\xi}^{D}_{N,bb}/m_b$, and for $C_7^{\rm eff} < 0$, upper and lower bounds almost coincide.

Figure 4 (5) represents $BR(t \to bWh^0(A^0))$ with respect to $m_{h^0}(m_{A^0})$ for $m_{H^\pm} = 400\,$GeV and $\bar{\xi}^{D}_{N,bb} = 30\, m_b$. Here BR increases with decreasing values of $m_{h^0}(m_{A^0})$. This can give a powerful information about the lower limit of the mass value $m_{h^0}(m_{A^0})$ with the help of possible future experimental measure-

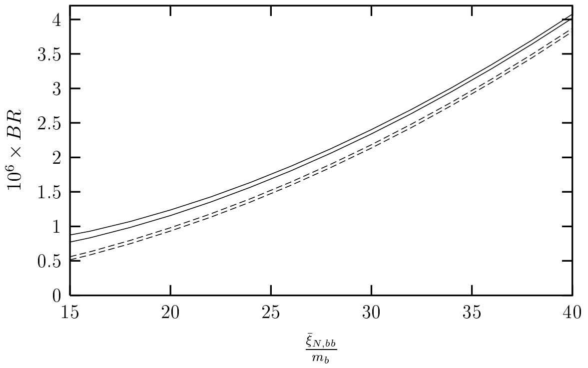

Fig. 2. BR($t \to bWh^0$) as a function of $\frac{\bar{\xi}^D_{N,bb}}{m_b}$ for $m_{H^\pm} = 400\,\mathrm{GeV}$, $m_{h^0} = 85\,\mathrm{GeV}$. Here BR is restricted in the region bounded by solid lines for $C_7^{\mathrm{eff}} > 0$ and by dashed lines for $C_7^{\mathrm{eff}} < 0$.

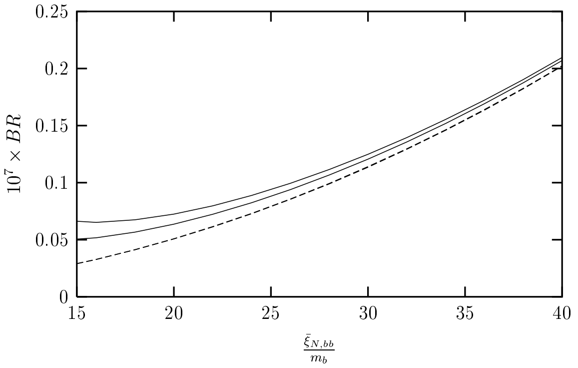

Fig. 3. The same as Fig. 2 but for the decay $t \to bWA^0$.

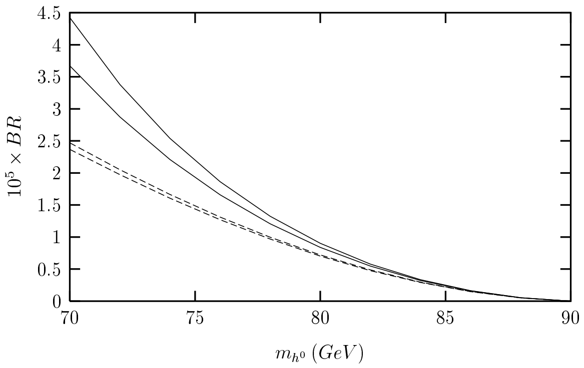

Fig. 4. BR($t \to bWh^0$) as a function of m_{h^0} for $\bar{\xi}^D_{N,bb} = 30\,m_b$, $m_{H^\pm} = 400\,\mathrm{GeV}$. Here BR is restricted in the region bounded by solid lines for $C_7^{\mathrm{eff}} > 0$ and by dashed lines for $C_7^{\mathrm{eff}} < 0$.

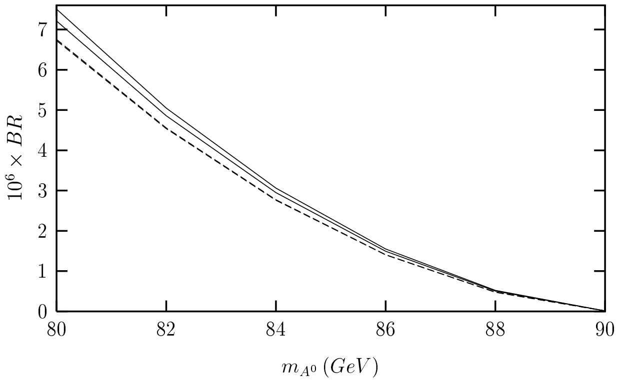

Fig. 5. The same as Fig. 4 but for the decay $t \to bWA^0$.

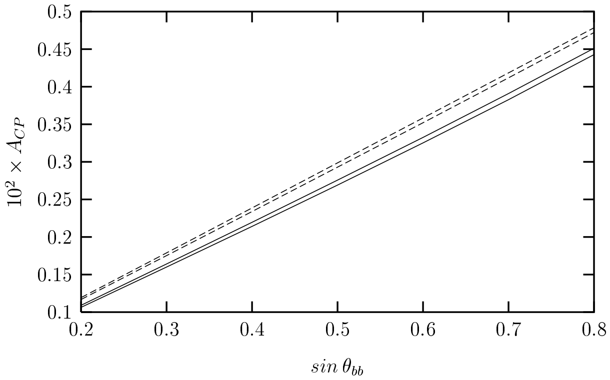

Fig. 6. $A_{CP}(t \to bWh^0)$ as a function of $\sin\theta_{bb}$ for $|\bar{\xi}^{D}_{N,bb}| = 30\,m_b$, $m_{H^\pm} = 400\,\mathrm{GeV}$, $|\chi| = 10^{-2}$, $\sin\theta_\chi = 0.5$. Here A_{CP} is restricted in the region bounded by solid lines for $C_7^{\mathrm{eff}} > 0$ and by dashed lines for $C_7^{\mathrm{eff}} < 0$.

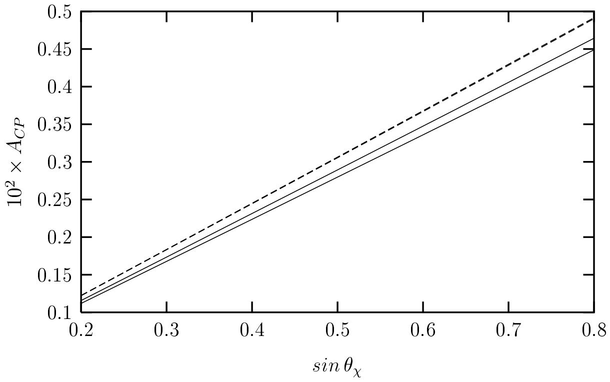

Fig. 7. The same as Fig. 6 but for the decay $t \to bWA^0$.

ment of the process under consideration. Notice that with the increasing values of $m_{h^0}(m_{A^0})$, the restriction regions for $C_7^{\rm eff} > 0$ and $C_7^{\rm eff} < 0$ become narrower and coincide.

Now, we would like to analyse the CP asymmetry, A_{CP}, of the decay $t \to bWh^0(A^0)$. To obtain a nonzero A_{CP}, we take the coupling $\bar{\xi}^D_{N,bb}$ complex and introduce a new complex parameter χ due to the physics beyond the model III (see Sect. 2).

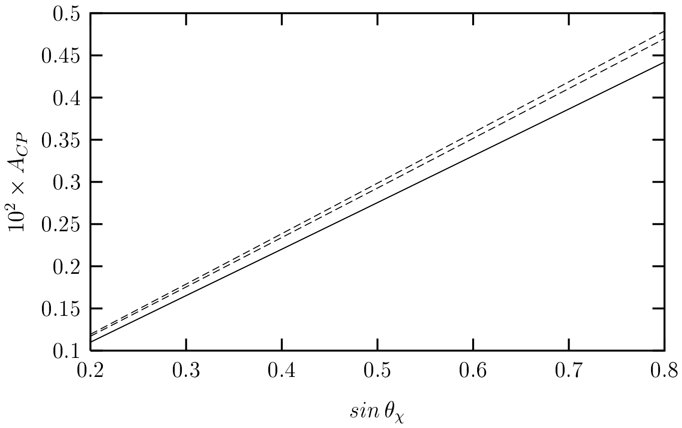

Fig. 8. $A_{CP}(t \to bWh^0)$ as a function of $\sin\theta_\chi$ for $|\bar{\xi}^D_{N,bb}| = 30\,m_b$, $m_{H^\pm} = 400\,\mathrm{GeV}$, $|\chi| = 10^{-2}$, $\sin\theta_{bb} = 0.5$. Here A_{CP} is restricted in the region bounded by solid lines for $C_7^{\rm eff} > 0$ and by dashed lines for $C_7^{\rm eff} < 0$.

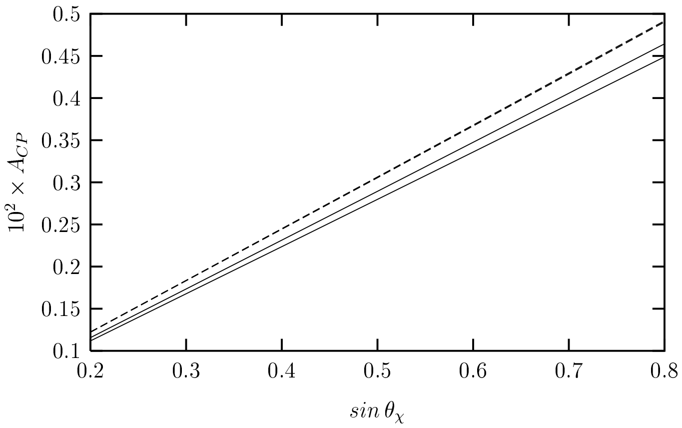

Fig. 9. The same as Fig. 8 but for the decay $t \to bWA^0$.

In Fig. 6 (7) we present the $\sin\theta_{bb}$ dependence of $A_{CP}(t \to bWh^0(A^0))$ for $|\bar{\xi}^D_{N,bb}| = 30\,m_b$, $|\chi| = 10^{-2}$, the intermediate value of $\sin\theta_\chi = 0.5$ and $m_{h^0} =$

85 GeV, ($m_{A^0} = 90$ GeV). A_{CP} is of the order of the magnitude of $10^{-3} - 10^{-2}$ and slightly larger for $C_7^{\rm eff} < 0$ compared to the one for $C_7^{\rm eff} > 0$. Figure 8 (9) represents the $\sin\theta_\chi$ dependence of $A_{CP}(t \to bWh^0(A^0))$ for $|\bar\xi^D_{N,bb}| = 30\, m_b$, $|\chi| = 10^{-2}$, the intermediate value of $\sin\theta_{bb} = 0.5$ and $m_{h^0} = 85$ GeV ($m_{A^0} = 90$ GeV). The behavior of A_{CP} is similar to the one obtained in Fig. 6 (7).

Now we summarize our results:

- There is a difficulty in the measurement of the decays $t \to bWh^0(A^0)$, especially for the output A^0, since the kinematical regions for the processes are narrow due to the large masses of W, h^0 and A^0. This is the reason for the strong suppression of the *BR* for the $t \to bWA^0$ decay. A similar suppression has been discussed in [16] for $t \to bWH^0$ decay, where H^0 is the SM Higgs boson. In the model III, we predict the *BR* of the process $t \to bWh^0(A^0)$ of the order of 10^{-6} (10^{-8}). As it is seen from the numerical values, one needs at least $10^9 - 10^{10}$ top quark pairs for the measurement of $t \to bWA^0$ and this is beyond the expectations of the future planned accelerators. Furthermore, the W boson is unstable and it is not observable directly. This forces one to make new definition of *BR* such as

$$BR(t \to bWh^0(A^0)) = \frac{BR(t \to b\mu\nu_\mu h^0(A^0))}{BR(W \to \mu\nu_\mu)}$$

  and to eliminate the background effects which come from $H^\pm \to \mu\nu_\mu$ decays, where $H^\pm$ is the charged Higgs boson assumed in the model III. In addition to this, the possible instability of A^0 makes it necessary to study some of its decay products since the final state can be affected much more from the background. Therefore, there is a possibility to measure the process $t \to bWh^0$, however the situation is much worse for $t \to bWA^0$. As a result, the measurement of the *BR* for the decay $t \to bWh^0$ can ensure a crucial test for the new physics beyond SM.

- *BR* is sensitive to $\bar\xi^D_{N,bb}$ and the mass value m_{h^0} (m_{A^0}). This is important in the prediction of the lower limit of the mass m_{h^0} with the possible future experimental measurement of $t \to bWh^0$.

- A_{CP} is at the order of the magnitude of 10^{-2} for the intermeditate values of $\sin\theta_{bb}$ and and $\sin\theta_\chi$. The measurement of A_{CP} for $t \to bWh^0$ needs almost $(A_{CP}^2 \times BR)^{-1} \sim 10^{10}$ events for a 1-sigma signal, and, therefore, $\sim 10^{10}$ top pairs should be obtained assuming high efficiency. This number is also beyond the predictions of the future planned accelerators. A_{CP} can be increased by taking larger values of the complex parameter χ, which comes from new physics beyond the model III. This means that the radiative corrections to the $b \to b$ transition are larger compared to those we have choosen in this work. To be more optimistic, we take $|\chi| \sim 10^{-1}$ and the large coupling $\bar\xi^D_{N,bb}$ to obtain $BR \sim 10^{-5}$ for the decay $t \to bWh^0$. In this case, A_{CP} increases to ~ 0.1 and one needs $\sim 10^7$ top pairs for the measurement of this quantity. This is a better scenario to measure the A_{CP} of the process $t \to bWh^0$ and if it could be measured, it would give strong clues about the possible physics beyond the SM and also more beyond. For the process $t \to bWA^0$, the A_{CP} measurement seems hopeless.

Therefore, experimental investigation of the process $t \to bWh^0(A^0)$ will be effective for the understanding of the physics beyond the SM.

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RASPADI $t \to bWh^0$ I $t \to bWA^0$ I MOGUĆI UČINCI NARUŠENJA CP

Proučavamo nabojne raspade $t \to bWh^0$ i $t \to bWA^0$ u okviru modela dvojnog Higgsovog dubleta, tzv. modela III i njegovog proširenja. Uzimamo kompleksna Yukawina vezanja i uvodimo nov kompleksan parametar radi proširenja modela III, kako bi se uključili učinci narušenja CP. Predviđamo omjere grananja $BR(t \to bWh^0) \sim 10^{-6}$ i $BR(t \to bWA^0) \sim 10^{-8}$. Nadalje, nalazimo asimetriju CP od oko 10^{-2} za oba raspada.