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NEW METHOD FOR SOLVING THE Nd BREAKUP SCATTERING PROBLEM IN CONFIGURATION SPACE

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A new computational method for solving the configuration-space Faddeev equations for a three-nucleon system has been developed. This method is based on the spline-decomposition in the angular variable and on a generalization of the Numerov method for the hyperradius. The s-wave calculations of the inelasticity and phase-shift, as well as of the breakup amplitudes for **nd** and **pd** breakup scattering for lab energies 14.1 and 42.0 MeV have been performed using the Malfliet-Tjon MT I-III potential. In the case of **nd** breakup scattering, the results are in good agreement with those of the benchmark solution (J. L. Friar et al., Phys. Rev. C **42** (1990) 1838 and J. L. Friar et al., Phys. Rev. C **51** (1995) 2356). In the case of **pd** quartet breakup scattering, disagreement for the inelasticities reaches up to 6% when compared with the results of the Pisa group (A. Kievsky et al., Phys. Rev. C **64** (2001) 024002). The calculated **pd** amplitudes fulfill the optical theorem with a good precision.

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1. *Faddeev equations in configuration space*

This paper deals with the s-wave breakup scattering in three-nucleon systems. Our approach is based on the method of the Faddeev equations [1], which was modified by Merkuriev to incorporate the Coulomb force [2]. The Faddeev components Ψ_α for three-body Coulomb systems satisfy the following set of differential Faddeev equations

$$\{-\Delta_{\overline{x}_\alpha} - \Delta_{\overline{y}_\alpha} + V_c + V_\alpha(|\overline{x}_\alpha|) - E\}\Psi_\alpha(\overline{x}_\alpha, \overline{y}_\alpha) = -V_\alpha(|\overline{x}_\alpha|)\sum_{\beta\neq\alpha}\Psi_\beta(\overline{x}_\beta, \overline{y}_\beta), \quad (1)$$

where V_c and V_α are the Coulomb and nuclear potentials, respectively. The Coulomb potential has the following form

$$V_c = \sum_\alpha \frac{n}{|x_\alpha|} \prod_{i\subset\alpha} \frac{1}{2}(1+\tau_z^i), \qquad n = \frac{me^2}{\hbar^2}, \tag{2}$$

where $e^2 = 1.44$ MeV fm and $\hbar^2/m = 41.47$ MeV fm^2. The sum runs over α =1,2,3 for the three possible pairs, and the product of the isospin projection operators runs over the indices i of the particles belonging to the pair α. As independent coordinates, we take the Jacobi vectors $\overline{x}_\alpha, \overline{y}_\alpha$. For the pair $\alpha = 1$, they are related to particle coordinates by the formulae

$$\overline{x}_1 = \overline{r}_2 - \overline{r}_3, \qquad \overline{y}_1 = \frac{\overline{r}_2 + \overline{r}_3}{2} - \overline{r}_1; \tag{3}$$

for α=2,3 one has to make cyclic permutations of the indices in Eq. (3). The Jacobi vectors with different α's are linearly related by the orthogonal transformation

$$\begin{pmatrix} \overline{x}_\alpha \\ \overline{y}_\alpha \end{pmatrix} = \begin{pmatrix} C_{\alpha\beta} & S_{\alpha\beta} \\ -S_{\alpha\beta} & C_{\alpha\beta} \end{pmatrix} \begin{pmatrix} \overline{x}_\beta \\ \overline{y}_\beta \end{pmatrix}, \quad C_{\alpha\beta}^2 + S_{\alpha\beta}^2 = 1 \tag{4}$$

where

$$C_{\alpha\beta} = -\sqrt{\frac{m_\alpha m_\beta}{(M-m_\alpha)(M-m_\beta)}}, \;\; S_{\alpha\beta} = (-)^{\beta-\alpha}\mathrm{sgn}(\beta-\alpha)\sqrt{1-C_{\alpha\beta}^2}, \;\; M = \sum_{\alpha=1}^{3} m_\alpha. \tag{5}$$

To derive the equations to be used in numerical computations, we perform the partial wave decomposition of Eq. (1) and separate the spin-isospin and angular variables (see, for instance, Refs. [3] and [4]). As a result, in the s-wave doublet case, we obtain a system of integro-differential equations which, in the polar coordinates $\rho^2 = x^2 + \frac{4}{3}y^2$ and $\tan\theta = \frac{2}{\sqrt{3}}y/x$ (here we omit the index 1) have the form

$$\begin{aligned}
&\left\{-\frac{\partial^2}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + V_c^t(\rho,\theta) + V^t(\rho,\theta) - \frac{1}{4\rho^2} - E\right\} U^t(\rho,\theta) = -\frac{1}{4}V^t(\rho,\theta) \\
&\times \int_{-1}^{+1} \mathrm{d}u \frac{\sin\theta\cos\theta}{\sin\theta'\cos\theta'} (U^t(\rho,\theta') - 3U^s(\rho,\theta')) \\
&\left\{-\frac{\partial^2}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + V_c^s(\rho,\theta) + V^s(\rho,\theta) - \frac{1}{4\rho^2} - E\right\} U^s(\rho,\theta) = -\frac{1}{4}V^s(\rho,\theta) \\
&\times \int_{-1}^{+1} \mathrm{d}u \frac{\sin\theta\cos\theta}{\sin\theta'\cos\theta'} (-3U^t(\rho,\theta') + U^s(\rho,\theta')),
\end{aligned} \tag{6}$$

where

$$\cos^2\theta'(u,\theta) = \frac{1}{4}\cos^2\theta - \frac{\sqrt{3}}{2}\,u\,\cos\theta\sin\theta + \frac{3}{4}\sin^2\theta, \tag{7}$$

and the first derivative in the radius is eliminated by means of the substitution $\Psi^{(t,s)} = \rho^{-1/2}U^{(t,s)}$. In Eq. (6), the s-wave Coulomb potential $V_c^{(t,s)}(\rho,\theta)$ is given by [4]

$$\begin{aligned} V_c^t(\rho,\theta) &= \frac{n\mu^t(\theta)}{\rho}, \qquad \mu^t(\theta) = \begin{cases} \dfrac{2}{\sqrt{3}\sin\theta} & \theta > 30^\circ\,, \\ \dfrac{2}{\cos\theta} & \theta \le 30^\circ\,, \end{cases} \\ V_c^s(\rho,\theta) &= \frac{n\mu^s(\theta)}{\rho}, \qquad \mu^s(\theta) = \frac{1}{3}\left(\frac{2}{\cos\theta} + \mu^t(\theta)\right). \end{aligned} \tag{8}$$

The Malfliet-Tjon MT I-III potential (see Refs. [5] and [6]) was chosen as the nuclear potential $V^{t,s}(\rho,\theta)$. The set of partial differential equations (6) must be solved for the functions satisfying the regularity conditions

$$U^{t,s}(0,\theta) = U^{t,s}(\rho,0) = U^{t,s}(\rho,\pi/2) = 0 \tag{9}$$

and the following asymptotic conditions [2]:

$$\begin{aligned} U^t\,|_{\rho\to\infty} &\sim \sqrt{\rho}\varphi_d(\rho,\cos(\theta))\{F_o(\gamma,p\frac{\sqrt{3}}{2}\rho\sin(\theta)) + a(p)[G_o(\gamma,p\frac{\sqrt{3}}{2}\rho\sin(\theta)) \\ &+\mathrm{i}F_o(\gamma,p\frac{\sqrt{3}}{2}\rho\sin(\theta))]\} + \mathcal{A}^t(\theta)\exp[\mathrm{i}\sqrt{E}\rho - \mathrm{i}\frac{n\mu^t(\theta)}{2\sqrt{E}}\ln(2\sqrt{E}\rho)]\,, \\ U^s\,|_{\rho\to\infty} &\sim \mathcal{A}^s(\theta)\exp[\mathrm{i}\sqrt{E}\rho - i\frac{n\mu^s(\theta)}{2\sqrt{E}}\ln(2\sqrt{E}\rho)]\,. \end{aligned} \tag{10}$$

Here, φ_d is the deuteron wave function, F_o and G_o are the regular and irregular Coulomb functions, $\gamma = 2n/(3p)$ is the Coulomb parameter with p the momentum in the center-of-mass system and $\mu^{t,s}(\theta)$ is defined in Eqs. (8). The unknown functions $a(p)$ and $\mathcal{A}^{t,s}$ are the elastic and breakup scattering amplitudes,

$$a(p) = \frac{\eta\exp(\mathrm{i}2\delta) - 1}{2\mathrm{i}}, \tag{11}$$

and η and δ are the inelasticity and phase shift, respectively. In the case of **nd** breakup scattering the asymptotic conditions retain the functional form of Eq. (10) but the Coulomb functions F_o and G_o should be replaced by sine and cosine, respectively.

2. *Numerov's method and spline approximation*

Our previous calculations of the elastic amplitudes for **nd** and **pd** breakup scattering, in which the reduction of the Faddeev equations to an algebraic problem was performed by means of finite-difference approximation for the hyperradius, have demonstrated a weak dependence of the results on the choice of the matching radius [7]. Nevertheless, to get accurate results for breakup amplitudes, one has to considerably increase the cutoff radius. So, to obtain accurate results at the same time, we have applied the Numerov method for solving partial differential equations. The idea of the Numerov method consists in using the initial differential equation to calculate higher derivatives in the expansion of the unknown function in Taylor's series (for instance, see Ref. [8]). According to Numerov, one has to keep all terms up to the sixth derivative in this expansion. Summing the equations for points $\rho - \Delta\rho$ and $\rho + \Delta\rho$, we get the following finite-difference approximation of the second radial derivative

$$\left.\frac{\partial^2 U(\rho,\theta)}{\partial\rho^2}\right|_{\rho_i} = \frac{U(\rho_{i+1},\theta) - 2U(\rho_i,\theta) + U(\rho_{i-1},\theta)}{\Delta\rho^2} - \frac{\Delta\rho^2}{12} U_\rho^{IV}(\rho_i,\theta) + O(\Delta\rho^4). \tag{12}$$

The fourth radial derivative of the Faddeev component has to be found by differentiating the second derivative in the corresponding Faddeev equation. From here on we carry out the analysis for the spin-quartet Faddeev equation. In the s-wave approach, this equation in polar coordinates has the form [4]

$$\begin{aligned}&\left\{-\frac{\partial^2}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + V_c(\rho,\theta) + V^t(\rho,\theta) - \frac{1}{4\rho^2} - E\right\} U(\rho,\theta)\\ &= \frac{2}{\sqrt{3}} V^t(\rho,\theta) \int\limits_{\theta^-}^{\theta^+} \mathrm{d}\theta' U(\rho,\theta'),\end{aligned} \tag{13}$$

where $\theta^- = |\theta - \pi/3|$, $\theta^+ = \pi/2 - |\theta - \pi/6|$. Thus we obtain the following formula for the fourth derivative of the Faddeev component

$$\begin{aligned}\frac{\partial^4 U(\rho,\theta)}{\partial\rho^4} &= -\frac{\partial^2}{\partial\rho^2}\left[\left\{\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} - V_c(\rho,\theta) - V^t(\rho,\theta) + \frac{1}{4\rho^2} + E\right\} U(\rho,\theta)\right.\\ &\left.+\frac{2}{\sqrt{3}} V^t(\rho,\theta) \int\limits_{\theta^-}^{\theta^+} \mathrm{d}\theta' U(\rho,\theta')\right].\end{aligned} \tag{14}$$

Substituting this expression into Eq. (12), we obtain the finite difference approximation for the second derivative up to the $4th$ order in $\Delta\rho$

$$\left.\frac{\partial^2 U(\rho,\theta_j)}{\partial\rho^2}\right|_{\rho_i} = \frac{U(\rho_{i+1},\theta_j) - 2U(\rho_i,\theta_j) + U(\rho_{i-1},\theta_j)}{\Delta\rho^2} + \frac{\Delta\rho^2}{12}\frac{\partial^2}{\partial\rho^2}\Bigg[\Bigg\{\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}$$

$$-V_c(\rho,\theta) - V^t(\rho,\theta) + \frac{1}{4\rho^2} + E\Bigg\}U(\rho,\theta) + \frac{2}{\sqrt{3}}V^t(\rho,\theta)\int\limits_{\theta^-}^{\theta^+}\mathrm{d}\theta' U(\rho,\theta')\Bigg]_{\rho_i\theta_j} + O(\Delta\rho^4). \tag{15}$$

Finally, replacing the second radial derivative in the Faddeev equations by the expression obtained, we have the analog of Numerov's method for the spin-quartet Faddeev equation as follows

$$-\Bigg[\frac{U(\rho_{i+1},\theta) - 2U(\rho_i,\theta) + U(\rho_{i-1},\theta)}{\Delta\rho^2} - \left(1 + \frac{\Delta\rho^2}{12}\frac{\partial^2}{\partial\rho^2}\right)\Bigg\{-\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + V_c(\rho,\theta)$$

$$+V^t(\rho,\theta) - \frac{1}{4\rho^2} - E\Bigg\}U(\rho,\theta)\Bigg]_{\rho_i,\theta_j} = \frac{2}{\sqrt{3}}\left(1 + \frac{\Delta\rho^2}{12}\frac{\partial^2}{\partial\rho^2}\right)\Bigg[V^t(\rho,\theta)\int\limits_{\theta^-}^{\theta^+}\mathrm{d}\theta' U(\rho,\theta')\Bigg]_{\rho_i,\theta_j}. \tag{16}$$

Generalization of the Numerov method for the set of differential equations (6) does not present serious difficulties. Thus, due to the complexity of the corresponding formulae, we do not show them here. To ensure an accuracy of order $(\Delta\theta)^4$ for the approximation in the angular variable, we used Hermitian splines of the fifth degree (see Ref. [9]). These splines are local, and each spline $S_{\sigma i}(x)$ is defined for x belonging to two adjacent subintervals $[x_{i-1}, x_i]$ and $[x_i, x_{i+1}]$. Their analytical form is fixed by the following smoothness conditions

$$S_{\sigma i}(x_{i-1}) = 0, \quad S_{\sigma i}(x_{i+1}) = 0, \quad \sigma = 0, 1, 2, \tag{17}$$

and

$$\begin{aligned} &S_{0i}(x_i) = 1, \quad &S'_{0i}(x_i) = 0, \quad &S''_{0i}(x_i) = 0, \\ &S_{1i}(x_i) = 0, \quad &S'_{1i}(x_i) = 1, \quad &S''_{1i}(x_i) = 0, \\ &S_{2i}(x_i) = 0, \quad &S'_{2i}(x_i) = 0, \quad &S''_{2i}(x_i) = 1. \end{aligned} \tag{18}$$

To reduce the resulting equation (16) to an algebraic problem, one must explicitly calculate the derivatives with respect to ρ in Eq. (16) using the following spline expansion

$$U(\rho,\theta) = \sum_{\sigma=0}^{2}\sum_{j=0}^{N_\theta+1} C_j^\sigma(\rho) S_{\sigma j}(\theta), \tag{19}$$

where $N_\theta + 1$ is the number of internal subintervals for the angular variable $\theta \in [0, \pi/2]$. Upon substituting the spline expansion (19) into the Faddeev equation, we use a collocation procedure with three Gaussian quadrature points per subinterval. As the number of internal breakpoints for angular variable θ is equal to N_θ, the basis of quintic splines consists of $3N_\theta+6$ functions. Three of them can be excluded using the last two regularity conditions from (9) and the continuity of the first derivative in θ of the Faddeev component at either $\theta = 0$ or $\theta = \pi/2$, and the collocation procedure yields $3N_\theta + 3$ equations.

3. Method of partial inversion

Using the spline approximation in the angular variable, and Numerov's method for the hyperradius, we obtain an algebraic problem for the unknown coefficients $C_j^\sigma(\rho_k)$. It is convenient to transform this problem back to the set of linear equations for the Faddeev components $U(\rho_i, \theta_k)$ by means of Eq. (19). Thus we reduce Eq. (16) to a matrix form

$$(D * U)_i = -\delta_{in} D^+ U_{n+1}, \quad n = N_\rho. \tag{20}$$

The matrices D and D^+ are of dimension $N_\rho N_c \times N_\rho N_c$ and $N_c \times N_c$, respectively. Here, N_ρ is the number of breakpoints in the hyperradius ρ and $N_c = 3N_\theta + 3$ is the number of collocation points in the angular variable θ.

The matrix D has the three-block-diagonal structure that optimizes considerably the inversion problem. Index $n+1$ stands for hyperradius $\rho_{n+1} = R_{\max}$, where $R_{\max}$ is the cutoff radius at which the asymptotic conditions Eq. (10) are implemented. By formal inversion of the matrix D in Eq. (19), we can write the solution of the problem

$$U_j = -D^{-1}_{jn} D^+ U_{n+1}, \qquad j = 1, 2....N_\rho. \tag{21}$$

The form of this equation results from keeping the incoming wave in the asymptotic conditions (10). As a consequence, the right hand part of Eq. (21) has a single nonzero term marked with index $n+1$. In Eq. (21), we consider the last two components of the vector U

$$\begin{aligned} U_{n-1} &= -D^{-1}_{n-1n} D^+ U_{n+1} \\ U_n &= -D^{-1}_{nn} D^+ U_{n+1}. \end{aligned} \tag{22}$$

Provided $R_{\max}$ is large enough, one may substitute for U_{n-1}, U_n on the left hand sides of Eqs. (22) the corresponding vectors obtained by evaluating Eqs. (10) at the radii $\rho = \rho_{n-1}$ and $\rho = \rho_n$. Thus we obtain a set of linear equations for the unknown amplitudes a and $\mathcal{A}$

$$\begin{aligned} a \cdot v_{n-1} + m_{n-1} \cdot \mathcal{A} &= \mathcal{F}_{n-1} \\ a \cdot v_n \quad + m_n \cdot \mathcal{A} &= \mathcal{F}_n \, . \end{aligned} \tag{23}$$

Here we do not show the explicit forms of vectors $v, \mathcal{F}$ and matrices m. As $R_{\max} \to \infty$, this set of equations has a constant a as a solution. At finite $R_{\max}$, the solution is a vector a with generally different components corresponding to different angles. We follow the method of Merkuriev et al. [3], which consists in selecting the components of a in the region of the maximum of the deuteron wave function, where a turns out to be independent of the angle.

Furthermore, we propose a new method for a more adequate calculation of the amplitudes. The set of linear equations (23) is overdetermined, since the number of equations is $2 \cdot N_c$ and the number of unknowns is N_c+1. Therefore, it is natural to use the least-squares method (LSM). One can apply it in either of two ways. In the first one, we express the breakup amplitude $\mathcal{A}$ from the lower equation (23) and substitute it into the upper one. As a result we have the following expression

$$a \cdot \mathbf{v} = \mathbf{F}\,, \tag{24}$$

where the vectors are definded as follows: $\mathbf{v} = v_{n-1} - m_{n-1}\, m_n^{-1}\, v_n$ and $\mathbf{F} = \mathcal{F}_{\mathbf{n-1}} - m_{n-1}\, m_n^{-1}\, \mathcal{F}_n$. According to LSM, one has to minimize the following functional

$$\| a \cdot \mathbf{v} - \mathbf{F} \|^{\mathbf{2}} = \min\,. \tag{25}$$

Differentiating this expression in $\mathrm{Re}\, a$ and $\mathrm{Im}\, a$, we obtain

$$a = \frac{(\mathbf{v}^*, \mathbf{F})}{(\mathbf{v}^*, \mathbf{v})}\,, \tag{26}$$

where (ξ^*, f) is an ordinary scalar product.

In the second way, one has to express the elastic amplitude a from the lower equation (23) using the scalar product

$$a = \frac{(v_n^*, \mathcal{F}_n - m_n \mathcal{A})}{(v_n^*, v_n)}. \tag{27}$$

Substituting a from Eq. (27) into the upper equation (23), we obtain the set of linear equations

$$m_{n-1} \mathcal{A} - v_{n-1} \frac{(v_n^*, m_n \mathcal{A})}{(v_n^*, v_n)} = \mathcal{F}_{n-1} - v_{n-1} \frac{(v_n^*, \mathcal{F}_n)}{(v_n^*, v_n)}. \tag{28}$$

The explicit form of Eq. (28) is as follows

$$\sum_{j=1}^{N_c} \left\{ m_{n-1,ij} - \frac{v_{n-1,i}}{(v_n^*, v_n)} \sum_{k=1}^{N_c} v_{n,k}^* m_{n,kj} \right\} \mathcal{A}_j = \mathcal{F}_{n-1,i} - v_{n-1,i} \frac{(v_n^*, \mathcal{F}_n)}{(v_n^*, v_n)}, \quad i = 1, .., N_c. \tag{29}$$

Solving the set Eq. (29), we get the breakup amplitude $\mathcal{A}$. Substituting the calculated breakup amplitude into Eq. (27), we may compute the elastic amplitude a. Note that to calculate it, one can apply Eq. (27) either in component form or via a scalar product. In the first case, the components of a are practically equal to a constant for all angles $\theta \in (0, \pi/2)$, and this constant coincides with the value of a calculated by using the scalar product to the fourth decimal. It should also be noted that the elastic amplitudes calculated by the method from Ref. [3] and LSM coincide with this constant to the same accuracy. For control, we used both methods of computing the amplitudes.

4. *Results of the calculations*

The elastic amplitude a and breakup amplitude $\mathcal{A}$ for **nd** and **pd** scattering were computed at $E_{\rm lab}$=14.1 and 42.0 MeV. The following values were used for the parameters of the calculation: $N_\rho \sim 10000$, $N_\theta \sim 600$ and values of the hyperradius $R_{\rm max}$ as large as 800 fm. In Table 1, the elastic phase shifts δ and inelasticities η are presented for various energies and spin cases. As one can see from Table 1, our results for **nd** breakup are in very good agreement with calculations of other groups. However, for **pd** breakup, they differ from those of the Pisa group [10]. To see the influence of the Coulomb interaction on **pd** as compared to **nd** breakup scattering, we calculated the **nd** and **pd** breakup amplitudes $\mathcal{A}(\theta)$ for the total spin $S = 3/2$ (spin-quartet case) and $\mathcal{A}^{t,s}(\theta)$ for the total spin $S = 1/2$ (spin-doublet case). Our results are shown in Figs. 1−3. It is important to note that in our s-wave approach we have not included higher partial waves of the Coulomb potential. Nevertheless the accuracy of this approach is about 1% for energies exceeding 1 MeV, as was already pointed out by Merkuriev et al. in Ref. [11]. From Fig. 1, it follows that the Coulomb interaction has a noticeable effect on the real and imaginary parts of the **pd** quartet breakup amplitude at $E_{\rm lab}$ = 14.1 MeV, especially for angles in

TABLE 1. n-d *and* p-d *elastic phase shifts and inelasticities.*

		LA/Iowa ref. [1]	Bochum ref. [1]	Pisa ref. [3]	present work	Pisa ref. [3]	present work
14.1 MeV		Doublet **nd**				Doublet **pd**	
	Re(δ)	105.48	105.50	105.48	105.47	108.44	108.06
	η	0.4648	0.4649	0.4649	0.4649	0.4984	0.4929
		Quartet **nd**				Quartet **pd**	
	Re(δ)	68.95	68.96	68.952	68.93	72.604	73.64
	η	0.9782	0.9782	0.9782	0.9782	0.9795	0.9202
42 MeV		Doublet **nd**				Doublet **pd**	
	Re(δ)	41.34	41.37	41.341	41.34	43.667	43.47
	η	0.5024	0.5022	0.5022	0.5022	0.5056	0.5071
		Quartet **nd**				Quartet **pd**	
	Re(δ)	37.71	37.71	37.722	37.70	39.947	39.19
	η	0.9035	0.9033	0.9033	0.9034	0.9046	0.866

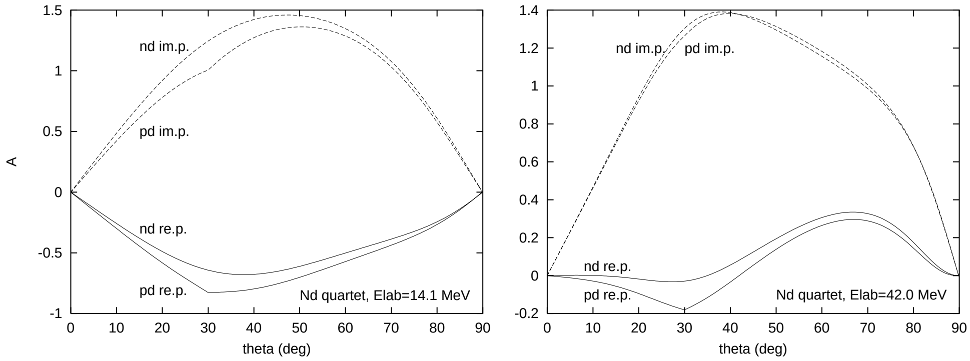

Fig. 1. Spin-quartet **nd** *and* **pd** *breakup amplitudes.*

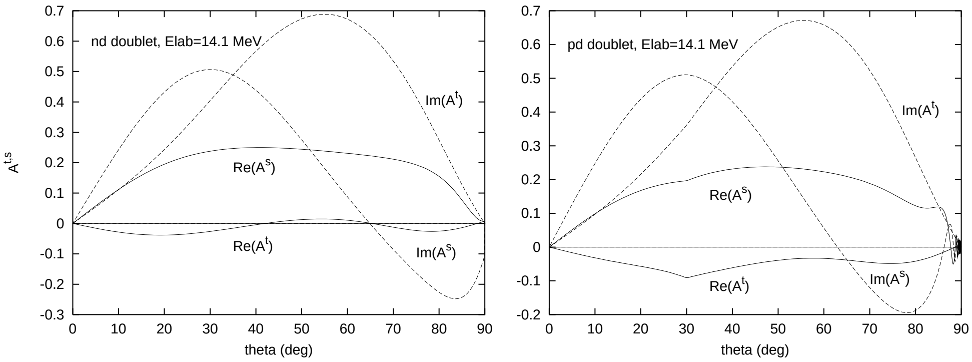

Fig. 2. Spin-doublet **nd** *and* **pd** *breakup amplitudes for* $E_{\rm lab} = 14.1$ MeV. A^s *is the singlet (pair spin* $s = 0$*) breakup amplitude and* A^t *is the triplet (pair spin* $s = 1$*) one.*

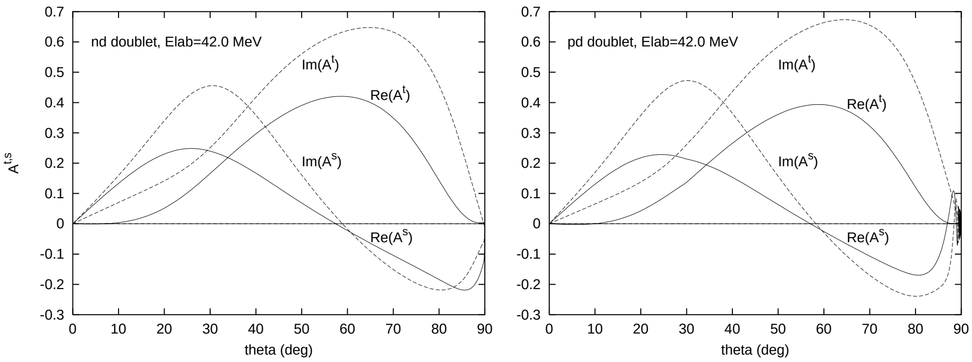

Fig. 3. Spin-doublet **nd** *and* **pd** *breakup amplitudes for* $E_{\rm lab} = 42.0$ MeV. *Notation is the same as in Fig. 2.*

the vicinity of $\pi/6$. At $E_{\rm lab}$ = 42.0 MeV, the effect of the Coulomb interaction is noticeable only in the real part of amplitude. In the spin doublet case, a substantial effect of the Coulomb interaction persists at the energy $E_{\rm lab}$ = 14.1 MeV. As one can see in Fig. 2, a large enough influence of the Coulomb force is noticeable for the real part of both the singlet and triplet breakup amplitudes. At $E_{\rm lab}$ = 42.0 MeV, a small influence of the Coulomb interaction is felt in the behavior of the singlet breakup amplitude for angles exceeding $\pi/3$ (see Fig. 3). The oscillations of the singlet breakup amplitudes for angles in a small vicinity of $\pi/2$ reflect the behavior of the breakup part in the singlet asymptotic condition in Eq. (10), as the essential singularity occurs at the angle θ=$\pi/2$.

In Tables 2 – 3, our results are presented for reduced quartet breakup amplitudes

TABLE 2. **nd** *and* **pd** *spin-quartet reduced breakup amplitude presented in the format x.xx ≡ x.xx* 10^{-1}, $E_{\rm lab}$ = 14.1 MeV.

θ (deg)	0	10	20	30	40	50	60	70	80
Present work, **nd** results, $R_{\rm max} \to \infty$									
Re(^{3}S$_1$)	-1.91	-1.93	-1.94	-1.89	-1.75	-1.58	-1.47	-1.51	-1.78
Im(^{3}S$_1$)	3.65	3.67	3.70	3.72	3.73	3.81	4.00	4.32	4.62
LA/Iowa, **nd** results, $R_{\rm max} \to \infty$, Ref. [6]									
Re(^{3}S$_1$)	-1.92	-1.93	-1.94	-1.89	-1.75	-1.58	-1.47	-1.51	-1.78
Im(^{3}S$_1$)	3.65	3.67	3.70	3.72	3.73	3.81	4.00	4.31	4.62
Present work, **pd** results, $R_{\rm max} \to \infty$									
Re(^{3}S$_1$)	-2.21	-2.24	-2.31	-2.44	-2.07	-1.82	-1.68	-1.71	-1.96
Im(^{3}S$_1$)	3.21	3.19	3.14	2.99	3.34	3.57	3.83	4.14	4.39

TABLE 3. **nd** *and* **pd** *spin-quartet reduced breakup amplitude presented in the format x.xx ≡ x.xx* 10^{-n}, $E_{\rm lab}$ = 42.0 MeV.

θ (deg)	0	10	20	30	40	50	60	70	80
Present work, **nd** results, $R_{\rm max} \to \infty$									
Re(^{3}S$_1$)	1.48-2	1.65-3	-3.11-2	-3.12-2	7.76-2	2.52-1	4.51-1	6.53-1	6.98-1
Im(^{3}S$_1$)	1.69-0	1.74-0	1.87-0	1.92-0	1.80-0	1.67-0	1.70-0	1.94-0	2.52-0
LA/Iowa, **nd** results, $R_{\rm max} \to \infty$, Ref. [6]									
Re(^{3}S$_1$)	1.48-2	9.22-4	-3.21-2	-3.09-2	7.70-2	2.52-1	4.51-1	6.53-1	6.93-1
Im(^{3}S$_1$)	1.69-0	1.74-0	1.87-0	1.92-0	1.80-0	1.68-0	1.70-0	1.95-0	2.52-0
Present work, **pd** results, $R_{\rm max} \to \infty$									
Re(^{3}S$_1$)	-8.22-2	-1.09-1	-1.83-1	-2.56-1	-3.83-2	1.81-1	3.90-1	5.78-1	5.85-1
Im(^{3}S$_1$)	1.67-0	1.72-0	1.83-0	1.86-0	1.79-0	1.70-0	1.74-0	1.99-0	2.54-0

defined in Ref. [6] as follows

$$\mathcal{A}_{\rm red}(\theta) = \frac{\mathcal{A}(\theta) K^2}{\sin(\theta)\cos(\theta)}, \quad K^2 = mE/\hbar^2. \tag{30}$$

TABLE 4. **nd** *and* **pd** *spin-doublet reduced breakup amplitude presented in the format x.xx-n* $\equiv$ *x.xx* 10^{-n}.

θ (deg)	0	10	20	30	40	50	60	70	80
$E_{\rm lab}$ = 14.1 MeV — Present work, **nd** results, $R_{\rm max} \to \infty$									
Re(^{1}S$_0$)	8.81-2	8.61-2	8.04-2	7.29-2	6.65-2	6.42-2	6.84-2	8.42-2	1.11-1
Im(^{1}S$_0$)	1.84-1	1.81-1	1.72-1	1.50-1	1.14-1	7.18-2	2.59-2	-3.49-2	-1.76-1
Re(^{3}S$_1$)	-2.44-2	-2.21-2	-1.59-2	-7.77-3	-3.46-4	4.75-3	5.21-3	-2.31-3	-1.82-2
Im(^{3}S$_1$)	8.00-2	8.44-2	9.79-2	1.20-1	1.48-1	1.76-1	2.00-1	2.14-1	2.09-1
LA/Iowa, **nd** results, $R_{\rm max} \to \infty$, Ref. [6]									
Re(^{1}S$_0$)	8.79-2	8.59-2	8.03-2	7.28-2	6.65-2	6.41-2	6.84-2	8.43-2	1.11-1
Im(^{1}S$_0$)	1.84-1	1.82-1	1.72-1	1.50-1	1.14-1	7.19-2	2.60-2	-3.49-2	-1.78-1
Re(^{3}S$_1$)	-2.43-2	-2.21-2	-1.60-2	-7.89-3	-4.11-4	4.68-3	5.10-3	-2.40-3	-1.82-2
Im(^{3}S$_1$)	8.01-2	8.45-2	9.80-2	1.20-1	1.48-1	1.76-1	1.99-1	2.14-1	2.09-1
Present work, **pd** results, $R_{\rm max} \to \infty$									
Re(^{1}S$_0$)	7.97-2	7.65-2	6.96-2	6.06-2	6.18-2	6.24-2	6.62-2	7.42-2	7.45-2
Im(^{1}S$_0$)	1.87-1	1.86-1	1.75-1	1.52-1	1.12-1	6.65-2	1.59-2	-5.10-2	-1.49-1
Re(^{3}S$_1$)	-2.57-2	-2.48-2	-2.34-2	-2.73-2	-1.53-2	-8.79-3	-7.87-3	-1.55-2	-3.11-2
Im(^{3}S$_1$)	6.95-2	7.37-2	8.66-2	1.07-1	1.40-1	1.70-1	1.95-1	2.10-1	2.02-1
$E_{\rm lab}$ = 42 MeV — Present work, **nd** results, $R_{\rm max} \to \infty$									
Re(^{1}S$_0$)	5.01-1	4.94-1	4.59-1	3.63-1	2.19-1	8.78-2	-3.49-2	-2.10-1	-7.04-1
Im(^{1}S$_0$)	5.56-1	5.90-1	6.70-1	6.67-1	4.63-1	2.08-1	-2.58-2	-2.99-1	-8.13-1
Re(^{3}S$_1$)	-1.30-2	1.41-2	1.01-1	2.41-1	3.85-1	5.08-1	6.20-1	7.00-1	5.69-1
Im(^{3}S$_1$)	2.64-1	2.66-1	2.85-1	3.69-1	5.39-1	7.23-1	9.34-1	1.25-0	1.70-0
LA/Iowa, **nd** results, $R_{\rm max} \to \infty$, Ref. [6]									
Re(^{1}S$_0$)	5.01-1	4.94-1	4.59-1	3.62-1	2.19-1	8.78-2	-3.50-2	-2.10-1	-7.05-1
Im(^{1}S$_0$)	5.56-1	5.91-1	6.70-1	6.66-1	4.63-1	2.09-1	-2.57-2	-2.99-1	-8.14-1
Re(^{3}S$_1$)	-1.30-2	1.33-2	1.00-1	2.42-1	3.85-1	5.07-1	6.20-1	7.00-1	5.69-1
Im(^{3}S$_1$)	2.63-1	2.66-1	2.85-1	3.70-1	5.39-1	7.23-1	9.34-1	1.25-0	1.70-0
Present work, **pd** results, $R_{\rm max} \to \infty$									
Re(^{1}S$_0$)	-2.61-2	6.74-4	8.23-2	1.97-1	3.44-1	4.70-1	5.82-1	6.48-1	4.87-1
Im(^{1}S$_0$)	2.43-1	2.47-1	2.77-1	3.84-1	5.63-1	7.55-1	9.73-1	1.29-0	1.74-0
Re(^{3}S$_1$)	4.95-1	4.84-1	4.35-1	3.24-1	2.02-1	8.06-2	-4.02-2	-2.17-1	-6.76-1
Im(^{3}S$_1$)	5.94-1	6.27-1	7.05-1	6.93-1	4.71-1	2.03-1	-4.41-2	-3.39-1	-8.90-1

As one can see from Tables 2 and 3, in the spin-quartet case agreement between our results and those of the Los-Alamos and Bochum groups Ref. [6] is excellent. It should be noted that one can not explicitly calculate the reduced breakup amplitude for the angle $\theta = \pi/2$ because one has to resolve an uncertainty in Eq. (30) for this angle. That is impossible numerically, as the breakup amplitude for the angle $\theta = \pi/2$ cannot be calculated using the Faddeev equations with sufficient accuracy in principle and one has to use another way to calculate it (for example, to exploit an integral representation from Ref. [3]). Unfortunately, we have no possibility to compare our results for the **pd** quartet amplitudes because of their absence in literature. In Table 4, the reduced doublet breakup amplitudes are presented.

The agreement between our results and those of Los-Alamos and Bochum groups [6] is again excellent. From this table, we again see a large enough influence of the Coulomb interaction for the **pd** doublet reduced amplitudes. In view of the large enough Coulomb effects in the case of spin-quartet breakup scattering, it is undoubtedly of interest to see it in more detail. In Fig. 4, the spin-quartet reduced breakup amplitudes are presented. Obviously, the Coulomb interaction effects are noticeable in the behavior of these amplitudes at all angles, especially for the lab energy $E_{\rm lab} = 14.1$ MeV. This effect should be important for the calculation of the total breakup amplitude, which should be quite different for **nd** and **pd** breakup scattering.

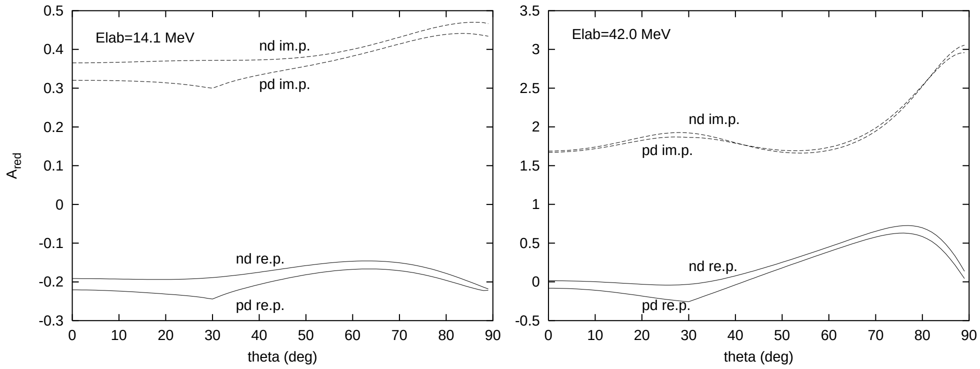

Fig. 4. Reduced quartet breakup amplitudes.

To verify our results, we applied the optical theorem [3]. In the spin-quartet case for the s-wave approach, it reads

$$\mathrm{Im}\,{}^4a_0(p) = |{}^4a_0(p)|^2 + \frac{K}{p}\int\limits_0^{\pi/2} \mathrm{d}\theta A^*_{\rm tot}\mathcal{A}, \qquad A_{\rm tot}(\theta) = \mathcal{A}(\theta) - \frac{2}{\sqrt{3}}\int\limits_{\theta^-}^{\theta^+} \mathrm{d}\theta' \mathcal{A}(\theta^{'}). \tag{31}$$

For the spin-doublet case in the s-wave approach (ref.[3]), the optical theorem reads

$$\begin{aligned}
\mathrm{Im}\,{}^2a_0(p) &= |{}^2a_0(p)|^2 + \frac{K}{p}\Bigg\{ \int\limits_0^{\pi/2} \mathrm{d}\theta [A^{*t}_{\rm tot}\mathcal{A}^t + A^{*s}_{\rm tot}\mathcal{A}^s] \Bigg\}, \\
A^t_{\rm tot}(\theta) &= \mathcal{A}^t(\theta) + \frac{1}{\sqrt{3}}\Bigg\{ \int\limits_{\theta^-}^{\theta^+} \mathrm{d}\theta' [\mathcal{A}^t(\theta') - 3\mathcal{A}^s(\theta')] \Bigg\}, \qquad (32) \\
A^s_{\rm tot}(\theta) &= \mathcal{A}^s(\theta) + \frac{1}{\sqrt{3}}\Bigg\{ \int\limits_{\theta^-}^{\theta^+} \mathrm{d}\theta' [\mathcal{A}^s(\theta') - 3\mathcal{A}^t(\theta')] \Bigg\}.
\end{aligned}$$

In Table 5, our optical theorem results are presented. Table 5 clearly confirms the accuracy of our results, and the estimation of Merkuriev et al. [11] of the contribution of higher Coulomb partial waves under an s-wave approach, which is less than 1%. In Figs. 5 and 6, we show the physical **nd** and **pd** breakup amplitudes. The physical **pd** quartet breakup amplitudes clearly demonstrate the influence of the Coulomb interaction, though they themselves have a small magnitude. For the doublet scattering, the Coulomb interaction has a smaller effect on the breakup amplitudes. The magnitude of these amplitudes is large as compared with the quartet ones. Therefore, the differential cross-sections of the **nd** and **pd** processes should

TABLE 5. The optical theorem results.

$E_{\rm lab}$	**nd**–quartet		**pd**–quartet		**nd**–doublet		**pd**–doublet	
MeV	l.h.s	r.h.s	l.h.s	r.h. s	l.h.s	r.h.s	l.h.s	r.h.s
14.1	0.8626	0.8626	0.8871	0.8776	0.6994	0.6994	0.6991	0.7061
42.0	0.3860	0.3860	0.4127	0.4086	0.4679	0.4679	0.4864	0.4881

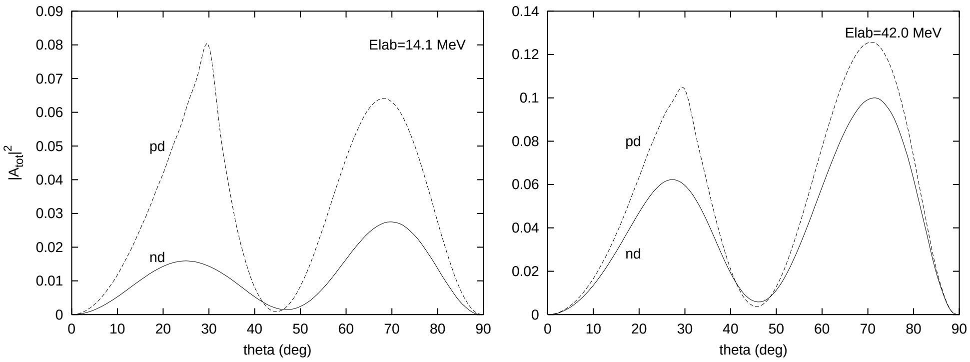

Fig. 5. Squares of the moduli of the physical quartet breakup amplitudes.

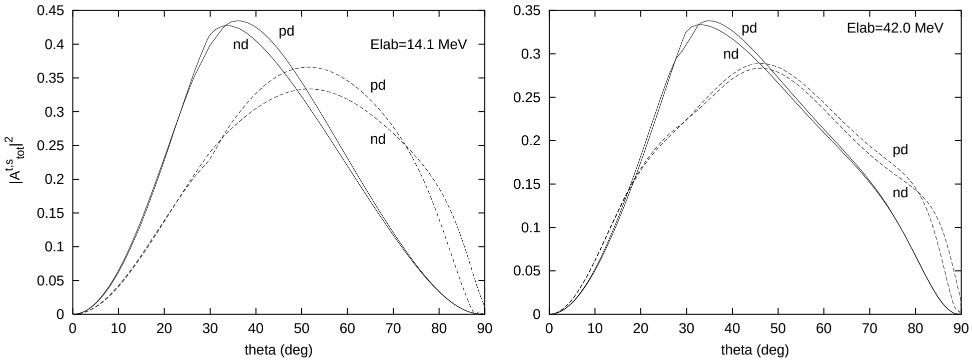

Fig. 6. Squares of the moduli of the physical doublet breakup amplitudes. The solid lines correspond to the spin-triplet amplitudes (pair spin $s = 1$). The dashed lines correspond to the spin-singlet amplitudes (pair spin $s = 0$).

have some difference. To directly study the dependence of the optical theorem results on the inelasticity in the spin-quartet case, we rewrite Eq. (31) in another form using the definition of the elastic amplitude Eq. (11)

$$1 = \eta^2 + 4\frac{K}{p}\int_0^{\pi/2} \mathrm{d}\theta A^*_{\mathrm{tot}}\mathcal{A}. \tag{33}$$

The results presented in Table 6 show that our **nd** amplitudes fulfill the optical theorem with a very high accuracy. In the case of **pd** breakup scattering, the accuracy is a little bit worse. It is a consequence of our truncation of the partial wave decomposition of the Coulomb potential. Analyzing Eq. (33) and the **nd** and **pd** quartet results for inelasticities of the Pisa group from Table 1, one inevitably comes to the conclusion about the equality of the contributions from integral terms of the optical theorem for the **nd** and **pd** breakup processes. On the other hand, our

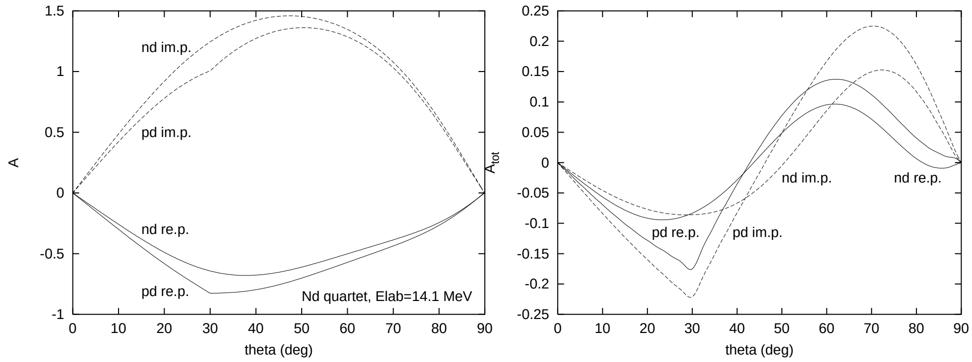

Fig. 7. The quartet breakup and total amplitudes for $E_{\mathrm{lab}} = 14.1$ MeV.

TABLE 6. The quartet optical theorem results.

$E_{\rm lab}$ (MeV)	**nd**	**pd**
14.1	0.9999	0.9621
42.0	0.9999	0.9835

quartet breakup amplitudes are quite different for these two reactions as one can see in Fig. 7, which is hardly compatible with the equality of these contributions.

This casts some doubts on the Pisa results for the **pd** quartet breakup scattering.

5. *Conclusion*

We have shown that by using the Numerov method very accurate calculations can be performed with minimal computation resources (PC).

By retaining the incident wave in the asymptotics for the Faddeev components, unnecessary additional computations are eliminated casting the problem in a form that allows partial inversion.

The stability of our solutions for relatively large values of $R_{\rm max}$ illustrates its advantages for the investigation of the asymptotic behavior of the solutions. In fact, the Numerov method enables us to compute the breakup amplitudes as well as the Faddeev components with a high accuracy for $R_{\rm max} = 800$ fm and more.

The disagreement of our results for the phase shifts and inelastisities with those of the Pisa group in the **pd** spin-quartet case reaches up to 6%, as one can see from Table 1. It can not be explained by truncation of the partial wave expansion of the Coulomb interaction in our calculation since the error introduced by neglecting higher partial Coulomb waves should not exceed 1% as it follows from the optical theorem results in Table 5.

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NOVA METODA RJEŠAVANJA PROBLEMA LOMNOG RASPRŠENJA Nd U KONFIGURACIJSKOM PROSTORU

Razvili smo novu računalnu metodu za rješavanje Faddeevih jednadžbi u konfiguracijskom prostoru za sustav tri nukleona. Ona se zasniva na razvoju "spline" po kutnoj varijabli i na poopćenoj Numerovoj metodi za hiperradijus. Računi neelastičnosti i faznih pomaka kao i amplituda loma za lomno raspršenje **nd** i **pd** na laboratorijskim energijama 14.1 i 42.0 MeV izvedeni su uz primjenu potencijala Malfliet -Tjon MT I-III. Za lomno raspršenje **nd**, ishodi računa su u dobrom skladu s ranijim rješenjima (J. L. Friar et al., Phys. Rev. C **42** (1990) 1838 i J. L. Friar et al., Phys. Rev. C **51** (1995) 2356). Za četvorno lomno raspršenje **pd**, razlike u neelastičnosti dosižu 6% u usporedbi s ishodima grupe u Pisi (A. Kievsky et al., Phys. Rev. C **64** (2001) 024002). Izračunate amplitude **pd** zadovoljavaju optički teorem s dobrom točnošću.