ISSN 1330–0016
CODEN FIZBE7

REALISTIC CALCULATIONS OF CORRELATIONS AND FINAL-STATE INTERACTION EFFECTS IN THE A(e,e′p)X PROCESS OFF COMPLEX NUCLEI

M. ALVIOLI[a], C. CIOFI degli ATTI[a] and H. MORITA[b]

[a]*Department of Physics, University of Perugia, and Istituto Nazionale di Fisica Nucleare, Sezione di Perugia, Via A. Pascoli, I–06100 Perugia, Italy*

[b]*Sapporo Gakuin University, Bunkyo-dai 11, Ebetsu 069, Hokkaido, Japan*

Received 20 September 2003; Accepted 26 April 2004
Online 22 November 2004

A linked cluster expansion for the calculation of ground-state observables of complex nuclei with realistic interactions has been used to calculate the ground-state energy, density and momentum distribution of ^{16}O and ^{40}Ca. Using the same cluster expansion and the wave function and correlation parameters obtained from the energy calculation, we have evaluated the semi-inclusive reaction A(e,e′p)X taking final-state interaction (FSI) into account by a Glauber-type approach; the comparison between the distorted and undistorted momentum distributions provides an estimate of the transparency of the nuclear medium to the propagation of the hit proton. The effect of color transparency is also included by considering the finite formation time (FFT) that the hit hadron needs to reach its asymptotic physical state.

PACS numbers: 25.20.-x, 21.10.Dr, 25.30.Fj, 24.10.-i, 21.60.-n UDC 539.143

Keywords: ^{16}O and ^{40}Ca, ground state, density and momentum distribution, final-state interaction, color transparency

1. *Introduction*

The exclusive, A(e,e′p)B, and semi-inclusive, A(e,e′p)X, electro-disintegrations of nuclei represent a powerful tool to investigate various aspects of nuclear structure (e.g., single-particle motion and mean field effects, nuclear correlations), as well as QCD motivated effects (e.g., color transparency). The accuracy of recent and forthcoming experimental data requires realistic theoretical calculations to be performed, based as much as possible on a description of the nucleus stemming from *first principle* calculations, which means that the nuclear wave functions which appear in the calculation of various matrix elements, used either to predict or interpret the experimental data, should in principle result from many-body calculations and realistic interactions. The problem has been solved in the case of few-body system, for which realistic wave functions are currently being used in the interpretation of electro-disintegration processes, but this problem still needs a solution in case of complex nuclei. As a matter of fact, calculations of the ground state observables for complex nuclei represent still a hard task, and even in those cases when approximate many-body calculations for the ground state energy can be performed, the structure of the wave function is so complicated that its use for calculations of matrix elements of operators, different from the potential and kinetic energies ones, is very involved. For such a reason, a simpler, but still realistic, method which would allow one to calculate various kinds of matrix elements with nuclear wave functions which correctly incorporate the most relevant features of a realistic wave function, in particular its correlation structure resulting from the main features of modern two-nucleon interactions, would be extremely useful. Cluster expansion techniques, when the expectation value of different operators can be calculated to a certain order, may represent a valid and practicable alternative to the full complex "exact" solution of the many body problem. It is the aim of the present paper to illustrate a cluster expansion approach to the calculation of ground state energy properties (energy, density and momentum distribution) and various types of electro-disintegration processes.

2. *Cluster expansion and the nuclear wave function*

In our linked-cluster expansion approach, the expectation value of a certain operator $\hat{\mathcal{O}}$

$$\langle \hat{\mathcal{O}} \rangle \,=\, \frac{\langle \Psi_{\rm A} |\, \hat{\mathcal{O}}\, | \Psi_{\rm A} \rangle}{\langle \Psi_{\rm A} | \Psi_{\rm A} \rangle} \tag{1}$$

is evaluated with correlated wave functions of the following "classical" form

$$\Psi_{\rm A} \,=\, \hat{F}(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)\, \Phi_{\rm A}(\boldsymbol{r}_1, ..., \boldsymbol{r}_A)\,, \tag{2}$$

where $\Phi_{\rm A}$ is a mean field (Slater determinant) wave function, and $\hat{F}$ a symmetrized (by the symmetrization operator $\hat{S}$) correlation operator which generates *correlations* into the mean field wave function; it has the following general form

$$\hat{F} \,=\, \hat{S} \prod_{i<j}^{A} \hat{f}(r_{ij}), \tag{3}$$

with

$$\hat{f}(r_{ij}) = \sum_p \hat{f}^{(p)}(r_{ij}) \qquad \text{and} \qquad \hat{f}^{(p)}(r_{ij}) = f^{(p)}(r_{ij})\, \hat{O}^{(p)}_{ij}\,, \tag{4}$$

where the operators $\hat{O}^{(p)}$ are the same which appear in the two-nucleon interaction, having the form (e.g. in case of a $V8$-type interaction)

$$\hat{O}_{ij}^{p=1,8} = [1,\, \sigma_i \cdot \sigma_j,\, S_{ij},\, (\boldsymbol{L}\cdot\boldsymbol{S})_{ij}] \otimes [1,\, \tau_i \cdot\ \tau_j]\,. \tag{5}$$

The central parts $f^{(p)}(r_{ij})$'s of the correlation function $\hat{f}^{(p)}$, reflect the radial behaviour of the various components, and their actual form is determined either by the minimization of the ground state energy, or by other criteria.

The cluster expansion of Eq. (1) is carried out in terms of the quantity $\hat{\eta}_{ij} = \hat{f}_{ij}^2 - 1$, whose integral plays the role of a small expansion parameter; we expand the numerator and the denominator in the terms of the same order n in η_{ij}, $\hat{\mathcal{O}}_n$, obtaining $\langle\hat{\mathcal{O}}\rangle = \mathcal{O}_0 + \mathcal{O}_1 + \mathcal{O}_2 + ...$, with

$$\begin{aligned}
\mathcal{O}_0 &= \langle\hat{\mathcal{O}}\rangle, \\
\mathcal{O}_1 &= \langle\sum_{ij}\hat{\eta}_{ij}\hat{\mathcal{O}}\rangle - \mathcal{O}_0\langle\sum_{ij}\hat{\eta}_{ij}\rangle, \\
\mathcal{O}_2 &= \langle\sum_{ij<kl}\hat{\eta}_{ij}\,\hat{\eta}_{kl}\,\hat{\mathcal{O}}\rangle - \langle\sum_{ij}\hat{\eta}_{ij}\,\hat{\mathcal{O}}\rangle\,\langle\sum_{ij}\hat{\eta}_{ij}\rangle\, + \\
&\qquad +\mathcal{O}_0\left(\langle\sum_{ij<kl}\hat{\eta}_{ij}\,\hat{\eta}_{kl}\,\rangle - \langle\sum_{ij}\hat{\eta}_{ij}\rangle^2\right),
\end{aligned} \tag{6}$$

where $\langle[...]\rangle \equiv \langle\Phi_{\mathrm{A}}\,|[...]|\,\Phi_{\mathrm{A}}\rangle$. From now on, our approach will consist in obtaining the parameters characterizing the correlation functions and the mean-field single-particle wave function which correspond to an acceptable value of the ground state energy, we will then use the obtained wave function Ψ_{A} to calculate the transition matrix elements entering in the theoretical description of electro-disintegration processes using the same cluster expansion employed to calculate the energy. We have calculated the ground state energy of ^{16}O and ^{40}Ca using the Argonne $V8'$ [1] potential and adopting, as in Ref. [2], the so called f_6 approximation consisting in considering only the first six components of Eq. (5). The expectation value of the many-body non-relativistic Hamiltonian of the nucleus was obtained by calculating the average values of the kinetic and potential energies, i.e.

$$\langle\hat{T}\rangle \;=\; -\frac{\hbar^2}{2m}\int \mathrm{d}\boldsymbol{k}\,k^2\,n(k), \tag{7}$$

where $n(k)$ is the nucleon momentum distribution ($k \equiv |\mathbf{k}|$),

$$n(k) \;=\; \frac{1}{(2\pi)^3}\int \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_1'\,\mathrm{e}^{-\mathrm{i}\,\boldsymbol{k}\cdot(\boldsymbol{r}_1-\boldsymbol{r}_1')}\,\rho^{(1)}(\boldsymbol{r}_1,\boldsymbol{r}_1') \tag{8}$$

and

$$\langle\hat{V}\rangle \;=\; \frac{1}{2}\sum_{i<j}\langle\hat{v}_{ij}\rangle = \frac{A(A-1)}{2}\sum_p\int \mathrm{d}\boldsymbol{r}_1\,\mathrm{d}\boldsymbol{r}_2\;v^{(p)}(r_{12})\rho_{(p)}^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2). \tag{9}$$

The calculations have been performed by cluster-expanding the expectation value of the non-diagonal one-body, $\hat{\rho}^{(1)}$, and diagonal two-body, $\hat{\rho}^{(2)}(\boldsymbol{r}_1, \boldsymbol{r}_2)$ density matrix operators. The six correlation functions $f^{(p)}(r_{ij})$ have been taken from Ref. [2], whereas harmonic oscillator (HO) and Saxon-Woods (SW) spwf's have been used to describe the mean field. As in Ref. [2], we found that the charge densities corresponding to the minimum of the energy, appreciably disagree with the corresponding experimental quantities, therefore, in view of the mild dependence of the energy around the minimum upon the mean-field parameters, following Ref. [2], we have changed the latter to obtain agreement between theoretical and experimental

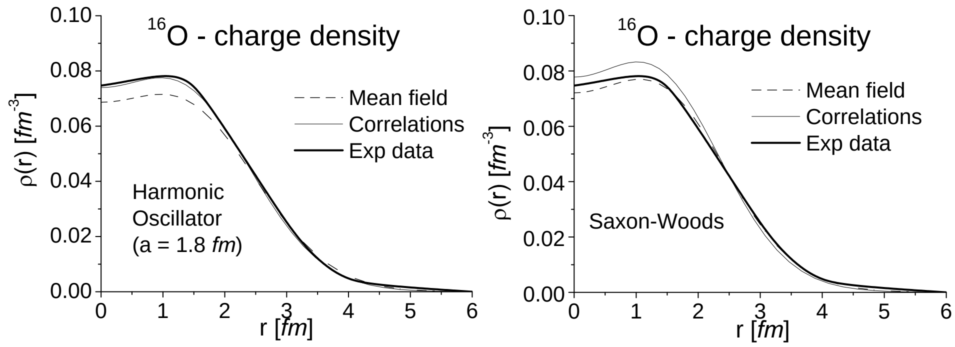

Fig. 1. The charge densities of ^{16}O *calculated using the cluster expansion (6) with harmonic oscillator (HO) (left panel) and Saxon-Woods (SW) (right panel) single-particle wave functions (spwf). Dashed line: mean-field wave functions used in the cluster expansion calculations; full line: results of the cluster expansion; thick full line: experimental data [4]. The HO and the SW spwf's parameters corresponding to the full line have been chosen such as to provide a good description of the density without appreciably changing the minimum value of the ground state energy. The normalization of the density is* $4\pi \int \rho(r)\, r^2\, \mathrm{d}r = Z$, Z *being the number of protons.*

charge densities. The results for the charge densities and momentum distributions, which are shown in Figs. 1 and 2, deserve the following comments:

1) the agreement between our cluster expansion and FHC/SOC result of Ref. [2] is very good;

2) both approaches predict momentum distributions which do not appreciably differ from the ones obtained in Ref. [3], where the variational Monte Carlo method and the AV18 interaction have been used;

3) the high momentum part of $n(k)$ is almost entirely exhausted by *non-central*, *long-range* correlations, with the *central*, *short-range*, Jastrow correlations under-predicting the high momentum part of $n(k)$ by about one order of magnitude;

4) the dominant non-central correlations are the isospin, $f_4 = f^{(4)}(r_{ij})\tau_i \cdot \tau_j$, and isospin-tensor, $f_6 = f^{(6)}(r_{ij})\tau_i \cdot \tau_j S_{ij}$, correlations.

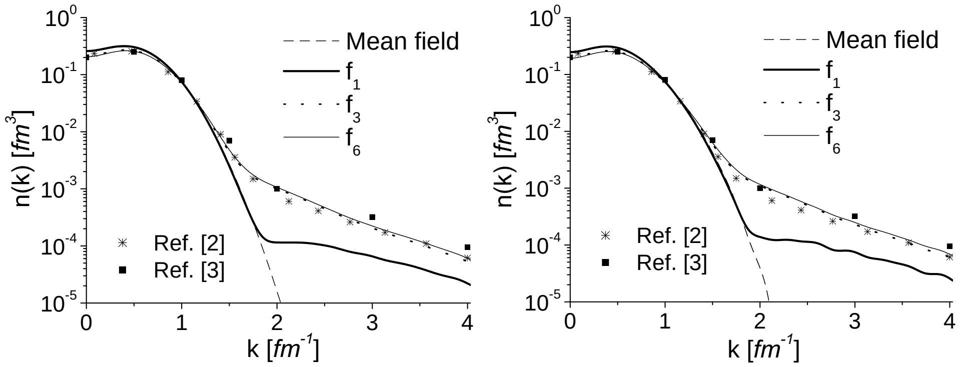

Fig. 2. The momentum distributions of ^{16}O *calculated with the cluster expansion (6) and harmonic oscillator (HO) (upper panel) and Saxon-Woods (SW) (lower panel) spwf's as in Fig. 1. Dashed line: mean field approximation; thick full line:* f_1 *approximation (only central correlations:* $(\hat{f} = \hat{f}^{(1)})$*; dotted line:* f_3 *approximation* $(\hat{f} = \hat{f}^{(1)} + \hat{f}^{(4)} + \hat{f}^{(6)})$*; full line:* f_6 *approximation* $(\hat{f} = \sum_{p=1}^{6} \hat{f}^{(p)})$*. The asterisks and the full squares are the results from Ref. [2] and [3], respectively. The values of the kinetic energy calculated by Eq. (7) are as follows:* $\langle T \rangle = 230.28$ MeV *(dashes), 287.23* MeV *(thick full), 435.94* MeV *(dots), 458.29* MeV *(full) for HO spwf's, and* $\langle T \rangle = 244.94$ MeV *(dashes), 306.98* MeV *(thick full), 469.83* MeV *(full), 494.47* MeV *(dots) for SW spwf's. The normalization of* $n(k)$ *is* $4\pi \int n(k)\, k^2 \, \mathrm{d}k = 1$.

3. *The final-state interaction in* A(e,e′p)X *reactions off complex nuclei: Glauber approach*

Using the results obtained in the previous section, we have calculated the semi-inclusive A(e,e′p)X process in which an electron with 4-momentum $k_1 \equiv \{\boldsymbol{k}_1, \mathrm{i}\epsilon_1\}$, is scattered off a nucleus with 4-momentum $P_\mathrm{A} \equiv \{\boldsymbol{0}, \mathrm{i}M_\mathrm{A}\}$ to a state $k_2 \equiv \{\boldsymbol{k}_2, \mathrm{i}\epsilon_2\}$ and is detected in coincidence with a proton p with 4-momentum $p \equiv \{\boldsymbol{p}, \mathrm{i}E_\mathrm{p}\}$; the final $(A-1)$ nuclear system with 4-momentum $P_\mathrm{X} \equiv \{\boldsymbol{P}_\mathrm{X}, \mathrm{i}E_\mathrm{X}\}$ is undetected. The cross section for the exclusive process A(e,e′p)B can be written as follows

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}\nu\,\mathrm{d}\boldsymbol{p}} = K\sigma_\mathrm{ep} P_\mathrm{D}(E_\mathrm{m}, \boldsymbol{p}_\mathrm{m}) \,, \tag{10}$$

where K is a kinematical factor, σ_ep the off-shell electron-nucleon cross section and $Q^2 = |\boldsymbol{q}|^2 - \nu^2$ the four momentum transfer. The quantity $P_\mathrm{D}(E_\mathrm{m}, \mathbf{p}_\mathrm{m})$ is the distorted nucleon spectral function which depends upon the observable *missing momentum* $\boldsymbol{p}_\mathrm{m} = \boldsymbol{q} - \boldsymbol{p}$ ($\boldsymbol{p}_\mathrm{m} = \boldsymbol{k}$ when the FSI is absent) and *missing energy* $E_\mathrm{m} = \nu - T_\mathrm{p} - T_{\mathrm{A}-1}$. In the semi-inclusive A(e,e′p)X process, the cross section (10) is integrated over the missing energy E_m, at fixed value of $\boldsymbol{p}_\mathrm{m}$ and becomes directly proportional to the *distorted* momentum distribution

$$n_\mathrm{D}(\boldsymbol{p}_\mathrm{m}) = (2\pi)^{-3} \int \mathrm{e}^{\mathrm{i}\boldsymbol{p}_\mathrm{m}\cdot(\boldsymbol{r}_1 - \boldsymbol{r}_1')} \rho_\mathrm{D}(\boldsymbol{r}_1, \boldsymbol{r}_1') \mathrm{d}\boldsymbol{r}_1\, \mathrm{d}\boldsymbol{r}_1' \,, \tag{11}$$

where

$$\rho_{\rm D}(\boldsymbol{r}_1, \boldsymbol{r}'_1) = \frac{\langle \Psi_{\rm A}\, S^\dagger\, \hat{O}(\boldsymbol{r}_1, \boldsymbol{r}'_1)\, S'\, \Psi'_{\rm A}\rangle}{\langle \Psi_{\rm A} \Psi_{\rm A}\rangle} \tag{12}$$

is the distorted one-body mixed density matrix, S the S-matrix describing FSI, and the primed quantities have to be evaluated at $\boldsymbol{r}'_i$ with $i = 1, ..., A$. The integral of $n_{\rm D}(\boldsymbol{p}_{\rm m})$ gives the nuclear transparency T

$$T = \frac{\int n_{\rm D}(\boldsymbol{p}_{\rm m}) {\rm d}\boldsymbol{p}_{\rm m}}{\int n(k) {\rm d}\boldsymbol{k}} = \int \rho_{\rm D}(\boldsymbol{r}) {\rm d}\boldsymbol{r} = 1 + \Delta T\,, \tag{13}$$

where $\rho_{\rm D}(\boldsymbol{r}) = \rho_{\rm D}(\boldsymbol{r}_1 = \boldsymbol{r}'_1 \equiv \boldsymbol{r})$, and ΔT originates from the FSI. In Ref. [5], Eq. (11) has been evaluated using a Glauber representation for the scattering matrix S, *viz*

$$S \to S_{\rm G}(\boldsymbol{r}_1 \ldots \boldsymbol{r}_A) = \prod_{j=2}^{A} G(\boldsymbol{r}_1, \boldsymbol{r}_j) \equiv \prod_{j=2}^{A} \left[1 - \theta(z_j - z_1)\Gamma(\boldsymbol{b}_1 - \boldsymbol{b}_j)\right], \tag{14}$$

where $\boldsymbol{b}_j$ and z_j are the transverse and the longitudinal components of the nucleon coordinate $\boldsymbol{r}_j \equiv (\boldsymbol{b}_j, z_j)$, $\Gamma(\boldsymbol{b})$ the Glauber profile function for elastic proton nucleon scattering, and the function $\theta(z_j - z_1)$ takes care of the fact that the struck proton "1" propagates along a straight-path trajectory so that it interacts with nucleon "j" only if $z_j > z_1$. The same cluster expansion described in Sect. 2 has been used taking Glauber rescattering exactly into account at the given order n, and using the approximation $|\Psi_{A-3}|^2 = \prod_3^A \rho(i)$. Using the mean-field and correlation

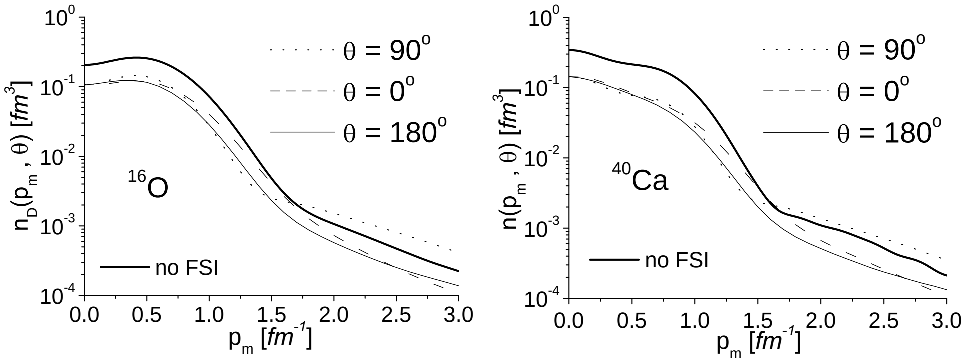

Fig. 3. The distorted momentum distribution, $n_{\rm D}(\boldsymbol{p}_{\rm m}) = n_{\rm D}(p_{\rm m}, \theta)$ ($\theta = \widehat{\boldsymbol{q}\boldsymbol{p}}_{\rm m}$), of ^{16}O and ^{40}Ca, obtained by Eq. (11) with correlated wave functions, harmonic oscillator spwf$'$s and the Glauber S matrix (14). The value of the integrated nuclear transparency (13) for ^{16}O is 0.5.

parameters obtained from the energy calculation, we have obtained the *distorted* nucleon momentum distributions $n_{\rm D}(\boldsymbol{p}_{\rm m}) = n_{\rm D}(p_{\rm m}, \theta)$, where θ is the angle between $\boldsymbol{q}$ and $\boldsymbol{p}_{\rm m}$; the results for ^{16}O and ^{40}Ca are presented in Fig. 3.

4. Finite formation time effects

Recently [6], the effects of color transparency in quasi-elastic lepton scattering off nuclei have been introduced by explicitly considering the finite formation time (FFT) that the hit hadron needs to evolve to its asymptotic physical state. It has been shown that at the values of the Bjorken scaling variable $x = Q^2/2m\nu \simeq 1$, FFT effects can be treated in a simple way, i.e. by replacing the Glauber operator (Eq. (14)) with

$$S_{\rm FFT}(\boldsymbol{r}_1, ..., \boldsymbol{r}_A) \quad = \quad \prod_{j=2}^{A} \Big(1 - J(z_1 - z_j)\Gamma(\boldsymbol{b}_1 - \boldsymbol{b}_j)\Big), \tag{15}$$

where

$$J(z) \quad = \quad \theta(z)\,{\rm e}^{-zxmM^2/Q^2}, \tag{16}$$

m being the nucleon mass and $M^2 = m^{*\,2} - m^2$ is a parameter describing the average excitation energy of the ejectile. It can be seen that at sufficiently high values of Q^2, $J \to 1$ and the FSI vanishes. The effects of FFT on the distorted momentum distribution are shown in Fig. 4.

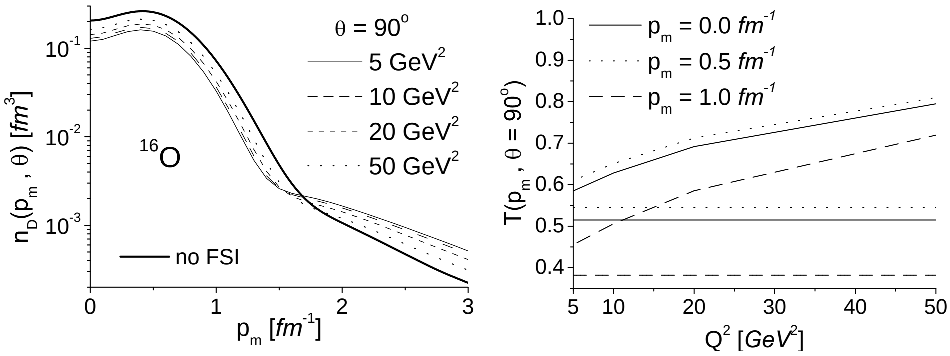

Fig. 4. Left panel: the Q^2- and $p_{\rm m}$-dependence of FFT effects on the distorted momentum distribution of ^{16}O calculated by Eqs. (11) and (14) at $\theta = 90°$. Right panel: the ratio of the distorted to the undistorted momentum distributions, i.e. the non-integrated nuclear transparency for various values of p_m and $\theta = 90°$. The horizontal lines represent the Q^2-independent Glauber results.

5. Summary and conclusions

We have obtained fully correlated wave functions by calculating the average value of the nuclear Hamiltonian by means of a linked cluster expansion and using realistic two-nucleon interactions. The wave functions have been used to obtain the ground state density and momentum distribution. By introducing FSI effect by a Glauber-type approach, the distorted momentum distributions appearing in the semi-inclusive A(e,e′p)X processes have been calculated. By such a procedure, a consistent treatment of initial-state correlations and final-state interactions has been achieved. Color transparency effects have also been investigated by the finite formation time approach. Comparison with available experimental data is in progress and will be reported elsewhere.

Acknowledgements

This work is part of the research activity of M. A. performed, under the supervision of C. d. A., for the fulfillment of the PhD title. H. M. thanks the Department of Physics, University of Perugia, and INFN, Sezione di Perugia, for hospitality and support. Partial support by the Italian Ministero dell'Istruzione, Universitá e Ricerca (MIUR), through the funds COFIN01, is acknowledged. We are indebted to A. Fabrocini for providing the correlation functions reported in Ref. [2].

References

[1] B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper and R. B. Wiringa, Phys. Rev. C **56** (1997) 1720.

[2] A. Fabrocini and G. Co', Phys. Rev. C **61** (2000) 044302 and private communication.

[3] S. C. Pieper, R. B. Wiringa and V. R. Pandharipande, Phys. Rev. C **46** (2000) 1741.

[4] H. de Vries, C. W. de Jager and C. de Vries, *Atom. Data Nucl. Data Tabl.* **36** (1987) 495.

[5] C. Ciofi degli Atti and D. Treleani, Phys. Rev. C **60** (1999) 024602.

[6] M. A. Braun, C. Ciofi degli Atti and D. Treleani, Phys. Rev. C **62** (2000) 034606.

[7] H. Morita, M. A. Braun, C. Ciofi degli Atti and D. Treleani, Nucl. Phys. A **699** (2002) 328c.

[8] M. Alvioli, C. Ciofi degli Atti and H. Morita, in preparation.

REALISTIČNI RAČUNI UČINAKA KORELACIJA I MEĐUDJELOVANJA U KONAČNOM STANJU U PROCESIMA A(e,e′p)X U KOMPLEKSNIM JEZGRAMA

Primijenili smo razvoj po vezanim nakupinama u kompleksnim jezgrama s realističnim međudjelovanjem za računanje osnovnog stanja, gustoće i impulsne raspodjele u ^{16}O i ^{40}Ca. S tim razvojem, te valnim funkcijama i korelacijskim parametrima dobivenim u računu energije, izveli smo poluinkluzivnu reakciju A(e,e′p)X, uzimajući u obzir međudjelovanje u konačnom stanju Glauberovim pristupom; usporedba distordiranih i nedistordiranih impulsnih raspodjela daje ocjenu prozirnosti nuklearne tvari za gibanje udarenog protona. Također smo uključili učinak prozirnosti boje, razmatrajući konačno vrijeme tvorbe koje udareni hadron treba da postigne svoje asimptotsko fizičko stanje.