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# HARD EXCLUSIVE REACTIONS AND THE TWO-GLUON COMPONENTS OF $\eta$ AND $\eta'$ MESONS

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The formalism for treating the leading-twist contributions of the two-gluon Fock components occurring in hard exclusive processes that involve $\eta$ and $\eta'$ mesons is reviewed. The calculation of the $\eta$, $\eta'$–photon transition form factor in next-to-leading order in $\alpha_{\rm s}$, as well as, the analysis of the $g^* g^* \eta'$ vertex and the electro- and photoproduction of $\eta$, $\eta'$ mesons are presented. Applications of this formalism to other relevant quantities such as glueballs are also discussed.

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## 1. Introduction

Within the framework for analyzing exclusive processes at large momentum transfer developed in the late seventies [1], the description of hard exclusive processes involving light mesons is based on the factorization of short- and long-distance dynamics and on the application of perturbative QCD. The former dynamics is represented by the process-dependent and perturbatively calculable parton-level subprocess amplitude, i.e. elementary hard-scattering amplitude, in which the meson is replaced by its Fock states, while the latter is described by the process-independent meson distribution amplitude (DA), which represents the probability of finding the corresponding Fock state in a meson and encodes the soft physics. Although the DA is essentially a nonperturbative quantity, its evolution is subject to a perturbative treatment. In the standard hard-scattering approach, the leading-twist contributions are obtained by regarding the meson as consisting only of valence Fock states, transverse parton momenta are neglected (collinear approximation) as well as the masses.

This work is focused on hard reactions involving $\eta$ and $\eta'$ mesons. In the formalism explained above, these particles are naturally described in terms of the $\mathrm{SU(3)_F}$ octet and singlet valence quark-antiquark Fock components and the two-gluon component, which also carries the flavour-singlet quantum numbers. A separate distribution amplitude corresponds to each of the three components. In comparison with the reactions involving the "pure" flavour-nonsinglet mesons ($\pi$, K, ...), the following novel features should be properly taken into account. First, owing to the $\mathrm{SU(3)_F}$ symmetry breaking and $\mathrm{U(1)_A}$ anomaly, the well-known flavour mixing is present in the $\eta$-$\eta'$ system (for a recent review, see Ref. [2]). Second, there are three valent components that contribute to $\eta$ and $\eta'$ in leading-twist and two of them are connected by evolution, i.e. the mixing of the singlet and gluon DAs under evolution should be properly taken into account. The latter feature has been investigated in a number of papers [3 – 6] However, although most of the results are in agreement [3, 5, 6] up to differences in the conventions used, a consistent set of conventions necessary for the calculation of both the elementary hard-scattering amplitude and the DA is not transparent from these works. Recently, the treatment of the two-gluon component and its mixing with the singlet one has been reexamined in Ref. [7]. A detailed analysis of the next-to-leading order (NLO) calculation of the $\eta$, $\eta'$-photon transition form factor was performed, making it possible to introduce and test the conventions for all ingredients of a leading-twist calculation for any process that involves $\eta$ or $\eta'$ mesons. The results were then applied to the $\eta$, $\eta'$–gluon transition form factor and the electroproduction of $\eta$ and $\eta'$ mesons. In this work, we briefly review the basic steps of that analysis, stress the important points occasionally still overlooked in the literature and extend the application of this formalism to the photoproduction of $\eta$ and $\eta'$ mesons and a possible description of glueballs.

## 2. *Formalism*

As the valence Fock components of the pseudoscalar mesons P$= \eta, \eta'$, we choose the $\mathrm{SU(3)_F}$ octet and singlet combinations of quark-antiquark states

$$|q\bar{q}_8\rangle = |(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}\rangle\,, \qquad |q\bar{q}_1\rangle = |(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}\rangle\,, \tag{1}$$

and the two-gluon state

$$|gg\rangle\,. \tag{2}$$

The corresponding DAs are denoted by $\Phi_{\mathrm{P}8,1,\mathrm{g}}$ and parameterized as

$$\begin{aligned} \Phi_{\mathrm{P}8}(x,\mu^2) &= \frac{f_{\mathrm{P}}^8}{2\sqrt{2N_{\mathrm{c}}}}\,\phi_8(x,\mu^2)\,, \\ \Phi_{\mathrm{P}1}(x,\mu^2) &= \frac{f_{\mathrm{P}}^1(\mu^2)}{2\sqrt{2N_{\mathrm{c}}}}\,\phi_1(x,\mu^2)\,, \qquad \Phi_{\mathrm{Pg}}(x,\mu^2) = \frac{f_{\mathrm{P}}^1(\mu_2)}{2\sqrt{2N_{\mathrm{c}}}}\,\phi_{\mathrm{g}}(x,\mu^2)\,, \end{aligned} \tag{3}$$

where the DAs $\phi_8$ and $\phi_1$ are normalized to unity

$$\int_0^1 \mathrm{d}x\,\phi_i(x,\mu^2) = 1\,. \tag{4}$$

However, since

$$\int_0^1 \mathrm{d}x\, \phi_\mathrm{g}(x,\mu^2) = 0\,, \tag{5}$$

there is no such a natural way to independently normalize the gluon DA[1]. Since the flavour-singlet quark and gluon DAs mix under evolution, it is convenient to pull out of the gluon DA the same factor as for the flavour-singlet quark one. In (3) the particle dependence and the mixing behaviour is solely embedded in the decay constants, while in a more general approach different distribution amplitudes $\phi_{\mathrm{P}8}$ and $\phi_{\mathrm{P}1}$ could be assumed for P$=\eta,\eta'$. The decay constants are parametrized in a two-angle octet-singlet mixing scheme [9, 8]

$$\begin{aligned} f_\eta^8 &= f_8\cos\theta_8\,, \qquad & f_\eta^1 &= -f_1\sin\theta_1\,,\\ f_{\eta'}^8 &= f_8\sin\theta_8\,, \qquad & f_{\eta'}^1 &= \phantom{-}f_1\cos\theta_1\,. \end{aligned} \tag{6}$$

The numerical values[8] $f_8 = 1.26 f_\pi$, $f_1 = 1.17 f_\pi$, $\theta_8 = -21.2^\circ$, and $\theta_1 = -9.2^\circ$ are used in this work, along with $f_\pi = 0.131$ GeV.

We note here that alternatively to the octet-singlet basis (1) and the two-angle octet-singlet mixing scheme (6), the phenomenologically better suited quark-flavour basis ($|q\overline{q}\rangle = |(u\overline{u}+d\overline{d})/\sqrt{2}\rangle$ and $|s\overline{s}\rangle$) and quark-flavour mixing scheme [8] were recently suggested. However, since the two-gluon state carries the flavour-singlet quantum numbers and mixes under evolution with the flavour-singlet component, octet-singlet basis turns out to be the natural one for the hard-scattering leading-twist analysis which includes the two-gluon components as well. When the two-gluon states are taken into account, the DA evolution introduces the appearance of "opposite" Fock components in the quark-flavour basis states[2], making the calculation unnecessarily difficult and untransparent. Furthermore, one has to remember that the one-angle quark-flavour mixing scheme has been derived under the assumption that the OZI violating effects, and among them the contributions of the two-gluon components, can be neglected [2]. Hence, for the calculations involving two-gluon states, one should use the octet-singlet basis. The mixing should then be implemented by the two-angle octet-flavour mixing scheme whose relation to the quark-flavour scheme is demonstrated in Ref. [8]. On the other hand, from the phenomenological success of the quark-flavour mixing scheme and the approximate validity of the OZI rule, one should expect that the effects of the two-gluon components are not excessively large in the $\eta$-$\eta'$ system.

The evolution of the octet DA $\phi_8$, being fully analogous to the pion case, is governed by the evolution equation of the form

$$\mu^2\frac{\partial}{\partial\mu^2}\phi_8(x,\mu^2) = V(x,u,\alpha_\mathrm{S}(\mu^2))\,\otimes\,\phi_8(u,\mu_\mathrm{F}^2)\,, \tag{7}$$

[1] As we shall explicitly see later on, the DAs satisfy the following symmetry properties in respect to the longitudinal momentum fractions $x$: $\phi_i(x,\mu^2)=\phi_i(1-x,\mu^2)$, $\phi_g(x,\mu^2) = -\phi_g(1-x,\mu^2)$

[2] For details, see Sec. III of Ref. [7].

while the singlet and gluon DAs mix under evolution and the evolution equation takes the matrix form

$$\mu^2 \frac{\partial}{\partial \mu^2} \begin{pmatrix} \phi_1(x,\mu^2) \\ \phi_{\mathrm{g}}(x,\mu^2) \end{pmatrix} = \begin{pmatrix} V_{\mathrm{qq}} & V_{\mathrm{qg}} \\ V_{\mathrm{gq}} & V_{\mathrm{gg}} \end{pmatrix} (x,u,\alpha_{\mathrm{S}}(\mu^2)) \otimes \begin{pmatrix} \phi_1(u,\mu^2) \\ \phi_{\mathrm{g}}(u,\mu^2) \end{pmatrix}. \tag{8}$$

Here $\otimes$ denotes the usual convolution symbol, kernels $V$ possess a well defined expansion in $\alpha_{\mathrm{S}}$ and in this work we are interested only in the leading-order (LO) evolution[3].

The solutions of the LO evolution equation (7) are given in terms of expansion in Gegenbauer polynomials $C_n^{3/2}$

$$\phi_8(x,\mu^2) = 6x(1-x)\left[1 + \sum_{n=2,4,\dots} B_n^8(\mu^2)\, C_n^{3/2}(2x-1)\right], \tag{9}$$

where the coefficients $B_n^8(\mu^2)$ evolve according to [1]

$$B_n^8(\mu^2) = B_n^8(\mu_0^2) \left(\frac{\alpha_{\mathrm{s}}(\mu_0^2)}{\alpha_{\mathrm{s}}(\mu^2)}\right)^{\gamma_n^{(0)}/\beta_0}. \tag{10}$$

$\gamma_n^{(0)}$ are LO anomalous dimensions, while $B_n^8(\mu_0^2)$ represent nonperturbative input at the scale $\mu_0^2$. The LO solutions of (8) take the more involved form

$$\begin{aligned} \phi_1(x,\mu^2) &= 6x(1-x)\left[1 + \sum_{n=2,4,\dots} B_n^1(\mu^2)\, C_n^{3/2}(2x-1)\right] \\ \phi_g(x,\mu^2) &= x^2(1-x)^2 \sum_{n=2,4,\dots} B_n^g(\mu^2)\, C_{n-1}^{5/2}(2x-1)\,, \end{aligned} \tag{11}$$

where

$$\begin{aligned} B_n^1\,(\mu_{\mathrm{F}}^2) &= B_n^{(+)}(\mu_0^2) \left(\frac{\alpha_{\mathrm{s}}(\mu_0^2)}{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}}^2)}\right)^{\gamma_n^{(+)}} /\beta_0 + \rho_n^{(-)}\, B_n^{(-)}\,(\mu_0^2) \left(\frac{\alpha_{\mathrm{s}}(\mu_0^2)}{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}}^2)}\right)^{\gamma_n^{(-)}/\beta_0}, \\ B_n^g\,(\mu_{\mathrm{F}}^2) &= \rho_n^{(+)}\, B_n^{(+)}\,(\mu_0^2) \left(\frac{\alpha_{\mathrm{s}}(\mu_0^2)}{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}}^2)}\right)^{\gamma_n^{(+)}/\beta_0} + B_n^{(-)}\,(\mu_0^2) \left(\frac{\alpha_{\mathrm{s}}(\mu_0^2)}{\alpha_{\mathrm{s}}(\mu_{\mathrm{F}}^2)}\right)^{\gamma_n^{(-)}/\beta_0}. \end{aligned} \tag{12}$$

Here the coefficients $B_n^{\pm}(\mu_0^2)$, i.e., $B_n^{\mathrm{q,(g)}}(\mu_0^2)$, represent nonperturbative input at scale $\mu_0^2$, while $\gamma_n^{(\pm)}$, $\rho_n^{(+)}$, $\rho_n^{(-)}$, are defined in terms of LO anomalous dimensions

[3] The evolution of the singlet decay constant $f_{\mathrm{P}}^1$ is also to be neglected in that case.

(see, for example, Ref. [7]): $\gamma_n^{\rm qq} = \gamma_n^{(0)}$, $\gamma_n^{\rm gg}$, and

$$\gamma_n^{\rm qg} = C_F \, \frac{n(n+3)}{3(n+1)(n+2)} \, , \qquad \gamma_n^{\rm gq} = n_f \, \frac{12}{(n+1)(n+2)} \, . \tag{13}$$

Finally, note that, in the limit $\mu^2 \to \infty$, the octet and singlet DAs evolve into the asymptotic form $\phi(x) = 6x(1-x)$ and the gluon one to zero.

When calculating the elementary hard-scattering amplitude, the projection of a collinear $q\bar{q}_i$ state onto a pseudoscalar meson state is achieved by replacing the quark and antiquark spinors by

$$\mathcal{P}^{i,q}_{\alpha\beta,rs,kl} = \mathcal{C}_{i,rs} \frac{\delta_{kl}}{\sqrt{N_c}} \left( \frac{\gamma_5 \not{p}}{\sqrt{2}} \right)_{\alpha\beta} \, , \tag{14}$$

where $\alpha$ ($r$, $k$) and $\beta$ ($s$, $l$) represent Dirac (flavour, colour) labels of the quark and antiquark, respectively, and $p$ denotes the meson momentum ($p^2 = 0$). The flavour content is taken into account by the matrices $\mathcal{C}_8 = \lambda_8/\sqrt{2}$ and $\mathcal{C}_1 = \mathbf{1_f}/\sqrt{\mathbf{n_f}}$, where $n_f = 3$ denotes the number of flavours contained in $q\bar{q}_1$.

The projection of a collinear $gg$ state onto a pseudoscalar meson state is achieved by replacing the gluon polarization vectors $\epsilon^\mu(xp, \lambda)$ and $\epsilon^\nu((1-x)p, -\lambda)$ by

$$\mathcal{P}^{\rm g}_{\mu\nu,ab} = \frac{\rm i}{2} \sqrt{\frac{C_F}{n_f}} \, \frac{\delta_{ab}}{\sqrt{N_c^2 - 1}} \, \epsilon^{\mu\nu\alpha\beta} \, \frac{n_\alpha p_\beta}{n \cdot p} \, \frac{1}{x(1-x)} \, , \tag{15}$$

where $a$, $b$ represent colour indices, and any vector having the space components opposite to $p$ can be taken as $n$ here. The projector (15) corresponds to the definition of $\phi_{\rm g}$, i.e. the anomalous dimensions $\gamma_n^{\rm qg}$ and $\gamma_n^{\rm gq}$, given by (13).

Owing to (5), there exists a freedom in defining the gluon DA. Suppose we change $\phi_{\rm g}$ by a factor $\sigma$

$$\phi^\sigma_{\rm Pg} = \sigma \, \phi_{\rm Pg} \, . \tag{16}$$

Inspection of Eq. (8), (or equivalently of Eqs. (11-12) ) reveals the following. Since the singlet and gluon DA are connected by evolution and in order to leave the quark DA $\phi_1$ unchanged, the change of the definition of the gluon DA (16) has to be converted into a change of $V_{\rm qg}$ and $V_{\rm gq}$, or equivalently into a change of the off-diagonal anomalous dimensions $\gamma_{\rm qg}$ and $\gamma_{\rm gq}$ and the $B^{(-)}_{{\rm P}n}$. Hence, (16) is equivalent to

$$\gamma_n^{{\rm qg},\sigma} = \frac{1}{\sigma} \, \gamma_n^{\rm qg} \, , \qquad \gamma_n^{{\rm gq},\sigma} = \sigma \, \gamma_n^{\rm gq} \, , \tag{17}$$

and $B^{(-)\,\sigma}_{{\rm P}n}(\mu_0^2) = \sigma B^{(-)}_{{\rm P}n}(\mu_0^2)$, which then implies $B^{{\rm g}\,\sigma}_{{\rm P}n}(\mu^2) = \sigma \, B^{\rm g}_{{\rm P}n}(\mu^2)$ and $B^{{\rm q}\,\sigma}_{{\rm P}n}(\mu^2) = B^{\rm q}_{{\rm P}n}(\mu^2)$. On the other hand, since any physical quantity must be independent of the choice of the convention, any change of the definition of the gluon DA is naturally accompanied by a corresponding change in the elementary hard-scattering amplitude. Namely, the projection (15) of the $gg$ state onto a pseudoscalar meson state is to be modified by a factor $1/\sigma$, i.e.

$$\mathcal{P}^{{\rm g}\,\sigma}_{\mu\nu} = \frac{1}{\sigma} \, \mathcal{P}^{\rm g}_{\mu\nu} \, , \tag{18}$$

and the elementary hard-scattering amplitude becomes altered accordingly. In the literature [3, 5, 6] one encounters various conventions for $\gamma_n^{\rm qg}$ and $\gamma_n^{\rm gq}$, but the corresponding definition of the gluon projector $\mathcal{P}^{\rm g}$ was often omitted, and it is crucial that these two ingredients of the leading-twist calculation are consistently defined. In Ref. [7] a consistent set of conventions (13) and (15) was fixed and tested on the NLO calculation of the $\eta$, $\eta'$-photon transition form factor. The relations (17) and (18) then enable us to make a connection with other conventions (note that the input coefficients $B_n^{(-)}(\mu_0^2)$, i.e. $B_n^{\rm g}(\mu_0^2)$, are also convention dependent).

## 3. *Applications*

First, we turn to the NLO calculation of the $\eta$, $\eta'$–photon transition form factor, i.e. to the evaluation of the $\gamma^*\gamma \to \eta(\eta')$ hard-scattering amplitude. The form factor can be expressed as a sum

$$F_{{\rm P}\gamma} = F_{{\rm P}\gamma}^{8}(Q^2) \, + \, F_{{\rm P}\gamma}^{1g}(Q^2)\,, \tag{19}$$

where $Q^2$ represents the photon virtuality. The flavour-octet contribution $F_{{\rm P}\gamma}^{8}(Q^2)$ can be obtained from the pion–photon transition form factor (see Ref. [10] and references therein); one only has to take into account the proper flavour factor. The contributions of the flavour-singlet and two-gluon components contained in

$$F_{{\rm P}\gamma}^{1{\rm g}}(Q^2) = \big(T_{{\rm H},1}(x,Q^2,\mu_{\rm F}^2)\ T_{{\rm H},{\rm g}}(x,Q^2,\mu_{\rm F}^2)\big) \otimes \begin{pmatrix} \Phi_{{\rm P}1}(x,\mu_{\rm F}^2) \\ \Phi_{{\rm P}{\rm g}}(x,\mu_{\rm F}^2) \end{pmatrix}, \tag{20}$$

were calculated in Ref. [7]. Following the recent analysis of the pion–photon transition form factor [10], a detailed NLO analysis was performed taking into account both the hard-scattering part and the perturbatively calculable DA part. The cancellation of the collinear singularities present in the parton-subprocess amplitude with the ultraviolet (UV) singularities appearing in the unrenormalized DAs[4] offered the most crucial test of the consistency of our set of conventions for singlet and gluon DAs and projectors. Using the mixing scheme defined in Eq. (6), the NLO leading-twist prediction for the $\eta$ and $\eta'$ transition form factors was obtained. Owing to the quality and quantity of experimental data [11], the Gegenbauer series (10) and (11) were truncated at $n = 2$, and the results were then fitted to the data. For $Q^2 \geq 2$ GeV$^2$ and $\mu_0 = 1$ GeV, the results of the fits read

$$B_2^8(\mu_0^2) = -0.04 \pm 0.04, \quad B_2^1(\mu_0^2) = -0.08 \pm 0.04, \quad B_2^g(\mu_0^2) = 9 \pm 12\,. \tag{21}$$

The existing experimental data and their quality allow us to obtain not more than a constraint on the value of $B_2^g$. As expected, we have observed a strong correlation

[4]Note that the renormalization introduces mixing of the composite operators $\bar\Psi(-z)\,\gamma^+\gamma_5\,\Omega\,\Psi(z)$ and $G^{+\alpha}(-z)\,\Omega\,\widetilde{G}_\alpha^{\ +}(z)$ in terms of which the quark singlet and gluon DAs are defined, respectively.

between $B_2^1$ and $B_2^g$. The quality of the fit as well as the sensitivity of the results on the size of two-gluon components[5] can be seen from Fig. 1.

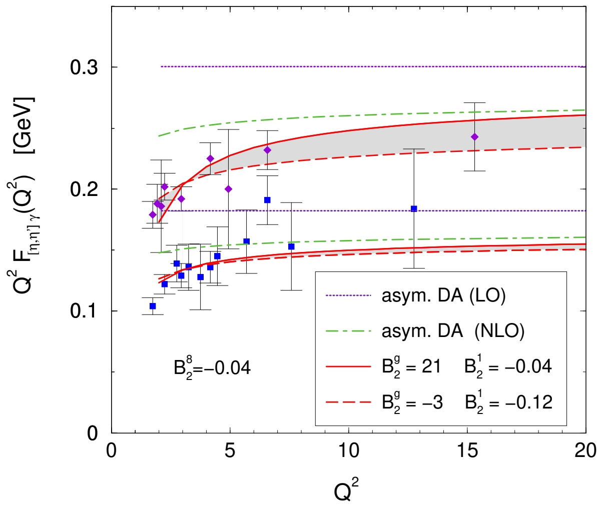


*Fig. 1. $\eta$ (below) and $\eta'$ (above) transition form factors. The shaded area corresponds to the range of $B_2^1(\mu_0^2)$ and $B_2^g(\mu_0^2)$ given in Eq. (21).*

As a next application, the $\eta'$–gluon transition form factor, i.e. $g^*g^*\eta'$ vertex, turns to be a natural choice. In contrast to the $\eta$, $\eta'$–photon transition form factor, the two-gluon components contribute to $g^*g^* \to \eta(\eta')$ already at LO in $\alpha_s$ and the contribution of the $q\bar{q}_8$ component vanishes. A consequence of the latter is that the $\eta$–gluon transition form factor is much smaller than the $\eta'$ one. A reliable determination of the $g^*g^*\eta'$ vertex is of importance for the calculation of a number of decay processes such as B→ $\eta'$X [12], B→ $\eta'$K, or of the hadronic production process pp→ $\eta'$X. To leading-twist order the $g^*g^*\eta'$ vertex has been first calculated in Refs. [13]. In Ref. [7], it was reanalyzed using our set of conventions, the previous calculations were examined and corrected, and the numerical predictions using the Gegenbauer coefficients (21) were provided. As expected, it was shown that the $g^*g^*\eta'$ vertex is quite sensitive to the two-gluon components.

In Ref. [7], our formalism was applied also to the deeply-virtual and wide-angle electroproduction of $\eta$ and $\eta'$ mesons with longitudinal photons. It was found that in the former, the two-gluon contributions were suppressed, while in the latter, they could be important depending on the value of the $B_2^g$ coefficient. Here we have extended this analysis to the photoproduction of $\eta$ and $\eta'$ mesons calculated in the handbag approach in which the process $\gamma$p $\to \eta(\eta')$p factorizes in the subprocess amplitude $\gamma q \to \eta(\eta')q$ and soft proton matrix elements. The meson is again generated by the leading-twist mechanism. As in the case of the wide-angle elec-

[5] Since $B_2^1$ and $B_2^g$ are correlated, the shaded area in Fig. 1 corresponds to the change of both of these coefficients. Nevertheless, the variation of $B_2^g$ is numerically dominant.

troproduction, the two-gluon contributions could be substantial and we illustrate that by displaying the ratio of the $gg$ and $q\bar{q}_1$ contributions (see Fig. 2).

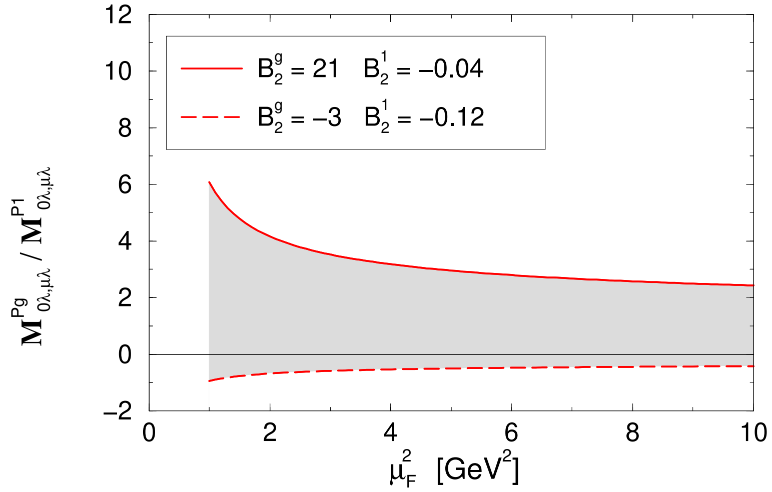


*Fig. 2. Ratio of the gluon and singlet quark amplitudes for the photoproduction of $\eta'(\eta)$ mesons as a function of $\mu_F^2$. The shaded area corresponds to the range of $B_2^1(\mu_0^2)$ and $B_2^g(\mu_0^2)$ given in Eq. (21).*

Finally, we mention some further applications that can be found in recent literature: in Ref. [14], the previously explained formalism was applied to the B→ $\eta'$K$^{(*)}$ process, the process $\Upsilon(1S) \to \eta'$X was analyzed in [15] obtaining further restrictions on the $B_2^g$ coefficient, while some modifications of the leading-twist formalism were introduced in Ref. [16].

## 4. *Description of glueballs*

Last but not least, we would like to comment on the possible application of this formalism to the description of glueballs.

The pioneering work was done in Ref. [17], were the pseudoscalar glueballs were described in the standard hard-scattering approach, while the gluon DA was obtained from the QCD sum rules. The results were then applied to the $\gamma\gamma \to \mathrm{G}\pi$ (G=glueball) process, but the mixing with the $q\bar{q}_1$ state and the evolution were neglected. However, in a consistent approach, the mixing of $gg$ and $q\bar{q}_1$ components under evolution should be taken into account.

Let us examine from the purely theoretical point of view the possible description of the glueball states in leading-twist formalism. In the pseudoscalar case, one should describe the glueball using the $\Phi_{\mathrm{Pg}}$ and $\Phi_{\mathrm{P1}}$ DAs of the form given by Eqs. (3) and (11), where P now denotes the pseudoscalar glueball. The decay constant $f_{\mathrm{P1}}$ as well as the $B_n^{\mathrm{g}}$ and $B_n^1$ coefficients are unknown. For simplicity reasons, let us again take only $n = 2$ and then compare $\phi_1$ and $\phi_g$ from (11). One can easily see that in order to have the dominantly glueball state, $B_2^g$ should be much larger than 1, i.e., than the "leading" term in the expansion of $\phi_1$. This is a condition which may not be trivially satisfied, especially since $B_2^g$ decreases with $\mu^2$ and, in the limit $\mu^2 \to \infty$, the pseudoscalar gluon DA vanishes, leaving us only with the $q\bar{q}_1$ contribution.

The situation looks more favourable in the scalar case where we describe glueballs in terms of $\Phi_{Sg}$ and $\Phi_{S1}$, $S$ being the scalar glueball state. The equivalent of Eq. (11) is given for the scalar case by

$$\begin{aligned} \phi_{S1}(x,\mu^2) &= x(1-x) + \sum_{n=2,4,\ldots} B_{Sn}^1(\mu^2)\, C_{n-1}^{3/2}(2x-1)\,, \qquad (22) \\ \phi_{Sg}(x,\mu^2) &= 30x^2(1-x)^2 \left[1 + \sum_{n=2,4,\ldots} B_{Sn}^g(\mu^2)\, C_n^{5/2}(2x-1)\right]. \end{aligned}$$

One can see that, in a sense, the role of the gluon and quark singlet DAs is here reversed. The gluon DA is now symmetric and well normalized (compare $\int_0^1 \mathrm{d}x\phi_{Sg} = 1$ and $\int_0^1 \mathrm{d}x\phi_{S1} = 0$ with Eqs. (4) and (5)), and in the limit of $\mu^2 \to \infty$, the quark singlet DA vanishes, while the gluon one takes the asymptotic form $\phi_g(x) = 30x^2(1-x)^2$. In order to have the dominantly glueball state, it is now sufficient that $B_{S2}^1$ is sufficiently smaller than 1 and this may be expected, especially since $B_{S2}^1$ decreases with $\mu^2$.

The analysis of both the pseudoscalar and scalar glueballs along these lines is underway.

## 5. *Conclusions*

In this work we have reviewed the leading-twist hard-scattering formalism for the description of $\eta$ and $\eta'$ mesons with two-gluon components included. The theoretical and numerical results of Ref. [7] have been summarized and applied further to the photoproduction of $\eta$ and $\eta'$ mesons as well as to the possible description of pseudoscalar and scalar glueballs. The processes such as $g^*g^* \to \eta'$, wide-angle electroproduction and photoproduction of $\eta$ and $\eta'$ mesons show sensitivity to two-gluon contributions. Future data should allow to pin down the gluon DA, while the description of glueballs offers a new interesting area of application of this formalism.

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## TVRDE EKSKLUZIVNE REAKCIJE I DVO-GLUONSKE KOMPONENTE $\eta$ I $\eta'$ MEZONA


Opisuje se formalizam za izračunavanje doprinosa vodećeg "twista" dvo-gluonskih Fock-ovih komponenti, koje se pojavljuju u tvrdim ekskluzivnim procesima, i koji uključuju $\eta$ i $\eta'$ mezone. Izlaže se račun $\eta$ i $\eta'$–$\gamma$ funkcije strukture prijelaza u redu do vodećeg u razvoju po $\alpha_s$, kao i analiza $g^*g^*\eta'$ verteksa, te elektro- i fotoprodukcije $\eta$ i $\eta'$ mezona. Također se raspravlja primjena ovog formalizma na druge relevantne veličine poput gluonskih lopti.