ISSN 1330–0016
CODEN FIZBE7

CLUSTER CALCULATION FOR ${}^{9}_{\Lambda}$Be HYPERNUCLEUS IN FADDEEV APPROACH

VLADIMIR SUSLOV[a,b], IGOR FILIKHIN[a,b] and BRANISLAV VLAHOVIĆ[b,1]

[a]*Department of Mathematical and Computational Physics, Sankt-Petersburg State University, 198504, Petrodvorets, Russia*

[b]*Department of Physics, North Carolina Central University, Durham, NC 27707, USA*

Received 15 October 2003; Accepted 10 October 2004
Online 14 November 2004

Configuration-space Faddeev calculations are performed for cluster model $\alpha\alpha\Lambda$ of the ${}^{9}_{\Lambda}$Be hypernucleus using various $\alpha\Lambda$ potentials. For the $\alpha\alpha$ interaction, the nuclear component is only taken into account (phenomenological Ali-Bodmer potential). The binding energy of the ${}^{9}_{\Lambda}$Be hypernuclei is calculated for two different potential models. In the first model, the s-wave $\alpha\Lambda$ potential acting in all partial waves in the $\alpha\Lambda$ subsystem is used. In the second model, a recent more realistic $\alpha\Lambda$ potential having the s- and p-partial components is employed. The core effect of nuclear $\alpha\alpha$ potential is also studied.

PACS numbers: 21.80.+a, 11.80.Jy, 21.45.+v UDC 539.128.3

Keywords: Λ hypernuclei, cluster model, $\Lambda\alpha$ interaction, Faddeev equations, few-body systems

1. Introduction

Configuration-space Faddeev calculations are performed to study the ${}^{9}_{\Lambda}$Be hypernucleus considered as two α clusters and a Λ hyperon. We estimated higher partial-wave contribution to the binding energy of the ${}^{9}_{\Lambda}$Be ground state ($\frac{1}{2}^{+}$) and particularly contribution coming from higher partial waves of the $\alpha\Lambda$ interaction. The Coulomb force between the α's is not taken into account. The phenomenological Ali-Bodmer potential [1] is used for the description of the $\alpha\alpha$ interaction. For the $\alpha\Lambda$ potential, both form and parameters are uncertain, because $\alpha\Lambda$ interaction data are limited by the experimental value of binding energy of the ${}^{5}_{\Lambda}$He hypernucleus, which is considered as the bound s-wave state of the $\alpha\Lambda$ system. To calculate the binding energy of the $\alpha\alpha\Lambda$ system, an s-wave potential model was

[1]Corresponding author, e-mail address: branko@jlab.org

used in Ref. [2]. In this model, one and the same s-wave potential acts in all partial waves of decomposition of the $\alpha\Lambda$ interaction. It leads to unrealistic large p-wave contribution to the binding energy. In a recent work [3], a new $\alpha\Lambda$ potential having the s- and p-wave components was proposed. This potential was constructed on the basis of the NSC97 model of ΛN interaction [4]. In the present work, we use both $\alpha\Lambda$ potential models and compare p-wave contributions to the binding energy. We also study the core effect of nuclear $\alpha\alpha$ potential.

2. *Theory*

The bound state of the $\alpha\alpha\Lambda$ system is calculated by solving the differential Faddeev equations [5], which have the following general form:

$$\{H_0 + V_\alpha(|\overline{x}_\alpha|) - E\}\Psi_\alpha(\overline{x}_\alpha, \overline{y}_\alpha) = -V_\alpha(|\overline{x}_\alpha|) \sum_{\beta\neq\alpha} \Psi_\beta(\overline{x}_\beta, \overline{y}_\beta),$$

where V_α is the short-range pair interaction in the channel α, $H_0 = -\Delta_{\overline{x}_\alpha} - \Delta_{\overline{y}_\alpha}$ is the kinetic energy operator, E is the total energy and the wave function of the three-body system Ψ is given as a sum over the three Faddeev components, $\Psi = \sum_{\alpha=1}^{3} \Psi_\alpha$.

When two particles of the three-body system are identical as α particles in ${}^{9}_{\Lambda}$Be ($\alpha\alpha\Lambda$), the coupled set of Faddeev equations reduces to two equations:

$$(H_0+V_{\alpha\alpha}-E)U = -V_{\alpha\alpha}(W+P_{12}W), \;\; (H_0+V_{\alpha\Lambda}-E)W = -V_{\alpha\Lambda}(U+P_{12}W), \quad (1)$$

where P_{12} is the permutation operator for the α particles (particles 1,2), $V_{\alpha\alpha}$ and $V_{\alpha\Lambda}$ are the nuclear potentials of $\alpha\alpha$ and $\alpha\Lambda$ interactions, respectively, U is the Faddeev component corresponding to the rearrangement channel $(\alpha\alpha) - \Lambda$ and W corresponds to the rearrangement channel $(\alpha\Lambda) - \alpha$. The total wave function is expressed by the components U and W: $\Psi = U + (1 + P_{12})W$. The total orbital angular momentum is given by $\mathbf{L} = \boldsymbol{\ell}_{\alpha\alpha} + \boldsymbol{\lambda}_{(\alpha\alpha)-\boldsymbol{\Lambda}} = \boldsymbol{\ell}_{\alpha\boldsymbol{\Lambda}} + \boldsymbol{\lambda}_{(\alpha\boldsymbol{\Lambda})-\alpha}$, corresponding to two available channels of rearrangements.

The ground state of the $\alpha\alpha\Lambda$ system, having zero total angular momentum L, is considered as the $(\frac{1}{2})^+$ state of the ${}^{9}_{\Lambda}$Be hypernucleus. Possible combinations for relative momenta $\ell_{\alpha\alpha}$ and $\lambda_{(\alpha\alpha)-\Lambda}$ are (0,0), (2,2), (4,4), ..., and for $\ell_{\alpha\Lambda}$, $\lambda_{(\alpha\Lambda)-\alpha}$ are (0,0), (1,1), (2,2), (3,3), (4,4), ... Thus the pair of quantum numbers $\ell_{\alpha\alpha}$ and $\ell_{\alpha\Lambda}$ describe completely the angular states of subsystems.

To describe interactions in the $\alpha\alpha\Lambda$ system, local pairwise potentials are used. The nuclear $\alpha\alpha$ interaction is given by version "a" of the phenomenological Ali-Bodmer potential [1] having s-, d- and g-wave components which was modified in Ref. [6]. This potential has the following form, $V_{\alpha\alpha}(r) = \sum_{l=0,2,4} V^l_{\alpha\alpha}(r)P_l$, where P_l is a projector onto the state of the $\alpha\alpha$ pair with the orbital momentum l and the functions $V^l_{\alpha\alpha}(x)$ have the form of one or two ranges Gaussians

$$V^l_{\alpha\alpha}(r) = V^l_{\rm rep}\exp(-r/\beta^l_{\rm rep})^2 - V^l_{\rm att}\exp(-r/\beta^l_{\rm att})^2\,. \quad (2)$$

The parameters of partial components $V^l_{\alpha\alpha}(r)$ are given in the Table 1. The s-wave component $V^0_{\alpha\alpha}(r)$ has strong repulsive core which simulates Pauli blocking effect for the α's at short distances.

TABLE 1. Parameters of the $\alpha\alpha$ and $\alpha\Lambda$ potentials for different pair angular momenta l.

System	Potential	l	$V^l_{\rm rep}$ (MeV)	$\beta^l_{\rm rep}$ (fm)	$V^l_{\rm att}$ (MeV)	$\beta^l_{\rm att}$ (fm)
$\alpha\alpha$	Ali-Bodmer [1]	0	125.0	1.53	30.18	2.85
		2	20.0	1.53	30.18	2.85
		4	–	–	30.0	0.35
$\alpha\Lambda$	TH [7]	0	–	–	60.17	1.2729
	Gibson [8]		–	–	43.48	1.5764
	Isle(DA) [11]		450.4	1.25	404.9	1.41
	MSA [3]		91.0	1.3	95.0	1.7
		1	33.4	1.3	39.4	1.7

For the $\alpha\Lambda$ interaction, a number of various potentials is used. These potentials have different shape and parameters, but reproduce well the experimental value of the binding energy of the ${}^{5}_{\Lambda}$He hypernucleus which is considered as an s-wave bound state of the $\alpha\Lambda$ system. The Tang-Herndon (TH) potential [7] and Gibson I (Gibson) potential [8, 9] have the form of one-range Gaussian. The Maeda-Schmid potential (MS) [10] is a sum of two Woods-Saxon type functions

$$V_{\alpha\Lambda}(r) = V_{\rm rep}(\exp((r - r_{\rm rep})/a_{\rm rep}) + 1)^{-1} - V_{\rm att}(\exp((r - r_{\rm att})/a_{\rm att}) + 1)^{-1},$$

where $V_{\rm rep} = 18.09$ MeV, $V_{\rm att} = 35.98$ MeV, $r_{\rm rep} = 0.88$ fm, $r_{\rm att} = 1.72$ fm, $a_{\rm rep} = 0.2353$ fm, $a_{\rm att} = 0.3541$ fm. The Isle potential [11] has a form of two range Gaussian Eq.(2). In addition to these s-wave $\alpha\Lambda$ potentials, we use potential of Myint, Shinmura and Akaishi (MSA) proposed in Ref. [3], having p-wave component and two rank Gaussian form. General formula for $\alpha\Lambda$ potential is written by

$$V_{\alpha\Lambda}(r) = \sum_{l=0,1,\dots} V^l_{\alpha\Lambda}(r) P_l\,, \tag{3}$$

where P_l is a projector onto the state of the $\alpha\Lambda$ system having the orbital momentum l, $V^l_{\alpha\Lambda}(r)$ are partial components of the potential. The parameters of the s- and p -wave components are listed in Table 1.

3. Results

The coupled configuration-space Faddeev equations, Eq. (1), for the $\alpha\alpha\Lambda$ system are solved numerically, applying the method given in Ref. [12]. We use two different potential models. In the first model, the s-wave $\alpha\Lambda$ potential acts in all partial waves of the $\alpha\Lambda$ subsystem: $V^l_{\alpha\Lambda}(r) = V^0_{\alpha\Lambda}(r)$, $l = 1, 2, 3, 4$. In the second model, we use two partial components of the $\alpha\Lambda$ potential Eq. (3). These are the s- and p-components $V^l_{\alpha\Lambda}(r)$, $l = 0, 1$, where $V^0_{\alpha\Lambda}(r)$ can be any potential from potentials considered above, and $V^1_{\alpha\Lambda}(r)$ is the p-wave component of potential that is given in Ref. [3]. The results of the calculations are listed in Table 2. For the first model, they are in good agreement with those of a recent Faddeev calculation [2]. In our

TABLE 2. Calculated binding energy $E_{\rm B}$ (in MeV) of the $\alpha\alpha\Lambda$ system for the Ali-Bodmer $\alpha\alpha$ potential and different models of the $\alpha\Lambda$ interaction. The Coulomb potential between α's is not taken into account. Here $l_{\alpha\Lambda}$ and $l_{\alpha\alpha}$ are relative angular momenta of subsystems, $l_{\max} = \max\{l_{\alpha\Lambda}, l_{\alpha\alpha}\}$.

Model	$l_{\max}$	$l_{\alpha\Lambda}$	$l_{\alpha\alpha}$	TH	MS	MSA	Isle(DA)
First	0	0	0	–6.844	–7.365	–8.392	–8.429
	1	0,1	0	–7.363	–8.278	–9.631	–9.768
	2	0,1,2	0,2	–7.786	–8.494	–9.758	–9.920
	3	0,1,2,3	0,2	–7.799	–8.503	–9.764	–9.935
	4	0,1,2,3,4	0,2,4	–7.813	–8.505	–9.765	–9.950
Cravo et al. [2]	–	0,1,2	0,2,4	–7.72	–8.49	–	–10.04
Second	4	0,1	0,2,4	–7.788	–8.050	–8.905	–8.948

calculations, all partial waves up to $l = 4$ were taken into account, but the first three waves ($l_{\alpha\Lambda} \leq 2$, $l_{\alpha\alpha} \leq 2$) give the main contribution. By comparing the first and the second model, one can conclude that the p-wave contribution to the $\alpha\alpha\Lambda$ binding energy is too large. The more realistic $\alpha\Lambda$ potential model of Myint at al. [3], which has the s- and p-wave components, reduces p-wave contribution of the $\alpha\Lambda$ interaction.

TABLE 3. Binding energy $E_{\rm B}$ (in MeV) of the $\alpha\alpha\Lambda$ system, rms distance between two α's $r_{\alpha\alpha}$ (in fm), rms distance between the center of α's pair mass and hyperon $r_{(\alpha\alpha)-\Lambda}$, the most probable distance between two α's $x_{\max}$ for different models of the $\alpha\Lambda$ interaction computed for $l_{\alpha\Lambda} = l_{\alpha\alpha} = 0$ (s-wave approximation). The binding energy $E_{\rm B}(V_{\alpha\alpha} = 0)$ in the case $V_{\alpha\alpha} = 0$ and the interaction energy of α pair is $\Delta B_{\alpha\alpha}$.

Potential	$E_{\rm B}$	$r_{\alpha\alpha}$	$r_{(\alpha\alpha)-\Lambda}$	$E_{\rm B}(V_{\alpha\alpha} = 0)$	$x_{\max}$	$\Delta B_{\alpha\alpha}$
TH	–6.844	3.75	2.65	–9.601	2.4	2.757
Gibson	–7.429	3.69	2.67	–8.934	2.5	1.510
MS	–7.364	3.68	2.72	–8.079	2.5	0.715
MSA	–8.392	3.65	2.80	–8.056	2.6	-0.335
Isle(DA)	–8.429	3.64	2.82	–7.839	2.8	-0.590

To explain substantial disagreements of the binding energies calculated with different $\alpha\Lambda$ potentials, we computed the interaction energy between the α particles for the bound $\alpha\alpha\Lambda$ system, which is defined by the formula

$$\Delta B_{\alpha\alpha} = E_{\mathrm{B}} - E_{\mathrm{B}}(V_{\alpha\alpha} = 0),$$

where the binding energy $E_{\mathrm{B}}(V_{\alpha\alpha} = 0)$ of the $\alpha\alpha\Lambda$ system is calculated when $\alpha\alpha$ interaction is absent ($V_{\alpha\alpha} = 0$). Results of calculation, together with other physical characteristics, are presented in Table 3. In addition, we calculate the most probable configuration of the $\alpha\alpha\Lambda$ system. In Fig. 1, the distances between particles and their localization are shown. From the table and the figure, one can see that the $\alpha\Lambda$ potentials having a weak core (MS), or not having it at all (TH, Gibson), bring the α particles close together. However, at small distances, a repulsion starts between the α's due to the presence of the strong repulsive core in the $\alpha\alpha$ potential, which decreases the $\alpha\alpha\Lambda$ binding energy.

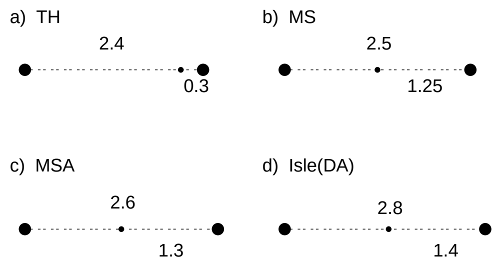

Fig. 1. Most probable configuration of the $\alpha\alpha\Lambda$ system for various $\alpha\Lambda$ potentials. The upper numbers are distances between the α particles (in fm*) and the lower ones are distances between α particle and hyperon.*

4. Conclusion

The $^9_\Lambda$Be hypernucleus is considered in the framework of the $\alpha\alpha\Lambda$ cluster model. We have studied the contributions of higher partial waves of the $\alpha\Lambda$ interactions to the binding energy of the ground state. We found that the p-wave contribution of $\alpha\Lambda$ interaction is unnatural large when potential model is applied in which the s-wave of the potential acts in all partial waves of the $\alpha\Lambda$ subsystem. This contribution can be reduced by including a realistic p-wave component of the $\alpha\Lambda$ interaction. The differences between the $\alpha\alpha\Lambda$ binding energies calculated for various $\alpha\Lambda$ potentials can be explained by influence of a repulsive core of nuclear $\alpha\alpha$ interaction.

Acknowledgements

This work was partially supported by the Department of Defenses through the grant No.DAAD 19-01-1-0795. The work of V.M.S and I.N.F was supported by the RFFI under Grant No. 02-02-16562.

References

[1] S. Ali and A. R. Bodmer, Nucl. Phys. **80** (1966) 99.

[2] E. Cravo, A. C. Fonseca and Y. Koike, Phys. Rev. C **66** (2002) 014001-7.

[3] K. S. Myint, S. Shinmura and Y. Akaishi, Eur. Phys. J. A **16** (2003) 21; nucl-th/0209090.

[4] Th. A. Rijken, V. G. J. Stoks and Y. Yamamoto, Phys. Rev. C **59** (1999) 21.

[5] L. D. Faddeev and S. P. Merkuriev, *Quantum Scattering Theory for Several Particle Systems*, Kluwer Academic, Dordrecht (1993).

[6] D. V. Fedorov and A. S. Jensen, Phys. Lett. B **389** (1996) 631.

[7] C. Tang and R. C. Herndon, Phys. Rev. **138** (1965) B637.

[8] B. F. Gibson, A. Goldberg and M. S. Weiss, Phys. Rev. C **6** (1972) 741.

[9] S. Oryu, H. Kamada, H. Sekine, H. Yamashita and M. Nakazawa, Few-Body Systems **28** (2000) 103.

[10] S. Maeda and E. W. Shmid, *Few-Body Problem in Physics*, Vol. II, Elsever Science Publisher B.V. (1984) 379.

[11] Y. Kurihara, Y. Akaishi and H. Tanaka, Phys. Rev. C **84** (1985) 971.

[12] J. Bernabeu, V. M. Suslov, T. A. Strizh and S. I. Vinitsky, Hyperfine Interaction **101/102** (1996) 391.

NAKUPINSKI RAČUN $^{9}_{\Lambda}$Be HIPERJEZGRE FADDEEVOM METODOM

Načinili smo račune Faddeevom metodom u konfiguracijskom prostoru za nakupinski model $\alpha\alpha\Lambda$ hiperjezgre $^{9}_{\Lambda}$Be primjenom različitih potencijala. Za nuklearnu sastavnicu međudjelovanja $\alpha\alpha$ primijenili smo samo fenomenološki Ali-Bodmerov potencijal. Energiju vezanja hiperjezgre $^{9}_{\Lambda}$Be smo računali dvama različitim potencijalnim modelima. U prvome se rabi s-valni potencijal $\alpha\Lambda$ koji djeluje na sve parcijalne valove u podsustavu $\alpha\Lambda$. U drugome, rabi se nov realističniji potencijal $\alpha\Lambda$ koji sadrži s- i p-parcijalne sastavnice. Proučavali smo također učinak sredice u potencijalu $\alpha\alpha$.