

NUCLEAR FORCES FROM CHIRAL DYNAMICS

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We made an analysis of time-ordered diagrams for the two-nucleon T -matrix in chiral perturbation theory. From the analysis, it follows that low-energy nucleon dynamics is governed by the generalized dynamical equation [derived in J. Phys. A **32** (1999) 5657] with a nonlocal-in-time interaction operator. We also present an operator that parametrizes the nucleon-nucleon interaction in the 1S_0 channel at leading order of the Weinberg power counting.

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1. Introduction

One of the most fundamental problems of nuclear physics is to derive nuclear forces from the principles of QCD. A first attempt to construct a bridge between QCD and low-energy nuclear physics was made by Weinberg [1]. He suggested to derive a nucleon-nucleon (NN) potential in time-ordered chiral perturbation theory (ChPT). However, such a potential is singular and the Schrödinger (Lippmann-Schwinger) equation can not be used without regularization and renormalization. This means that in the effective field theory (EFT) of nuclear forces, which following the pioneering work of Weinberg has become very popular in nuclear physics (for a review, see Ref. [2]), the Schrödinger equation is not valid. On the other hand, quantum mechanics is one of the basic ingredients of the whole formalism of fields and particles, and hence in the nonrelativistic limit, QCD must produce low-energy nucleon physics consistent with the basic principles of quantum mechanics. However, as follows from the Weinberg analysis, QCD leads, through ChPT, to the low-energy theory in which the Schrödinger equation is not valid. This means that either there is something wrong with QCD and ChPT or the Schrödinger equation is not the basic dynamical equation of quantum theory. Meanwhile, in Ref. [3], it has been shown that the Schrödinger equation is not the most general equation consistent with the current concepts of quantum physics, and a more general equation of motion has been derived as a consequence of the basic postulates of

the Feynman [4] and canonical approaches to quantum theory. Being equivalent to the Schrödinger equation in the case of instantaneous interactions, this generalized dynamical equation permits the generalization to the case where the dynamics of a system is generated by a nonlocal-in-time interaction. The generalized quantum dynamics (GQD) developed in this way has proved to be a useful tool for solving various problems in quantum theory [5, 6].

The Weinberg program for low-energy nucleon physics employs the analysis of time-ordered diagrams for the two-nucleon T -matrix in ChPT to derive a NN potential and then to use it in the Lippmann-Schwinger (LS) equation for constructing the full NN T -matrix. Obviously, the starting point for this program is the assumption that in the nonrelativistic limit ChPT leads to low-energy nucleon dynamics which is Hamiltonian and is governed by the Schrödinger equation. However, as we have noted, this is not the case because the Schrödinger equation with chiral potentials constructed in this way makes no sense without renormalization. The GQD provides a new insight into this problem: The above may mean that the low-energy nucleon dynamics, which results from the analysis of diagrams in ChPT, is governed by the generalized dynamical equation with a nonlocal-in-time interaction operator when this equation cannot be reduced to the Schrödinger equation. In Ref. [5], it has been shown that such a dynamical situation takes place in the case of the pionless EFT which is valid at extreme low energies: After renormalization, low-energy nucleon dynamics is governed by the generalized dynamical equation with a nonlocal-in-time interaction operator. Moreover, at leading order this dynamics is just the same as in the model of Refs. [3] and [7], developed as a test model illustrating the possibility of the extension of quantum dynamics provided by the formalism of the GQD. The aim of the present paper is to show that the same situation takes place at higher energies when one must include the π -field explicitly. We will show that from the analysis of diagrams for the two-nucleon T -matrix in ChPT it follows that low-energy dynamics is governed by the generalized dynamical equation with a nonlocal-in-time interaction operator which is well defined and does not give rise to UV divergences. At leading order of the Weinberg power counting, we will construct a chiral NN interaction operator for the 1S_0 channel. It will be shown that the generalized dynamical equation with this interaction operator is well defined and allows one to construct the T -matrix and the evolution operator without regularization and renormalization.

2. Generalized quantum dynamics

In the GQD the evolution operator $U(t, t_0)$ in the interaction picture is represented in the form [3]

$$\langle \psi_2 | U(t, t_0) | \psi_1 \rangle = \langle \psi_2 | \psi_1 \rangle + \int_{t_0}^t dt_2 \int_{t_0}^{t_2} dt_1 \langle \psi_2 | \tilde{S}(t_2, t_1) | \psi_1 \rangle, \quad (1)$$

where $\langle \psi_2 | \tilde{S}(t_2, t_1) | \psi_1 \rangle$ is the probability amplitude that if at time t_1 the system was in the state $|\psi_1\rangle$, then the interaction in the system will begin at time t_1 and

will end at time t_2 , and at this time the system will be in the state $|\psi_2\rangle$. This equation represents the Feynman superposition principle according to which the probability amplitude of an event which can happen in several different ways is a sum of contributions from each alternative way. Here subprocesses with definite instants of the beginning and end of the interaction in the system are used as alternative ways of realization of the corresponding evolution process, and $\langle \psi_2 | \tilde{S}(t_2, t_1) | \psi_1 \rangle$ represents the contribution to the evolution operator from the subprocess in which the interaction begins at time t_1 and ends at time t_2 . As has been shown in Ref. [3], the condition of unitarity for the evolution operator given by Eq. (1) is that for any t and t_0 , the operator $\tilde{S}(t_2, t_1)$ must satisfy the equation

$$(t_2 - t_1)\tilde{S}(t_2, t_1) = \int_{t_1}^{t_2} dt_4 \int_{t_1}^{t_4} dt_3 (t_4 - t_3)\tilde{S}(t_2, t_4)\tilde{S}(t_3, t_1). \quad (2)$$

A remarkable feature of this relation is that it works as a recurrence relation and allows one to obtain the operators $\tilde{S}(t_2, t_1)$ for any t_1 and t_2 , if $\tilde{S}(t'_2, t'_1)$ corresponding to infinitesimal time intervals $\tau = t'_2 - t'_1$ of interaction are known. It is natural to assume that most of the contribution to the evolution operator in the limit $t_2 \rightarrow t_1$ comes from the processes associated with the fundamental interaction in the system under study. Denoting this contribution by $H_{\text{int}}(t_2, t_1)$, we can write

$$\tilde{S}(t_2, t_1) \xrightarrow{t_2 \rightarrow t_1} H_{\text{int}}(t_2, t_1) + o(\tau^\epsilon), \quad (3)$$

where $\tau = t_2 - t_1$. The parameter ϵ is determined by demanding that $H_{\text{int}}(t_2, t_1)$, called the generalized interaction operator, must be so close to the solution of Eq. (2) in the limit $t_2 \rightarrow t_1$ that this equation has a unique solution having the behavior (3) near the point $t_2 = t_1$. If $H_{\text{int}}(t_2, t_1)$ is specified, Eq. (2) allows one to find the operator $\tilde{S}(t_2, t_1)$, and hence the evolution operator. Thus Eq. (2), which is a direct consequence of the principle of the superposition, can be regarded as an equation of motion for states of a quantum system. This equation allows one to construct the evolution operator by using the contributions from fundamental processes as building blocks. In the case of Hamiltonian dynamics, the fundamental interaction is instantaneous. The generalized interaction operator describing such an interaction is of the form

$$H_{\text{int}}(t_2, t_1) = -2i\delta(t_2 - t_1)H_I(t_1), \quad (4)$$

where the delta function $\delta(t_2 - t_1)$ emphasizes that the interaction is instantaneous. In this case Eq. (2) is equivalent to the Schrödinger equation [3], and the operator $H_I(t)$ is an interaction Hamiltonian. At the same time, Eq. (2) permits the generalization to the case where the fundamental interaction in a quantum system is nonlocal in time, and hence the dynamics is non-Hamiltonian [3].

By using Eq. (1) for $U(t, t_0)$, we can write

$$U(t, t_0) = \mathbf{1} + \frac{i}{2\pi} \int_{-\infty}^{\infty} dx \exp[-i(z - H_0)t]$$

$$\times (z - H_0)^{-1} T(z) (z - H_0)^{-1} \exp[i(z - H_0)t_0], \quad (5)$$

where $z = x + iy$, $y > 0$, and H_0 is the free Hamiltonian. The operator $T(z)$ is defined by

$$T(z) = i \int_0^\infty d\tau \exp(i z \tau) \tilde{T}(\tau), \quad (6)$$

where $\tilde{T}(\tau) = \exp(-i H_0 t_2) \tilde{S}(t_2, t_1) \exp(i H_0 t_1)$, and $\tau = t_2 - t_1$. In terms of the T-matrix defined by Eq. (6), the generalized dynamical equation (2) can be rewritten in the form [3]

$$\frac{d\langle \psi_2 | T(z) | \psi_1 \rangle}{dz} = - \sum_n \frac{\langle \psi_2 | T(z) | n \rangle \langle n | T(z) | \psi_1 \rangle}{(z - E_n)^2}, \quad (7)$$

where n stands for the entire set of discrete and continuous variables that characterize the system in full, and $|n\rangle$ are the eigenvectors of H_0 . As follows from Eq. (4), the boundary condition for this equation is of the form

$$\langle \psi_2 | T(z) | \psi_1 \rangle \xrightarrow{|z| \rightarrow \infty} \langle \psi_2 | B(z) | \psi_1 \rangle + o(|z|^{-\beta}), \quad (8)$$

where

$$B(z) = i \int_0^\infty d\tau \exp(i z \tau) H_{\text{int}}^{(s)}(\tau), \quad (9)$$

and $H_{\text{int}}^{(s)}(t_2 - t_1) = \exp(-i H_0 t_2) H_{\text{int}}(t_2, t_1) \exp(i H_0 t_1)$ is the interaction operator in the Schrödinger picture. As can be seen from Eqs. (8) and (9), the operator $B(z)$ represents the contribution which $H_{\text{int}}^{(s)}(\tau)$ gives to the operator $T(z)$.

3. Chiral dynamics and the NN interaction operator

The Weinberg program implies the employment of the analysis of diagrams in ChPT for deriving the chiral NN potential, which is assumed to be a sum of irreducible diagrams involving only two external nucleons. Here irreducible diagrams are two-nucleon irreducible: Any intermediate state contains at least one pion or isobar. The motivation for this is as follows. If we assume that the two-nucleon T-matrix satisfies the LS equation with the interaction potential $\langle \mathbf{p}_2 | V | \mathbf{p}_1 \rangle$, then in the limit $|z| \rightarrow \infty$ the operator $T(z)$ must behave as

$$\langle \mathbf{p}_2 | T(z) | \mathbf{p}_1 \rangle \xrightarrow{|z| \rightarrow \infty} \langle \mathbf{p}_2 | V | \mathbf{p}_1 \rangle. \quad (10)$$

Since any reducible diagram can be constructed from irreducible ones by connecting the latter with intermediate two-nucleon states whose evolution is described by the free Green operator $G_0(z) = (z - H_0)^{-1}$, reducible diagrams tend to zero as $|z| \rightarrow \infty$ and hence do not contribute to the potential $\langle \mathbf{p}_2 | V | \mathbf{p}_1 \rangle$. For this reason, it is natural to use the sum of irreducible diagrams as a potential describing a “fundamental” interaction in the two-nucleon system. In addition, as follows from Eq. (6), the large- z behavior of the T -matrix relates to the behavior of the operator $\tilde{S}(t_2, t_1)$ in the limit of infinitesimal duration times of interaction in the system when this operator describes a fundamental interaction in the system. At leading order of Weinberg power counting, we have two types of irreducible diagrams: The one-pion-exchange diagram and the diagram describing the contact interaction. The corresponding chiral potential is of the form

$$\langle \mathbf{p}_2 | V | \mathbf{p}_1 \rangle = V_{\text{OPE}}(\mathbf{p}_2, \mathbf{p}_1) + C_S + C_T \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2, \quad (11)$$

where $V_{\text{OPE}}(\mathbf{p}_2, \mathbf{p}_1)$ is the one-pion exchange (OPE) potential

$$V_{\text{OPE}}(\mathbf{p}_2, \mathbf{p}_1) = - \left(\frac{g_A^2}{2f_\pi^2} \right) \frac{\mathbf{q} \cdot \boldsymbol{\sigma}_1 \mathbf{q} \cdot \boldsymbol{\sigma}_2}{\mathbf{q}^2 + m_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2,$$

with $\mathbf{q} \equiv \mathbf{p}_2 - \mathbf{p}_1$. The coupling g_A is the axial coupling constant, m_π is the pion mass, f_π is the pion decay constant, and $\sigma(\tau)$ are the Pauli matrices acting in spin (isospin) space. In the 1S_0 channel, the chiral potential can be rewritten in the form

$$\langle \mathbf{p}_2 | V | \mathbf{p}_1 \rangle = C_0 + V_\pi(\mathbf{p}_2, \mathbf{p}_1), \quad (12)$$

where

$$C_0 = C_S - 3C_T, \quad V_\pi(\mathbf{p}_2, \mathbf{p}_1) = - \frac{4\pi\alpha_\pi}{q^2 + m_\pi^2} \quad \text{and} \quad \alpha_\pi = \frac{g_A^2 m_\pi^2}{8\pi f_\pi^2}. \quad (13)$$

However, the potential (11) is singular and the Schrödinger equation makes no sense without regularization and renormalization. The reason for this is that the chiral Lagrangian is only of formal importance. In addition, one has to specify a renormalization scheme to make the physical predictions finite. The formal chiral Lagrangian determines the structure of the theory. In particular, it determines the structure of the two-nucleon T -matrix, i.e., its dependence on the momenta of nucleons. On the other hand, this T -matrix must satisfy Eq. (7), and there is the one-to-one correspondence between the large-momentum behavior of the T -matrix and the character of the dynamics in the two-nucleon system [3]. As will be shown below, in order to establish such a behavior, one need not enter into detail and perform renormalization of diagrams: This behavior directly follows from the form of the chiral Lagrangian. This gives us the hope that, using the structure of the two-nucleon diagrams, which follows from ChPT together with the requirement that the T -matrix satisfy the generalized dynamical equation, may allow one to find a true asymptotic behavior for $T(z)$. By using this behavior, one can then derive

the NN interaction operator. With such an operator, the generalized dynamical equation permits one to construct the two-nucleon T -matrix without resorting to regularization and renormalization.

The form of the chiral Lagrangian specifies the dependence of the two-nucleon T -matrix on the momenta of nucleons. From the analysis of the time-ordered diagrams in ChPT, it follows that the leading order two-nucleon T -matrix should be of the form

$$\langle \mathbf{p}_2 | T(z) | \mathbf{p}_1 \rangle = t_{00}(z) + t_{01}(z, \mathbf{p}_1) + t_{01}(z, \mathbf{p}_2) + t_{11}(z, \mathbf{p}_1, \mathbf{p}_2). \quad (14)$$

In fact, there are two kinds of vertices in the leading order diagrams: The vertex which corresponds to the contact interaction without derivatives and is a constant, and the π NN vertex. The term $t_{00}(z)$ describes the contribution from the time-ordered diagrams in which the initial and final vertices are the above contact ones. The term $t_{01}(z, \mathbf{p}_1)$ is the contribution from the diagrams in which the initial and final vertices are the π NN and contact ones, respectively. Correspondingly, the initial and final vertices in the diagrams which contribute to the term $t_{11}(z, \mathbf{p}_1, \mathbf{p}_2)$ are the π NN ones. Thus Eq. (14) represents the structure of the leading order two-nucleon T -matrix which results from the form of the chiral Lagrangian. As we have noted, the character of the dynamics of a system is determined by the behavior of $T(z)$ in the limit $|z| \rightarrow \infty$. Let us examine this behavior of the two-nucleon T -matrix (14) using the fact that this T -matrix must satisfy Eq. (7). The main contribution to $t_{11}(z, \mathbf{p}_1, \mathbf{p}_2)$ in the limit $|z| \rightarrow \infty$ comes from the two-nucleon irreducible diagram describing the one-pion-exchange processes, and, for the 1S_0 channel we have

$$t_{11}(z, \mathbf{p}_1, \mathbf{p}_2) \xrightarrow{|z| \rightarrow \infty} V_\pi(\mathbf{p}_1, \mathbf{p}_2). \quad (15)$$

Since there are no irreducible diagrams which contribute to the terms $t_{01}(z, \mathbf{p})$ and $t_{10}(z, \mathbf{p}_1)$, it is natural to assume that $\lim_{|z| \rightarrow \infty} t_{01}(z, \mathbf{p}) = \lim_{|z| \rightarrow \infty} t_{10}(z, \mathbf{p}) = 0$. At the same time, one might think that in this limit the function $t_{00}(z)$ tends to a nonzero constant C_0 playing the role of a contact potential. However, such a potential is singular and leads to UV divergences. This is a manifestation of the fact that the large-momentum behavior of the T -matrix shown in Eq. (14) does not satisfy the requirements of Hamiltonian formalism. There are no potentials which could generate the T -matrix of such a form, and hence the dynamics of the system is non-Hamiltonian. On the other hand, the form of the two-nucleon T -matrix shown in Eq. (14) is not at variance with the requirements of the GQD. Such a form of the T -matrix corresponds to the case of a nonlocal-in-time interaction when the dynamics is non-Hamiltonian. In this case, the contact term must tend to zero as $|z| \rightarrow \infty$ [5].

The large- z behavior of $T(z)$ is determined by Eq. (7). From this equation, it follows that $t_{00}(z)$ and $t_{ij}(z, \mathbf{p})$ satisfy the equations:

$$\frac{dt_{00}(z)}{dz} = - \int \frac{d^3k}{(2\pi)^3} \frac{(t_{00}(z) + t_{01}(z, \mathbf{k}))(t_{00}(z) + t_{10}(z, \mathbf{k}))}{(z - E_k)^2}, \quad (16)$$

$$\frac{dt_{01}(z, \mathbf{p})}{dz} = - \int \frac{d^3k}{(2\pi)^3} \frac{(t_{00}(z) + t_{01}(z, \mathbf{k})) (t_{01}(z, \mathbf{p}) + t_{11}(z, \mathbf{k}, \mathbf{p}))}{(z - E_k)^2} \quad (17)$$

$$\text{and } \frac{dt_{10}(z, \mathbf{p})}{dz} = - \int \frac{d^3k}{(2\pi)^3} \frac{(t_{10}(z, \mathbf{p}) + t_{11}(z, \mathbf{p}, \mathbf{k})) (t_{00}(z) + t_{10}(z, \mathbf{k}))}{(z - E_k)^2}. \quad (18)$$

Since $t_{01}(z, \mathbf{p})$ and $t_{10}(z, \mathbf{p})$ describe the contribution from diagrams which consist of connected diagrams corresponding to the terms $t_{00}(z)$ and $t_{11}(z, \mathbf{p}_1, \mathbf{p}_2)$, in the limit $|z| \rightarrow \infty$ the terms $t_{01}(z, \mathbf{p})$ and $t_{10}(z, \mathbf{p})$ must tend to zero faster than the term $t_{00}(z)$. Taking this fact into account, from Eq. (16) we get

$$\frac{dt_{00}(z)}{dz} \sim -(t_{00}(z))^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{(z - E_k)^2}, \quad |z| \rightarrow \infty.$$

From this equation it follows that

$$t_{00}(z) = b_1(-z)^{-1/2} + o(|z|^{-1/2}), \quad |z| \rightarrow \infty, \quad (19)$$

where $b_1 = -4\pi/(M\sqrt{M})$. Substituting this expression into Eq. (17) and taking into account Eq. (15), one obtains

$$\frac{dt_{01}(z, \mathbf{p})}{dz} = - \frac{b_1}{\sqrt{-z}} \int \frac{d^3k}{(2\pi)^3} \frac{(V_\pi(\mathbf{k}, \mathbf{p}) + t_{01}(z, \mathbf{p}))}{(z - E_k)^2} + o(|z|^{-2}).$$

From this equation we get

$$t_{01}(z, \mathbf{p}) = B_{01}(z, \mathbf{p}) + o(|z|^{-1}), \quad |z| \rightarrow \infty,$$

where

$$B_{01}(z, \mathbf{p}) = \frac{b_1}{\sqrt{-z}} \int \frac{d^3k}{(2\pi)^3} \frac{V_\pi(\mathbf{k}, \mathbf{p})}{(z - E_k)}.$$

In the same way, for $t_{10}(z, \mathbf{p})$, we get $t_{10}(z, \mathbf{p}) = B_{10}(z, \mathbf{p}) + o(|z|^{-1})$, where $B_{10}(z, \mathbf{p}) = B_{01}(z, \mathbf{p})$. Equation (19) represents the first term in the asymptotic expansion of $t_{00}(z)$. In order to obtain further terms in this expansion, let us write the equation for $\tilde{t}_{00}(z) \equiv t_{00}(z) - b_1(-z)^{-1/2}$

$$\begin{aligned} \frac{d\tilde{t}_{00}(z)}{dz} &= - \frac{2b_1^2}{z} \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{V_\pi(\mathbf{k}_1, \mathbf{k}_2)}{(z - E_{k_1})^2 (z - E_{k_2})} \\ &- \frac{2b_1}{\sqrt{-z}} \int \frac{d^3k}{(2\pi)^3} \frac{\tilde{t}_{00}(z)}{(z - E_k)^2} + o(|z|^{-2}) = - \frac{\tilde{t}_{00}(z)}{z} - \frac{b_3}{z^2} + o(|z|^{-2}), \end{aligned} \quad (20)$$

where

$$b_3 = -2b_1^2 4\pi\alpha_\pi \int \frac{d^3q_1 d^3q_2}{(2\pi)^6} (\mathbf{q}_1 - \mathbf{q}_2)^{-2} (1 - q_1^2/M)^{-2} (1 - q_2^2/M)^{-1},$$

with $\mathbf{q}_i = \mathbf{p}_i/\sqrt{z}$. Here we have used Eq. (16) and the above expressions for $t_{00}(z)$, $t_{01}(z, \mathbf{p})$ and $t_{10}(z, \mathbf{p})$. The solution of Eq. (20) is of the form $t_{00}(z) = -b_2/z - b_3 \ln z/z + o(|z|^{-1})$, where b_2 is a free parameter, and hence

$$t_{00}(z) = \frac{b_1}{\sqrt{-z}} - \frac{b_2}{z} - b_3 \frac{\ln z}{z} + o(|z|^{-1}), \quad |z| \rightarrow \infty.$$

Thus the requirement that the T -matrix be of the form (14) and satisfy the generalized dynamical equation determines its behavior for $|z| \rightarrow \infty$ up to one free parameter, b_2 ,

$$\begin{aligned} \langle \mathbf{p}_2 | T(z) | \mathbf{p}_1 \rangle &= \frac{b_1}{\sqrt{-z}} - \frac{b_2}{z} - b_3 \frac{\ln z}{z} + B_{01}(z, \mathbf{p}_1) + B_{10}(z, \mathbf{p}_2) \\ &+ V_\pi(\mathbf{p}_1, \mathbf{p}_2) + o(|z|^{-1}), \quad |z| \rightarrow \infty. \end{aligned} \quad (21)$$

Knowing the terms in the asymptotic expansion of $\langle \mathbf{p}_2 | T(z) | \mathbf{p}_1 \rangle$ represented in Eq. (21) is sufficient to construct the T -matrix, and hence they determine the effective interaction operator, which governs the dynamics of a two-nucleon system. Thus in the 1S_0 channel, this operator is of the form

$$\langle \mathbf{p}_2 | B(z) | \mathbf{p}_1 \rangle = \frac{b_1}{\sqrt{-z}} - \frac{b_2}{z} - b_3 \frac{\ln z}{z} + B_{01}(z, \mathbf{p}_1) + B_{10}(z, \mathbf{p}_2) + V_\pi(\mathbf{p}_1, \mathbf{p}_2). \quad (22)$$

The corresponding NN interaction operator $H_{\text{int}}^{(s)}(\tau)$ can be obtained by using Eq. (9). The generalized dynamical equation with this nonlocal-in-time interaction operator is well defined and allows one to construct the T -matrix without regularization and renormalization. It can be shown that the solution of Eq. (7) with this interaction operator is of the form

$$\langle \mathbf{p}_2 | T(z) | \mathbf{p}_1 \rangle = t_L(z, \mathbf{p}_1, \mathbf{p}_2) + \chi(z, \mathbf{p}_2)\chi(z, \mathbf{p}_1) \left[C^{-1} - G_z^{(R)}(0, 0) \right]^{-1}, \quad (23)$$

where

$$\begin{aligned} G_z^{(R)}(0, 0) &= (4\pi)^{-1} M^{3/2} \sqrt{-z} - 2 \int_0^z ds \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} \frac{V_\pi(\mathbf{k}_1, \mathbf{k}_2)}{(s - E_{k_1})^2 (s - E_{k_2})} \\ &+ \int \frac{d^3 k_1 d^3 k_2 d^3 k_3}{(2\pi)^9} \frac{V_\pi(\mathbf{k}_1, \mathbf{k}_2) T_L(z, \mathbf{k}_2, \mathbf{k}_3)}{(z - E_{k_1})(z - E_{k_2})(z - E_{k_3})} \end{aligned}$$

$$\text{and} \quad \chi(z, \mathbf{p}) = 1 + \int \frac{d^3 k}{(2\pi)^3} \frac{T_L(z, \mathbf{p}, \mathbf{k})}{(z - E_k)}.$$

$C = -(4\pi)^2 M^{-3} b_2^{-1}$ and $T_L(z, \mathbf{k}_2, \mathbf{k}_1)$ is the solution of the LS equation with the Yukawa potential $\tilde{V}_\pi(\mathbf{k}_2, \mathbf{k}_1)$. For the scattering amplitude, we then get

$$A(p) = A_\pi - \{\chi(E_p, \mathbf{p})\}^2 \left[C^{-1} - G_{E_p}^{(R)}(0, 0) \right]^{-1}, \quad (24)$$

where A_π is just the amplitudes one finds in the pure Yukawa theory with the potential $V_\pi(\mathbf{p}_2, \mathbf{p}_1)$. This expression for the scattering amplitude is exactly what Kaplan, Savage and Wise have obtained in Ref. [8] by summing and renormalizing the relevant diagrams in the EFT of nuclear forces. Equations (5) and (23), can then be used for constructing the evolution operator describing the dynamics of the two-nucleon system in the 1S_0 channel.

4. Conclusion

We have shown that the formalism of the GQD provides a new way to realize the Weinberg program for deriving the forces between nucleons from the analysis of diagrams in ChPT. The Weinberg proposal was based on the assumption that the low energy nucleon dynamics is governed by the Schrödinger (LS) equation, and hence what one has to derive from the analysis of diagrams in ChPT is an NN potential. However, as we have seen, this equation with the NN chiral potentials derived in this way makes no sense without renormalization. At the same time, there is no reason to expect that low-energy nucleon dynamics is necessarily governed by the Schrödinger equation. In principle it may be governed by the generalized dynamical equation with a nonlocal-in-time interaction operator when this equation is not equivalent to the Schrödinger equation. In the present paper, we have shown that really at leading order of the Weinberg power counting, low-energy nucleon dynamics is governed by the generalized dynamics equation with the nonlocal-in-time interaction operator (22) which is well defined and does not require renormalization. The problem with the UV divergences arise when we try to extract from the analysis of the diagrams in ChPT an NN potential, while really in the low-energy limit ChPT leads to nucleon dynamics that is generated by a nonlocal-in-time NN interaction. This means that in order to realize the Weinberg program in a consistent way, instead of the LS equation, one should use the generalized dynamical equation being a more general equation of motion than the LS equation.

It should be noted that, from the physical point of view, the fact that the chiral potentials lead to UV divergences means that the LS equation with such potentials does not provide a good separation of the low-energy scale from the scale of the underlying high-energy physics. Such a separation of the scales is needed for the low-energy theory in which nucleons and pions emerge as the only effective degrees of freedom to be self consistent. In the standard EFT approach, the needed separation of the scales is achieved by using renormalization. The advantage of the use of the generalized dynamical equation is that this equation provides the good separation of the scale of the low-energy nuclear physics from the scale of the underlying high-energy physics without renormalization. This allows one to formulate the effective theory of nuclear forces as a perfectly consistent theory, keeping all advantages of the traditional nuclear physics approach based on the use of “realistic” potentials: In this case we deal with a well defined equation of motion that, in contrast with the standard EFT of nuclear forces, allows one to construct not

only the scattering amplitudes, but also the off-shell T -matrix and the evolution operator. At the same time, the advantage of such a formulation of the theory of nuclear forces over the traditional approach is that it allows one to construct the NN interaction operator as an inevitable consequence of the symmetries of QCD and the basic principles of quantum mechanics without employing any *ad hoc* form factors. The leading order NN interaction operator in the 1S_0 channel, for example, should be of the form (22). The solution of Eq. (7) with this interaction operator yields the two-nucleon T -matrix given in Eq. (23). This T -matrix can be used not only for calculating the scattering amplitude (in this case we reproduce the results obtained in Ref. [8] by summing the relevant diagrams and performing regularization and renormalization), but also the evolution operator (5). Note that Eq. (23) determines the off-shell behavior of the two-nucleon T -matrix at leading order. In other words, our approach allows one to find constraints on the off-shell behavior of the two-nucleon T -matrix that are placed by the symmetries of QCD. This is very important because this off-shell behavior may play a crucial role in solving the many-nucleon problem and is an important factor in calculating in-medium observables and in microscopic nuclear structure calculations. As is well known, the “realistic” potentials constructed within the traditional approach cannot guarantee a reliable off-shell extrapolation of the two-nucleon T -matrix since they are all constrained by the two-nucleon phase shifts analysis.

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NUKLEARNE SILE IZ KIRALNE DINAMIKE

Načinili smo analizu vremenski-uređenih dijagrama dvonukleonske T -matrice u kirralnoj teoriji smetnje. Iz te analize slijedi da nukleonsku dinamiku na niskim energijama određuje poopćena dinamička jednačba (izvedena u J. Phys. A **32** (1999) 5657) s vremenski nelokalnim operatorom međudjelovanja. Predstavljamo i operator koji parametrizira međudjelovanje nukleon-nukleon u kanalu 1S_0 za vodeći član Weinbergovog brojanja potencija.