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LEPTOPRODUCTION OF CUMULATIVE NUCLEONS

M. A. BRAUN[a,b], V. V. VECHERNIN[a] and B. VLAHOVIĆ[b]

[a]*Department of High-Energy Physics, S.Petersburg University, S.Petersburg, Russia*

[b]*North Carolina Central University, Durham, USA*

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Leptoproduction of nucleons on nuclear targets into the backward hemisphere is studied at relativistic subasymptotic energies and momenta. Spins are neglected. The relativistic internucleon potential is extracted from the appropriate photoproduction data. Different production mechanisms are shown to work together and interfere. Calculations show that whenever rescattering is possible, it gives the bulk of the contribution, except at very high Q^2. The Weizsäcker-Williams approximation is found to generally reproduce only a small part of the total cross-section. Comparison with the data for A(e,e′p) reaction at $E = 2.4$ GeV shows a reasonable agreement.

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1. *Introduction*

Production of particles off nuclei in the cumulative domain $x > 1$ presents great interest, since it is related to the nuclear structure at small distances, where one may expect formation of multiquark states and even droplets of a cold quark- gluon plasma. In the picture where the nucleus consists of nucleons interacting with some potentials, cumulative particle production allows to study the high-momentum asymptotics of these potentials and multi-nucleon forces, otherwise hidden due to the low probability for several nucleons to be located at the same point.

The proposed models for the description of cumulative production off nuclei can roughly be divided into three categories. The first model that was introduced ascribes the production to the presence of states with a much higher nuclear density than the average in the initial nucleus [1, 2]: one just assumes several nucleons at the same point ("many-body correlations" [3]) or multi-quark states [4]). The second model assumes that such high-density states are formed in the course of the collision (”compressed tube” models [5]). Finally, one can assume that the high nuclear densities are irrelevant, and cumulative particles are produced as a result of simple rescattering [6].

Microscopic studies have shown that in reality all three mechanisms work together and interfere in the cumulative production [7]. Note that the contribution from the rescattering depends differently on the target properties that the other two mechanisms. That prevents simple estimates of the cross-sections on heavy nuclei based on those with only few nucleons, as suggested in Ref. [8]. Only for sufficiently high momentum transfers to the nucleus, of all contributions, only the first one remains, corresponding to the few-nucleon correlations or multi-quark states inside the nucleus. To see it clearly against the background of other components, one has to select events with high enough Q^2, when the already small cross-sections drastically diminish. The exact values of Q^2 at which one can neglect the contributions from other two mechanisms are not known *apriori*.

For this reason, it is important to carry out calculations of cumulative production at finite Q^2 which take into account all three above mentioned mechanisms. In this way, one expects to obtain predictions for the inclusive cross-sections, integrated over all Q^2, and thus directly related to the bulk of experimental data. Moreover, studying the relative weight of the contributions at different Q^2 one may determine the minimum Q^2 value, starting from which rescattering and compressed tube contributions can be safely neglected.

In this paper we do these calculations in the framework of the nucleon degrees of freedom. Estimates have shown that this description gives reasonable results at not too large x, provided one uses the correct relativistic kinematics [3]. We limit ourselves to the cumulative region $1 < x < 2$, the easiest from both the experimental and theoretical points of view. In this region, it is commonly assumed that one has to study hard interactions of only two nucleons inside the nucleus. Since inclusion of relativistic spins presents serious difficulties for nuclei with $A > 2, 3$, we simplify our approach by taking nucleons scalar. This approximation may certainly change the results by a factor of the order 2 to 3. However, in the cumulative region, as x grows from 1 to 2, the cross-sections fall by several orders of magnitude. On this scale, a factor of the order of unity plays a minor role.

2. Short- and long-range contributions

It will be convenient to start with a reaction $\mathrm{e} + \mathrm{A} \to \mathrm{e} + \mathrm{X}$ whose cross-section can be standardly expressed via nuclear structure functions $F_{1,2}(x, Q^2)$, related to the well-known hadronic tensor $W_{\mu\nu}$. To find the inclusive cross-section for the leptoproduction of cumulative nucleons, we shall integrate the cross-section over the final lepton momenta, present both structure functions as integrals over intermediate particles and then lift the integration over the momenta of the observed nucleon.

As mentioned above, at $1 < x < 2$, the incoming photon practically interacts with two nucleons (approximation of “pair correlations” for the short-range part), which will generally have large momenta, whereas the rest of intermediate particles will be slow, and their momenta will be neglected wherever possible.

The total contribution to the amplitude for the reaction $\gamma^* + \mathrm{A} \to \mathrm{N}(p_1) + \mathrm{N}(p_2) + \mathrm{X}$ will be given by a sum of four diagrams, shown in Figs. 1a–d. An important point is that they involve both short- and long-range distances in the nucleus, the latter corresponding to the rescattering contribution. We separate the long-range part following Ref. [7].

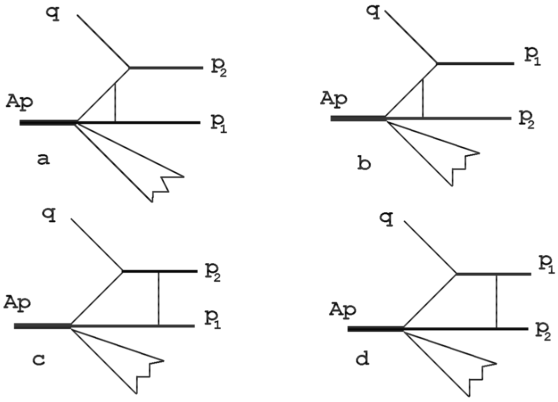

Fig. 1. Diagrams illustrating different mechanisms of cumulative production: (a) spectator, (b) direct and (c and d) compressed tube plus rescattering.

The high-energy factor, H_μ, depending on the initial active nucleon momenta $k_{1,2}$, contains denominators coming from the virtual nucleon propagators. In the amplitudes a and b in Fig. 1, these denominators do not vanish. However, the common denominator $m^2 - (q + k_1)^2$ in amplitudes c and d can vanish. It can be presented in the form $a(u + \alpha - \mathrm{i}\,0)$, where a and u are finite at $\mathbf{k}_{1,2} = 0$. a is positive, u can vanish and α is linear in $\mathbf{k}_{1,2} = 0$. Thus we find the high-momentum factor H_+ as

$$H_+ = \tilde{H}_+ + \frac{B}{u + \alpha - \mathrm{i}\,0}\,, \tag{1}$$

where $\tilde{H}_+$ and B are finite at $\mathbf{k}_{1,2} = 0$. Therefore, W_{++} will contain a singular term

$$B^2 \frac{1}{u + \alpha - \mathrm{i}\,0}\;\frac{1}{u + \alpha' + \mathrm{i}\,0}\,, \tag{2}$$

where α' is linear in $\mathbf{k}'_{1,2} = 0$ on which the conjugated amplitude depends. We transform (2) into

$$B^2 \Big[\mathrm{Re}\,\frac{1}{(u - \mathrm{i}\,0)^2} + 2\pi^2 E(q) \delta(u) \delta(\mathbf{q}, \mathbf{k}_1 - \mathbf{k}_2) \Big]\,, \tag{3}$$

where $E(q)$ is the energy of the nucleon with momentum $\mathbf{q}$.

The first term in (3) and the rest of W_{++}, and also W_{yy}, do not depend on small nuclear momenta. Integration over them will then produce a factor ρ_0 with a meaning of the probability for the two active nucleons to be at the same point in the nucleus. This contribution corresponds to the short-range part of the cumulative mechanism.

The second term in (3) describes the long-range mechanism, corresponding to the rescattering. In this case, integration over small nuclear momenta will give a factor l, which has the meaning of the average dimension of the nucleus travelled by the nucleon in rescattering.

3. Photoproduction of cumulative protons and the choice of the potentials

Expressions for H contain the relativistic potential $v(k)$ which describes the pair interaction between the nucleons. A variety of relativistic potentials have been proposed which correctly describe the nucleon-nucleon scattering data at moderate energies. However, we actually need the high-energy asymptotics of the potential, which is not fixed by these data well enough. So we shall recur to a different approach and use the fact that in the production of cumulative protons by real photons the cross-sections are directly related to the internucleon potential. Therefore, using the experimental data on the photoproduction, one can extract an effective internucleon potential in a straightforward manner.

In our approximation, we find the inclusive cross-section for the photoproduction of a proton off the deuteron in the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \psi^2(0)\frac{e^2 p_{1\perp}^2}{64\pi m^3 E^2}\left(\frac{v(t_1/2)}{t_1}\right)^2 , \tag{4}$$

where $t_1 = t - m^2$ and $\psi(r)$ is the deuteron wave function in the coordinate space. Of course, one must remember that our formulae may pretend to be valid only for the values of x sufficiently higher than unity. This is achieved in the emission of protons in the backward hemisphere relative to the direction of the incoming photon. Present data mostly refer to forward directions. Only the data in Ref. [9] on the deuteron at the incident photon energy 4.0 GeV and $\theta_1 = 90^\circ$ involve sufficiently high values of x. These data show that with more or less standard nuclear potentials borrowed from low energy physics one cannot desribe the experiment above $E = 1$ GeV. The experimental data for $s^{11}\mathrm{d}\sigma/\mathrm{d}t$ show a clear plateau above 1.4 GeV in accordance with the QCD scaling law, whereas the predictions from the meson-exchange potentals steadily rise [9]. This is not unexpected, since one cannot hope that the relatively low-energy parametrizations for the potential will work sufficiently well at high momentum transfers involved.

We determine $v(t)$ from Eq. (4) putting the experimental data from [9] into its left-hand side. The latter can be satisfactorily fit by the expression

$$s^{11}\frac{\mathrm{d}\sigma}{\mathrm{d}t}\,(kbn * \mathrm{GeV}^{20}) = 0.4 + 0.147741\,\mathrm{e}^{-E^2} + 1.47116\,\mathrm{e}^{-2E^2} \qquad (5)$$

valid for $E = 0.2$ to 4.0 GeV. We take the value for the deuteron wave function at the origin as $\psi(0) = 3.131\,10^{-2}$ GeV$^{3/2}$.

4. *Numerical results*

Using the effective potential extracted from the photoproduction data, we calculated the total leptoproduction cross-sections for the initial lepton energy of 6 GeV in the backward hemisphere on various nuclear targets. To cure the divergence at $Q^2 = 0$, we retained the small lepton mass m_l and separated the leading $\log(1/m_l)$ contribution as the standard Weiszäcker-Williams approximation term. The A dependence of our cross-sections is concentrated in factors ρ_0 and l for short- and long-range contributions, correspondingly. These factors were calculated in the standard manner, neglecting correlations in the nuclear ρ-matrix and using the Woods-Saxon nuclear density.

Our results for the inclusive cross-section for Pb target are presented in Fig. 2 for various angles in the backward hemisphere. To see the relative importance of

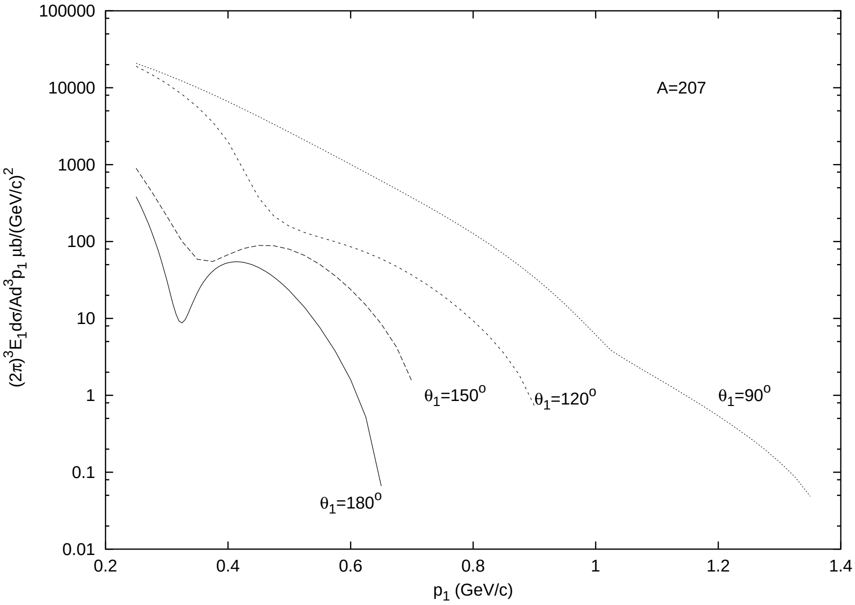

Fig. 2. Inclusive nucleon leptoproduction cross-sections on Pb *at the incident energy 6* GeV *into the backward hemisphere.*

different production mechanisms, we show in Figs. 3 – 6 differential cross-sections in both $\mathbf{p}_1$ and Q^2 at $Q^2 = 1$ and 4 $(\mathrm{GeV}/c)^2$ and 90° and 180° on the Pb target. Solid and dashed lines correspond to the total and spectator contributions. The

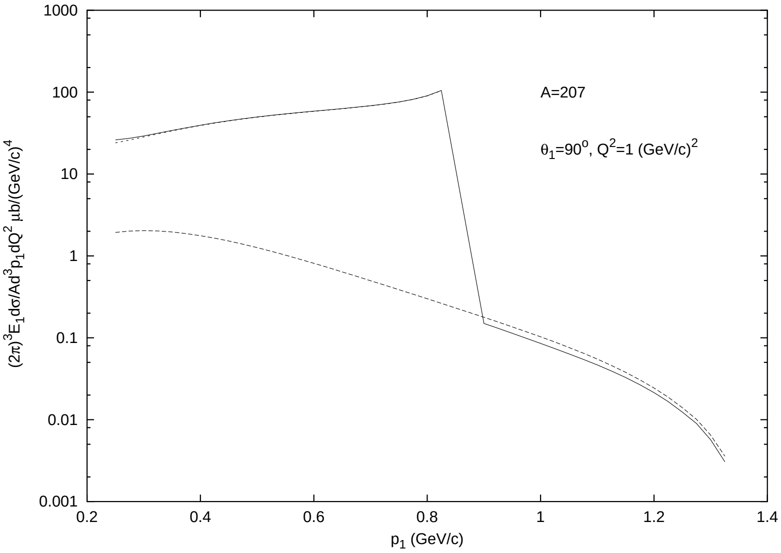

Fig. 3. Double differential cross-sections in $\mathbf{p}_1$ *and* Q^2 *for nucleon leptoproduction on* Pb *at the incident energy 6* GeV, *emission angle* $\theta_1 = 90^\circ$ *and* $Q^2 = 1$ $(\mathrm{GeV}/c)^2$. *See notations in the text.*

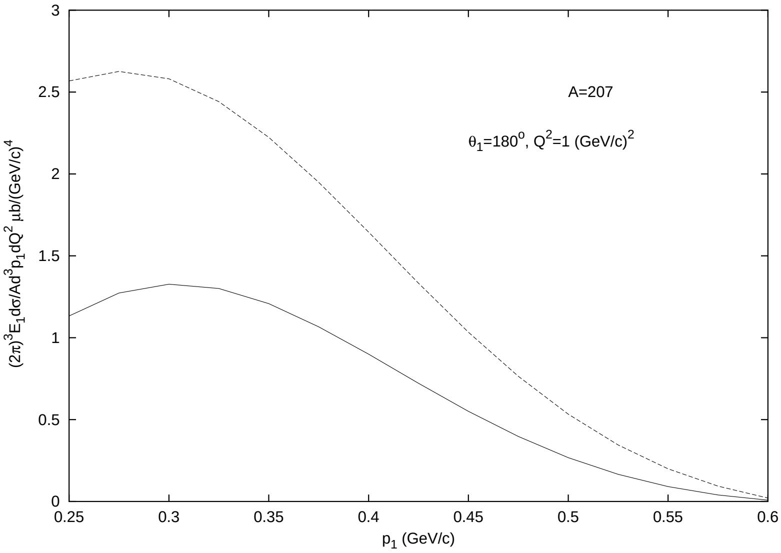

Fig. 4. Same as Fig. 3 but at $\theta_1 = 180^\circ$ *and* $Q^2 = 1$ $(\mathrm{GeV}/c)^2$.

rescattering contribution (shown by short-dashed lines) practically coincides with the total when it is not zero (at 90° and $p_1 < 0.8$ GeV/c). The sharp drop of the curves at the point where rescattering vanishes is an *artefact* of our neglecting the binding, so that our results cannot be trusted in this region.

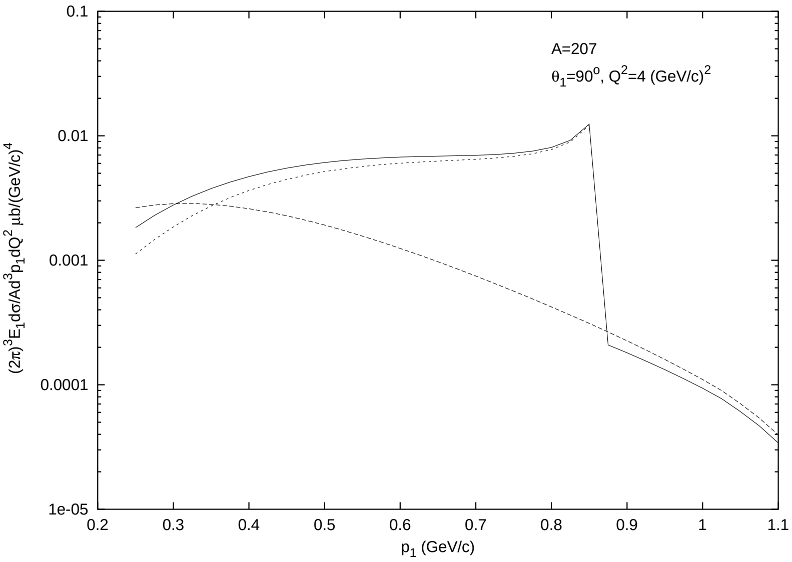

Fig. 5. Same as Fig. 3 but at $\theta_1 = 90°$ *and* $Q^2 = 4$ (GeV/c)^2.

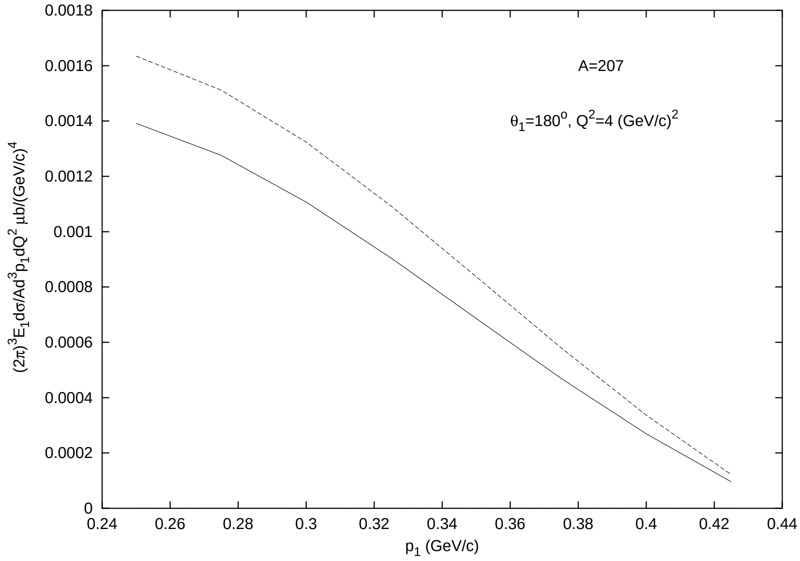

Fig. 6. Same as Fig. 3 but at $\theta_1 = 180°$ *and* $Q^2 = 4$ (GeV/c)^2.

As a general result, we find that for the studied values of Q^2 in the kinematical region where the rescattering is possible, it either completely dominates (at $Q^2 \sim 1$ $(\mathrm{GeV}/c)^2$ or lower) or gives a contribution of the same order as the spectator mechanism. The contribution of the direct mechanism has been always found quite small. At comparatively low values of Q^2, the short range contribution from the final interaction ("compressed tube" mechanism) is also of the same order as the spectator mechamism and negative (see Fig. 4). However, with the growth of Q^2, its relative weight diminishes, as expected (see Fig. 6). We have also found that the relative contribution from the Weizsäcker-Williams term (leading in $\log(1/m_l)$) is generally quite small, except at highest values of p_1 for a given angle, where it becomes of the same order or even several times larger than the rest short-range contribution.

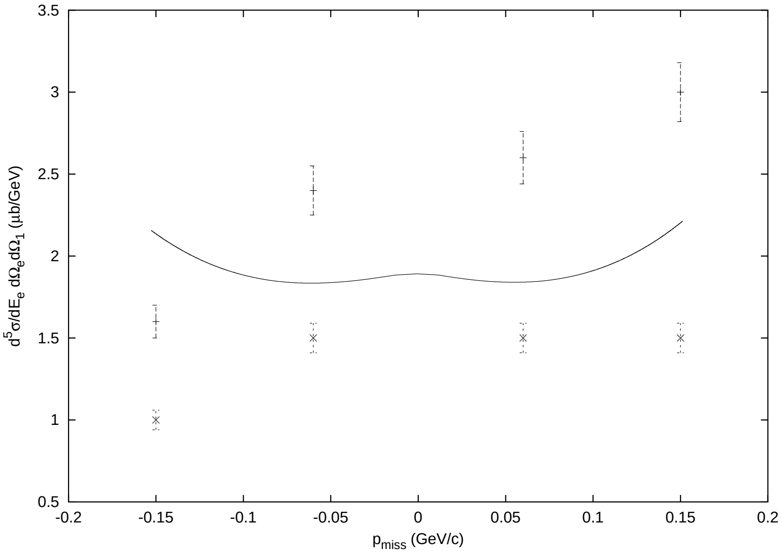

Fig. 7. Double inclusive cross-section for the reaction $\mathrm{e}+{}^{16}\mathrm{O} \to \mathrm{e}'(\mathrm{p}_e) + \mathrm{N}(p_1) + \mathrm{X}$ *at* $E = 2.4$ GeV, $Q^2 = 0.8$ $(\mathrm{GeV}/c)^2$ *and* $q_0 = 0.439$ GeV *for the nucleons ejected into the backward hemisphere. Positive* $p_{\mathrm{miss}} = |\mathbf{q} - \mathbf{p_1}|$ *correspond to* $\phi_1 = 180°$, *negative* p_{miss} *to* $\phi_1 = 0$. *Experimental points are from [10]. Lower (upper) points correspond to the* $1\mathrm{p}_{1/2}$ *(*$1\mathrm{p}_{3/2}$*) nuclear level.*

To have some comparison with existing experimental data on A(e,e′p) reaction, we also calculated the double-inclusive cross-section $\mathrm{d}\sigma/(\mathrm{d}E'\mathrm{d}\Omega\mathrm{d}\Omega_1)$ on the ${}^{16}\mathrm{O}$ target, where Ω and Ω_1 are angular variables for the final lepton and nucleon, respectively. We have taken $E = 2.4$ GeV, $Q^2 = 0.8$ $(\mathrm{GeV}/c)^2$ and $q_0 = 0.439$ GeV corresponding to the experimental kinematics in Ref. [10]. Our results for the backward hemisphere are shown in Fig. 7 together with experimental points for protons knocked out of the $1\mathrm{p}_{1/2}$ shell (lower points) and $1\mathrm{p}_{3/2}$ shell (upper points). The recoil nucleon momenta are small in this kinematics, so that our approximation of neglecting the binding does not seem to be well justified. However, the agreement is unexpectedly good: our calculated cross-sections well correspond to the experimental ones averaged over the two nuclear levels.

5. *Conclusions*

Our results show that in the kinematical region where rescattering is possible, its contribution cannot be neglected unless Q^2 is fixed and large. For the inclusive cross-section integrated over Q^2, rescattering dominates the cross-section in this region. Of the other mechanisms, the direct one has been found to be completely unimportant at $Q^2 > 1$ $(\mathrm{GeV}/c)^2$. However, at smaller Q^2 its contribution interferes with the other mechanisms to make the amplitude vanish at $Q^2 = 0$. The short range part of the final state interaction (the compressed tube mechanism) gives a negative contribution, of the same order as that of the spectator contribution, except at high values of Q^2, when it diminishes.

On the one hand, our results confirm the standard expectations that at very high Q^2 only the spectator mechanism survives, which allows to relate the cumulative production on different nuclear targets in a simple manner [8]. On the other hand, they show that for realistic not too large Q^2 and certainly for the inclusive cross-sections integrated over all Q^2, all mechanisms give contributions of comparable order and interfere. In particular, in the kinematical region accessible for rescattering, the latter gives a sizable (or even dominating) part of the contribution, which prevents using simple scaling arguments to fix the A-dependence.

Another important result is that the Weizsäcker-Williams approximation generally constitutes only a small part of the total cross-section. Thus leptoproduction cross-sections cannot be simply reduced to photoproduction ones.

The comparison with the existing data on A(e,e′p) reactions reveals a reasonable agreement, which supports validity of our approach.

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References

[1] D. I. Blokhintzev, JETP **33** (1957) 1295.

[2] I. A. Schmidt and R. Blankenbecler, Phys. Rev. D **15** (1977) 3321; **16** (1977) 1318.

[3] L. L. Frankfurt and M. I. Strikman, Phys. Rep. **76** (1981) 215.

[4] A. V. Efremov, Yad. Fiz. **24** (1976) 1208; V. V. Burov, V. K. Lukianov and A. I. Titov, Phys. Lett. B **67** (1977) 46; A. V. Efremov, A. B. Kaidalov, V. T. Kim, G. I. Lykasov and N. V. Slavin, Yad. Fiz. **47** (1988) 1364.

[5] M. I. Gorenshtein, G. M. Zinoviev and V. P. Shelest, Yad. Fiz. **26** (1977) 788.

[6] L. A. Kondratiuk and V. B. Kopeliovich, JETP Lett. **21** (1975) 40; V. B. Kopeliovich, JETP Lett. **23** (1976) 313, Phys. Rep. **139** (1986) 51; M. A. Braun and V. V. Vechernin, Sov. J. Nucl. Phys. **25** (1977) 676, **28** (1978) 753.

[7] M. A. Braun and V. V. Vechernin, J. Phys. G **19** (1993) 517 and 531.

[8] L. L. Frankfurt, M. I. Strikman, D. B. Day and M. Sargsyan, Phys. Rev. C **48** (1993) 2451.

[9] C. Bochna et al., Phys. Rev. Lett. **81** (1998) 4576.

[10] J. Gao et al., Phys. Rev. Lett. **84** (2000) 3265.

VIŠENUKLEONSKA LEPTOTVORBA

Proučavamo leptotvorbu nukleona u nukleanrim metama u stražnju polusferu na relativističkim podasimptotskim energijama i impulsima. Zanemarujemo spinove. Relativistički međunukleonski potencijal izvodimo iz odnosnih podataka za fototvorbu. Pokazujemo da je djelotvorno više mehanizama koji interferiraju. Ako je višestruko raspršenje moguće, računi pokazuju da ono i prevladava, izuzev za vrlo velike Q^2. Nalazimo da Weizsäcker-Williamsovo približenje dobro opisuje samo mali dio ukupnog udarnog presjeka. Usporedba s podacima za reakciju A(e,e$'$p) na $E = 2.4$ GeV pokazuje dobro slaganje.