

## CHIRAL PERTURBATION ON THE LIGHTFRONT<sup>1</sup>

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A new geometrical interpretation of chiral perturbation theory based on a topological QCD model is explained in pictures.

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### 1. Introduction

Chiral perturbation provides a framework to calculate corrections to nuclear observables such as nucleon masses, charge radii and form factors. When chiral symmetry is broken, intermediate mesons are created as Goldstone bosons which contribute to the tree-level values of relevant physical observables. Traditional chiral perturbation theory based on Feynman diagrams provides a means to estimate the corrections that come from the Goldstone bosons. Unfortunately, the calculations become intractable rapidly beyond the one-loop level. The current doctrine of particle physics is a bottom-up approach that tries to analyze the most fundamental units of nature in terms of elementary particles with all the associating dynamics and symmetries. The bottom-up approach of the particle picture (particularly in the framework of perturbative QCD) cannot solve the nucleons because such systems are characterized by highly non-linear many-body problems. Recent developments of non-critical dimension string theory in  $AdS^5 \times S^5$  with  $\mathcal{N} = 1$  shows some promises in solving the nucleons by virtue of avoiding the the difficult perturbative QCD calculations by effectively summing the infinite perturbative QCD series on the string world sheets. This approach is a paradigm shift from traditional wisdom in a sense that the bottom-up approach of particle physics that begins from elementary symmetries is replaced by the top-down approach of string theory that starts with global symmetries that hide the details on the particle level. Although supersymmetry is a beautiful symmetry, it may pose too many

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<sup>1</sup>This work is a written summary, with minor changes, of a talk presented at NAPP 2003, Dubrovnik, Croatia.

constraints when modeling the nucleons. The present work suggests an alternative top-down approach by summing the infinite series on the string worldsheets with no assumption of any supersymmetry nor extra dimensions. This research is work in progress. This paper explains the logic of this new approach in picture format. The introductory remarks, such as a brief review of chiral perturbation theory and recent works related to  $\chi$ pt on the lightfront, are skipped in this paper so that the main concept, i.e. topological QCD, is addressed right away.

## 2. Topological QCD

Geometrical interpretation of QCD is quite old. For instance, the gauge field of a nucleon is usually represented as a torus  $T^2$  in lattice QCD. In the same spirit, the disconnected graphs of the tadpoles can be modeled as  $S^2$  spheres. A gauge field is represented by a worldsheet called the gauge surface from now on. In the instant form, the geometry of the nucleon is represented in Fig. 1. The plane of initial data intersects the gaugeballs representing the vacuum. The Hamiltonian evolves the plane of initial data along the torus in the time-like direction. In this case, the plane sweeps through the gaugeballs so that the structure of the vacuum is not simple. On the other hand, the initial data in the front form are parameterized along a plane tangent to the the torus as shown in Fig. 2. The front form Hamiltonian evolves the plane of initial data around the torus in spiral directions with various winding modes. Since the gaugeballs are disconnected from the torus, the plane of initial data does not necessarily intersect the gaugeballs during the evolution. This picture recovers the typical result that lightfront QCD is uniquely endowed with a simpler vacuum.

At the end, the worldsheets of the gauge fields can be summed using path integrals. The integrals are categorized according to geni. A genus 0 surface has no hole, a genus 1 surface has 1 hole and so on. All topologically equivalent surfaces

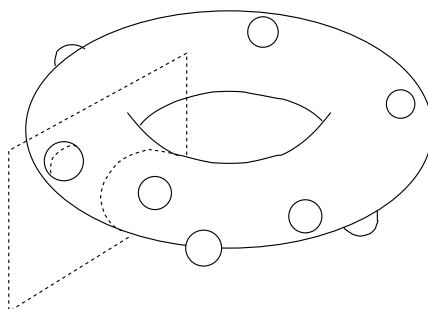


Fig. 1. The geometry of a nucleon in the instant form. The torus represents the nucleon and the spheres (gaugeballs) symbolize the vacuum. The plane is the surface of initial data that intersects the torus. The instant form Hamiltonian evolves plane of initial data along the torus in the time-like direction and sweeps it through the gaugeballs so that the instant form Hamiltonian sees a complicated vacuum.

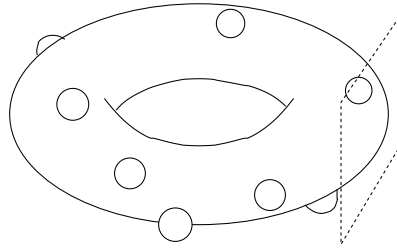


Fig. 2. The geometry of a nucleon in the front form. The torus represents the nucleon and the spheres (gaugeballs) symbolize the vacuum. The plane is the surface of initial data that is tangential to the torus. The front form Hamiltonian evolves the plane of initial data around the torus in a spiral direction so that the front form Hamiltonian sees a simpler vacuum.

are grouped together. Topological equivalence greatly simplifies the calculations. For instance, the geometry of all hairpins and sea quarks are equivalent to a plane and are effectively cancelled as shown in Figs. 3a and c.

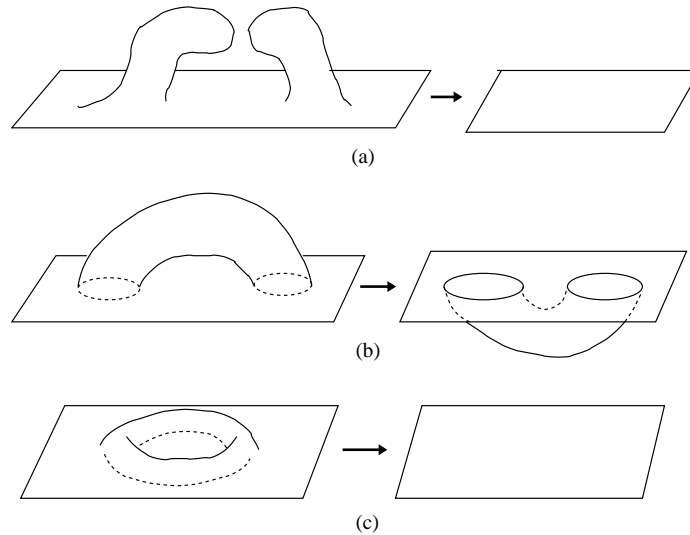


Fig. 3. Simplifications that comes from topological equivalence. (a) A hairpin, (b) a pion loop and (c) sea quarks.

### 3. Non-Abelian gauge as torsion

In non-Abelian gauge theory, the partial derivatives are modified by the addition of an extra term that comes from minimal coupling as in

$$D_\mu = \partial_\mu + i\tau_c A_\mu^c. \quad (1)$$

With the extra term, the total derivatives no longer commute,

$$D_\mu D_\nu - D_\nu D_\mu = \underbrace{\partial_\mu \partial_\nu - \partial_\nu \partial_\mu}_0 + i \tau_c F_{\mu\nu}^c. \quad (2)$$

The commutation of the total derivatives measures the shift of the gauge field around a loop. A non-zero shift indicates an implicit twist in the gauge configuration. In general relativity, the torsion-free condition is written as

$$\nabla_a \nabla_b f - \nabla_b \nabla_a f = 0, \quad (3)$$

where  $f$  is a scalar field. By analogy, we can interpret the right hand side of Eq. (2) geometrically as torsion. It is hoped that this torsion can be gauged away by an appropriate choice of transformation. Figure 4 shows heuristically how torsion can be canceled with a carefully chosen topology. In general relativity, the mass of the field is related to curvature as in

$$m \sim \frac{1}{r}, \quad (4)$$

so that the curvature of the edge of the folded surface is interpreted as the effective gluon mass. This interpretation is slightly different from that given in the

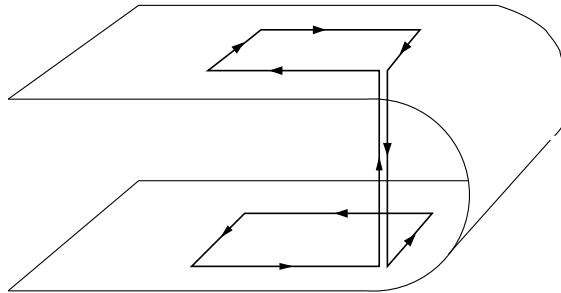


Fig. 4. The torsion vector from the loop on the upper surface is in the opposite direction of that on the lower surface so that torsion is effectively canceled in this topology.

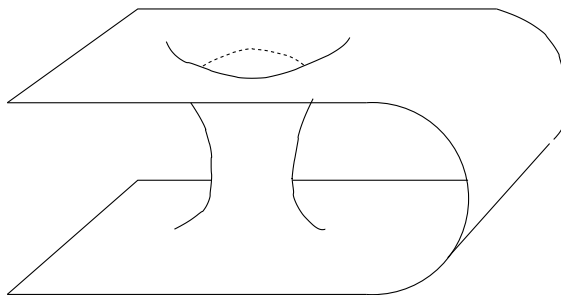


Fig. 5. A pion loop (shown as a wormhole) on a surface of torsion-free topology.

original talk at NAPP 2003 where the curvature was interpreted as the mass of the constituent quark. Figure 3b suggests the possibility that the pion loop can be constructed inside the gauge surface as shown in Fig. 5.

#### 4. Global torsion-free QCD topology

It is reasonable to assume that the presence of constituent quarks must affect the geometry of the gauge field. A good guess is that the global topology of a meson has two edges and that of a baryon has three. Although torsion (the non-Abelian part of the gauge theory) is gauged away by a suitable transform, it survives as a twist on the transformed field configuration. Figure 6 shows a sketch of the topological structures of the gauge fields of a meson and a baryon. The twist causes

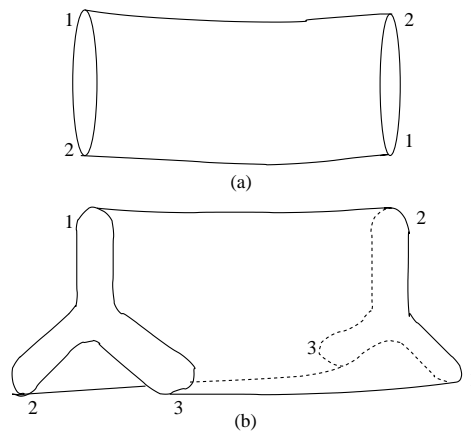


Fig. 6. The topological structures of the gauge fields of a meson and a baryon. The identifications of the boundaries are labeled.

the topology of the gauge field to have an unusual boundary condition similar to a Möbius strip. The topology of the baryon has been called the 3-Möbius strip and the “triniton” in previous presentations. In this picture, chiral perturbation on the lightfront is represented by a sum of these graphs of different geni as shown in Fig. 7.

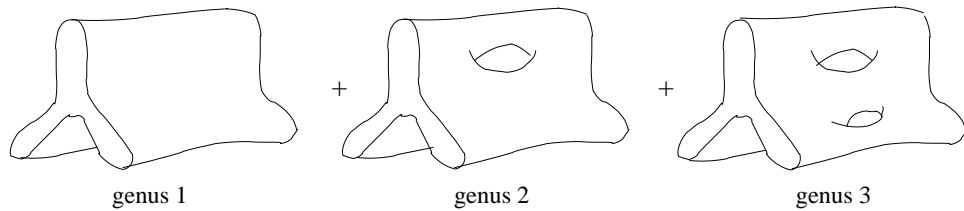


Fig. 7. Chiral perturbation on the lightfront is reduced to a sum of graphs of various geni.

## 5. Conclusion

The present work illustrates the concept of topological QCD in pictures. This research work is in progress. Many claims made in this presentation are based on intuition but are now being subjected to the rigor of mathematics. The contributions of the constituent quarks have not yet been included in the pictures of this work. These deficiencies will be amended in a future publication. Some colleagues have also pointed out that this research should aim at producing relevant predictions. It suffices to say that so far topological QCD is suggestive and is promising as a candidate for the analysis of the structure of the hadrons. Currently, a search is underway to find a suitable transform that maps the non-Abelian gauge field configuration to the torsion-free twisted  $n$ -Möbius surface. Then the mathematics of the path integrals over the surfaces of the twisted tori needs to be formulated. And lastly, it is hoped that this model can explain traditionally difficult theoretical problems such as confinement and asymptotic freedom. It is also hoped that new technology of non-perturbative QCD calculations will be discovered in the process.

### References

- [1] A. Tang, hep-th/0309206.

## KIRALNA SMETNJA NA VALNOJ FRONTI

Objašnjava se slikama novo geometrijsko tumačenje kiralne teorije smetnje zasnovano na topološkem modelu QCDA.