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VIRTUAL COMPTON SCATTERING AT LOW ENERGY AND THE GENERALIZED POLARIZABILITIES OF THE NUCLEON

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Virtual Compton scattering (VCS) $\gamma^{*}\mathrm{p} \rightarrow \gamma\mathrm{p}$ at low CM energy gives access to the generalized polarizabilities of the nucleon. These observables generalize the concept of electromagnetic polarizabilities to the case of a virtual photon. Dedicated VCS experiments have been performed at MAMI, Jefferson Lab and MIT-Bates. The experimental status is reviewed, including methods of analysis and physical results. The measurement of absolute ($\mathrm{ep} \rightarrow \mathrm{ep}\gamma$) cross sections allows the extraction of the two unpolarized VCS structure functions $P_{\mathrm{LL}} - P_{\mathrm{TT}}/\epsilon$ and P_{LT}, which are combinations of the generalized polarizabilities of the proton. Future prospects in the field of VCS at low energy are also presented.

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1. *Real and virtual Compton scattering and the concept of generalized polarizabilities*

We recall that the electric and magnetic polarizabilities α_{E} and β_{M} that are studied in real Compton scattering (RCS) [1] are a measure of the deformation of the nucleon structure under an applied electromagnetic (EM) field. Their smallness indicates that the proton is a very rigid object. The smallness of β_{M} relative to α_{E} is usually interpreted in terms of a cancellation between a diamagnetic (pion cloud) and a paramagnetic (quark core) contribution.

The concept of generalized polarizabilities (GP) was first established by P. Guichon et al. in 1995 [2], introducing observables that are defined not only at $Q^2 = 0$ but at any photon virtuality Q^2. Thus the GPs measure the EM deformation locally inside the nucleon, with a scale given by Q^2, and they can be seen as “deformed form factors". The full formalism [3, 4] leads to six independent GPs in the lowest order: two scalar ones $\alpha_{\rm E}(Q^2)$ and $\beta_{\rm M}(Q^2)$ (giving the above mentioned RCS polarizabilities at $Q^2 = 0$) and four spin GPs.

Virtual Compton scattering (VCS) can be accessed via photon electroproduction $\mathrm{ep} \to \mathrm{ep}\gamma$. The amplitude of this process is decomposed into the coherent sum of the Bethe-Heitler (BH), the VCS Born, and the VCS non-Born amplitudes. The (BH+Born) cross section is known and depends only on the proton EM form factors. The VCS non-Born term contains the unknown physics, i.e. the GPs which are the goal of the measurements.

2. *Methods of analysis*

Up to now, our knowledge of GPs comes from the measurement of cross section for the (unpolarized) $\mathrm{ep} \to \mathrm{ep}\gamma$ process. The main kinematic variables for physical analysis are the four-momentum transfer squared Q^2, the available energy $\sqrt{s}$ in the (γp) centre-of-mass (CM) and the CM variables of the outgoing photon: its momentum q' and its spherical angles θ and φ with respect to the virtual photon momentum vector $\vec{q}$. There are presently two methods to extract GPs, depending on the range of CM energy. A short description of these methods is given below.

2.1. *The Low energy theorem*

Below the pion production threshold ($\sqrt{s} \leq (m_{\rm p} + m_\pi)$), one can use the Low energy theorem (LET), first applied to VCS by P. Guichon et al. [2]. In this context, one obtains the following expression for the (ep→epγ) cross section:

$$\mathrm{d}^5\sigma^{\rm EXP} = \mathrm{d}^5\sigma^{\rm BH+Born} + (\phi q') \times \left[v_1 \left(P_{\rm LL} - \frac{1}{\epsilon} P_{\rm TT} \right) + v_2 P_{\rm LT} \right] + \mathcal{O}(q'^2) \qquad (1)$$

where $\mathrm{d}^5\sigma^{\rm BH+Born}$ represents the Bethe-Heitler+Born cross section, $(\phi q')$ a phase space factor, and v_1 and v_2 are known kinematic factors [3]. $P_{\rm LL} - \frac{1}{\epsilon} P_{\rm TT}$ and $P_{\rm LT}$ are the structure functions to be measured, and ϵ is the virtual photon polarization. These structure functions are combinations of the GPs (see Ref. [3] for details); they contain the electric and magnetic generalized polarizabilities $\alpha_{\rm E}(Q^2)$ and $\beta_{\rm M}(Q^2)$ in the form

$$\begin{array}{lcl} P_{\rm LL} & \sim & \alpha_{\rm E}(Q^2) \\ P_{\rm LT} & \sim & \beta_{\rm M}(Q^2) + \text{spin GPs.} \end{array} \qquad (2)$$

To extract the structure functions from the data at fixed $|\vec{q}|$ and fixed ϵ, one basically makes a fit of the quantity $(\mathrm{d}^5\sigma^{\rm EXP} - \mathrm{d}^5\sigma^{\rm BH+Born})/(\phi q')$ to the expression of the bracketed term in Eq. (1), thus adjusting two free parameters. The main difficulty of the method is that the effect of GPs is small below the pion threshold; it amounts to no more than 10-15% of the cross section. Hence, the systematics of the experiments have to be well under control.

2.2. *The approach of dispersion relations*

Both below and above pion threshold and up to the $\pi\pi$N threshold, one can use the dispersion relation (DR) formalism developed by B. Pasquini et al. for real and virtual Compton scattering [5]. This allows VCS measurements in the $\Delta(1232)$ resonance region to be interpreted in terms of GPs. In this model, the GPs $\alpha_{\mathrm{E}}(Q^2)$ and $\beta_{\mathrm{M}}(Q^2)$ contain free parameters; experimentally, we fit their values at a given Q^2. The advantage of this method is that the (epγ) cross section is more sensitive to the GPs in the resonance region than below pion threshold.

3. *Experiments and results*

Table 1 summarizes the experiments performed as of today in the (ep$\to$epγ) channel to investigate the GPs. All of them use magnetic spectrometers to detect the final electron and proton, and they identify the single photon electroproduction reaction by the missing mass technique. These difficult experiments usually have to meet the following requirements: *i)* a good spectrometer resolution to separate the nearby missing mass peak due to (ep$\to$epπ^0); *ii)* measurements of small absolute cross sections with good accuracy, implying a precise calculation of the solid angle; *iii)* a careful study of experimental cuts (data are not free of background).

TABLE 1. Status of performed VCS experiments.

Ref.	Lab	Q^2 (GeV^2)	CM energy	Data taking	Status
[6]	MAMI (I)	0.33	$< (m_{\mathrm{N}} + m_\pi)$	1995+97	published
[7]	JLab E93-050	0.9, 1.8	up to 1.9 GeV	1998	final stage
[8]	Bates E97-03	0.05	$< (m_{\mathrm{N}} + m_\pi)$	2000	analysis
[9]	MAMI, $\vec{e}$ (II)	0.33	at $\Delta(1232)$	2002-2003	analysis

Figure 1 shows the fit of the structure functions using the LET approach, in the experiments performed at MAMI [6] and in the Hall A of the Thomas Jefferson National Accelerator Facility (JLab) [10]. The reasonable goodness of the fit confirms the validity of the LET even at high Q^2 ($\sim$ 0.9 and 1.8 GeV^2 at JLab). Figure 2 shows (ep$\to$epγ) cross sections from the JLab experiment [11], scanning the resonance region for the first time. The data are well reproduced by the DR model in the region of the $\Delta(1232)$ resonance. These data are used to fit the free parameters of the model and subsequently extract the structure functions $P_{\mathrm{LL}} - (1/\epsilon)P_{\mathrm{TT}}$ and P_{LT} at $Q^2 = 1$ GeV^2.

Figure 3 gives a representation of the structure functions measured by the mentioned experiments as a function of Q^2. On the left plot, the structure function P_{LL} is obtained from the measurement of $(P_{\mathrm{LL}} - P_{\mathrm{TT}}/\epsilon)$ by adding the value of

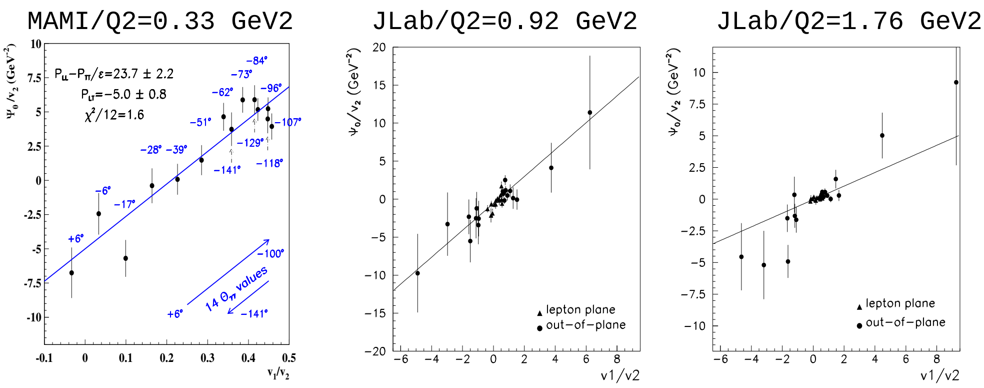

Fig. 1. A graphical representation of the polarizability fit by the LET method. The ordinate is the quantity $(\mathrm{d}^5\sigma^{\mathrm{EXP}} - \mathrm{d}^5\sigma^{\mathrm{BH+Born}})/(\phi q')/v_2$ *extrapolated to* $q' = 0$ *in each bin in* (θ, φ)*. The abscissa is the ratio of the kinematic factors* v_1 *and* v_2*. The fitted slope yields* $P_{\mathrm{LL}} - P_{\mathrm{TT}}/\epsilon$ *and the intercept yields the* P_{LT} *structure function.*

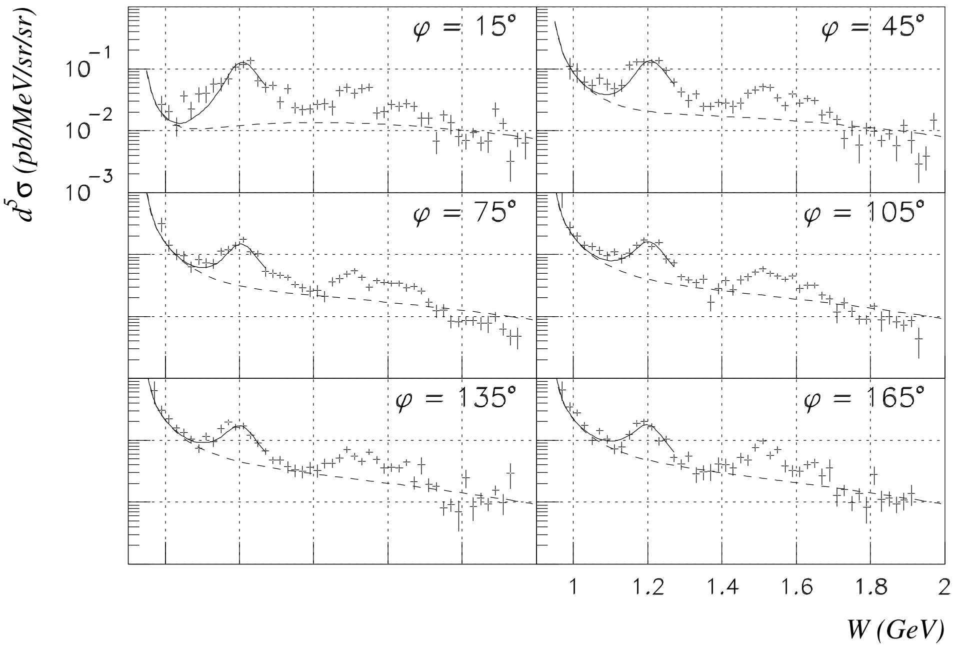

Fig. 2. Photon electroproduction cross section $\mathrm{d}^5\sigma/\mathrm{d}k'_{\mathrm{lab}}\mathrm{d}[\Omega_{\mathrm{e}}]_{\mathrm{lab}}\mathrm{d}[\Omega_{\mathrm{p}}]_{\mathrm{CM}}$ *measured in the JLab experiment [11] for six bins in* φ*, at* $Q^2 = 1$ *GeV*^{2} *and* $\cos\theta_{\gamma^*\gamma\mathrm{CM}} = -0.975$*. The solid curve is the DR model prediction, and the dashed curve is the (BH+Born) calculation.*

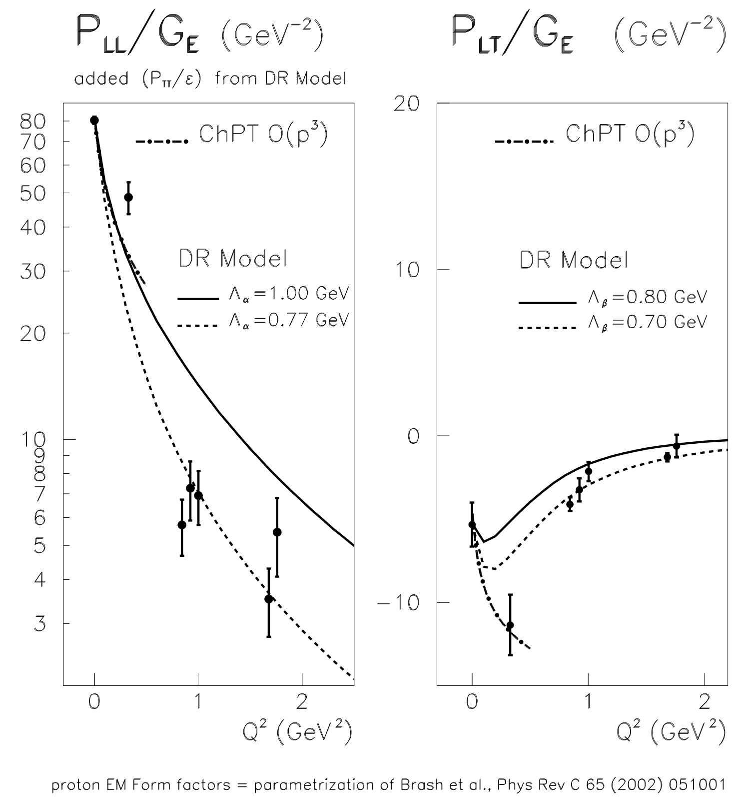

Fig. 3. World results on VCS structure functions (statistical errors only). The data points at $Q^2 = 0$ are deduced from the TAPS experiment [1], the ones at $Q^2 = 0.33$ GeV^2 are from MAMI [6] and the other ones are from the JLab VCS E93-050 experiment including both analysis methods [10, 11]. Some points have been shifted in abscissa for clarity. The dash-dotted curve is the calculation of the chiral perturbation theory [13], the solid and dashed curves are the DR model predictions for different values of the free parameters (Λ_α and Λ_β, related to the GPs $\alpha_{\mathrm{E}}(Q^2)$ and $\beta_{\mathrm{M}}(Q^2)$; see Ref. [5] for more details).

P_{TT}/ϵ calculated in the DR model, at the value of Q^2 and ϵ of each experiment[1]. After dividing by the proton electric form factor G_{E}^p, one gets the Q^2-behaviour of the generalized polarizability $\alpha_{\mathrm{E}}(Q^2)$, up to a constant factor. On the right plot, $P_{\mathrm{LT}}/G_{\mathrm{E}}^p$ is proportional to the generalized polarizability $\beta_{\mathrm{M}}(Q^2)$, neglecting a small term containing the spin GPs. The JLab data [12] are in good agreement with the DR model (and parameter values of the dashed curve). At lower Q^2, the chiral perturbation theory calculation to order p^3 [13] is in good agreement with the MAMI data for P_{LT} and $P_{\mathrm{LL}} - P_{\mathrm{TT}}/\epsilon$. Measurements tend to confirm

[1]This is a model-dependent operation, since P_{TT} has not been measured yet.

the turn-over of $P_{\rm LT}$ that is predicted by most models in the low Q^2 region. This turn-over reflects the competing effects of para- and dia-magnetism in $\beta_{\rm M}(Q^2)$, and the expected Bates measurement at $Q^2 = 0.05$ GeV^2 [8] is of great interest in this regard.

4. Future prospects and conclusions

Table 2 summarizes the foreseen experiments in the field of VCS at low energy. Most projects aim at a deeper disentangling of the individual GPs, by polarization measurements or Rosenbluth-type separations.

TABLE 2. Experimental prospects of VCS at low energies. $SSA(DSA)$= single (double) spin asymmetry.

Type of experiment	Lab	Meas. $\rightarrow$ structure functions	$\sqrt{s}$	Q^2 (GeV^2)
Double polarization	MAMI	DSA $\rightarrow$ separate the six GPs	$< \pi$N	0.33
	& Bates	DSA	$< \pi$N	0.20
Polarized beam	JLab	$\mathrm{d}^5\sigma \rightarrow P_{\rm LL} - P_{\rm TT}/\epsilon$ and $P_{\rm LT}$	Δ	≤ 4
		$SSA \rightarrow$ test inputs to DR model		
Unpolarized	MAMI	$\mathrm{d}^5\sigma \rightarrow \epsilon$ separation $\rightarrow P_{\rm TT}, P_{\rm LL}$	$< \pi$N	0.33
	& Bates		$< \pi$N	0.20

Virtual Compton scattering is an active field of research; generalized polarizabilities are new observables providing an original way to study nucleon structure, and there is an ongoing effort to learn more about them both experimentally and theoretically. Experiments make use of low- and high-energy machines (Bates, MAMI, JLab), of polarization degrees of freedom, and they exploit the versatility of methods to extract GPs at low and high Q^2. These observables are also predicted by many theoretical approaches: chiral perturbation theory [13, 14], quark models [15], linear σ-model [16], dispersion relations [5], effective lagrangian model [17], ..., most calculations being valid at rather low Q^2. In that view, the results of the JLab VCS E93-050 experiment should stimulate new calculations of the GPs at high Q^2.

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VIRTUALNO COMPTONOVO RASPRŠENJE NA NISKIM ENERGIJAMA I POOPĆENE POLARIZIVOSTI NUKLEONA

Virtualno Comptonovo raspšenje (VCR) $\gamma^* \mathrm{p} \to \gamma \mathrm{p}$ na niskim energijama u CMS pruža mogućnost određivanja poopćenih polarizivosti nukleona. Te opservable poopćuju pojam elektromagnetske polarizivosti za slučaj virtualnih fotona. Usmjerena mjerenja VCR načinjena su u MAMI, Jefferson Labu i u MIT-Batesu. Dajemo pregled mjerenja, te metoda analize podataka i ishode. Mjerenja apsolutnih udarnih presjeka reakcije (ep$\to$epγ) dozvoljava izvođenje dviju strukturnih funkcija VCR bez polarizacije $P_{\mathrm{LL}} - P_{\mathrm{TT}}/\epsilon$ i P_{LT}, koje su kombinacije poopćenih polarizivosti protona. Predstavljaju se izgledi budućih istraživanja VCR a niskim energijama.