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RESCATTERING CONTRIBUTIONS TO FINAL STATE INTERACTIONS IN (e,e′p) REACTIONS

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A semiclassical model is employed to study the effects of rescattering on (e,e′p) cross sections. We consider a two-step process with the propagation of an intermediate nucleon and use Glauber theory to account for the effects of N–N scattering. This calculation has relevance for the analysis of data at high missing energies. Of particular interest is the E97-006 experiment done at JLab. It is found that rescattering is strongly reduced in parallel kinematics and that the excitation of nucleon resonances is likely to give important contributions to the final-state interactions in the correlated region.

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1. Introduction and motivations

Short-range correlations strongly influence the dynamics of nuclear systems. The repulsive core at small internucleon distances has the effect of removing the nucleons from their shell-model orbitals, producing pairs of nucleons with high and opposite relative momenta. This results in spreading out a sizable amount of spectral strength, about 10 − 15% [1], to very high missing energies and momenta and in increasing the binding energy of the system. Theoretical studies of the distribution of short-range correlated nucleons for finite nuclei have been carried out in Ref. [2] and by Benhar et al. [3]. These calculations suggest that most of this strength is found along a ridge in the momentum-energy ($k-E$) plane that spans several hundreds of MeV/c (and MeV). The most probable energy is the one of a free moving nucleon but shifted by a constant term that represents the two-hole potential well in which the correlated pair moves. Evidence of two-nucleon structures, similar to the deuteron, inside finite nuclei has also been discussed using the variational approach in Ref. [4] and found to be driven by short-range and tensor correlations. We note that short-range effects alone do not completely explain the depletion of single-particle orbitals near the Fermi energy [5], which require a proper description of low-energy collective modes [6]. However, a proper understanding of short-range effects remains of great importance for the understanding of nuclear structure [7].

Experimentally, (e,e′p) reactions have been used for a long time to determine spectral functions at small missing energy. Past measurements in the region of interest to short-range correlations have been limited due to the enormous background that is generated by final-state interactions (FSI). The issue of how to minimize the FSI has been addressed in Ref. [8]. There, it is shown that exclusive (e,e′p) cross sections are dominated by two-step processes like the one depicted in Fig. 1. This becomes particularly relevant when perpendicular kinematics is employed to probe the regions of small spectral strength. In Ref. [8], it was suggested that the contribution of rescattering can be diminished in parallel kinematics. New data was subsequently taken in these conditions at Jefferson Lab [9].

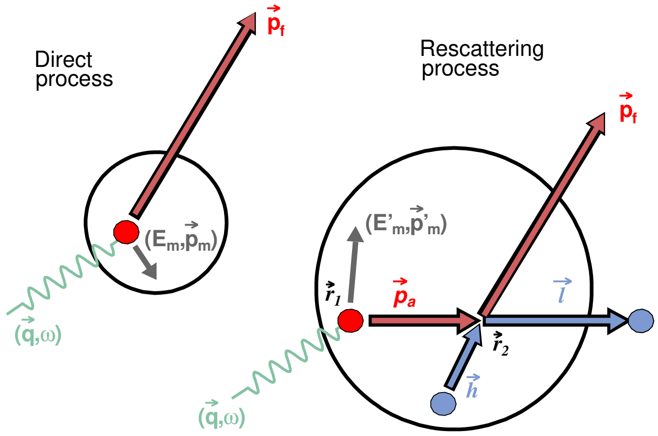

Fig. 1. Schematic representation of the direct knockout of a proton, given by the PWIA (left) and the contribution from a two-step rescattering (right). In the latter a proton or neutron is emitted with momentum $\boldsymbol{p}_a$ and different missing energy and momentum $(E'_{\rm m}, \boldsymbol{p}'_{\rm m})$. Due to a successive collision, a proton is finally detected with the same momentum $\boldsymbol{p}_{\rm f}$ seen in the direct process.

The theoretical calculation of the rescattering yield has been addressed in Ref. [10]. This calculation was based on the semiclassical model of Ref. [11] and intended to be applied at lower missing energies. This contribution reports about an ongoing work aimed to extend these calculations to the kinematics of interest for the study of short-range correlations.

2. Model

At large E_m, appreciable contributions to the experimental yield come from two-step mechanisms, in which a reaction (e,e′a) is followed by a scattering process from a nucleon in the nuclear medium, N′(a,p)N″, eventually leading to the emission of the detected proton. In general, a may represent a nucleon or another possible intermediate particle, as a Δ excitation. In the following we will also use the letter a to label the different open channels.

Following the semiclassical approach proposed in Refs. [11] and [10], we write the contribution to the cross section coming from rescattering through the channel a as

$$\frac{\mathrm{d}^6\sigma^{(\mathrm{a})}_{\mathrm{rescat}}}{\mathrm{d}E_0\ \mathrm{d}\Omega_{\hat{k}_\mathrm{o}}\mathrm{d}E_\mathrm{f}\ \mathrm{d}\Omega_{\hat{p}_\mathrm{f}}} = \int\limits_V \mathrm{d}\boldsymbol{r}_1 \int\limits_V \mathrm{d}\boldsymbol{r}_2 \int\limits_0^\omega \mathrm{d}T_\mathrm{a}\rho_\mathrm{N}(\boldsymbol{r}_1)\, \frac{K\ S^h_\mathrm{N}(p_\mathrm{m},E_\mathrm{m})\ \sigma^{cc1}_{\mathrm{eN}}}{A\ (\boldsymbol{r}_1-\boldsymbol{r}_2)^2} \times P_\mathrm{T}(p_\mathrm{a},\boldsymbol{r}_1,\boldsymbol{r}_2)\rho_{\mathrm{N}'}(\boldsymbol{r}_2)\, \frac{\mathrm{d}^3\sigma_{\mathrm{aN}'}}{dE_\mathrm{f}\ d\Omega_{\hat{p}_\mathrm{f}}}\ P_\mathrm{T}(p_\mathrm{f},\boldsymbol{r}_2,\infty)\ , \quad (1)$$

where $(E_\mathrm{o},\boldsymbol{k}_\mathrm{o})$ and $(E_\mathrm{f},\boldsymbol{p}_\mathrm{f})$ represent the four-momenta of the outgoing electron and proton, respectively. Equation (1), assumes that the intermediate particle a is generated in plane-wave impulse approximation (PWIA) by the electromagnetic current at a point $\boldsymbol{r}_1$ inside the nucleus. Here $K = |\boldsymbol{p}_\mathrm{a}|E_\mathrm{a}$ is a phase space factor, $S^h_\mathrm{N}(p_\mathrm{m},E_\mathrm{m})$ the spectral function of the hit nucleon a and $\sigma^{cc1}_{\mathrm{eN}}$ is the off-shell electron-nucleon cross section [12]. The transparency factor $P_\mathrm{T}(p,\boldsymbol{r}_1,\boldsymbol{r}_2)$ gives the probability that the struck nucleon a propagates to a second point $\boldsymbol{r}_2$, where it scatters from the nucleon N′ with cross section $\mathrm{d}^3\sigma_{\mathrm{aN}'}$. The whole process is depicted in Fig. 1.

In the calculation described in Sec. 3, we will only consider the channels in which a is either a proton or a neutron. It is clear that other channels are expected to be important. In particular, the excitation of the Δ resonance is also seen to contribute from the preliminary data of the E97-006 experiment [9].

Equation (1) is a seven-fold integral that can be conveniently evaluated with Monte Carlo techniques, once the terms in the integrand are known. In the following, we describe the calculation of the cross section $\mathrm{d}^3\sigma_{\mathrm{aN}'}$ and of the transmission probability P_T.

2.1. Evaluation of the in-medium nucleon-nucleon rate

For the present purposes, the spectral distribution of the hit nucleon, N′, can be appropriately described by the free Fermi gas distribution. The cross section is, therefore, computed for a nucleon a travelling in symmetric nuclear matter at a given density ρ_{NM}. The nucleon N′ must have a momentum $\boldsymbol{h}$ smaller than the Fermi momentum $k_\mathrm{f} = (3\pi^2\rho_{\mathrm{NM}}/2)^{1/3}$. Among the nucleons involved in the process, $\boldsymbol{p}_\mathrm{f}$ will refer to the outgoing proton while the others can be either neutrons or protons depending on the channel a. The finite size effects are eventually included in Eq. (1) using the local density approximation, that is, by evalutaing the cross section for the density at the point $\boldsymbol{r}_2$.

The probability per unit time of an event leading to the emission of a proton with momentum $\boldsymbol{p}_{\rm f}$ is obtained by imposing the Pauli constraints and integrating over the unobserved momenta $\boldsymbol{h}$ and $\boldsymbol{l}$. Employing a relativistic notation,

$$\begin{aligned}
\frac{{\rm d}^3P}{{\rm d}p_{\rm f}{\rm d}\Omega_{\hat{p}_{\rm f}}} &= 2\,\vartheta(p_{\rm f}-k_{\rm f})\; L^3 \int\!\!\int_{L^3} \frac{{\rm d}\boldsymbol{h}\,{\rm d}\boldsymbol{l}}{(2\pi)^6}\;\vartheta(k_{\rm f}-h)\vartheta(l-k_{\rm f})\;W_{\rm I} \\
&= 2\,p_{\rm f}^2\;\vartheta(p_{\rm f}-k_{\rm f}) \int_{L^3} \frac{{\rm d}\boldsymbol{h}}{(2\pi)^3}\vartheta(k_{\rm f}-h)\vartheta(l-k_{\rm f})\frac{m_{\rm a}\,m_{\rm h}}{E_{\rm a}(p_{\rm a})E_{\rm h}(h)} \qquad (2)\\
&\qquad \times\frac{|\mathcal{M}(s,t,u)|^2}{4\pi^2}\frac{m_{\rm p}\,m_{\rm l}}{E_{\rm p}(p_{\rm f})E_{\rm l}(l)}\delta(E_{\rm a}+E_{\rm h}-E_{\rm f}-E_{\rm l})\bigg|_{\boldsymbol{l}=\boldsymbol{p}_{\rm a}+\boldsymbol{h}_{\rm f}-\boldsymbol{p}_{\rm f}}\,,
\end{aligned}$$

where L^3 is the volume of a normalization box and $E_{\rm N}(p) = (p^2 + m_{\rm N}^2)^{1/2}$. In Eq. (2), $W_{\rm I}$ is the probability per unit time for the event $p_{\rm a}^\mu + h^\mu \rightarrow p_{\rm f}^\mu + l^\mu$ which is expressed in terms of the Lorentz invariant amplitude $\mathcal{M}(s,t,u)$ [13]. In the present work, we use the in vacuum values for $\mathcal{M}(s,t,u)$ and extract them from the SAID phase shift data analysis [14].

The in-medium scattering rate is finally related to Eq. (2) by

$$\frac{{\rm d}^3\;\sigma_{\rm aN'}}{dE_{\rm f}\;d\Omega_{\hat{p}_{\rm f}}} = \frac{E_{\rm a}}{\rho_{\rm N'}p_{\rm a}}\frac{E_{\rm f}}{p_{\rm f}}\frac{{\rm d}^3P}{{\rm d}p_{\rm f}{\rm d}\Omega_{\hat{p}_{\rm f}}}\,. \qquad (3)$$

2.2. Transparency factor

According to Glauber theory, the probability $P_{\rm T}$ that a proton struck at $\boldsymbol{r}_1$ will travel with momentum $\boldsymbol{p}$ to the point $\boldsymbol{r}_2$ without being rescattered is given by

$$\begin{aligned}
P_{\rm T}(p,\boldsymbol{r}_1,\boldsymbol{r}_2) = \;\; &\exp\bigg\{-\int_{z_1}^{z_2}{\rm d}z\Big[g_{\rm pp}(|\boldsymbol{r}_1-\boldsymbol{r}|)\;\tilde{\sigma}_{\rm pp}(p,\rho(\boldsymbol{r}))\;\rho_{\rm p}(\boldsymbol{r}) \qquad (4)\\
&+\;g_{\rm pn}(|\boldsymbol{r}_1-\boldsymbol{r}|)\;\tilde{\sigma}_{\rm pn}(p,\rho(\boldsymbol{r}))\;\rho_{\rm n}(\boldsymbol{r})\Big]\bigg\}\,,
\end{aligned}$$

where the z axis is chosen along the direction of propagation $\boldsymbol{p}$, the impact parameter $\boldsymbol{b}$ is defined so that $\boldsymbol{r} = \boldsymbol{b} + z\hat{p}$, and z_1 (z_2) refer to the initial (final) position. The in medium total cross sections $\tilde{\sigma}_{\rm pp}(p,\rho)$ and $\tilde{\sigma}_{\rm pn}(p,\rho)$ have been computed in Ref. [11] up to energies of 300 MeV and account for the effects of Pauli blocking, Fermi spreading and the velocity dependence of the nuclear mean field. For energies above 300 MeV, these have been extended to incorporate effects of pion emission [15]. The pair distribution functions $g_{\rm pN}(|\boldsymbol{r}-\boldsymbol{r}_1|)$ account for the joint probability of finding a nucleon N in $\boldsymbol{r}$ and a proton at $\boldsymbol{r}_1$.

In Glauber theory, the nuclear transparency is defined as the average over the nucleus of the probability that the struck proton emerges from the nucleus without any collision. This is related to $P_{\rm T}$ by

$$T = \frac{1}{Z} \int {\rm d}\boldsymbol{r} \rho_{\rm p}(\boldsymbol{r}) P_{\rm T}(p, \boldsymbol{r}, \infty) \ . \tag{5}$$

For the nucleus of ^{12}C and an outgoing proton of energy $E_{\rm f} \sim 1.8$ GeV, which is of interest for the present application, we find $T = 0.62$.

3. *Results*

At energies close to the Fermi level, the hole spectral function is dominated by the contribution from the mean field orbitals in s- and p-shells. These are known from ^{12}C(e,e′p) experiments and describe about 60 % of the total distribution [16]. It is, therefore, convenient to split the spectral function in a mean field and a correlated part,

$$S^{\rm h}_{\rm p}(p_{\rm m}, E_{\rm m}) = S^{\rm h}_{\rm MF}(p_{\rm m}, E_{\rm m}) + S^{\rm h}_{\rm corr}(p_{\rm m}, E_{\rm m}) \ , \tag{6}$$

in which $S^{\rm h}_{\rm corr}(p_{\rm m}, E_{\rm m})$ also contains the short-range correlated tail at very high missing energies and momenta [2, 3]. In the following, we parametrize this as

$$S^{\rm h}_{\rm corr}(p_{\rm m}, E_{\rm m}) = \frac{C \ e^{-\alpha \, p_{\rm m}}}{[E_{\rm m} - e(p_{\rm m})]^2 + [\Gamma(p_{\rm m})/2]} \tag{7}$$

where $e(p_{\rm m})$ and $\Gamma(p_{\rm m})$ are smooth functions and all parameters were chosen to give an appropriate fit to the available data in parallel kinematics [9]. The solid line in Fig. 2 shows the model spectral function (6) employed in the calculation in that part of the $k-E$ plane where $S_{\rm corr}$ dominates.

We have performed calculations of the rescattering contribution by employing both parallel and perpendicular kinematics. In the first case, the angle between the momenum transfered by the electron and the momentum of the final proton was chosen to be $\vartheta_{\rm qf} \sim 5$ deg and the energy of the final proton was $E_{\rm f} \sim 1.6$ GeV. For the perpendicular kinematics, $\vartheta_{\rm qf} \sim 30$ deg and $E_{\rm f} \sim 1$ GeV. In both cases the four momentum transfered by the electron was $Q^2 \sim 0.40$ GeV^2.

The results for the total cross section ($\sigma_{\rm PWIA} + \sigma_{\rm rescat}$) have been converted to a reduced spectral function representation by dividing them by $|p_{\rm f} E_{\rm f}| T \sigma^{\rm cc1}_{\rm eN}$, evaluated for the kinematics of the direct process (see Fig. 2). As can be seen, FSI from nucleon-nucleon rescattering gives a small contribution to the total cross section in parallel kinematics, and the resulting reduced spectral function is close to the true one. For perpendicular kinematics, more sizable contributions are found and they tend to fill the region at higher missing energies where the spectral function is small. This confirms the trend of FSI expected for parallel kinematics that strength

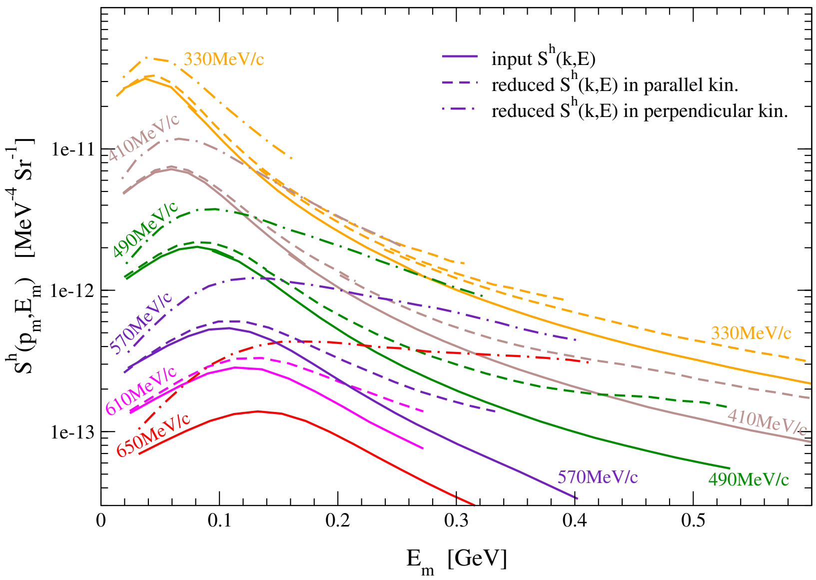

Fig. 2. Theoretical results for the reduced spectral strength in the correlated region obtained in parallel (dashed line) and perpendicular (dot-dashed line) kinematics. The full line shows the model spectral function, Eq. (6), employed in the calculations.

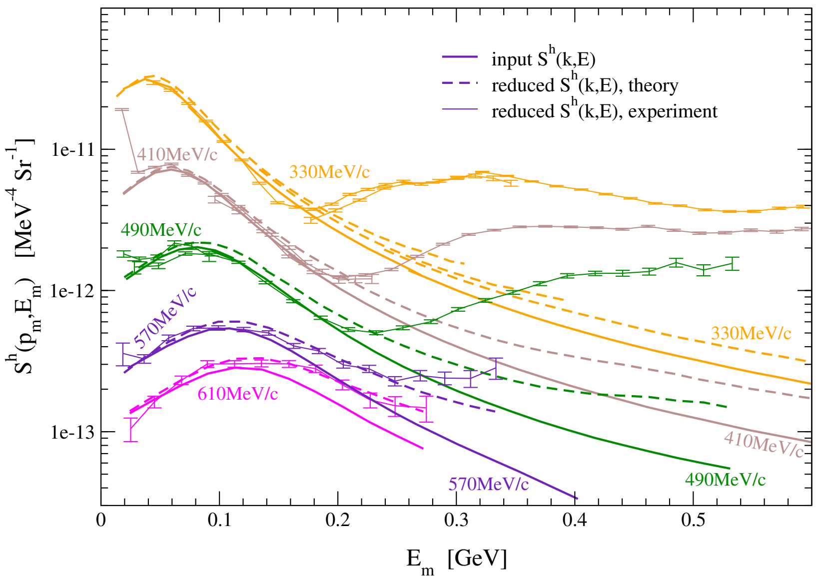

Fig. 3. Theoretical results for the reduced spectral strength obtained in parallel kinematics (dashed line) compared to the experimental results of Ref. [9]. The full line shows the model spectral function of Eq. (6) employed in the calculations.

primarly is moved from places where $S^{\mathrm{h}}(p_{\mathrm{m}}, E_{\mathrm{m}})$ is small to places where it is large, and thus gives a small relative effect [8]. To check that the contribution found is actually coming from the correlated part of the spectral function itself, we have repeated the calculation in perpendicular kinematics by neglecting S_{corr} in Eq. (6). For missing momenta above 400 MeV no rescattering from nucleons was found in the mean field region. One should note that since little reliable experimental information for S_{corr} is available to date, the correlated strength can be extracted from the experimental data only in a self-consistent fashion. This of course requires a proper treatment of the FSIs.

Figure 3 compares the model spectral function (6) and the theoretical reduced one, with preliminary results from the E97-006 collaboration [9]. An enhancement of the cross section is found esperimentally at very high missing energies that is presumably generated by the excitation of a Δ resonance. This effect is not included in the present calculation yet. Contributions from rescattering through this channel are expected to fill up the valley between the correlated and Δ regions more substantially for heavier nuclei. Therefore, these additional degrees of freedom need to be included in the present model.

4. *Conclusions*

This contribution reports about an ongoing work aimed to study the effects of final state interactions in (e,e′p) reactions, as generated by rescattering effects. The two-step rescattering processes that involve the intermediate propagation of a nucleon have been approached by using a semiclassical model. A preliminary calculation has been reported for ^{12}C(e,e′p). It is found that the contribution from final-state interactions in parallel kinematics is much smaller than in perpendicular ones. In the latter case, a large amount of strength is shifted from regions where the spectral function is big to regions where it is smaller, thus overwhelming the experimental yield from the direct process. This confirms the studies of Ref. [8].

At $E_{\mathrm{m}} > 250$ MeV, the present experimental results exceed the calculated direct plus rescattering contributions by about an order of magnitude. This is presumably due to the excitation of Δ resonances. It suggests that rescattering through this channel also affects the measurements in the correlated region. The inclusion of these effects in the present model will be the topic of future work.

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DOPRINOSI DODATNIH RASPRŠENJA MEĐUDJELOVANJU U KONAČNOM STANJU U (e,e′p) REAKCIJAMA

Primjenjujemo poluklasičan model za proučavanje učinaka dodatnih raspršenja u udarnim presjecima reakcija (e,e′p). Razmatramo dvostruki proces s gibanjem međunukleona, a učinke N–N raspršenja uključujemo prema Glauberovoj teoriji. Ovi su računi važni za analize podataka pri velikim manjkovima energije. Posebno je zanimljivo mjerenje E97-006 u JLaboratoriju. Nalazimo jako smanjenje dodatnih raspršenja u usporednoj kinematici te vjerojatno velike doprinose od uzbude nukleonskih rezonancija međudjelovanju u konačnom stanju u koreliranom području.