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THE KUGO–OJIMA CONFINEMENT CRITERION AND THE INFRARED BEHAVIOUR OF LANDAU GAUGE QCD

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Recent investigations of the Dyson–Schwinger equations and Monte–Carlo lattice calculations resulted in a coherent description of the fully dressed gluon, ghost and quark propagators in Landau gauge QCD. In the Dyson–Schwinger approach, the infrared behaviour of these propagators is determined analytically. For finite space-like momenta the gluon, ghost and quark propagators are compared to available corresponding results of lattice Monte–Carlo calculations. For all three propagators, an almost quantitative agreement is found. These results for the non-perturbative propagators allow an analytical verification of the Kugo–Ojima confinement criterion. Our numerical analysis clearly reveals positivity violation for the gluon propagator generated by a cut in the complex momentum plane. The non-perturbative strong running coupling resulting from these propagators possesses an infrared fixed point. The quark propagator obtained from quenched and unquenched calculations displays dynamical chiral symmetry breaking with quark masses close to "phenomenological" and lattice values. We confirm that linear extrapolations of the quark propagator for different bare masses to the chiral limit are inaccurate.

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1. *Some aspects of confinement*

Data taken at the Jefferson Lab over the last few years have uncovered many unexpected properties of hadrons. It seems that CEBAF operates in a very interesting energy range, and it is foreseeable that after its upgrade many more highly interesting results will be obtained. More or less, all of these investigations aim at an understanding of the structure of hadrons in terms of the underlying degrees of freedom, the quarks and gluons. To bring our current theory of strong interactions, quantum chromodynamics (QCD), into agreement with observations the hypothesis of confinement is needed. Even thirty years after the formulation of QCD a detailed understanding of the confinement mechanism(s) is still lacking.

Experimental data obtained at the Jefferson Lab pose a challenge to theoretical physicists working in this field: In principle, the requirement is to understand hadronic properties at intermediate and large momentum transfers in terms of QCD degrees of freedom. An adequate theoretical approach for such investigations has to be deeply rooted in quantum field theory to accommodate a description of confinement, it has to be Poincaré-covariant to be applicable at the employed momentum transfers, and, last but not least, it has to be manageable for quite complicated hadronic form factors and reactions.

Non-perturbative calculations of QCD correlation functions do fulfill all these requirements. Studies of their equations of motion, the Dyson–Schwinger equations (DSEs), have the potential to provide a successful phenomenology of hadrons in terms of quarks and gluons (for recent reviews see, e.g., Refs. [1 – 3]). Within this approach, dynamical breaking of chiral symmetry is obtained from first principles [4]. The description of meson properties and reactions based on their Bethe–Salpeter amplitudes has been quite successful (see Ref. [1] and references therein), and it seems viable that covariant Faddeev amplitudes will allow for a computation of baryonic reactions as well (see, e.g., Ref. [5]). On the other hand, the results of Ref. [5] make it clear that an understanding of confinement and its implications on the analytic structures of QCD Green's functions is a necessary prerequisite for further progress in this direction.

The phenomenon of confinement is truly non-perturbative in nature: There is a physical scale associated with it, and we know from asymptotic freedom of QCD that such scales are non-analytic in the coupling. Futhermore, as we will see shortly, QCD Green's functions exhibit infrared singularities related to confinement. Lattice calculations are very helpful because they provide a rigorous non-perturbative method, nevertheless, there is definite need for continuum-based methods. In the following, we discuss how the infrared behaviour of QCD propagators can be determined analytically, and what the corresponding results tell us about confinement in the covariant gauge.

Let us take a step back and discuss quantum electrodynamics (QED) first. In QED in the covariant gauge, the electromagnetic field can be decomposed into transverse, longitudinal and time-like photons, but the latter two are never observed. From a mathematical point of view, this can be understood from the representations of the Poincaré group: Massless particles have only two possible polarizations. This apparent contradiction is resolved by the fact that time-like and longitudinal photons cancel exactly in the S-matrix [6]. We can interpret this also otherwise: The time-like photon, being unphysical due to the Minkowski metric from the very beginning, "confines" the longitudinal photon.

In QCD, cancelations of unphysical degrees of freedom in the S-matrix also occur but are more complicated due to the self-interaction of the gluons. One obtains, e.g., amplitudes for the scattering of two transverse into one transverse and one longitudinal gluons to order α_S^2. A consistent quantum formulation in a functional integral approach leads to the introduction of ghost fields [7]. To order α_S^2, a ghost loop then cancels all gluon loops which describe scattering of transverse to longitudinal gluons. The proof of this cancelation to all orders in perturbation theory has been possible by employing the BRS symmetry of the covariantly gauge fixed theory [8]. At this point, one has achieved a consistent quantization. If one wants to describe confinement of coloured states in a similar efficacious way, one has to go only one slight step further: One has to require that only BRS singlets are allowed as physical states [9, 10].

The Kugo–Ojima confinement scenario [9, 10] describes a mechanism by which the physical state space contains only colourless states. The coloured states are not BRS singlets and, therefore, do not appear in S-matrix elements: They are confined. Transverse gluons are BRS-non-singlets states with gluon-ghost, gluon-antighost and gluon-ghost-antighost states in the same multiplet. Gluon confinement then occurs as destructive interference between these states. In the Landau gauge, a sufficient criterion for this type of confinement to occur is given by the infrared behaviour of the ghost propagator: If it is more singular than a simple pole, the Kugo–Ojima confinement criterion is fulfilled. It is important to note that the general properties of the ghost DSE and one additional assumption, namely that QCD Green's functions can be expanded in asymptotic series in the infrared, allow to prove this version of the Kugo–Ojima confinement criterion [11, 12].

2. *Gluon propagator and running coupling*

Recently, strong arguments have been provided in favour of infrared dominance of the gauge fixing part of the QCD action in the covariant gauge [13]. The related infrared dominance of ghost loops also occurs in truncation schemes of DSEs being self-consistent at the level of two-point functions [14]. These schemes have been refined and generalized [15] and then allowed to solve the coupled set of DSEs for the ghost, gluon and quark propagators [4].

In the Landau gauge, these momentum-space propagators, $D_G(p)$, $D_{\mu\nu}(p)$ and $S(p)$, renormalized at a scale μ, can be generically written as

$$D_G(p,\mu^2) = -\frac{G(p^2,\mu^2)}{p^2}\,, \tag{1}$$

$$D_{\mu\nu}(p,\mu^2) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\frac{Z(p^2,\mu^2)}{p^2}\,, \tag{2}$$

$$S(p,\mu^2) = \frac{1}{-\,\mathrm{i}\not{p}\,A(p^2,\mu^2) + B(p^2,\mu^2)} = \frac{Z_Q(p^2,\mu^2)}{-\,\mathrm{i}\not{p} + M(p^2)}\,. \tag{3}$$

Two renormalisation-scale-independent combinations built from these functions will be important for further discussion: $M(p^2) = B(p^2,\mu^2)/A(p^2,\mu^2)$ denotes the quark mass function, and a non-perturbative definition of the running coupling,

$\alpha_S(p^2) = \alpha_S(\mu^2)\, G^2(p^2,\mu^2)\, Z(p^2,\mu^2)$, is possible due to the non-renormalisation of the ghost-gluon vertex in the Landau gauge [14].

Employing asymptotic expansions for the propagators at small momenta, the ghost and gluon equations can be solved analytically. One finds simple power laws,

$$Z(p^2,\mu^2) \sim (p^2/\mu^2)^{2\kappa}, \qquad G(p^2,\mu^2) \sim (p^2/\mu^2)^{-\kappa}, \tag{4}$$

for the gluon and ghost dressing functions with exponents related to each other. Hereby κ is an irrational number, $\kappa = (93 - \sqrt{2101})/98 \approx 0.595$ [12, 16]. The product $G^2(p^2,\mu^2)\, Z(p^2,\mu^2)$ tends to a constant in the infrared. Correspondingly, we find an infrared fixed point for the running coupling,

$$\alpha_S(0) = \frac{4\pi}{6N_c} \frac{\Gamma(3-2\kappa)\Gamma(3+\kappa)\Gamma(1+\kappa)}{\Gamma^2(2-\kappa)\Gamma(2\kappa)} \approx 2.972$$

for the gauge group SU(3). This result depends slightly on the employed truncation scheme. Infrared dominance of the gauge fixing part of the QCD action [13] implies infrared dominance of ghosts which in turn can be used to show [12] that $\alpha_S(0)$ depends only weakly on the dressing of the ghost-gluon vertex and not at all on other vertex functions.

The running coupling, as it results from numerical solutions for the gluon, ghost and quark propagators, can be quite accurately fitted by the relatively simple function [4]

$$\alpha_{\rm fit}(p^2) = \frac{\alpha_S(0)}{1+p^2/\Lambda^2_{\tt QCD}} + \frac{4\pi}{\beta_0} \frac{p^2/\Lambda^2_{\tt QCD}}{1+p^2/\Lambda^2_{\tt QCD}} \left(\frac{1}{\ln(p^2/\Lambda^2_{\tt QCD})} - \frac{1}{p^2/\Lambda^2_{\tt QCD} - 1} \right) \tag{5}$$

with $\beta_0 = (11N_c - 2N_f)/3$. Note that, following Ref. [17], the Landau pole has been subtracted. The scale $\Lambda_{\tt QCD}$ is hereby determined by fixing the running coupling at a certain scale, e.g. $\alpha_S(M_Z^2) = 0.118$.

3. *Quark propagator*

In the quark DSE, as well as in the quark loop of the gluon equation, the quark-gluon vertex enters. It has been proven successful [4] to assume that the quark-gluon vertex factorizes,

$$\Gamma_\nu(q,k) = V_\nu^{abel}(p,q,k)\, W^{\neg abel}(p,q,k), \tag{6}$$

with p and q denoting the quark momenta and k the gluon momentum. The non-Abelian factor $W^{\neg abel}$ multiplies an Abelian part V_ν^{abel}, which carries the tensor structure of the vertex. For the latter we choose a construction [18] used widely in QED.

The Slavnov–Taylor identity for the quark-gluon vertex implies that the non-abelian part $W^{\neg abel}(p,q,k)$ has to contain factors of the ghost renormalization function $G(k^2)$. Due to the infrared singularity of the latter, the effective low-energy quark-quark interaction is enhanced as compared to the interaction generated by the exchange of an infrared suppressed gluon. Therefore, the effective kernel of the quark DSE contains an integrable infrared singularity. Further constraints imposed on $W^{\neg abel}(p,q,k)$ are such that the quark mass function is, as required from general principles, independent of the renormalization point, and the one-loop anomalous dimensions of all propagators are reproduced.

In Fig. 1, we compare our results for the quark propagator in quenched approximation, as well as for three massless flavours, with the lattice data [19]. These results nicely agree with the one from the lattice. Furthermore, for the considered number of flavours, the quenched approximation works well.

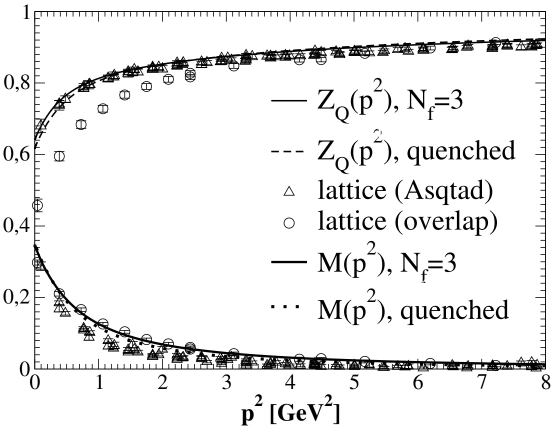

Fig. 1. The quark propagator functions in quenched approximation as well as for three massless flavours compared to the lattice data [19].

4. Spectral properties of the gluon propagator

The infrared exponent κ is an irrational number, and thus the gluon propagator possesses a cut on the negative real p^2 axis. It is possible to fit the solution for the gluon propagator quite accurately without introducing further singularities in the complex p^2 plane. The fit to the gluon renormalization function [21]

$$Z_{\rm fit}(p^2) = w \left(\frac{p^2}{\Lambda^2_{\tt QCD} + p^2}\right)^{2\kappa} \left(\alpha_{\rm fit}(p^2)\right)^{-\gamma} \tag{7}$$

is shown in Fig. 2. Here, w is a normalization parameter and $\gamma = (-13N_c + 4N_f)/(22N_c - 4N_f)$ is the one-loop value for the anomalous dimension of the gluon propagator. The corresponding discontinuity along the cut vanishes for $p^2 \to 0^-$, diverges to $+\infty$ at $p^2 = -\Lambda^2_{\tt QCD}$ and goes to zero for $p^2 \to \infty$.

The absolute value of the Fourier transform of the (transverse) gluon propagator

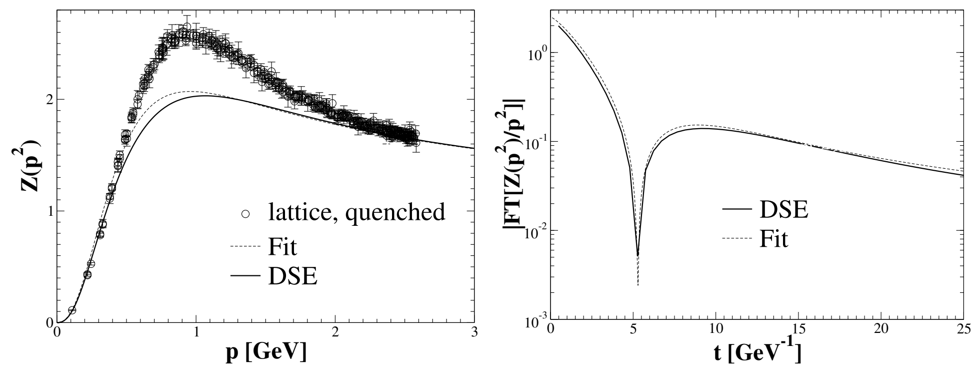

Fig. 2. The gluon propagator (left diagram) and its Fourier-transform (right diagram) compared to the fit, Eq.(7), and lattice data [20].

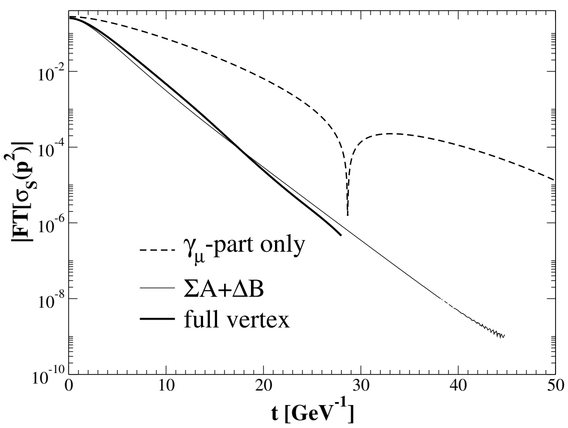

Fig. 3. The Fourier transform of the scalar part of the quark propagator for three different types of vertices. Positivity violations disappear once the scalar coupling, ΔB, is included in the vertex [21].

with respect to the Euclidean time is shown in Fig. 3. First, we clearly observe positivity violations in the gluon propagator. Second, the agreement of the numerical Schwinger function with the Fourier transformed fit is excellent. The crucial property of the gluon propagator is that it tends to zero for vanishing momentum. This can be seen from the relation $0 = D(p = 0) = \int \mathrm{d}^4x \, D(x)$ (with $D(p) = Z(p^2)/p^2$), which implies that a nontrivial propagator function, $D(x)$, in coordinate space must contain positive as well as negative norm contributions.

The function (7) contains only four parameters: the overall magnitude which, due to renormalization properties, is arbitrary (it is determined via the choice of the renormalization scale), the scale $\Lambda_{\mathtt{QCD}}$, the infrared exponent κ and the anomalous dimension of the gluon γ. The latter two are not free parameters: κ is determined from the infrared properties of the DSEs, and for γ, its one-loop value is used. Thus we have found a parameterization of the gluon propagator which has effectively only one parameter, the scale $\Lambda_{\mathtt{QCD}}$.

5. Spectral properties of the quark propagator

When discussing the results for the quark propagator, we already stated that a dressed quark-gluon vertex is mandatory. For the present discussion, it is important to note that such or similar solutions of the Slavnov–Taylor identity for the quark-gluon vertex will always result in the appearance of a quark-gluon coupling term ΔB proportional to the sum of quark momenta,

$$\begin{aligned} V_\nu^{abel}(p,q) &:= \Sigma A_\nu + \Delta B_\nu + ... \\ &= \frac{A(p^2)+A(q^2)}{2}\gamma_\nu + \mathrm{i}\frac{B(p^2,\mu^2)-B(q^2,\mu^2)}{p^2-q^2}(p+q)_\nu + \dots . \end{aligned} \quad (8)$$

Such a coupling, being effectively scalar, is *per se* not invariant under chiral transformations, contrary to the leading term, ΣA, of the quark-gluon vertex. It is important to realize that the term ΔB appears only in case chiral symmetry is already dynamically broken. Thus it is consistent with the chiral Ward identities. Its existence, on the other hand, provides a significant amount of self-consistent enhancement of dynamical chiral symmetry breaking. Fairly independent of the form of the gluon propagator, the resulting quark propagator respects positivity if the term ΔB is included in the quark-gluon vertex [21].

A peculiar feature of the Fourier transform of the scalar part of the quark propagator is the curvature appearing for small Euclidean times. There are at least three possible sources for this. A leading singularity on the real momentum axis may be accompanied by additional real singularities at larger masses, or by complex conjugate singularities with a larger real part of the mass, or it may be the starting point of a branch cut on the negative real momentum axis [21].

6. Extrapolation to the chiral limit

Lattice simulations of dynamical chiral symmetry breaking are facing problems from two sides. Finite volume effects tend to obfuscate results in the far infrared region. Furthermore, although in principle the Ginsparg-Wilson relation enables one to implement chiral quarks on a lattice, small quark masses are computationally very expensive. Therefore, lattice simulations are usually carried out at finite bare quark masses and then linearly extrapolated to the chiral limit. The DSE-approach, however, suggests that this linear extrapolation is inaccurate [22]. The DSE for the quark propagator also contains the nontrivial relation between the dynamically generated quark mass at small momenta and the renormalized quark mass at a perturbative renormalization point. To illustrate this point, we display the dynamical mass $M(p^2, m_\mu)$ from the DSEs for two different momenta, $p^2 = 0$ and $p^2 = 0.38\,\mathrm{GeV}^2$ in Fig. 4. One clearly observes the curvature in the DSE-results for both momenta, reflecting the nonlinear behaviour of the underlying equations. We compare these results with the lattice data taken from Ref. [19] and m_μ from Ref. [22]. The lattice data are consistent with a linear fit. However, by multiplying

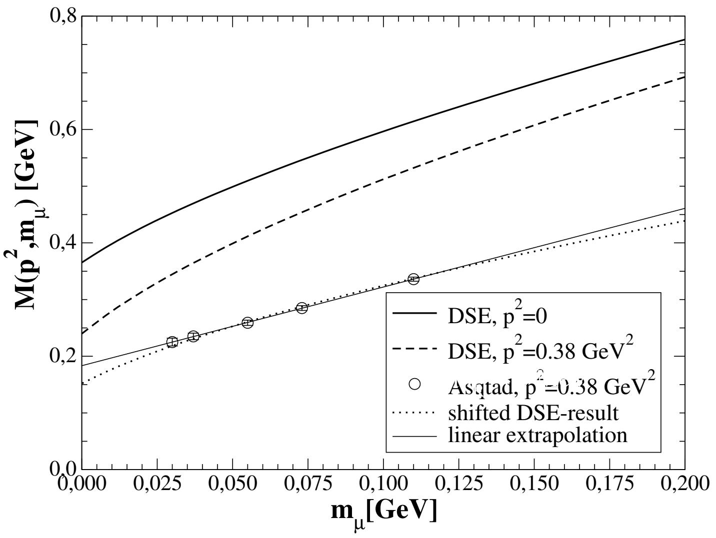

Fig. 4. Relation between the dynamically generated quark mass at $p^2 = 0.38$ GeV^2 *and* $p^2 = 0$ *to the renormalized quark mass* m_μ *at* $\mu = 19$ GeV *from the DSEs and from the lattice data of Ref. [19].*

our DSE-results with an overall factor, we obtain a curved mass function that mimics a DSE-model fit to the lattice data. The extrapolated chiral dynamical quark mass at $p^2 = 0.38$ GeV^2 is roughly 20% below the one obtained from the linear fit. Employing only the leading γ_μ-structure of the quark-gluon vertex, as done in Ref. [22], leads to even more drastic deviations. Thus we confirmed that the linear extrapolation of lattice data to the chiral limit cannot be trusted.

7. *Epilogue*

Recent years have seen a lot of progress in Strong QCD. In the Landau gauge, we gained an understanding of the infrared behaviour of gluon and quark propagators. We have verified the Kugo–Ojima scenario for gluon confinement and found dynamical chiral symmetry breaking in an *ab initio* calculation.

We have proposed relatively simple functions to describe the running coupling, the gluon and the quark propagators (see Ref. [21] for details) for all possible values of momenta. These have the potential to provide a basis for a hadron phenomenology based on quarks and gluons, even and especially in the non-perturbative regime.

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KUGO–OJIMAOV KRITERIJ OGRANIČENJA I INFRACRVENA SVOJSTVA LANDAUOVE BAŽDARNE QCD

Nedavna istraživanja Dyson–Schwingerovih jednadžbi i Monte–Carlo računa na rešetki dali su usklađen opis potpuno obućenih propagatora gluona, duhova i kvarkova u Landauovoj baždarnoj QCD. U Dyson–Schwingerovom pristupu, infracrvena svojstva tih propagatora određena su analitički. Za konačne vremenske impulse, propagatori gluona, duhova i kvarkova uspoređuju se dostupnim odgovarajućim ishodima Monte–Carlo računa na rešetki. Gotovo kvantitativno slaganje nalazi se za sva tri propagatora. Ti ishodi za neperturbativne propagatore dopuštaju analitičku provjeru Kugo–Ojimaovog kriterija ograničenja. Naša numerička analiza jasno pokazuje kršenje pozitivnosti za gluonski propagator uzrokovan rezom u ravnini kompleksnog impulsa. Neperturbativno jako vezanje, koje je posljedica tih propagatora, ima infracrvenu čvrstu točku. Kvarkovski propagator koji se dobije gašenim i negašenim računima pokazuje lomljenje dinamičke kiralne simetrije s kvarkovskim masama koje su blizu "fenomenološkim" i rešetkinskim vrijednostima. Utvrdili smo da su linearne ekstrapolacije kvarkovskih propagatora za razne gole mase vrlo netočne u kiralnoj granici.