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ANISOTROPIC FLUID DISTRIBUTION IN HIGHER-DIMENSIONAL ROSEN THEORY OF GRAVITATION

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We have obtained static and spherically symmetric solutions of the gravitational field equations for isotropic and anisotropic distribution of matter in the context of higher-dimensional bimetric theory of gravitation under the assumption that the physical metric admits a one-parameter group of conformal motion. The solutions agree with Einstein's general relativity for physical systems such as the solar system. This work is an extension of the previous work of Shri Ram and Pandey (Astrophys. Space Sci. **127** (1986) 9) for four-dimensional space time.

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1. Introduction

The isotropic fluid configurations in general relativity have been studied in several investigations due to their possible application in astrophysical studies of massive objects. Anisotropy in fluid pressure can be introduced by a solid core, by the pressure of type-3A superfluid or by other physical effects. Bowers and Liang [2] have investigated the possible importance of local anisotropies for a relativistic fluid sphere by generalizing the equation of hydrostatic equilibrium to include the effects of local anisotropy. Their study shows that anisotropy may have non-negligible effects on such parameters as maximum equilibrium mass and surface red-shift. Consenza et al. [3], Bayin [4], Krori et al. [5], Maharaj and Maartens [6] have obtained different exact solutions of Einstein's field equations describing the interior gravitational field of an anisotropic fluid sphere. Herrera et al. [7] studied the consequences of inclusion of a one-parameter group of conformal motions of anisotropic matter in Einstein's general relativity and obtained analytical solutions of field equations for static and spherically-symmetric distributions of isotropic and anisotropic matter.

The purpose of the present paper is to obtain static spherically-symmetric solutions of field equations in higher-dimensional bimetric theory of gravitation proposed by Rosen [8,9] for isotropic and anisotropic distributions of matter when the physical metric admits a one-parameter group of conformal motion. The solutions agree with the higher-dimensional Einstein's general relativity for physical system known in the Universe, such as the solar system.

2. Field equations

Bimetric theory of gravitation is a modification of Einstein's general relativity theory involving a background metric in addition to the usual physical metric. The background metric corresponds to the space-time of constant curvature and can be throught of as the geometry which the universe would have in the absence of matter (a de Sitter Universe). The physical metric $g_{\mu\nu}$ as in conventional general relativity and there is a background metric $\gamma_{\mu\nu}$ having the curvature tensor $P_{\lambda\mu\nu\sigma}$ given by

$$P_{\lambda\mu\nu\sigma} = \frac{1}{a^2}(\gamma_{\mu\nu}\gamma_{\lambda\sigma} - \gamma_{\mu\sigma}\gamma_{\lambda\nu}). \tag{1}$$

The field equations of bimetric general relativity are taken to be the same as in general relativity, except for the fact that ordinary derivatives of physical metric are replaced by covariant derivatives with respect to the background metric. It was found by Rosen [10] that these equations can be written in the form of Einstein's field equations but with an additional term on right-hand side

$$G_{\mu\nu} = S_{\mu\nu} - 8\pi T_{\mu\nu}, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein's tensor, $T_{\mu\nu}$ is the energy stress tensor and

$$S_{\mu\nu} = \frac{3}{a^2}(\gamma_{\mu\nu} - \frac{1}{2}g_{\mu\nu}g^{\alpha\beta}\gamma_{\alpha\beta}). \tag{3}$$

For a higher-dimensional spherically-symmetric system we take the physical metric as

$$\mathrm{d}s^2 = e^{\nu}\mathrm{d}t^2 - e^{\lambda}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{4}$$

where

$$\mathrm{d}\Omega^2 = \mathrm{d}\theta_1^2 + \sin^2\theta_1\mathrm{d}\theta_2^2 + \sin^2\theta_1\sin^2\theta_2\mathrm{d}\theta_3^2 + \ldots + \left[\prod_{i=1}^{n-1}\sin^2\theta_i\right]\mathrm{d}\theta_n^2,$$

and λ and ν are the functions of r alone. An anisotropic spherically symmetric matter distribution $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (\rho + P_\perp)U_\mu U_\nu - P_\perp g_{\mu\nu} + (P_r - P_\perp)\chi_\mu \chi_\nu, \tag{5}$$

where U^μ is the $(n+2)$-velocity, χ^μ a unit space-like vector orthogonal to U^μ, ρ the energy density, P_r the radial pressure in the direction of χ_μ and $P_\perp$ the pressure orthogonal to χ_μ. In the co-moving system, we choose

$$\begin{aligned} U^\mu &= (U^0, 0, 0, 0, \ldots (n+1) \text{ times}), \\ \chi^\mu &= (0, \chi^1, 0, 0, \ldots n \text{ times}). \end{aligned} \tag{6}$$

From $U^\mu U_\mu = -\chi^\mu \chi_\mu = 1$, we obtain

$$U^0 = e^{-\nu/2}, \qquad \chi^1 = e^{-\lambda/2}.$$

The non-vanishing components of energy momentum tensors are

$$T_0^0 = \rho, \qquad T_1^1 = -P_r, \qquad T_2^2 = T_3^3 = \ldots = T_{n+1}^{n+1} = -P_\perp. \tag{7}$$

If we take the background metric $\gamma_{\mu\nu}$, in a static de Sitter form, the line element is given by

$$\mathrm{d}\sigma^2 = \left(1 - \frac{r^2}{a^2}\right)\mathrm{d}t^2 - \left(1 - \frac{r^2}{a^2}\right)^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2. \tag{8}$$

For $r \ll a$, this line element on the flat space has the form

$$\mathrm{d}\sigma^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2\mathrm{d}\Omega^2. \tag{9}$$

Let us consider $S_{\mu\nu}$ in a region where $r \ll a$. If we neglect the quantities which are small everywhere, we can write for a non-vanishing component

$$S_0^0 = -S_1^1 = \ldots = -S_{n+1}^{n+1} = \frac{3}{2a^2}e^{-\nu}. \tag{10}$$

Following the procedure of Rosen [10], field equations (2) for $r \ll a$ are

$$e^{-\lambda}\left[\frac{n(n-1)}{2r^2} - \frac{n\lambda'}{2r}\right] - \frac{n(n-1)}{2r^2} = \frac{3}{2a^2}e^{-\nu} - 8\pi\rho, \tag{11}$$

$$e^{-\lambda}\left[\frac{n(n-1)}{2r^2} + \frac{n\nu'}{2r}\right] - \frac{n(n-1)}{2r^2} = -\frac{3}{2a^2}e^{-\nu} + 8\pi P_r, \tag{12}$$

$$\begin{aligned} e^{-\lambda}\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{(n-1)(\lambda'-\nu')}{2r} - \frac{\lambda'\nu'}{4} \right. &+ \left. \frac{(n-1)(n-2)}{2r^2}\right] \\ - \frac{(n-1)(n-2)}{2r^2} &= -\frac{3}{2a^2}e^{-\nu} + 8\pi P_\perp, \end{aligned} \tag{13}$$

where the primes denotes derivatives with respect to r. It is very difficult to obtain the solution of equations (11) – (13) due to the non-linearity of the field equations. Therefore, one has to make certain simplifying assumptions to derive useful results. In the next section, we assume that the physical metric (4) admits a one-parameter group of conformal motions and obtain analytical solutions of the field equations for static and spherically-symmetric fluid distributions of isotropic and anisotropic matter.

3. *Conformal motions and solutions of field equations*

The space-time admits a one-parameter group of conformal motions generated by the vector field ξ^μ if

$$L_\xi g_{\mu\nu} = g_{\mu\nu,\sigma}\xi^\sigma + g_{a\nu}\xi^a_{,\mu} + g_{\mu a}\xi^a_{,\nu} = \psi g_{\mu\nu}, \tag{14}$$

where the commas denote the covariant differentiation with respect to $\gamma_{\mu\nu}$, and ψ is an arbitrary function of coordinates. If the vector field ξ^μ is collinear with χ^μ, then by virtue of spherical symmetry and independence of the metric tensor on the time-like coordinates, the most general form of ξ^μ is

$$\xi^\mu = F(r)\chi^\mu. \tag{15}$$

From Eqs. (4), (14) and (15), we obtain (see Appendix)

$$\psi = F\nu' e^{-\lambda/2} = 2F' e^{-\lambda/2} = \frac{2Fe^{-\lambda/2}}{r}. \tag{16}$$

A straightforward calculation gives

$$e^{-\lambda} = \frac{\psi^2}{4C_2^2}, \qquad e^\nu = C_1 r^2, \qquad F = C_2 r, \tag{17}$$

where C_1 and C_2 are constants of integration. Without loss of generality, we can take $C_1 = 1$. Substituting (17) into Eqs. (11) – (13), we get

$$8\pi\rho = \frac{-n\psi\psi'}{4C_2^2 r} - \frac{n(n-1)\psi^2}{8C_2^2 r^2} + \frac{n(n-1)}{2r^2} + \frac{3}{2a^2}\frac{1}{r^2}, \tag{18}$$

$$8\pi P_r = \frac{n(n+1)\psi^2}{8C_2^2 r^2} - \frac{n(n-1)}{2r^2} + \frac{3}{2a^2}\frac{1}{r^2}, \tag{19}$$

$$8\pi P_\perp = \frac{n\psi\psi'}{4C_2^2 r} + \frac{n(n-1)\psi^2}{8C_2^2 r^2} - \frac{(n-1)(n-2)}{2r^2} + \frac{3}{2a^2}\frac{1}{r^2}. \tag{20}$$

If we wish to match any solution to the exterior metric on the boundary of the sources, the radial pressure should vanish for some finite values of the radial coordinate (say, $r = r_0$). The vanishing of the radial pressure gives

$$\psi^2(r_0) = \frac{8C_2^2}{n(n+1)} \left[\frac{n(n-1)}{2} - \frac{3}{2a^2} \right]. \tag{21}$$

After integrating (18), we get the total mass M with in the sphere of radius r_0. By using (18) and assuming $\psi(0) < \infty$, it can be shown that

$$\frac{M}{r_0} = \frac{n}{2(n+1)} \left[n - 1 + \frac{3}{a^2} \right]. \tag{22}$$

We observe from (22) that all solutions obtained from Eqs. (18) – (20) have the same gravitational potential, provided that the boundary is the surface of vanishing radial pressure for any choice of the function bounded in the interval $0 \leq r \leq r_0$. If we take $\psi = 2$, then from Eqs. (18) – (20) we obtain

$$\rho = \frac{1}{8\pi} \left[\frac{n(n-1)}{2} - \frac{n(n-1)}{2C_2^2} + \frac{3}{2a^2} \right] \frac{1}{r^2}, \tag{23}$$

$$P_r = \frac{1}{8\pi} \left[\frac{n(n+1)}{2C_2^2} - \frac{n(n-1)}{2} + \frac{3}{2a^2} \right] \frac{1}{r^2}, \tag{24}$$

$$P_\perp = \frac{1}{8\pi} \left[\frac{n(n-1)}{2C_2^2} - \frac{(n-1)(n-2)}{2} + \frac{3}{2a^2} \right] \frac{1}{r^2}. \tag{25}$$

From (24), it is clear that the radial pressure does not vanish for any finite value of r. Hence, the solution cannot be matched to any exterior metric. From Eqs. (23) – (25), we obtain the relation

$$nP_\perp - (n-1)P_r = \rho. \tag{26}$$

This relationship between the stresses and the density has been established by Herrera et al. [7] for $n = 2$ when the space-time admits the special conformal motion for which the function ψ satisfies the condition

$$\psi_{,\mu,\nu} = 0. \tag{27}$$

For $\psi = 2$, Eq. (27) is satisfied identically. When $C_2 = \sqrt{n/(n-1)}$, the equation of state becomes

$$\rho = P_r = P_\perp, \tag{28}$$

which is widely used in general relativity to obtain stellar and cosmological models for ultradense matter.

3.1. Perfect-fluid solution

From the condition $P_r = P_\perp$ and using Eqs. (19) and (20), we get the equation

$$r\psi\psi' + \frac{4(n-1)C_2^2}{n} - \psi^2 = 0, \tag{29}$$

the general solution of which is given by

$$\psi^2 = C_2^2 \left[\frac{4(n-1)}{n} + Cr^2\right], \tag{30}$$

where C is the constant of integration. If we wish to match the solution to exterior metric on the boundary $r = r_0$, the radial pressure must be zero, which implies that

$$C = \frac{-4}{n(n+1)} \left[n - 1 + \frac{3}{a^2}\right] \frac{1}{r_0^2}. \tag{31}$$

The metric of the solution can be written as

$$\mathrm{d}s^2 = r^2 \mathrm{d}t^2 - \left[\frac{n(n-1)}{2} - \frac{Mr^2}{r_0^3}\right]^{-1} \mathrm{d}r^2 - r^2 \mathrm{d}\Omega^2. \tag{32}$$

The expressions for pressure and density are

$$\rho = \frac{1}{16\pi} \left[n - 1 + \frac{3}{a^2}\right] \left[\frac{1}{r^2} + \frac{1}{r_0^2}\right], \tag{33}$$

$$P_r = P_\perp = \frac{1}{16\pi} \left[n - 1 + \frac{3}{a^2}\right] \left[\frac{1}{r^2} - \frac{1}{r_0^2}\right]. \tag{34}$$

From Eqs. (33) and (34) it is clear that

$$\rho \geq P_r \geq 0. \tag{35}$$

3.2. An anisotropic solution

We choose

$$\psi^2 = C_2^2 \left[Cr^2 + \frac{4(n-1)}{n}\right] + C_2^2 H, \tag{36}$$

where H is a constant which measures the anisotropy. Substituting (36) into Eqs. (18) – (20), we get

$$8\pi\rho = -\frac{n(n+1)}{8} C + \left[\frac{n-1}{2} + \frac{3}{2a^2} - \frac{n(n-1)}{8} H\right] \frac{1}{r^2}, \tag{37}$$

$$8\pi P_r = \frac{n(n+1)}{8}C + \left[\frac{n-1}{2} + \frac{3}{2a^2} + \frac{n(n+1)}{8}H\right]\frac{1}{r^2}, \tag{38}$$

$$8\pi P_\perp = \frac{n(n+1)}{8}C + \left[\frac{n-1}{2} + \frac{3}{2a^2} + \frac{n(n-1)}{8}H\right]\frac{1}{r^2}. \tag{39}$$

The radius of sphere r_0 is given by

$$r_0^2 = -\frac{4}{n(n+1)C}\left[n-1+\frac{3}{2a^2}+\frac{n(n+1)}{4}H\right], \qquad C<0. \tag{40}$$

The metric of the solution becomes

$$\mathrm{d}s^2 = r^2\mathrm{d}t^2 - \left[\left(\frac{n-1}{n}+\frac{H}{4}\right) - \left(n-1+\frac{3}{a^2}+\frac{n(n+1)}{4}H\right)\frac{r^2}{r_0^2}\right]^{-1}\mathrm{d}r^2 - r^2\mathrm{d}\Omega^2, \tag{41}$$

which possesses vanishing radial-pressure surface. For positiveness of energy-density and the pressure ($\rho + p_r \geq 0$), we must have

$$-\frac{4}{n}\left(n-1+\frac{3}{a^2}\right) \leq H \leq 0. \tag{42}$$

4. Conclusion

In the present paper we have obtained static spherically symmetric solutions for isotropic and anisotropic distribution of matter in the contexts of higher-dimensional bimetric theory of gravitation. It should be stresses that the assumption $P_r = 0$ has been made for the sake of mathematical simplicity. The solutions represent the gravitational field near the source of gravitation. For a physical system, the term $3e^{-\nu}/(2a^2)$ in the field equations (11) – (13) is negligible. This means that in such a case, the present equations give the agreement with the Einstien's field equations. The present results reduces to the Einstein general relativity, obtained by Herrera et al. [7] for $n = 2$. The interesting feature of the solution is that the energy density is larger than any of the stresses within the sphere.

Appendix

Higher dimensional spherically symmetric space time is

$$\mathrm{d}s^2 = e^\nu \mathrm{d}t^2 - e^\lambda \mathrm{d}r^2 - r^2\mathrm{d}\Omega^2. \tag{43}$$

Here

$$g_{00} = e^\nu, \quad g_{11} = -e^\lambda, \quad g_{22} = -r^2, \quad g_{33} = -r^2\sin^2\theta_1,$$

$$g_{44} = -r^2\sin^2\theta_1\sin^2\theta_2, \quad \dots \quad , \quad g_{nn} = -r^2\prod_{i=1}^{n-2}\sin\theta_i.$$

The corresponding flat space-time of (43) is

$$\mathrm{d}\sigma^2 \quad = \quad \mathrm{d}t^2 - \mathrm{d}r^2 - r^2 \mathrm{d}\Omega^2 \tag{44}$$

Here

$$\gamma_{00} = 1, \qquad \gamma_{11} = -1, \qquad \gamma_{22} = -r^2, \qquad \gamma_{33} = -r^2 \sin^2 \theta_1,$$
$$\gamma_{44} = -r^2 \sin^2 \theta_1 \sin^2 \theta_2, \quad \dots \quad , \qquad \gamma_{nn} = -r^2 \prod_{i=1}^{n-2} \sin \theta_i .$$

The non-vanishing Christoffel symbols for the metric (44) are

$$\Gamma^1_{22} = -r,$$
$$\Gamma^2_{12} = \Gamma^3_{13} = \Gamma^4_{14} = \cdots = \Gamma^n_{1n} = \cot \theta_1,$$
$$\Gamma^3_{23} = \Gamma^4_{24} = \Gamma^5_{25} = \cdots = \Gamma^n_{2n} = \cot \theta_2,$$
$$\Gamma^4_{34} = \Gamma^5_{35} = \Gamma^6_{36} = \cdots = \Gamma^n_{3n} = \cot \theta_3,$$
$$\Gamma^2_{33} = -\sin \theta_1 \cos \theta_1, \quad \Gamma^3_{44} = -\sin \theta_2 \cos \theta_2,$$
$$\Gamma^4_{55} = -\sin \theta_3 \cos \theta_3, \quad \dots \quad , \quad \Gamma^n_{(n+1)(n+1)} = -\sin \theta_{n-1} \cos \theta_{n-1},$$
$$\Gamma^1_{33} = -r \sin^2 \theta_1, \quad \Gamma^1_{44} = -r \sin^2 \theta_1 \sin^2 \theta_2, \quad \dots \quad , \Gamma^1_{(n+1)(n+1)} = -r \prod_{i=1}^{n-1} \sin^2 \theta_i,$$
$$\Gamma^2_{44} = -\sin \theta_1 \cos \theta_1 \sin^2 \theta_2, \quad \Gamma^2_{55} = -\sin \theta_1 \cos \theta_1 \sin^2 \theta_2 \sin^2 \theta_3, \quad \dots \quad ,$$
$$\Gamma^2_{(n+1)(n+1)} = -\sin \theta_1 \cos \theta_1 \prod_{i=2}^{n-1} \sin^2 \theta_i,$$
$$\Gamma^3_{55} = -\sin \theta_2 \cos \theta_2 \sin^2 \theta_3, \quad \Gamma^3_{66} = -\sin \theta_2 \cos \theta_2 \sin^2 \theta_3 \sin^2 \theta_4, \quad \dots \quad ,$$
$$\Gamma^3_{(n+1)(n+1)} = -\sin \theta_2 \cos \theta_2 \prod_{i=3}^{n-1} \sin^2 \theta_i,$$
$$\Gamma^4_{66} = -\sin \theta_4 \cos \theta_4 \sin^2 \theta_5, \quad \Gamma^4_{77} = -\sin \theta_3 \cos \theta_3 \sin^2 \theta_4 \sin^2 \theta_5, \quad \dots \quad ,$$
$$\Gamma^4_{(n+1)(n+1)} = -\sin \theta_3 \cos \theta_3 \prod_{i=4}^{n-1} \sin^2 \theta_i .$$

A one-parameter group of conformal motions generated by the vector field ξ^μ is admitted by a space-time if

$$\psi g_{\mu\nu} \quad = \quad g_{\mu\nu,\sigma} \xi^\sigma + g_{a\nu} \xi^a_{,\mu} + g_{\mu a} \xi^a_{,\nu} . \tag{45}$$

The function ξ^μ is defined as

$$\xi^\mu \quad = \quad F(r) \chi^\mu . \tag{46}$$

Equation (45) is written as

$$\begin{aligned}
\psi g_{\mu\nu} &= \left(\frac{\partial g_{\mu\nu}}{\partial x^\sigma} - g_{\mu\alpha}\Gamma^\alpha_{\nu\sigma} - g_{\nu\alpha}\Gamma^\alpha_{\mu\sigma}\right)\xi^\sigma + g_{a\nu}\left[\frac{\partial \xi^a}{\partial x^\mu} + \xi^\alpha \Gamma^a_{\alpha\mu}\right] \\
&\quad + g_{a\mu}\left[\frac{\partial \xi^a}{\partial x^\nu} + \xi^\alpha \Gamma^a_{\alpha\nu}\right],
\end{aligned}$$

$$\begin{aligned}
\text{(i)} \qquad \psi g_{00} &= \left(\frac{\partial g_{00}}{\partial x^\sigma} - g_{0\alpha}\Gamma^\alpha_{0\sigma} - g_{0\alpha}\Gamma^\alpha_{0\sigma}\right)\xi^\sigma + g_{a0}\left[\frac{\partial \xi^a}{\partial x^0} + \xi^\alpha \Gamma^a_{\alpha 0}\right] \\
&\quad + g_{a0}\left[\frac{\partial \xi^a}{\partial x^0} + \xi^\alpha \Gamma^a_{\alpha 0}\right] \\
&= \left(\frac{\partial g_{00}}{\partial x^1} - g_{0\alpha}\Gamma^\alpha_{01} - g_{0\alpha}\Gamma^\alpha_{01}\right)\xi^1 + g_{00}\left[\frac{\partial \xi^0}{\partial x^0} + \xi^1 \Gamma^0_{10}\right] \\
&\quad + g_{00}\left[\frac{\partial \xi^0}{\partial x^0} + \xi^1 \Gamma^0_{10}\right] = \left(\frac{\partial g_{00}}{\partial x^1}\right)\xi^1 .
\end{aligned}$$

That is,

$$\begin{aligned}
\psi(e^\nu) &= \frac{\partial}{\partial r}(e^\nu) \cdot F(r) e^{-\lambda/2} = \nu' e^\nu F(r) e^{-\lambda/2}, \\
\psi &= \nu' F(r) e^{-\lambda/2}, \\
\text{(ii)} \qquad \psi g_{11} &= \left(\frac{\partial g_{11}}{\partial x^\sigma} - g_{1\alpha}\Gamma^\alpha_{1\sigma} - g_{1\alpha}\Gamma^\alpha_{1\sigma}\right)\xi^\sigma + g_{a1}\left[\frac{\partial \xi^a}{\partial x^1} + \xi^\alpha \Gamma^a_{\alpha 1}\right] \\
&\quad + g_{a1}\left[\frac{\partial \xi^a}{\partial x^1} + \xi^\alpha \Gamma^a_{\alpha 1}\right] \\
&= \left(\frac{\partial g_{11}}{\partial x^1} - g_{1\alpha}\Gamma^\alpha_{11} - g_{1\alpha}\Gamma^\alpha_{11}\right)\xi^1 + g_{11}\left[\frac{\partial \xi^1}{\partial x^1} + \xi^1 \Gamma^1_{11}\right] \\
&\quad + g_{11}\left[\frac{\partial \xi^1}{\partial x^1} + \xi^1 \Gamma^1_{11}\right] = \left(\frac{\partial g_{11}}{\partial x^1}\right)\xi^1 + 2 g_{11}\frac{\partial \xi^1}{\partial x^1}.
\end{aligned}$$

That is,

$$\begin{aligned}
\psi(-e^\lambda) &= \frac{\partial}{\partial r}(-e^\lambda) F(r) e^{-\lambda/2} + 2(-e^\lambda)\frac{\partial}{\partial r}[F(r) e^{-\lambda/2}], \\
\psi &= 2F'(r) e^{-\lambda/2}, \\
\text{(iii)} \qquad \psi g_{22} &= \left(\frac{\partial g_{22}}{\partial x^\sigma} - g_{2\alpha}\Gamma^\alpha_{2\sigma} - g_{2\alpha}\Gamma^\alpha_{2\sigma}\right)\xi^\sigma + g_{a2}\left(\frac{\partial \xi^a}{\partial x^2} + \xi^\alpha \Gamma^a_{\alpha 2}\right) \\
&\quad + g_{a2}\left(\frac{\partial \xi^a}{\partial x^2} + \xi^\alpha \Gamma^a_{\alpha 2}\right)
\end{aligned}$$

$$= \left(\frac{\partial g_{22}}{\partial x^1} - g_{22}\Gamma^2_{21} - g_{22}\Gamma^2_{21}\right)\xi^1 + 2g_{22}(\xi^1\Gamma^2_{12}) = \left(\frac{\partial g_{22}}{\partial x^1}\right)\xi^1.$$

That is,

$$\psi(-r^2) = \frac{\partial}{\partial r}(-r^2)F(r)e^{-\lambda/2},$$

$$\psi = \frac{2F(r)e^{-\lambda/2}}{r}.$$

Similarly, we obtain

$$\psi g_{nn} = \frac{\partial g_{nn}}{\partial x^1}\xi^1 \quad \Rightarrow \quad \psi = \frac{2F(r)e^{-\lambda/2}}{r}. \tag{47}$$

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ANIZOTROPNA RASPODJELA TVARI U ROSENOVOJ VIŠEDIMENZIJSKOJ TEORIJI GRAVITACIJE

Dobili smo statička sferno simetrična rješenja jednadžbi gravitacijskog polja za izotropnu i neizotropnu raspodjelu tvari, na osnovi višedimenzijske bimetrijske teorije gravitacije, uz pretpostavku da fizička metrika dopušta jedno-parametarsku grupu konformnog gibanja. Rješenja se slažu s Einsteinovom općom relativnošću za sustave kao što je Sunčev sustav. Ovaj je rad proširenje ranijeg rada Shri Rama i Pandeya za četiridimenzijski prostor-vrijeme.