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PRE-EQUILIBRIUM HEAVY-ION COLLISIONS WITH SPIN: A TOY MODEL

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We present a simple model capable to give the initial exciton number for heavy-ion collisions at modest energies in the case when also spin variables are taken into account. This model is a generalization of the model proposed by Cindro et al. a decade ago.

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1. Introduction

The pre-equilibrium models became rather popular tool to analyse and understand nuclear reactions at excitation energies ranging from several tens of MeV up to the GeV regions. A whole variety of the models has been developed (see e.g. Refs. [1–3]). At the low-energy end, they usually employ some classification of the states according to their complexity, e.g. the number of excitons which develop during the course of a reaction. Higher energies prefer thermodynamical approaches and/or "classical-like" descriptions within the nuclear-molecular-dynamic family of the models.

A great variety of experimental data obtained at relatively low energies, i.e. below 100 MeV or so in the nucleon-induced reactions and not much above the Coulomb barrier for the heavy-ion-induced reactions, together with the ease of use of the phenomenological pre-equilibrium models, made rather popular the pre-equilibrium exciton model [4] or its younger brother, the hybrid model. They both are essentially based on the notion of an exciton, i.e. particle above or hole below the Fermi level. The initial stage of the reaction is the one which is responsible for the essential part of emission at the high-energy end of the spectrum. Here, the initial number of degrees of freedom – or, in the language of the exciton or hybrid models, the initial exciton number – plays a crucial role.

For a long time, there was not any reliable rule how to treat the pre-equilibrium initial stage of heavy-ion collisions. The original recipe of Blann [5], that the initial exciton number equals to the projectile mass number, did not work sufficiently well. A way out has been found by recognizing that the initial exciton number in the heavy-ion reactions, when treated as a parameter to yield a reasonable description of the observed data, shows some simple trends and under suitable scalling it becomes a universal function independent of the type of impacting ions, capable to be used for a wide range of (almost all possible) targets. These systematics [6–8] can be used as a suitable starting points for pre-equilibrium heavy ion collision calculations, as was really done (e.g. Refs. [8, 9]).

Though the universal behaviour of the initial states, describing a wide range of various combinations of projectiles and targets for energies starting from the Coulomb barrier up to several tens of MeV per nucleon, was successfully used, its understanding became clear only with a tandem of papers of Cindro et al. [10, 11], who recognized that (with a few exceptions for light nuclei) all nuclei have nearly the same size in the *momentum* space, and one can reasonably estimate the initial number of degrees of freedom by calculating their overlaps. The simple approach has been soon improved by Ma Yugang et al. [12, 13] using the Boltzmann-Uehling-Uhlenbeck dynamics. For practical reasons, however, even the original (and more simple) approach of Cindro et al. [10, 11] has been to a good degree approximated by using some simple derived trends.

Both Cindro et al. [10, 11] and Ma Yugang et al. [12, 13] gave only a single number for the initial number of degrees of freedom, without any specification for its possible spin dependence. However, the heavy-ion collisions are the type of reactions where the role of spin for many reasons plays a very important role. It is the aim of the present paper to give a very simple model of the initial number of degrees of freedom in heavy-ion collisions which can be used for calculations of reactions sensitive to the spin variables.

2. *Basis of the model*

The pre-equilibrium exciton model is governed by the set of master equations which describes both competing processes, the equilibration of the nucleus and the emission. In the most simple case, when we consider just one possible excitation energy and only the composite system prior to and including the first emission, this set reads [14, 15]

$$\begin{aligned}\frac{\mathrm{d}P(n,E,t)}{\mathrm{d}t} &= P(n-2,E,t)\,\lambda^{+}(n-2,E)+P(n+2,E,t)\,\lambda^{-}(n+2,E)\\ &\quad -P(n,E,t)\,[\lambda^{+}(n,E)+\lambda^{-}(n,E)+L(n,E)]\,, \qquad (1)\end{aligned}$$

where $P(n, E, t)$ is the occupation probability of finding the nucleus with the excitation energy E in an n-exciton state at time t, $\lambda^{\pm}(n, E)$'s are the decay rates from an n-exciton to the $(n \pm 2)$ exciton state and $L(n, E)$ is the emission rate (integrated over outgoing energies and summed over all possible ejectiles). The decay rates should be consistent with the minimal $n_{\min}$ and the maximal $n_{\max}$ possible exciton number. Their determination is trivial[1]. The set of master equations is solved with the initial condition, which is usually written in the form

$$P(n, E, t = 0) = \delta_{nn_0}. \tag{2}$$

The behaviour at distant times, i.e. after all excited nuclei decay,

$$P(n, E, t \to \infty) = 0 \qquad \text{for all } n, \tag{3}$$

simply follows from the properties of the equations themselves. As an alternative to (2), instead of using a single value n_0, one can use a distribution over some range of initial exciton numbers [17]. In such a case, the solution is simply an incoherent sum of properly weighted contributions arising from different initial exciton numbers. Solving the set of master equations (1), one obtains the time integrals $\tau(n, E)$

$$\tau(n, E) = \int\limits_0^\infty P(n, E, t)\mathrm{d}t, \tag{4}$$

i.e., the time spent by the excited nucleus in an n-exciton state. The cross sections and related quantities are now straightforwardly expressed as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\varepsilon_x} = \sigma_{\mathrm{R}} \sum_n \tau(n, E) \lambda_x^c(n, E, \varepsilon_x), \tag{5}$$

where $\lambda_x^c(n, E, \varepsilon_x)$ is the emission rate of the particle x with energy ε_x from the n-exciton state of the excitation energy E.

3. *Initial exciton number*

In the above scheme, the initial exciton number n_0, which is specific for each projectile-target combination and energy, plays the key role. Once we know reliably the initial exciton number, all the rest is governed by the master equations with the emission and decay rates which should *not* depend on the conditions of how the composite system has been created.

[1] The minimal possible exciton number, $n_{\min}$, is found, e.g. from the fact that no particle-hole pair can annihilate (and, therefore, decrease the number of excitons by $\Delta n = -2$), if there are no more particles (or no more holes); the maximal exciton number $n_{\max}$ can be determined by the energy necessary to cope with the Pauli principle (no two excitons of the same kind are allowed to share the same state), or simply by exhausting the number of nucleons in a nucleus [16].

If one ignores the structure effects, a good overall rule says that for reactions induced by nucleons and α-particles, the initial exciton number is equal or close to the mass number of the projectile [14, 18, 19] and that all these initial excitons are of particle type. For the nucleon-induced reactions, however, the meaning of the initial exciton configuration is taken over by that arisen after the first interaction, i.e. the 3-exciton state instead of the 1-exciton one[2]. The philosophy that the initial exciton number be close to the mass number of the projectile has been suggested to be used also for the reactions induced by heavy ions [5], however, the results have not been encouraging.

Several years later, Korolija et al. [6, 20] and Běták [7] recognized that the initial exciton number needed to yield reasonable results of particle spectra and cross sections observed in heavy-ion collisions – if properly scaled – follows some trends which are universal for all impacting ions.

4. *Spin-dependent formulation*

The spin-independent version of the pre-equilibrium model serves well for the calculations of particle spectra, cross sections and many other quantities. However, there are at least two types of processes where the spin effects play a very important role, namely the γ de-excitation to the discrete levels with given quantum numbers, and the heavy-ion reactions, where the momenta introduced by the projectile to the system are *much* higher than is the case of the reactions induced by nucleons.

At least for these two areas, it is reasonable to switch from the simple spin-independent description given above to the spin-dependent one. To do that, we have to add the spin variables into the set of master equations. Simultaneously with that, we include also some provision for description of longer de-excitation chains within the reaction, where different nuclei with various energies are coupled together. Instead of (1) we have now [21]

$$\begin{aligned} \frac{\mathrm{d}P(i,n,E,J,t)}{\mathrm{d}t} = & \; P(i,n-2,E,J,t)\,\lambda^{+}(i,n-2,E,J) \\ & +P(i,n+2,E,J,t)\,\lambda^{-}(i,n+2,E,J) \\ & -P(i,n,E,J,t)\left[\lambda^{+}(i,n,E,J)+\lambda^{-}(i,n,E,J)\right. \\ & \hspace{6cm} \left. +L(i,n,E,J)\right] \\ & + \sum_{i',J',n',x} \int P(i',n',E',J',t) \\ & \hspace{2cm} \times\lambda_x^c([i',n',E',J'] \xrightarrow{\varepsilon} [i,n,E,J])\mathrm{d}\varepsilon, \end{aligned} \tag{6}$$

[2] Both these "real initial" 1-exciton and "pseudo-initial" 3-exciton configurations lead to exactly the same results (if one does not consider γ-emission), as no emission is allowed from the 1-exciton state and all its strength is transferred to the 3-exciton one.

where i (or i') denotes the nuclei within the reaction chain and J and J' are the spins. The total emission is now

$$L(i,n,E,J) = \sum_x \int \lambda_x([i,n,E,J] \xrightarrow{\varepsilon} [\text{anything}])\mathrm{d}\varepsilon, \tag{7}$$

and the spin-dependent intranuclear transition rates

$$\lambda^{\pm}(E,J,n) = \frac{2\pi}{\hbar}|M|^2 {Y_n}^{\downarrow} {X_{nJ}}^{\downarrow} \tag{8}$$

have been derived by Obložinský [22]. Here, ${Y_n}^{\downarrow}$ is the energy part of the accessible final states, which remains the same as it was in the case of spin-independent calculations, and ${X_{nJ}}^{\downarrow}$ represents the angular momentum part (see Ref. [22]).

Together with that, one has to replace the reaction cross section σ_{R} by the cross section of the creation of composite system with spin J_{c} at the excitation energy E_{c}, which is

$$\sigma(E_{\mathrm{c}},J_{\mathrm{c}}) = \pi\lambda^2 - \frac{2J_{\mathrm{c}}+1}{(2s_a+1)(2I+1)} \sum_{S=|I-s_a|}^{I+s_a} \sum_{l=|J_{\mathrm{c}}-S|}^{J_{\mathrm{c}}+S} T_l(\varepsilon_{\mathrm{proj}}), \tag{9}$$

where the subscript a denotes the projectile and T_l's are the transmission coefficients.

Now, the primary energy spectrum (i.e. the first ejectile only) is

$$\left(\frac{\mathrm{d}\sigma_x}{\mathrm{d}\varepsilon_x}\right)_0 = \sum_{J_{\mathrm{c}},n} \sigma(E_{\mathrm{c}},J_{\mathrm{c}})\tau(0,n,E_{\mathrm{c}},J_{\mathrm{c}})\lambda_x([0,n,E_{\mathrm{c}},J_{\mathrm{c}}] \xrightarrow{\varepsilon_x} [\text{anything}]) \tag{10}$$

instead of (5). In (10), the index 0 denotes that we care for the primary emission only, i.e. τ and λ refer to the nucleus prior to any emission ($i=0$, $E=E_{\mathrm{c}}$). The total spectrum is obtained by summing the contributions from the whole course of a reaction over all possible spins and integrating over all energies of all intermediate composite systems, i.e.

$$\frac{\mathrm{d}\sigma_x}{\mathrm{d}\varepsilon_x} = \sum\!\!\!\!\!\!\int_{E,J,J_{\mathrm{c}},n,i} \sigma(i,E_{\mathrm{c}},J_{\mathrm{c}},E,J)\tau(i,n,E,J)\lambda_x([i,n,E,J] \xrightarrow{\varepsilon_x} [\text{anything}]). \tag{11}$$

Here, $\sigma(i,E_{\mathrm{c}},J_{\mathrm{c}},E,J)$ represents the (total) population of states of nucleus i in the reaction chain which have the energy E and spin J, if the composite system has been created with cross section $\sigma(E_{\mathrm{c}},J_{\mathrm{c}})$. To get $\sigma(i,E_{\mathrm{c}},J_{\mathrm{c}},E,J)$, one needs to follow all prior history of the system, i.e. to calculate all emissions (both of particles and γ's) which could lead to energy E and spin J. The symbol $\sum\!\!\!\!\!\!\int$ represents summation over discrete and integration over continuous variables.

5. *Pre-equilibrium heavy-ion collisions*

The interaction of heavy projectile with the target at incident energies, roughly starting from the Coulomb barrier and extending close to the Fermi energy, can be easily described using the proximity potential (see e.g. Ref. [23]). The calculation using code TRAJEC [24] shows that there are essentially three stages of this collision: *i)* fast approach phase of nearly free incoming ions, followed by *ii)* intense slowing down due to the deep-inelastic collisions of heavy ions accompanied by friction, and finally *iii)* a long phase of co-existing excited nuclear system [25].

A more sophisticated approach using time-dependent potential switching from the diabatic to adiabatic one [26] is even capable to describe the re-building of the potential in the third phase and the corresponding development of the system afterwards [7].

The original systematics [6, 7] did not present any suitable formulae to describe the observed dependence of the initial exciton number. Such a proposal, obtained as a fit to the empirical data, appeared in Ref. [20]

$$\frac{E}{n_0} = 0.74\ \frac{E_{\rm lab} - V_{\rm C}}{A_{\rm proj}}\,, \tag{12}$$

and was accompanied by a similar one some three years later [10],

$$\frac{E}{n_0} = 6.8 + 0.54\ \frac{E_{\rm cm} - V_{\rm C}}{A_{\rm proj}}\,, \tag{13}$$

which, in turn, was compared to the best-fit line describing the model values, namely [10]

$$\frac{E}{n_0} = 4.6 + 0.54\ \frac{E_{\rm cm} - V_{\rm C}}{A_{\rm proj}}\,. \tag{14}$$

In Eqs. (12) to (14), $E_{\rm lab}$ is the laboratory energy of the projectile, whereas $E_{\rm cm}$ is the center-of-mass one, $V_{\rm C}$ is the Coulomb barrier and $A_{\rm proj}$ is the mass number of the projectile. The difference in slopes between (12) and (13) is caused by the use of different systems for expressing the projectile energy. In addition to the three dependencies given above, two more have been presented by Ma, namely

$$\frac{E}{n_0} = 9.65 + 0.27\ \frac{E_{\rm cm} - V_{\rm C}}{A_{\rm proj}} \tag{15}$$

in the first paper of their tandem [12] and

$$\frac{E}{n_0} = 5.61 + 0.42\ \frac{E_{\rm lab} - V_{\rm C}}{A_{\rm proj}} \tag{16}$$

in the second one [13]. All these linear fits yield reasonable (and rather close) results for energies well above the Coulomb barrier. For lower energies, however, another approximation to the model results is to be used, namely [11]

$$\frac{n_0}{A_{\rm proj}} = 0.09 + \left(0.38 - 0.08\,\frac{A_{\rm targ} - A_{\rm proj}}{A_{\rm targ} + A_{\rm proj}}\right)\sqrt{\frac{E_{\rm cm} - V_{\rm C}}{A_{\rm proj}}}. \tag{17}$$

This form implicitly assumes that $A_{\rm proj} \leq A_{\rm targ}$. The "switching point" from (17) to the other formulae is about 5 or 10 MeV per nucleon above the Coulomb barrier. Of course, in the very vicinity of the Coulomb barrier (or even slightly below it) all the approximations become very dangerous and one has no other choice than properly evaluate the overlaps of nuclei as suggested by the model itself. For simplicity, all equations (12) to (17) are written for energies expressed in MeV (otherwise conversion coefficients are to be inserted). The model-derived initial exciton numbers (14) and (17) have been successfully used in analyses of γ spectra from several heavy-ion reactions [27, 28].

6. Spin-dependent initial configuration for heavy-ion collisions

In the preceding sections (especially in the last one), we have prepared all necessary ingredients. What needs to be done is the step from the spin-independent understanding of the initial configuration to the spin-dependent one.

We decompose the movement of the two colliding nuclei into their radial and tangential parts. The latter one is determined by the impact parameter, and it fully transforms into rotation of the double-nuclear system[3]. The rotational energy is

$$E_{\rm rot}(l) = \frac{l(l+1)\hbar^2}{2\mu R(l)^2}, \tag{18}$$

where μ is the reduced mass and $R(l)$ is the distance of the centers of nuclei at the moment of their contact. As has been shown [25], this distance can be approximated to a good degree by a constant independent of l. The energy $E_{\rm rot}(l)$ is the quantity which has to be subtracted from the incident energy. The energy diminished by the rotation

$$E'_{\rm cm}(l) = E_{\rm cm} - E_{\rm rot}(l) \tag{19}$$

is now the one which is responsible for approaching the nuclei in their radial coordinate and which has to be considered as a replacement of the original $E_{\rm cm}$ determining the overlap of nuclei and therefore the initial number of degrees of freedom, i.e. the initial exciton number n_0. Clearly, it is dependent on the angular momentum, so that we do not get any more a single value of the excitation energy E

[3]We have to keep in mind that the fusion of the two colliding nuclei occurs at times substantionally longer than is the period of rotation or the time characteristic for fast pre-equilibrium emission (see Refs. [7, 25]).

with one corresponding value of n_0, but rather an extensive table which attaches to each partial wave l its corresponding excitation energy $E(l)$ and the initial number of excitons $n_0(l)$, each of them entering the calculations with the corresponding cross section $\sigma(E_C, J_C)$. A possible future refinement of the approach may, in principle, yield not only single value of $n_0(l)$, but even a kind of their distribution. Current pre-equilibrium exciton-model codes based on the master equations, like DEGAS [21], can handle such distributions equally easily as a single value ascribed to a collision.

7. *Conclusions*

We have suggested a method how to estimate the spin-dependent initial exciton number for heavy-ion collisions at modest energies in a feasible way. It is based on the simple model suggested a decade ago by N. Cindro et al., where an additional dependence on the angular momentum has been incorporated. As a result, one does not get a single pair of E and n_0, but rather a wide distribution over the excitation energies, spins and initial exciton numbers. Current pre-equilibrium master-equation based exciton-model codes (e.g. Ref. [21]) can handle this situation with the same level of ease as was in the case for the spin-independent view on the heavy-ion collisions.

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MODEL PREDRAVNOTEŽNIH TEŠKOIONSKIH SUDARA SA SPINOM

Predstavljamo jednostavan model koji može dati početan broj eksitona u teškoionskim sudarima na srednjim energijama kad se razmatraju i spinske varijable. Ovaj je model generalizacija modela predloženog od Cindra i sur. prije deset godina.