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BIANCHI TYPE-I COSMOLOGICAL MODEL IN BIMETRIC RELATIVITY

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Bianchi type-I cosmological model is studied in the context of bimetric relativity, taking the source as perfect fluid distribution coupled with an incident magnetic field directed along the z-axis. The solution represents non-existence of Bianchi-I cosmological model in this theory.

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1. Introduction

Rosen [1] proposed the bimetric theory of relativity to remove some of the unsatisfactory features of the general theory of relativity. In the bimetric theory, there exist two metric tensors at each point of space-time, g_{ij} which describes gravitation, and the background metric γ_{ij} which enters into the field equations and interacts with g_{ij}, but does not interact directly with matter. One can regard γ_{ij} as describing the geometry that exists if no matter were present.

Accordingly, at each space-time point, one has two line elements

$$\mathrm{d}s^2 = g_{ij}\mathrm{d}x^i\mathrm{d}x^j$$

and

$$\mathrm{d}\sigma^2 = \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \, .$$

This work concludes that the Bianchi type-I cosmological model od bimetric relativity does not accomodate perfect fluid coupled with an incident magnetic field directed along the z-axis.

2. *Field equation*

The line element describing the Bianchi-I space-time is taken in the form

$$ds^2 = A^2(dx^2 - dt^2) + B^2 dy^2 + C^2 dz^2 \,, \tag{1}$$

where the metric potentials A, B and C are functions of time t.

The background metric of flat space-time is

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 \,. \tag{2}$$

The matter distribution consists of an electrically neutral perfect fluid with infinite electrical conductivity, coupled with a source-free magnetic field given by the momentum tensor

$$T_i^j = (\varepsilon + p)\nu_i \nu^j + p g_i^j + E_i^j \,, \tag{3}$$

together with an orthogonality condition

$$\nu_4 \nu^4 = -1 \,, \tag{4}$$

where

$$E_i^j = F_{ir}F^{jr} - \frac{1}{4} F_{ab}F^{ab} g_i^j \,. \tag{5}$$

Here p is isotropic pressure, ε the matter density, E_i^j the electromagnetic energy tensor, F_{ij} the electromagnetic field tensor and ν^i the four-velocities of the fluid which are chosen so that $\nu^1 = \nu^2 = \nu^3 = 0$ and $\nu^4 = A^{-1}$. The magnetic field is taken along the Z direction, so that the only non-zero component of F_{ij} is $F_{12} = -F_{21}$.

Maxwell's equation

$$F_{ij,\,k} + F_{jk,\,i} + F_{ki,\,j} = 0 \tag{6}$$

gives rise to

$$F_{12} = F(\text{a constant}) \,. \tag{7}$$

The field equations of bimetric relativity proposed by Rosen [1] are

$$K_i^j = N_i^j - \frac{1}{2} N g_i^j = -8\pi k T_i^j \,, \tag{8}$$

where

$$N_i^j = \frac{1}{2} \gamma^{\alpha\beta} (g^{hj} g_{hi|\alpha})_\beta \,, \tag{9}$$

and

$$N = N^{\alpha}_{\alpha} \,, \tag{10}$$

where the vertical bar (|) denotes the γ differentiation with respect to γ_{ij}.

Using Eqs. (1) to (10), the field equations are

$$\left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k(p+\rho) \,, \tag{11}$$

$$2\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = -16\pi k(p+\rho) \,, \tag{12}$$

$$2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = -16\pi k(p-\rho) \,, \tag{13}$$

$$\left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 16\pi k(\varepsilon+\rho) \,, \tag{14}$$

where

$$\rho = \frac{1}{2}\frac{F^2}{A^2B^2} \,, \tag{15}$$

and the suffix 4 following unknown functions A,B and C denotes ordinary differentiation with respect to t.

The conservation equation for the energy-momentum tensor T_i^j

$$T_{i_j} = 0$$

leads to

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}\tau} + (\varepsilon+p)\theta = 0 \,, \tag{16}$$

where θ is the expansion scalar defined by $\theta = 3R(\mathrm{d}R/\mathrm{d}\tau)$, while $R^3 = ABC$ and τ is the cosmic time given by $\tau = \int A\mathrm{d}t$.

Using Eqs. (11) and (14), we get

$$\varepsilon + p + 2\rho = 0 \,. \tag{17}$$

The dominant energy condition [2] gives rise to

$$\begin{array}{ll} \varepsilon + p + 2\rho \geq 0, & \varepsilon + p \geq 0 \\ \varepsilon - p + 2\rho \geq 0, & \varepsilon - p \geq 0 \,. \end{array}$$

Since $\rho > 0$, it is sufficient to have

$$\varepsilon + p \geq 0 \qquad \varepsilon - p \geq 0\,.$$

Equation (6) shows that the matter density of the Universe is decreasing with time during its expansion stage, provided the energy condition $\varepsilon + p > 0$ holds when p satisfies the barometric equation of state $p = (\gamma - 1)\varepsilon, \quad 1 \leq \gamma \leq 2$.

Equation (16) gives us

$$\varepsilon = \varepsilon_0 \frac{R_0^{3\gamma}}{R^{3\gamma}}\,, \tag{18}$$

where ε_0 and R_0 are the present-day values of ε and R, and this shows that the matter density ε varies as $R^{-3\gamma}$.

Equation (17) shows that each term on the left-hand side of the equation vanishes separately, what means

$$p = \varepsilon = \rho = 0\,, \tag{19}$$

i.e., there is no contribution from the perfect fluid with magnetic field along the Z-axis to the Bianchi-I cosmological model in bimetric relativity.

Using Eq. (19) in Eqs. (11) to (14), we obtain

$$\left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 0\,, \tag{20}$$

$$2\left(\frac{A_4}{A}\right)_4 - \left(\frac{B_4}{B}\right)_4 + \left(\frac{C_4}{C}\right)_4 = 0\,, \tag{21}$$

$$2\left(\frac{A_4}{A}\right)_4 + \left(\frac{B_4}{B}\right)_4 - \left(\frac{C_4}{C}\right)_4 = 0\,. \tag{22}$$

Solving Eqs. (20) to (22), we have

$$A = B = C = e^{k_1 t}\,, \tag{23}$$

where k_1 is an integration constant.

Thus, in view of Eq. (23), the metric (1) takes the form

$$\mathrm{d}s^2 = \exp 2k_1 t(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2 - \mathrm{d}t^2)\,. \tag{24}$$

This can be transformed through a proper choice of coordinates to

$$\mathrm{d}s^2 = \exp\left[2T(\mathrm{d}X^2 + \mathrm{d}Y^2 + \mathrm{d}Z^2 - \mathrm{d}T^2)\right]. \tag{25}$$

It is interesting to note that the vacuum model (25) has no singularity at $T = 0$.

3. Conclusion

We conclude that the Bianchi Type-I cosmological model does not accomodate the perfect fluid distribution coupled with an incident magnetic field directed along the z-axis in Rosen's bimetric theory of relativity. Thus the vacuum model obtained which is free from singularity.

References

[1] N. Rosen, Gen. Relat. Grav. **4** (1973) 435.

[2] S. W. Hawking and G. F. R. Ellis, *The Large Scale of Space*, Cambridge Univ. Press, Cambridge (1973).

BIANCHIJEV KOZMOLOŠKI MODEL TIPA-I U DVOMETRIČKOJ RELATIVNOSTI

Proučavamo Bianchijev kozmološki model tipa-I u okviru dvometričke relativnosti, uzimajući za izvor perfektnu raspodjelu tekućine vezanu s upadnim magnetskim poljem koje je usmjereno duž z osi. Rješenje pokazuje nepostojanje kozmološkog modela Bianchi-I u ovoj teoriji.