ISSN 1330–0016
CODEN FIZBE7

STUDY OF RADIATIVE B-DECAY CONSIDERING TWO-HIGGS-DOUBLET MODELS AND THE FOURTH GENERATION IN THE QUARK SECTOR

HARAPRASAD ROY, SURATH KUMAR BISWAS[1] and VINOD P. GAUTAM*

Theoretical Physics Department, Indian Association for the Cultivation of Science, Jadavpur, Kolkata-700032, India
E-mail address: tpdpb@mahendra.iacs.res.in

Received 15 December 2000 ; revised manuscript received 8 June 2002
Accepted 6 September 2002 Online 7 February 2003

Considering the nonstandard indirect effect of Yukawa couplings of the Higgs doublet to the quarks, we examine strong-interaction effects in weak radiative B-meson decay. The virtual effects of the Model I and Model II in the penguin-type diagram on the $b \to s\gamma$ decay have been tested considering the possibility of the fourth generation in quark sector, with corrections up to the leading QCD logarithms, using evolution of the fourth-generation CKM matrix, with CP violation phase equal to zero. Range of the masses of the fourth-generation down-type quark b' and up-type quark t' have been taken with due consideration of the constraint imposed by the present experimental value of the ρ parameter, keeping in view the mass difference of the fourth-generation quark doublet. As a by-product, it is observed that the CLEO bound clearly sets the lower bound on the Higgs boson mass at 220 GeV, which is above the top quark mass.

PACS numbers: 12.60.-i UDC 539.126
Keywords: weak radiative B-meson decay, fourth generation, Higgs boson mass

1. Introduction

Weak radiative B-meson decay is a very sensitive probe of new physics [1,2]. Heavy-quark effective theory gives us the idea that inclusive B-meson decay rate into charmless hadrons and photon is well approximated by the corresponding partonic decay rate

$$\Gamma(\mathrm{B} \to \mathrm{X}_s\gamma) \quad = \quad \Gamma(b \to \mathrm{X}_s\gamma). \tag{1}$$

[1]E-mail address: tpskb@vsnl.net

The accuracy of this approximation is expected to be better than 10% [3].

Experimental evidence for the flavour-changing neutral current (FCNC) process in B decay is at present based on the following quantities:

(i) Exclusive radiative decay $\mathrm{B} \to \mathrm{K}^*\gamma$ having branching ratio [4]

$$Br(\mathrm{B} \to \mathrm{K}^*\gamma) \quad = \quad (4.0 \pm 1.9) \times 10^{-5}\,. \tag{2}$$

(ii) The measurement of the photon energy spectrum in the decay $\mathrm{B} \to \mathrm{X}_s + \gamma$ yielding a branching ratio [5]

$$10^{-4} \quad < \quad Br(b \to s\gamma) < 4 \times 10^{-4}. \tag{3}$$

CLEO [6] found the rate

$$Br(b \to s\gamma) \quad = \quad (2.32 \pm 0.57 \pm 0.35) \times 10^{-4} \text{ (CLEO)}, \tag{4}$$

measuring from the end point of the inclusive photon spectrum in B-decay.

ALEPH used a lifetime tagged sample of $\mathrm{Z} - b\bar{b}$ events to search for high-energy photon in the hemisphere opposite to the tag. This allows them to measure the photon spectrum from B-decays which ultimately leads to [7]

$$Br(b \to s\gamma) \quad = \quad (3.11 \pm 0.80 \pm 0.72) \times 10^{-4} \text{ (ALEPH)}. \tag{5}$$

Our theoretical understanding of inclusive $b \to s\gamma$ transitions has been significantly enhanced by two new calculations that now include all terms up to next-to-leading order [8].

The expected standard model (SM) rate, while slightly larger now, is still consistent with both CLEO and ALEPH results. We expect much more precise measurements from the ungraded CLEO detector as well as from the B-factories at SLAC and KEK in near future.

In this paper we intend to explore the possibility of new physics, namely, to see the effects of considering the two-Higgs-doublet model (2HDM) and the fourth generation of quarks, restricting the calculation up to the leading logarithmic terms. Model I has both the up- and down-type quarks getting mass from Yukawa couplings to the same Higgs doublet H_2. The Higgs doublet H_1 has no Yukawa couplings to the quarks. Model II has the up-type quarks getting mass from Yukawa couplings of H_2 and the down-type quarks getting mass from Yukawa couplings to the other Higgs doublet H_1. Each doublet obtains a vacuum expectation value (vev) v_i, subject to the constraint $v_1^2 + v_2^2 = v^2$, where v is the usual vev present in the SM. We endeavour to find the constraints on the parameters taking the CLEO data on $b \to s\gamma$ given in Eq. (3).

In this paper we have use the procedure followed in Ref. [9]; we use all the equations of Sect. 2, and the introduction of the fourth generation is done in a way described in Sect. 2.2 of Ref. [9]. We also utilize all the current data of Ref. [10] which gives recent developments in the field of our study, especially the contribution of D. E. Groom et al. [11] for the calculations.

The paper is organized as follows: Sect. 2 summarises the formulation for the incorporation of the 2HDM. In Sect. 3 the results are discussed. For the calculation of the fourth-generation CKM mixing matrices from quark masses, Appendix to Ref. [9] has been used.

2. *Higgs scalar couplings*

The effects of Higgs doublet may be tested in the $b \to s\gamma$ decay in the following way.

General CP conserving Lagrangians for $b \to s\gamma$ interaction for model I and model II can be written as [12,13]

Model I

$$\mathcal{L}_{\mathcal{I}} = \frac{g_2}{\sqrt{2}M_W} H^+ \left[\left(\frac{v_1}{v_2}\right) (\bar{u}\bar{c}\bar{t})_R M_U V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L - \left(\frac{v_1}{v_2}\right) (\bar{u}\bar{c}\bar{t})_L V M_D \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \right] + h.c., \tag{6}$$

where v_1 and v_2 are the vacuum expectation values of H_1 and H_2, respectively; V represents the CKM matrix, M_U and M_D denote the diagonalized quark mass matrices.

Model II

$$\mathcal{L}_{\mathcal{II}} = \frac{g_2}{\sqrt{2}M_W} H^+ \left[\left(\frac{v_1}{v_2}\right) (\bar{u}\bar{c}\bar{t})_R M_U V \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + \left(\frac{v_2}{v_1}\right) (\bar{u}\bar{c}\bar{t})_L V M_D \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \right] + h.c. \tag{7}$$

For the calculation, we adopt the following procedure. The coupling presented in Eqs. (6) and (7) is put into relevant Feynman diagram in which a physical charged scalar can take the place of the W-boson and one extracts the pure dipole-like terms after performing the loop integrations. All other potential Lorentz structures vanish due to the electromagnetic gauge invariance and as the photon is on-shell. Now the operator O_7 of the Hamiltonian which is the coefficient of the dipole $b \to s$ transition operators is modified due to the presence of the Higgs scalar couplings. At the M_W scale, O_7 is the only operator mediating the $b \to s\gamma$ decay. But mixing occurs between various $b \to s$ transition operators during the evolution of the coefficient of O_7 to the b quark mass scale. In the M_W scale the Wilson coefficient $C_7(m_t, M_W)$ and $C_8(m_t, M_W)$ are only changed, so yielding an additional contribution. Rewriting the Wilson coefficient C_7 at the W boson scale incorporating the effects of Higgs doublet, one gets [13, 14]

For Model I

$$C_7^{\rm eff}(m_t,M_W,M_{H^\pm}) = C_7(m_t,M_W)_{\rm SM} + \left(\frac{v_1}{v_2}\right)^2 \left[B(m_t,M_{H^\pm})+\frac{1}{3}C_7(m_t,M_{H^\pm})_{\rm SM}\right] \tag{8}$$

For Model II

$$C_7^{\rm eff}(m_t,M_W,M_{H^\pm}) = C_7(m_t,M_W)_{\rm SM} - B(m_t,M_{H^\pm}) + \frac{1}{3}\left(\frac{v_1}{v_2}\right)^2 C_7(m_t,M_{H^\pm})_{\rm SM}\,, \tag{9}$$

where $C_i(m_t,M_{H^\pm})_{\rm SM}$, $i=7,8$ has the same expression as that of $C_i(m_t,M_W)_{\rm SM}$ with simply M_W replaced by $M_{H^\pm}$,

$$B(m_t,M_{H^\pm}) = \frac{y}{2}\left[\frac{\frac{5}{6}y-\frac{1}{2}}{(y-1)^2} - \frac{(y-\frac{2}{3})}{(y-1)^3}\ln y\right], \tag{10}$$

and $y = m_t^2/M_{H^\pm}^2$ where $M_{H^\pm}$ is the charged-Higgs-scalar mass.

The $H^\pm$ coupling in model II differs from that of model I in two ways:

(i) In model I, the ratios of vacuum expectation values are the same for the terms involving M_U and M_D, while in model II they are different.

(ii) In model I, the sign of the terms proportional to M_U and M_D is opposite while in model II it is the same.

Since the couplings of $H^\pm$ in (6) and (7) are proportional to the quark masses, the most important low-energy consequences of these couplings are likely to occur in the processes where virtual top quarks play an important role. For example, the CP violation in kaon decays, $B^0-\bar{B}^0$ mixing and rare B-meson decay under consideration here.

However, Model I and Model II give different results for $C_7^{\rm eff}(m_t,M_W)$. This is because, even though $m_t \gg m_b$, the coupling of charged scalar $H^\pm$ proportional to m_b is important since operator O_7 contains a factor of m_b.

In $\overline{MS}$, the operator C_8 also suffers change, and one has

For Model I

$$C_8^{\rm eff}(m_t,M_W,M_{H^\pm}) = C_8(m_t,M_W)_{\rm SM} + \left(\frac{v_1}{v_2}\right)^2 \left[E(m_t,M_{H^\pm})+\frac{1}{3}C_8(m_t,M_{H^\pm})_{\rm SM}\right. . \tag{11}$$

For Model II

$$C_8^{\rm eff}(m_t,M_W,M_{H^\pm}) = C_8(m_t,M_W)_{\rm SM} - E(m_t,M_{H^\pm}) + \frac{1}{3}\left(\frac{v_1}{v_2}\right)^2 C_8(m_t,M_{H^\pm})_{\rm SM}\,, \tag{12}$$

where
$$E(m_t, M_{H^\pm}) = +\frac{y}{2}\left[\frac{\frac{1}{2}y-\frac{3}{2}}{(y-1)^2}-\frac{\ln y}{(y-1)^3}\right]. \tag{13}$$

The expressions for $C_7(m_t, M_W)$ and $C_8(m_t, M_W)$ have been procured from Ref. [9]. Equations (8) and (11) immediately give the 2HDM predictions for Model I and Eqs. (9) and (12) for Model II for the B→ $X_s\gamma$ branching ratio. Using the lifetime of the B-meson [11] and the limits prescribed by the CLEO data [15], the constraints on the parameters, namely, the masses of $H^\pm$ and $\tan\beta \equiv v_2/v_1$, can be estimated.

3. Discussion of results and conclusion

Nowadays, it is customary to calculate the strong coupling constant with $\alpha_s(M_Z)$ as an initial condition. A straightforward calculation gives the solution [1] of the form
$$\alpha_s(\mu) = \frac{\alpha_s(M_Z)}{\alpha(\mu)}\left(1-\frac{\beta_1}{\beta_0}\frac{\alpha_s(M_Z)}{4\pi}\frac{\ln\alpha(\mu)}{\alpha(\mu)}\right), \tag{14}$$
$$\text{where}\quad \alpha(\mu) = 1-\beta_0\frac{\alpha_s(M_Z)}{2\pi}\ln\left(\frac{M_Z}{\mu}\right). \tag{15}$$

Thus λ=$\alpha_s(M_W)/\alpha_s(\mu)$ can now be calculated.

For the calculation in the instant case, $\beta_0 = \frac{1}{3}(11N-2n_f)$ and $\beta_1 = \frac{34}{3}N^2 - \frac{10}{3}Nn_f - 2c_f n_f$, where N is the number of colours, n_f the number of active flavours and $c_f = N^2 - 1/(2N)$. In our paper $N = 3$ and $n_f = 5$. We use $\alpha^{-1} = 130.3$ (Ref. [8]), and the following values were taken [10]: $G_F = 1.16639\times10^{-5}$ GeV^{-2}, $M_Z = 91.187$ and $M_W = 80.41$. Also we take $m_t = 175$ GeV, $m_b = 5$ GeV and we use $V_{ts} = 0.0566$, $V_{tb} = 0.9989$.

The value of $\alpha_s(M_Z^2)$ so far available is taken as 0.119 ± 0.002 considering the average of Refs. [10,16-18].

The values of $\alpha_s(\mu)$ at the M_Z mass scale for different values of m_b are calculated with the help of Eqs. (14) and (15). It may be worth mentioning that to the leading-order, the effect of the Higgs boson is simply to increase the strength of interquark coupling [19]
$$\frac{4}{3}\alpha_s \to \frac{4}{3}\alpha_s + \frac{m_t^2}{4\pi v}, \tag{16}$$

where v is the Higgs-field vacuum expectation value equal to 246 GeV (in minimal SM). The calculation of λ is trivial.

For the mean life of B [11], one has the total decay width $\Gamma_b = 4.274105\times10^{-13}$ GeV which is used to calculate the branching ratio. With these values chosen for different parameters, one obtains the SM value for the $Br(b\to s\gamma)$ as 2.218389×10^{-4}.

Following observations are in order.

For Model I, the results are given in Table 1. In the table, the following notations are used:

The values of the branching ratio $Br(b \to s\gamma)$ above the CLEO upper bound are denoted by "A", the values within the bounds are denoted by "W", the values below the CLEO lower bound are denotd by "B" and $Br(b \to s\gamma)$ is denoted by "Br".

TABLE 1. Model I. Values of $\tan\beta$ *versus* $M_{H^\pm}$ *denoted in the columns marked as A, W, B, and W.*

$M_{H^\pm}$ (GeV)	A	W		B		W	Limiting values
	up to	from	to	from	to	from	of $Br(b \to s\gamma)$ for large $\tan\beta$
40	0.4887	0.4888	0.5785	0.579	1.3	1.31	2.218389×10^{-4}
80	0.452	0.453	0.535	0.536	1.2	1.21	2.218389×10^{-4}
120	0.418	0.419	0.495	0.496	1.11	1.12	2.218389×10^{-4}
160	0.388	0.389	0.46	0.461	1.03	1.04	2.218389×10^{-4}
200	0.362	0.364	0.428	0.429	0.96	0.97	2.218389×10^{-4}
240	0.339	0.34	0.401	0.402	0.9	0.913	2.218389×10^{-4}
280	0.319	0.3195	0.376	0.377	0.85	0.855	2.218389×10^{-4}
300	0.309	0.31	0.366	0.367	0.82	0.83	2.218389×10^{-4}
400	0.27	0.28	0.321	0.322	0.72	0.73	2.218389×10^{-4}
500	0.241	0.242	0.285	0.286	0.64	0.65	2.218389×10^{-4}
600	0.21	0.22	0.25	0.26	0.58	0.585	2.218389×10^{-4}

Table 1 shows that Model I value for Br approaches the SM value at very large values of $\tan\beta$ for any $M_{H^\pm}$. We find two CLEO allowed zones in the $\tan\beta$ - $M_{H^\pm}$ plane; for any value of $M_{H^\pm}$, two allowed disjoint intervals for $\tan\beta$ are obtained.

(1) (a) For $M_{H^\pm} = 40$ GeV, Br remains A up to 0.4887 and for 600 GeV Br remains A up to 0.21, so with the incease of the Higgs mass, allowable region for $\tan\beta$ is pushed back.

(b) The width of the first allowed region in $\tan\beta$ space is 0.09 for $M_{H^\pm} = 40$ GeV, and the value is gradually decreased to 0.03 for $M_{H^\pm} = 600$ GeV.

(c) Decay width now decreased further and goes below the CLEO region and then after reaching a minimum it again increases and comes back to retrive the link with the allowed region and remains within the allowed zone. The seperation of two such regions may be called forbidden zone. Width of the forbidden zone is 0.7 in $\tan\beta$ space for $M_{H^\pm} = 40$ GeV and it reduces to 0.32 for $M_{H^\pm}$= 600 GeV.

(2) Br remains initially above the CLEO bound, then enters the CLEO bound, the allowed region, then goes below the lower bound, reaches a minimum value and then inreases and enters the CLEO allowable region. And for very large value of

$\tan\beta$ it approaches the SM value from below. This last spell starts at $\tan\beta = 1.31$ for $M_{H^\pm} = 40$ GeV, and pushed back to $\tan\beta = 0.585$ at $M_{H^\pm} = 600$ GeV.

For Model II, calculations up to $M_{H^\pm} = 200$ GeV keep the branching ratio above the CLEO upper bound for any value of $\tan\beta$; it enters within at 240 GeV for certain values of $\tan\beta$ but it never goes below the lower limit of the CLEO bound, it becomes stationary at very very large values of $\tan\beta$. The results are given in the Table 2.

TABLE 2. Model II values of $\tan\beta$ *in the columns marked as A and W versus* $M_{H^\pm}$.

$M_{H^\pm}$ (GeV)	A up to	W from	$Br(b \to s\gamma)$ for very large values of $\tan\beta$
80	For all values	never reached	5.744433×10^{-4}
120	For all values	never reached	5.049779×10^{-4}
160	For all values	never reached	4.536090×10^{-4}
200	For all values	never reached	4.151588×10^{-4}
240	1.4	1.5	3.857845×10^{-4}
280	0.79	0.8	3.628707×10^{-4}
300	0.67	0.68	3.532599×10^{-4}
400	0.41	0.42	3.178089×10^{-4}
500	0.3	0.31	2.955085×10^{-4}
600	0.24	0.25	2.804835×10^{-4}

We see that even if it goes within the bound from above, however, the value gradually decreases but never goes below the CLEO bound.

In view of the above observations, it is imperative to address the question on the point where the Model II result of the $Br(b \to s\gamma)$ comes within the CLEO bound. It has been observed that with the values of the parameters chosen, the bound is never reached for $M_{H^\pm}$ up to 219 GeV and it is reached only at 220 GeV (more correctly at 219.4 GeV). And the Br remains above up to $\tan\beta = 8.2$, and at $\tan\beta = 8.3$ it goes inside and remains within the CLEO bounds, and for large $\tan\beta$ Br it restricts itself to 3.995247×10^{-4}. We may note from the Model I results for $M_{H^\pm} = 220$ GeV, namely, A up to $\tan\beta = 0.35$, W from $\tan\beta = 0.351$ to 0.41, B from $\tan\beta = 0.42$ to 0.93 and W from $\tan\beta = 0.94$ onwards.

3.1. Fourth generation

Next we turn our attention to the effects of the fourth generation of quarks. First, we calculate the Br values for the fourth generation quark doublet with masses $(m_{t'}, m_{b'}) = (110$ GeV, 45 GeV$)$ with Model I; they are given in the Table 3. Here we have incorporated the results for $M_{H^\pm} = 220$ GeV. The limiting value of $Br(b \to s\gamma)$ for Model I with the effects of the fourth generation (denoted by I4Gn) for large $\tan\beta$ leads to a unique limit of 3.812745×10^{-4}, this value is reached for all values of $M_{H^\pm}$ and it is much above the SM value. The enhancement is evident.

TABLE 3. Model I with the effects of fourth generation with $m_{t'} = 110$ GeV, *and* $m_{b'} = 45$ GeV. *Values of* $\tan\beta$ *versus* $M_{H^\pm}$ *denoted in the columns marked as A, W, B and W.*

$M_{H^\pm}$ (GeV)	A	W		B		W	Limiting values
	up to	from	to	from	to	from	of $Br(b \to s\gamma)$ for large $\tan\beta$
40	0.59	0.6	0.82	0.9	1.315	1.4	3.812745×10^{-4}
80	0.624	0.625	0.736	0.737	1.2	1.21	The same
120	0.563	0.564	0.663	0.664	1.05	1.1	The same
160	0.513	0.514	0.603	0.604	1.0	1.01	The same
200	0.472	0.473	0.553	0.554	0.92	0.93	The same
220	0.454	0.455	0.53	0.54	0.89	0.9	The same
240	0.437	0.438	0.512	0.513	0.86	0.87	The same
280	0.407	0.408	0.475	0.48	0.8	0.805	The same
300	0.394	0.395	0.461	0.462	0.77	0.78	The same
400	0.339	0.34	0.39	0.4	0.67	0.68	The same
500	0.29	0.3	0.34	0.35	0.59	0.6	The same
600	0.26	0.27	0.312	0.313	0.53	0.54	The same

A few observations are in order.

(1) For any $M_{H^\pm}$, Br enters the first allowable zone later than Model I; for example, with $M_{H^\pm} = 40$ GeV, Br for Model I reaches at $\tan\beta = 0.4888$ and for I4Gn enters at $\tan\beta = 0.59$; the difference is 0.09 approximately. With $M_{H^\pm} = 600$ GeV, Br for Model I reaches at $\tan\beta = 0.22$, however, for I4Gn it enters at $\tan\beta = 0.27$ having difference of about 0.05. The value of $\tan\beta$ gets closer with the increase of $M_{H^\pm}$.

(2) For I4Gn, as the values of $\tan\beta$ sweeps Br initially remained above, but subsequently it goes within and then goes below the CLEO lower bound and then shoots up and goes within the allowable zone and reaches the saturation value, namely 3.812745×10^{-4}.

(3) For the entrance of Br to the CLEO bound in the last spell for $M_{H^\pm} = 40$ GeV, $\tan\beta$ is less for Model I than I4Gn. It coincides approximately at $M_{H^\pm} = 80$ GeV, but reversed at $M_{H^\pm} = 200$ GeV where Br enters at $\tan\beta = 0.97$ for Model I, and at $\tan\beta = 0.93$ for I4Gn.

For Model II, $Br(b \to s\gamma)$, considering the effects of the fourth generation (denoted by II4Gn), the limiting value never touches the CLEO bound, always remains above the upper bound for any finite $M_{H^\pm}$, even for large $\tan\beta$ as revealed from Table 4.

TABLE 4. SM value of $Br(b \to s\gamma) = 2.218389 \times 10^{-4}$ *and limiting value of* $Br(b \to s\gamma)$ *for very large* $\tan\beta$ (10^6), *with the fourth generation of quark masses:* $m_{t'} = 110$ GeV *and* $m_{b'} = 45$ GeV.

$M_{H^\pm}$ (GeV)	Model-I	I4Gn	Model-II	II4Gn
40	2.2184×10^{-4}	3.8127×10^{-4}	6.6556×10^{-4}	18.339×10^{-4}
80	The same	The same	5.7444×10^{-4}	14.26×10^{-4}
120	The same	The same	5.4977×10^{-4}	11.6379×10^{-4}
160	The same	The same	4.536×10^{-4}	9.9034×10^{-4}
200	The same	The same	4.1516×10^{-4}	8.7059×10^{-4}
220	The same	The same	3.9952×10^{-4}	8.2419×10^{-4}
240	The same	The same	3.8578×10^{-4}	7.8445×10^{-4}
280	The same	The same	3.6287×10^{-4}	7.2030×10^{-4}
300	The same	The same	3.5326×10^{-4}	6.9417×10^{-4}
400	The same	The same	3.1781×10^{-4}	6.0159×10^{-4}
500	The same	The same	2.9551×10^{-4}	5.4638×10^{-4}
600	The same	The same	2.8048×10^{-4}	5.1050×10^{-4}

From Table 4, it is observed that for Model II, $Br(b \to s\gamma)$ enters the CLEO bound at $M_{H^\pm} = 220$ GeV.

In Table 5a, we summarise the results for the limits of $\tan\beta$ for $M_{H^\pm} = 220$ GeV for different fourth-generation quark doublets.

TABLE 5a. For Model II and $M_{H^\pm} = 220$ GeV, *the limits of* $\tan\beta$ *for different pairs of fourth-generation quark doublets* $m_{t'}$, $m_{b'}$.

$m_{b'}$ (GeV)	$m_{t'}$ (GeV)	A $\tan\beta$ up to	W $\tan\beta$ from - to	B $\tan\beta$ from - to	W $\tan\beta$ from
45	110	0.454	0.455-0.53	0.54-0.89	0.9
50	110	0.46	0.47-0.54	0.55-0.89	0.9
	120	0.46	0.47-0.54	0.55-0.89	0.9
60	110	0.46	0.47-0.54	0.55-0.88	0.89
	130	0.47	0.48-0.55	0.56-0.89	0.9
85	130	0.47	0.48-0.55	0.56-0.89	0.9
	140	0.48	0.49-0.57	0.58-0.89	0.9
90	140	0.48	0.49-0.57	0.58-0.89	0.9
	150	0.49	0.5-0.58	0.59-0.89	0.9
400	500	0.44	0.45-0.51	0.52-0.93	0.94
500	540	0.44	0.45-0.51	0.52-0.93	0.94

In Table 5b, we present two results: (i) the value of the branching fraction for very large value of $\tan\beta$ (10^6) for $M_{H^\pm} = 220$ GeV, given in the 3rd and 4th columns for Model I and Model II for fourth generation, and (ii) the value of the branching ratio for very large value of $M_{H^\pm}(2 \times 10^6)$ GeV and large $\tan\beta$, in last two columns of the table.

TABLE 5b. Branching fractions for different pairs of fourth-generation quark doublets for studied models and fourth-generation formulation.

$m_{b'}$ (GeV)	$m_{t'}$ (GeV)	I4Gn $M_{H^\pm}$=220 GeV large $\tan\beta$	II4Gn $M_{H^\pm}$=220 GeV large $\tan\beta$	I4Gn large $M_{H^\pm}$ large $\tan\beta$	II4Gn large $M_{H^\pm}$ large $\tan\beta$
45	110	3.81×10^{-4}	8.24×10^{-4}	3.81×10^{-4}	3.81×10^{-4}
50	110	3.905×10^{-4}	8.492×10^{-4}	3.905×10^{-4}	3.905×10^{-4}
	120	3.935×10^{-4}	8.636×10^{-4}	3.935×10^{-4}	3.935×10^{-4}
60	110	4.05×10^{-4}	8.905×10^{-4}	4.05×10^{-4}	4.05×10^{-4}
	130	4.13×10^{-4}	9.2×10^{-4}	4.13×10^{-4}	4.13×10^{-4}
85	130	4.13×10^{-4}	9.2×10^{-4}	4.13×10^{-4}	4.13×10^{-4}
	140	4.48×10^{-4}	10.3×10^{-4}	4.48×10^{-4}	4.48×10^{-4}
90	140	4.48×10^{-4}	10.3×10^{-4}	4.48×10^{-4}	4.48×10^{-4}
	150	4.59×10^{-4}	10.8×10^{-4}	4.59×10^{-4}	4.59×10^{-4}
400	500	3.472×10^{-4}	7.747×10^{-4}	3.472×10^{-4}	3.472×10^{-4}
500	540	3.491×10^{-4}	7.838×10^{-4}	3.491×10^{-4}	3.491×10^{-4}

One may observe the following:

Firstly, for large values of $M_{H^\pm}$ and large values of $\tan\beta$, I4Gn and II4Gn tend to the same limit. And secondly, considering Model I and Model II, the effect of fourth generation is in the affirmative in respect of fourth-generation quark doublets, namely, $(m_{b'}, m_{t'})$ = (45, 110), (50, 110), (50, 120), (440, 500) and (500, 540), all masses are taken in GeV. For the remaining pair of masses, however, result is rather negative to the CLEO bound. What is interesting in this formulation is that we get a different region for the allowable range predicted by [2], but at the same time this procedure conforms to the CLEO data.

This formulation is done keeping in mind that a fourth generation is consistent with the LEP/SLC data as long as the fourth neutrino is heavy, i.e., $m_{\nu_4} \gtrsim M_Z/2$, and that such a heavy fourth neutrino could mediate a see-saw type mechanism thus generating a small mass for $\nu_{e,\mu,\tau}$. And the possibility of the fourth family of fermions may be taken as a popular potential extension.

Before we conclude, we add a few lines on the lower bound of the Higgs mass which may be relevant.

(1) In spite of the theoretical uncertainties, present experimental data still put a strong bound on the parameters of the two-Higgs-doublet model. Considering

Model II, with the values of the parameters stated herein before, and taking $m_t =$ 175 GeV, the CLEO bounds on the branching ratio for $b \to s\gamma$ clearly sets the lower bound for the Higgs mass at 220 GeV, or more correctly at 219.4 GeV, and the value of $\tan\beta = 8.3$ at which the branching ratio goes within the CLEO bound from above.

(2) Taking $m_t = 180$ GeV, the scenario is changed; the CLEO data sets lower bound for Higgs mass at 230.9 GeV. It is further observed that the value of $\tan\beta$ for the entrance is enhanced ten times over its value for m_t= 175 GeV. Branching ratio exactly coincides with the CLEO upper bound at $\tan\beta = 84$ and remains well within at $\tan\beta = 85$.

(3) Throughout our calculation, we have made corrections to the strong coupling constant $\alpha_s(q)$ for the quarks to incorporate the effects of Higgs boson (Eq.(16)); let us have a look into the situation when this effect is switched off. In this case, with $m_t = 175$ GeV, the lower bound of $M_{H^\pm}$ is set to 185.2 GeV which is far below the earlier one, namely, 219.4 GeV; but we must recognise that it is still above the top quark mass. The value of $\tan\beta$ is 37 at which the branching ratio goes into the allowable region through the upper bound of the CLEO limit. However, under these conditions, with $m_t = 180$ GeV, the lower bound of Higgs mass is set at 194.9 GeV and the lower bound for $\tan\beta$ has been calculated to be 26.

Lastly, we must mention one observation which is held as a by-product, particularly in the light of the CLEO bounds on the $b \to s\gamma$ decay, that there should be a restriction on the mass of Higgs boson sought by the virtual process under consideration. Besides ensuring the branching ratio in the appropriate interval, as well as enforcing no restriction on the value of $\tan\beta$, it has become essential that the lower bound of charged Higgs mass should be 220 GeV for $m_t = 175$ GeV and 231 GeV for $m_t = 180$ GeV, respectively, taking into consideration the effects of Higgs mechanism on the inter-quark coupling (enhanced approximately by 10 GeV for the enhancement of the top quark mass by 5 GeV). The lower bound is, however, diminished if the intervention be considered absent. And in that case, lower bounds of the $M_{H^\pm}$ become 185 GeV and 195 GeV for m_t 175 GeV and 180 GeV, respectively. In any case, the Higgs boson mass remains always above the top quark mass. We may note further that $\tan\beta \equiv v_1/v_2$, which is the ratio of the vacuum expectation values that may be taken to be the ratio of the third-generation up-type quark mass and the down-type quark mass [1], and in the present case its value is approximately 35. The lower bound on $\tan\beta$ is very close to the quoted value for the lower bound values of the Higgs boson mass predicted by our calculation. The theoretical predictions can be improved significantly when more experimental data will be available.

References

[1] A. J. Buras, M. Misiak, M. Münz and S. Pokorski, Nucl. Phys. B **424** (1994) 374.

[2] J. L. Hewett, *Top Ten Models Constrained by* $b \to s\gamma$, *Proc. Conf. on Spin Structure in High Energy Physics*, Stanford (1993) p. 463.

[3] A. F. Falk, M. Luke and M. Savage, Phys. Rev. D **51** (1994) 3367.

[4] Phys. Rev. **D 54** (1996) 498.

[5] D. Du, X. Li and Z. Xiao, Phys. Rev. D **51** (1995) 279.

[6] M. S. Alam et al., CLEO Collab., Phys. Rev. Lett. **74** (1995) 2885.

[7] R. Borate et al., ALEPH Collab., CERN-EP/98-044.

[8] K. Chetyrkin, M. Misiak and M. Münz, Phys. lett. B **400** (1997) 206.

[9] S. K. Biswas and V. P. Gautam, Fizika B **7** (1998) 105.

[10] C. Caso et al., *Review of Particle Physics*, Eur. Phys. J. C **3** (1998) 1.

[11] D. E. Groom et al., *Review of Particle Physics*, Eur. Phys. J. C **15** (2000) 1.

[12] B. Grinstein, R. Springer and M. B. Wise, Nucl. Phys. B **339** (1990) 269.

[13] B. Grinstein and M. B Wise, Phys. Lett. B **210** (1988) 274.

[14] W. -s Hou and R. Willey, Phys. Lett. B **202** (1988) 591.

[15] *The Albuquerque Meeting*, August 2-6, 1994, Univ. New Mexico, Volume 1, Ed. S. Seidel, p.915.

[16] S. Narison, Phys. Lett. B **341** (1994) 73.

[17] P. N. Burrows, T. Abrah, M. Samuel, E. Steinfelds and H. Masuda, Phys. Lett. B **341** (1994) 73.

[18] W. T. Giele, Talk given at the *5th International Workshop on Deep inelastic Scattering and QCD*, Chicago, April 1997, FERMILAB-Conf-97/240-T.

[19] M. J. Strassler and M. E. Peskin, Phys. Rev. D **43** (1991) 1500.

PROUČAVANJE RADIJATIVNOG RASPADA B-MEZONA U MODELIMA DVOJNI-HIGGS-DUBLET S ČETVRTOM GENERACIJOM KVARKOVA

Razmatramo nestandardni posredni učinak Yukawinog vezanja Higgsovog dubleta na kvarkove i ispitujemo učinke jakog međudjelovanja u slabom radijativnom raspadu B-mezona. Virtualni učinci u Modelu I i Modelu II u dijagramima pingvinskog tipa u raspadu $b \to s\gamma$ ispituju se razmatrajući mogućnost četvrte generacije u kvarkovskom sektoru, s ispravkama do vodećih QCD logaritama, primjenjujući CKM matricu četvrte generacije s fazom kršenja CP jednakom nuli. Područja masa četvrte generacije kvarka "dolje" b' i četvrte generacije kvarka "gore" t' su određena uzevši u obzir ograničenja sadašnjim eksperimentalnim vrijednostima parametra ρ, uz uvažavanje razlike masa kvarkovskog dubleta četvrte generacije. Dodatni je ishod da ograničenje CLEO daje jasnu donju granicu na masu Higgsovog bozona od 220 GeV, što je iznad mase kvarka "gore".