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AN EXPLANATION OF POSSIBLE NEGATIVE MASS-SQUARE OF NEUTRINOS

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The recent data of mass-square for the electron neutrino and the muon neutrino are probably negative. Motivated by this fact, we have further investigated the hypothesis that neutrinos are superluminal fermions. A tachyonic Dirac-type equation is further studied. This equation is equivalent to two Weyl equations coupled together via nonzero mass while respecting the maximum parity violation. It reduces to one Weyl equation when the neutrino mass becomes zero.

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1. Introduction

The square of the neutrino mass is measured in tritium beta-decay experiments by fitting the shape of the beta spectrum near the endpoint. In many experiments, it has been found to be negative. The most recent data are listed in "Review of Particle Physics, 2000" [1] and references therein. The weighted average from two experiments reported in 1999 [2–3] is

$$m^2(\nu_e) = (-2.5 \pm 3.3)\ e\mathrm{V}^2\,. \tag{1}$$

However, nine other measurements from different experiments in 1991 – 1995 are not used for the average. For instance, a value of $m^2(\nu_e) = (-130 \pm 20)\ e\mathrm{V}^2$ with the 95% confidence level was measured in LLNL in 1995 [4]. It is now broadly accepted that the old values of negative mass-square of the neutrino are originated from experimental shortcomings.

Interesting enough, the pion decay experiment also obtained a negative value for the muon neutrinos [5],

$$m^2(\nu_\mu) = (-0.016 \pm 0.023)\, MeV^2 \,. \tag{2}$$

The negative value of the neutrino mass-square simply means

$$E^2/c^2 - p^2 = m_\nu^2 c^2 < 0\,. \tag{3}$$

The right-hand side in Eq.(3) can be rewritten as $(-m_s^2 c^2)$, then m_s has a positive value. Equations (1) and (2) suggests that neutrinos might be particles moving faster than light, no matter how small m_s is. This hypothesis is further investigated in this paper.

Based on the special relativity and known as reinterpretation rule, superluminal particles were proposed by Bilaniuk et al. in 1960s [6–8]. The sign of the 4-D world line element, $\mathrm{d}s^2$, is associated with three classes of particles. For simplicity, let $\mathrm{d}y = \mathrm{d}z = 0$, then

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 \quad \begin{array}{cll} > & 0 & \text{Class I (subluminal particles)} \\ = & 0 & \text{Class II (photon)} \\ < & 0 & \text{Class III (superluminal particles)} \end{array} \tag{4}$$

For Class III particles, i.e. superluminal particles, the relation of momentum and energy is given in Eq. (3). The negative value on the right-hand side of Eq. (3) for superluminal particles means that p^2 is greater than $(E/c)^2$. The velocity of a superluminal particle, u_s, is greater than the speed of light. The momentum and energy in terms of u_s are as follows

$$p = \frac{m_s u_s}{\sqrt{u_s^2/c^2 - 1}}, \quad E = \frac{m_s c^2}{\sqrt{u_s^2/c^2 - 1}}\,, \tag{5}$$

where the subscript s means superluminal particle, i.e. tachyon. From Eq. (5), it is easily seen that when u_s is increased, both p and E would be decreased. This property is the opposite of the Class I particle.

Any physical reference system is built of Class I particles (atoms, molecules etc.), which requires that any reference frame must move slower than light. On the other hand, once a superluminal particle is created in an interaction, its speed is always greater than the speed of light. The neutrino is the most likely candidate for a superluminal particle because it has left-handed spin in any reference frame. On the other hand, the anti-neutrino always has right-handed spin.

The first step in this direction is usually to introduce an imaginary mass, but these efforts could not reach the point of constructing a consistent quantum theory. Some early investigations of a Dirac-type equation for tachyonic fermions can be found in Ref. [9]. An alternative approach was investigated by Chodos et al. [10]. They examined the possibility that neutrinos might be tachyonic fermions. A form of the Lagrangian density for tachyonic neutrinos was proposed. Although they did not obtain a satisfactory quantum theory for tachyonic fermions, they suggested that more theoretical work would be needed to determine a physically acceptable tachyonic theory.

2. A Dirac-type equation for tachyonic neutrinos

In this paper, we will start with a different approach to derive a Dirac-type equation for tachyonic neutrinos. In order to avoid introducing the imaginary mass, Eq. (3) can be rewritten as

$$E = \pm(c^2p^2 - m_s^2c^4)^{1/2}\,, \tag{6}$$

where m_s is called proper mass or tachyonic mass, for instance, $m_s(\nu_e) = 1.6$ eV from Eq. (1). To follow Dirac's search [11], the Hamiltonian must be first order in the momentum operator $\hat{p}$

$$\hat{E} = -c(\vec{\alpha}\cdot\hat{p}) + \beta_s m_s c^2\,, \tag{7}$$

with ($\hat{E} = \mathrm{i}\hbar\partial/\partial t, \hat{p} = -\mathrm{i}\hbar\nabla$), where $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$ and β_s are $4\times\,4$ matrices, which are defined as

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \quad \beta_s = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \tag{8}$$

where σ_i are 2×2 Pauli matrices and I is the 2×2 unit matrix. Notice that β_s is a new matrix, which is different from the one in the traditional Dirac equation. The relation between the matrix β_s and the traditional matrix β is as follows

$$\beta_s = \beta\gamma_5\,, \tag{9}$$

where

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{9a}$$

When we take the square of both sides of Eq. (7) and consider the following relations

$$\begin{gathered} \alpha_i\alpha_j + \alpha_j\alpha_i = \delta_{ij}\,, \\ \alpha_i\beta_s + \beta_s\alpha_i = 0\,, \\ \beta_s^2 = -1\,, \end{gathered} \tag{10}$$

the expressions in Eq. (3) and Eq. (6) are reproduced. Since Eq. (6) is related to Eq. (5), this means that β_s is the right choice to describe neutrinos as superluminal particles.

Denote the wave function as

$$\Psi = \begin{pmatrix} \varphi(\vec{x},t) \\ \chi(\vec{x},t) \end{pmatrix}, \quad \text{with} \quad \varphi = \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{11}$$

From Eq. (7), the complete form of the Dirac-type equation for tachyonic neutrinos becomes

$$\hat{E}\Psi = -c(\vec{\alpha}\cdot\hat{p})\Psi + \beta_s m_s c^2 \Psi\,. \tag{7a}$$

Since $\beta_s^2 = -1$ in the Eq. (7a), this Dirac-type equation is different from the traditional Dirac equation in any covariant representation in terms of the γ matrices.

We now study the spin-1/2 property of the neutrino as a tachyonic fermion. Equation (7a) can be rewritten as a pair of two-component equations

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial\varphi}{\partial t} &= \mathrm{i}c\hbar\vec{\sigma}\cdot\nabla\chi + m_s c^2\chi\,, \\ \mathrm{i}\hbar\frac{\partial\chi}{\partial t} &= \mathrm{i}c\hbar\vec{\sigma}\cdot\nabla\varphi - m_s c^2\varphi\,. \end{aligned} \tag{12}$$

Equation (12) is invariant under the space-time inversion transformation, i.e., for $\vec{x}\to-\vec{x}, t\to -t$ [12],

$$\varphi(-\vec{x},-t)\to\chi(\vec{x},t), \qquad \chi(-\vec{x},-t)\to\varphi(\vec{x},t)\,. \tag{13}$$

From Eq. (12), the continuity equation can be derived

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\vec{j} = 0\,, \tag{14}$$

and we have

$$\rho = \varphi^\dagger\chi + \chi^\dagger\varphi, \quad \vec{j} = -c(\varphi^\dagger\vec{\sigma}\varphi + \chi^\dagger\vec{\sigma}\chi)\,, \tag{15}$$

where ρ and $\vec{j}$ are the probability density and current; $\varphi^\dagger$ and $\chi^\dagger$ are the Hermitian adjoints of φ and χ, respectively.

Equation (15) can be rewritten as

$$\rho = \Psi^\dagger\gamma_5\Psi, \quad \vec{j} = -c(\Psi^\dagger\gamma_5\vec{\alpha}\Psi)\,. \tag{15a}$$

Considering a plane wave along the z axis for a particle with negative helicity, $(\vec{\sigma}\cdot\vec{p})/p = -1$, Eq. (12) yields the following solution

$$\chi = \frac{cp - m_s c^2}{E}\varphi\,. \tag{16}$$

We now consider a linear combination of φ and χ,

$$\xi = \frac{1}{\sqrt{2}}(\varphi + \chi)\,, \quad \eta = \frac{1}{\sqrt{2}}(\varphi - \chi)\,, \tag{17}$$

where $\xi(\vec{x},t)$ and $\eta(\vec{x},t)$ are two-component spinor functions. In terms of ξ and η, Eq. (15) is given by

$$\rho = \xi^\dagger\xi - \eta^\dagger\eta, \qquad \vec{\jmath} = -c(\xi^\dagger\vec{\sigma}\xi + \eta^\dagger\vec{\sigma}\eta)\,. \tag{18}$$

It is easy to see that the probability density ρ is positive definite when $\xi^\dagger\xi > \eta^\dagger\eta$. Since ρ can be negative, it may be interpreted as the charge density as suggested by a referee. More discussion on the physical meaning of ρ is needed. A comparison between the tachyonic Dirac equation and the ordinary Dirac equation can be found in Ref. [13].

In terms of Eq. (17), Eq. (12) can be rewritten in the Weyl representation

$$\begin{aligned} \mathrm{i}\hbar\frac{\partial\xi}{\partial t} &= \mathrm{i}c\hbar\vec{\sigma}\cdot\nabla\xi - m_s c^2\eta\,, \\ \mathrm{i}\hbar\frac{\partial\eta}{\partial t} &= -\mathrm{i}c\hbar\vec{\sigma}\cdot\nabla\eta + m_s c^2\xi\,. \end{aligned} \tag{19}$$

In Eq. (19), both ξ and η are coupled via mass m_s.

In order to compare Eq. (19) with the well known two-component Weyl equation, we take the limit $m_s = 0$, then the two equations in (19) are decoupled, and the first one reduces to

$$\frac{\partial\xi_\nu}{\partial t} = c\vec{\sigma}\cdot\nabla\xi_\nu\,, \tag{20}$$

while the second equation in Eq. (19) vanishes because $\varphi = \chi$ for the massless limit.

Equation (20) is the two-component Weyl equation for describing neutrinos, which is related to the maximum parity violation discovered in 1956 by Lee, Yang and Wu [14,15]. They pointed out that no experiment had shown parity to be a good symmetry for weak interactions. Now we see that, in terms of Eq. (19), if a neutrino has some mass, no matter how small it is, the two equations are coupled together via the mass term while still respecting maximum parity violation.

From experimental point of view, the shape of the allowed β-spectrum near the endpoint is mainly determined by the statistical factor, $p_e E_e p_\nu E_\nu$, which is related to the mass term of the neutrino. For the tachyonic neutrino, it has zero energy at the endpoint in the Kurie plot. Therefore, the shape near the endpoint and the location at the endpoint of the β-spectrum are different for tachyonic neutrino and the neutrino with a rest mass. In addition, the shape of the tritium β-spectrum for tachyonic neutrinos may depend on more parameters than the tachyonic mass m_s alone [16].

3. *Remarks*

In this paper, we have further investigated the hypothesis that neutrinos are tachyonic fermions. A Dirac-type equation is further studied for tachyonic neutrinos. It may help to solve the puzzle of negative mass-square of neutrinos.

Notice that the matrix β_s in the Dirac-type equation (7a) is not a 4×4 Hermitian matrix. However, based on the above study, we now realize that violation of the Hermitian property is related to violation of parity. Though a non-Hermitian Hamiltonian is not allowed for a subluminal particle, it does work for superluminal neutrinos. The tachyonic Dirac-type equation (7a) can be rewritten in covariant forms by multiplying matrices β and γ_5. A covariant form has been discussed in Refs. [10] and [16] except that the sign for the momentum operator is negative in Eq. (7a). The negative sign in Eq. (7a) may be not trivial because it is related to the fixed helicity of the neutrino. A more general form of Dirac equation with two mass parameters has also been studied in Ref. [17], which covers the case discussed in this paper.

On the other hand, the ordinary Dirac equation is valid for positive-mass electron, which can be both left-handed and right-handed. When writing the ordinary Dirac equation in the Weyl representation [18], one sign for the mass term is opposite to the tachyonic Dirac-type equation (19). Under the transformation of space reflection, the ordinary Dirac equation has invariant property, but Eq. (19) does not. Therefore, the results derived from the ordinary and tachyonic Dirac equation have major differences, especially for the symmetry of parity.

In the light of the above theoretical framework, the results from the tachyonic Dirac-type equation agree with the fact that the symmetry of parity is violated in the weak interactions. Therefore, neutrinos might be tachyonic fermions with permanent helicity if the neutrino mass-square is negative. Of course, more accurate tritium beta-decay experiments are needed to further determine the neutrino mass-square.

According to the special relativity [19], if there is a superluminal particle, it might travel backward in time. However, a reinterpretation rule has been introduced since 1960s [6–8]. Another approach is to introduce a kinematic time under a non-standard form of the Lorentz transformation where the kinematic time always goes forward in any inertial frame [20–23]. Therefore, special relativity can be extended to the space-like region, and superluminal particles are allowed without violation of causality.

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OBJAŠNJENJE MOGUĆEG NEGATIVNOG KVADRATA MASE NEUTRINA

Nedavni su ishodi mjerenja kvadrata masa elektronskog i mionskog neutrina vjerojatno negativne vrijednosti. Ti su nas podaci pobudili nastaviti istraživanja hipoteze da su neutrini nadsvjetlosne čestice (tahioni). Proučavamo Diracovu jednadžbu tahionskog tipa. Ta je jednadžba jednakovrijedna dvjema Weylovim jednadžbama vezanim preko mase različite od nule, uz zadržavanje najvećeg kršenja parnosti. One se svode na jednu Weylovu jednadžbu ako je masa neutrina jednaka nuli.