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WEAK MESON VERTICES AND THE HYPERNUCLEAR POTENTIAL

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Dedicated to Professor Kseno Ilakovac on the occasion of his 70$^{\text{th}}$ birthday

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This paper is a continuation of an earlier paper on hypernuclear potentials. A novel derivation of the hypernuclear strangeness-violating potential due to the pseudoscalar meson exchanges is presented. Comparison with the earlier method shows that the theoretical uncertainty is less than 30%. Relative signs of pseudoscalar meson exchanges and vector (axial vector) exchanges are discussed in detail. Additional comments on the nonrelativistic approximation are included.

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1. Introduction

Our recently published review paper [1] left unexplored some details concerning the derivation of the weak hypernuclear potentials. All these items are well known to experts, as many previously published details were. Nevertheless, it might be useful to present them systematically from an unified viewpoint. Moreover, in many discussions with numerous colleagues, additional questions, not addressed in Ref. [1], were rised. Thus the present paper should be read in conjunction with the earlier one.

A general quantum field theory and weak interaction background can be found in numerous publications (e.g., see [2–9]). The strong interaction input can be taken from Refs. [10–14]. The SU(3) and $SU(6)_W$ symmetries are used and/or described in Refs. [2–9, 15–23]. It turned out that theoretical description of the hypernuclear decays depends very much on the relative signs and phases of various pieces of the weak strangeness violating potential [5,20,24–27]. Therefore, it seemed useful to review and collect all theoretical arguments which determine those signs.

In Appendices A and B, all standard knowledge about effective weak Hamiltonian and about the semileptonic weak decays is reviewed. The sign and phase conventions, all experimentally confirmed and tested [2–9], will serve as a foundation for the following deductions. Section 2 connects the separable contributions to the hyperon nonleptonic decays (see Sect. 3. in Ref. [1]) to the induced pseudoscalar in semileptonic decays. The relative phases of the $K^\star$ exchange and the π exchange PV contributions are connected to the semileptonic hyperon decays and thus determined. The current algebra (CA) contribution has, as shown in Sect. 3, the same phase, as the one required by the earlier discussion. As can be seen in Sect. 4, the relative phases can be determined quite generally using CP properties and Hermiticity. Those argeements are specified for the $SU(6)_W$ classified PV Hamiltonian in Sect. 5. It turns out that the Hermiticity requirement introduces the factor i^{1-S_M}. Here $S_M = 0, 1$ is the meson spin. As discussed in Sect. 6, the relative sign depend also on the relative signs of the strong coupling constants [5,10–14] and on the signs of the weak $\overline{B}B\pi$ amplitudes, which have to be correctly read form the experimental tables [11].

With all that knowledge, one can determine the relative signs and phases of the parity-conserving pseudoscalar and vector meson exchange potentials (Sect. 7). One can easily make the same predictions for the axial-vector meson exchanges [26], discussed in Sect. 8. Further elucidation of the separable contributions in the framework of the $SU(6)_W$ calculational scheme can be found in Sect. 9.

The weak NNK [28] and $N\Lambda\eta$ vertices are used here to illustrate the theoretical uncertainties which appear in their determination. While the $N\Lambda\pi$ weak vertex can be read from the experimental data (Sect. 6), the other pseudoscalar meson vertices must be determined theoretically [4–9,12–21,29,30]. The weak PV NNK amplitudes can be determined by using SU(3) based sum rules and/or CA. Some uncertainties in those procedures are described in Sect. 10. The complete calculation of A and B amplitudes appearing in $\mathcal{M} = \langle N'K| H_W |N\rangle = \mathrm{i}\overline{u}(A - \gamma_5 B)u$ was performed using two separate schemes. In Ref. [1], the CA (A_{cc}), separable (SEP) and octet baryon pole (B_8) contribution were introduced, i.e.

$$\begin{aligned} A &= A_{cc} + A_{SEP} \\ B &= B_8 + B_{SEP} \end{aligned}$$

while Ref. [22,30] relied on the decouplet pole contributions ($A_{10},\ B_{10}$), i.e.

$$\begin{aligned} A &= A_{cc} + A_{10} + A_{\Lambda'} \\ B &= B_8 + B_{10} + B_{\Lambda'}. \end{aligned}$$

All details are given in Sect. 12 where one can find the definitions of the terns $A_{\Lambda'}$ and $B_{\Lambda'}$.

Finally, in the Sect. 10, the complete pseudoscalar meson exchanges $\Delta S = 1$ potentials are discussed. They have the same form as shown in Ref. [1], i.e., they contain both $\Delta I = 1/2$ and $\Delta I = 3/2$ parts. Their strengths are displayed in Table 11.2 which can be compared with Table 9.1 from Ref. [1]. The differences found by using various calculational schemes serve as indicators of theoretical uncertainties.

In addition, Appendix G contains some useful remarks concerning the nonrelativistic approximation (NRA) of the weak ($\Delta S = 1$) potential, which was briefly discussed in Sect. 8 of Ref. [1]. The NRA form of the effective interaction is the one which is usually used. All potential pieces, with their relative signs and phases, are written in NRA before being confronted with experimental data.

2. *The separable contribution to the PV hyperon nonleptonic decay*

In the following we show:

(i) The separable contribution to the PV hyperon nonleptonic decays (SCHN) is connected with the induced pseudoscalar (IP) term, i.e. with g_P term in (B.3) and (B.6).

(ii) This will fix relative phase between PV potential terms due to the vector meson and pion (or kaon) exchange contribution to the strangeness-violating (SV) and PV effective weak potential [4]. This will be illustrated by calculating a separable contribution (SEP) to the process

$$\Lambda + p \to n + p. \tag{2.1}$$

Of all possible SEP terms only one, needed to show i) and ii) is selected. All factors inessential for that proof, like $\sin\theta_C$, $\cos\theta_C$ etc. are not openly displayed. One starts with

$$\begin{aligned}(-\mathrm{i})V \quad &= \quad (-\mathrm{i})\tfrac{G_F}{\sqrt{2}}\overline{u}(n_f)[g_A\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5]u(p_i)\cdot \\ &\qquad \overline{u}(p_f)g_V(q^2)\gamma^\mu u(p_\Lambda) \\ g_A \quad &= const. \qquad g_P(q^2) = \frac{\kappa_P}{q^2 - m_\pi^2} \\ &\qquad g_V(q^2) = \frac{m_{K^*}^2}{m_{K^*}^2 - q^2}\end{aligned} \tag{2.2a}$$

The term V is one of SEP's obtained from

$$W = \frac{G_F}{\sqrt{2}}\langle n_f|\,\mathcal{J}_\mu^i\,|p_i\rangle\langle p_f|\,\mathcal{J}_i^{\mu\,k}\,|p_\Lambda\rangle. \tag{2.3}$$

Here $\mathcal{J}_\mu^k$ are general $V-A$ weak currents appearing in the effective weak Hamiltonian. As said above, the enhancement coefficient C [1] is also omitted. Only the pertinent form factors, listed in (2.2) are kept in V. With (B.10)

$$\kappa_p = -\sqrt{2} g_\pi f_\pi \tag{2.2b}$$

and by using Dirac equation (Appendix B), one finds

$$V = V_{K^*} + V_\pi \tag{2.4}$$

$$V_{K^*} = \frac{G_F}{\sqrt{2}} g_V g_A \frac{m_{K^*}^2}{m_{K^*}^2 - q^2} \overline{u}(n_f)\gamma_\mu\gamma_5 u(p_i)\, \overline{u}(p_f)\gamma^\mu u(p_\Lambda)\,, \tag{2.5}$$

$$V_\pi = \frac{-\sqrt{2} g_\pi f_\pi}{q^2 - m_\pi^2} \overline{u}(n_f)\gamma_5 u(p_i)\,(m_\Lambda - \overline{m}_p) u(p_f) u(p_\Lambda). \tag{2.6}$$

The above expressions[1] can be identified as parts of a general K* and π exchanges produced by general Hamiltonians [5]

$$\begin{aligned}
\mathcal{H}_{K^*}^S &= g_S^V \overline{\psi}_p \gamma^\mu \psi_\Lambda V_\mu^{K^*}\,,\\
\mathcal{H}_{K^*}^W &= \epsilon_{K^*} \overline{\psi}_n \gamma^\mu \gamma_5 \psi_p V_\mu^{\overline{K^*}}\,,\\
\mathcal{H}_\pi^S &= \mathrm{i}\sqrt{2} g_\pi \overline{\psi}_n \gamma_5 \psi_p \pi^-\,,\\
\mathcal{H}_\pi^W &= \mathrm{i} A \overline{\psi}_p \psi_\Lambda \pi^+\,.
\end{aligned} \tag{2.7}$$

The second-order contributions comming from (2.7), written in the same sceleton form as (2.5) and (2.6), are

$$\begin{aligned}
(-\mathrm{i})^2 \mathcal{H}_{K^*}^S \mathcal{H}_{K^*}^W &= (-) g_S^V \epsilon_{K^*} (-\mathrm{i}) \frac{1}{q^2 - m_{K^*}^2} \overline{u}(n_f)\gamma_\mu\gamma_5 u(p_i)\ \overline{u}(p_f)\gamma^\mu u(p_\Lambda)\\
&\sim (-\mathrm{i}) V_{K^*}.
\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}
(-\mathrm{i})^2 \mathcal{H}_\pi^S \mathcal{H}_\pi^W &= (-)(-)\sqrt{2} g_\pi A i \frac{1}{q^2 - m_\pi^2} \overline{u}(n_f)\gamma_5 u(p_i)\ \overline{u}(p_f) u(p_\Lambda)\\
&\sim (-\mathrm{i}) V_\pi.
\end{aligned} \tag{2.9}$$

The contribution (2.8) corresponds to SEP term and according to Ref. [5], one can connect it to the SEP contributions a_T appearing in SU(6)$_W$ based sum rules[2]

$$g_S^V \epsilon_{K^*} \sim g_S^V a_T. \tag{2.10}$$

[1]Here $q = n_f - p_i = p_\Lambda - p_f$, $p_f + n_f = p_i + p_\Lambda$ Expression (2.2a) is Hermitian conjugate of equation (B.3a). Note $(\overline{u}(p) g_P k_\mu \gamma_5 u(n))^\dagger = -\overline{u}(n) g_P k_\mu \gamma_5 u(p)$, with $k = p - n = -q = -(n-p)$.

[2]See Sect. 9 below.

The term (2.9) obviously corresponds to SEP contributions to the PV hyperon nonleptonic decay amplitude A [1]. In order to find correspondence with (2.6), one must have iA. The reader must keep in mind that terms (2.5) and (2.6) were found from a standard SEP approximation (2.3). In that approximation, the current matrix elements $\langle\beta|\,\mathcal{J}_\mu\,|\alpha\rangle$ are experimentally determined from semileptonic decays (as for example $\Lambda \to p+e^-+\overline{\nu}$) [6]. Thus, the relative phase of the terms (2.5) and (2.6) is determined experimentally. When this is compared with the effective Hamiltonian description (2.7) that also fixes the relative phases of coupling constants $g_V^S\,\epsilon_{K^*},\ g_\pi$ and A which appear there. One should also keep in mind:

(a) The relative phases of g_ρ, g_{K^*} and g_π are already fixed by the baryon-baryon scattering experiments [10,11,13];

(b) Thus, the results (2.8) and (2.9) determine the relative phases among weak Hamiltonians;

(c) As soon as the relative phase between $\mathcal{H}^W_{K^*}$ and $\mathcal{H}^W_\pi$ is fixed, everything else follows via SU(3) (or SU(6)$_W$) symmetry;

(d) PC couplings of $\pi,\ K$ and η, where strengths are labeled as B_i's have their phases fixed through hyperon nonleptonic decays. Experiment gives the effective interaction of the form [11]

$$\overline{u}_f[A+B\gamma_5]u_i\phi_M \tag{2.11}$$

As a final illustration, here is the calculation of SEP contribution to $\Lambda \to p+\pi^-$ decay [1]. One has

$$\begin{aligned}
\langle p\pi^-|\,\mathcal{H}_W^{PV}\,|\Lambda\rangle \quad &\simeq (-)\frac{G_F}{\sqrt{2}}\langle p|\,\mathcal{J}_\mu\,|\Lambda\rangle\langle\pi^-|\,\mathcal{J}^{\mu\,5}\,|0\rangle \\
&= -\frac{G_F}{\sqrt{2}}g_V\overline{u}(p)\gamma_\mu u(\Lambda)(-\mathrm{i})q^\mu f_\pi \\
&\qquad (q=\Lambda-p; \qquad \Lambda=p+q) \\
&= \frac{G_F f_\pi}{\sqrt{2}}(m_\Lambda-m_p)\overline{u}_p u_\Lambda \to \mathrm{i}A(SEP)\overline{u}_p u_\Lambda .
\end{aligned} \tag{2.12}$$

The same phase as used in (2.7) is obtained. It will be shown in the next section that a general current algebra (CA) term has the same phase.

3. Current algebra contribution to PV hyperon nonleptonic decay amplitude A

One starts with the LSZ reduction of the general matrix element Eq. (2.12). In the vanishing ($k \to 0$) pion momentum limit, as shown earlier in Refs. [1,2,4], one can write

$$B \to B' + M_A$$

$$\begin{aligned}
&\langle B' M_A(k;out)|\, \mathcal{H}_W(0)\,|B\rangle \\
&\quad = \mathrm{i}\int \mathrm{d}^4x \frac{\mathrm{e}^{\mathrm{i}kx}}{(2\pi)^3 2\omega_k}\overrightarrow{(\Box_x + m_M^2)}\langle B'|\,\mathcal{T}[M_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)]\,|B\rangle \\
&\quad = \mathrm{i}\frac{C_M}{f_M}\left(1-\frac{k^2}{m_M^2}\right)\int \mathrm{d}^4x\mathrm{e}^{\mathrm{i}kx}\langle B'|\,\mathcal{T}[\partial_\mu A^\mu_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)]\,|B\rangle .
\end{aligned} \tag{3.1}$$

Eventually [1], one obtains

$$\begin{aligned}
&\langle B' M_A(k;out)|\, \mathcal{H}_W(0)\,|B\rangle \\
&= \mathrm{i}\frac{C_M}{f_M}\left(1-\frac{k^2}{m_M^2}\right)\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}kx}\{-\mathrm{i}k_\lambda\langle B|\,\mathcal{T}[A^\lambda_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)\,|B\rangle \\
&\qquad -\delta(x^0)\langle B'|\,[A^0_{a+\mathrm{i}b}(x),\mathcal{H}_W(0)]\,|B\rangle\}.
\end{aligned} \tag{3.2}$$

By taking the limit $k \to 0$ (*soft pion limit* for off-shell pions) one finds a typical CA relation

$$\mathcal{M}(q\to 0) = -\mathrm{i}\frac{C_M}{f_M}\langle B'|\,[F^5_{a+\mathrm{i}b}(0),\mathcal{H}_W(0)]\,|B\rangle = \mathrm{i}A\,\overline{u}_{B'}\,u_B. \tag{3.3}$$

Here the SU(3) (axial) charge is defined by

$$F^5_{a+\mathrm{i}b}(t) = \int \mathrm{d}^3x\, A^0_{a+\mathrm{i}b}(t,\boldsymbol{x}). \tag{3.4}$$

The following relations introduce (vector) charges $F_{a+\mathrm{i}b}$

$$\begin{aligned}
[F^5_{a+\mathrm{i}b},\mathcal{H}_W] &= [F_{a+\mathrm{i}b},\mathcal{H}_W^{PV}]\,, \\
[F^5_{a+\mathrm{i}b},\mathcal{H}_W^{PC}] &= [F_{a+\mathrm{i}b},\mathcal{H}_W^{PV}]\,, \\
[F^5_{a+\mathrm{i}b},\mathcal{H}_W^{PV}] &= [F_{a+\mathrm{i}b},\mathcal{H}_W^{PC}]\,, \\
F_{a+\mathrm{i}b} &= \textstyle\int \mathrm{d}^3x\, V^0_{a+\mathrm{i}b}(t,\boldsymbol{x})\,.
\end{aligned} \tag{3.5}$$

Omitting $\overline{u}_{B'}u_B$ spinors (3.3), one finds the transition amplitudes which are the *current algebra* contributions to A

$$
\begin{aligned}
A(\Lambda^0_-) &= -\frac{\sqrt{2}}{f_\pi}\frac{1}{\sqrt{2}}\langle p|\,[F_+,\mathcal{H}_W^{PC}(0)]\,|\Lambda\rangle = -\frac{1}{f_\pi}\langle n|\,\mathcal{H}_W^{PC}(0)\,|\Lambda\rangle = -\frac{1}{f_\pi}a_{\Lambda n}\,,\\
A(\Lambda^0_0) &= -\frac{\sqrt{2}}{f_\pi}\frac{1}{\sqrt{2}}\langle n|\,[F_3,\mathcal{H}_W^{PC}(0)]\,|\Lambda\rangle = \frac{1}{\sqrt{2}f_\pi}a_{\Lambda n}\,,\\
A(\Xi^-_-) &= -\frac{\sqrt{2}}{f_\pi}\frac{1}{\sqrt{2}}\langle \Lambda|\,[F_+,\mathcal{H}_W^{PC}(0)]\,|\Xi^-\rangle = -\frac{1}{f_\pi}\langle \Lambda|\,\mathcal{H}_W^{PC}\,|\Xi^0\rangle = -\frac{1}{f_\pi}a_{\Xi^0\Lambda}\,,\\
A(\Xi^0_0) &= -\frac{1}{\sqrt{2}f_\pi}a_{\Xi^0\Lambda}\\
A(\Sigma^-_-) &= \frac{\sqrt{2}}{f_\pi}a_{\Sigma^0 n}\,,\\
A(\Sigma^+_+) &= -\frac{1}{f_\pi}\left[a_{\Sigma^+p}+\sqrt{2}a_{\Sigma^0 n}\right]\,,\\
A(\Sigma^+_0) &= \frac{1}{\sqrt{2}f_\pi}a_{\Sigma^+p}\,.
\end{aligned}
\qquad ,(3.6)
$$

As the matrix elements

$$
\langle B'|\,\mathcal{H}_W^{PC}\,|B\rangle = a_{BB'} \tag{3.7}
$$

(calculable in bag models [12]) are real quantities, the matrix element (3.3) is imaginary, as was the matrix element (2.12). This result is consistent with the comparisson with semileptonic matrix elements (Sec. 2). It agrees with the most general argument wich is displayed in the next section.

For completeness sake let us show that PC amplitudes B have the same phase as PV amplitudes A, as they should according to the analysis of the empirical data on hyperon nonleptonic decays [11]. Those amplitudes obtain contributions from baryon poles. A typical contribution to the transition $\Lambda \to p+\pi^-$ can be written as [3]

$$
\begin{gathered}
(-\mathrm{i})\overline{u}(p_2)a_{\Sigma p}\frac{\mathrm{i}(\not p_1-\not q)+m_\Sigma}{(p_1-q)^2-m_\Sigma^2}\mathrm{i}\gamma_5 g_{\Sigma\pi\Lambda}u(p_1) = \mathrm{i}\widetilde{B}(\Lambda^0_-)\overline{u}(p_2)\gamma_5 u(p_1),\quad (p_1\to p_2+q)\\
\widetilde{B}(\Lambda^0_-) = g_{\Lambda\Sigma\pi^-}\frac{a_{\Sigma^+p}}{\Sigma^+-p}\\
B(\Lambda^0_-) = \widetilde{B}(\Lambda^0_-) + \text{crossed term contribution.}
\end{gathered}
\tag{3.8}
$$

Again B is real, as it should be [11].

[3] In Ref. [5] i is associated with perturbation: (-i), with the baryon propagator. Eq. (3.8) requires (-i) with respect to (3.1). There we calculate a matrix element in the lowest order, while (3.2) contains explicitly the strong vertex and thus belongs to a higher-order of perturbation.

4. *General arguments for PV amplitudes*

The relative phase of PV $\Delta S = 1$ pseudoscalar meson and vector effective weak Hamiltonians can be determined quite generally. One can use PC invariance and Hermiticity.

In order to implement that, let us first list a behaviour of a bilinear combination

$$K = \overline{\psi}_2 \Gamma \psi_1 .$$

Here Γ is any combination of Dirac gamma matrices. One has

$$\overline{\psi}_2 \Gamma \psi_1 \xrightarrow{\mathcal{P}} \overline{\psi}_2 \gamma^0 \Gamma \gamma^0 \psi_1 \tag{4.1}$$

and

$$\overline{\psi}_2 \Gamma \psi_1 \xrightarrow{\mathcal{C}} -\psi_2^T C^{-1} \Gamma C \overline{\psi}_1^T . \tag{4.2}$$

Here

$$\begin{gathered} C = \mathrm{i}\gamma_2\gamma_0 = -C^{-1} = -C^\dagger - C^T , \\ C^{-1}\gamma^\mu C = -(\gamma^\mu)^T, \qquad C^{-1}\gamma_5 C = (\gamma_5)^T = \gamma_5 . \end{gathered} \tag{4.3}$$

Using (4.1), (4.2) and (4.3) one finds [2]

$$\begin{array}{lcllcl} \overline{\psi}_2\gamma^\mu\psi_1 & \xrightarrow{\mathcal{C}} & -\overline{\psi}_1\gamma^\mu\psi_2 , & \overline{\psi}_2\gamma^\mu\gamma_5\psi_1 & \xrightarrow{\mathcal{C}} & -\overline{\psi}_1\gamma^\mu\gamma_5\psi_2 , \\ \overline{\psi}_2\psi_1 & \xrightarrow{\mathcal{C}} & \overline{\psi}_1\psi_2 , & \overline{\psi}_2\gamma_5\psi_1 & \xrightarrow{\mathcal{C}} & \overline{\psi}_1\gamma_5\psi_2 . \end{array} \tag{4.4}$$

Here an additional (-) sign was introduced as in the transposition ψ_1 and ψ_2 are interchanged and they are fermion operators. One also finds [2,7]

$$\begin{array}{lcl} \overline{\psi}_2\gamma^\mu\psi_1 & \xrightarrow{\mathcal{P}} & -\overline{\psi}_2\boldsymbol{\gamma}\psi_1 , \\ & \xrightarrow{\mathcal{P}} & +\overline{\psi}_2\gamma^0\psi_1 , \\ \overline{\psi}_2\gamma^\mu\gamma_5\psi_1 & \xrightarrow{\mathcal{P}} & +\overline{\psi}_2\boldsymbol{\gamma}\gamma_5\psi_1 , \\ & \xrightarrow{\mathcal{P}} & -\overline{\psi}_2\gamma^0\gamma_5\psi_1 , \\ \overline{\psi}_2\psi_1 & \xrightarrow{\mathcal{P}} & \overline{\psi}_2\psi_1 , \\ \overline{\psi}_2\gamma_5\psi_1 & \xrightarrow{\mathcal{P}} & -\overline{\psi}_2\gamma_5\psi_1 . \end{array} \tag{4.5}$$

The meson fields behave as [2,7]

$$\begin{array}{lcllcl} \phi^M & \xrightarrow{\mathcal{C}} & \phi^{\overline{M}} , & \phi^M & \xrightarrow{\mathcal{P}} & -\phi^M , \\ V_\mu^\alpha & \xrightarrow{\mathcal{C}} & -V_\mu^{\overline{\alpha}} , & V_\mu^\alpha & \xrightarrow{\mathcal{P}} & -\boldsymbol{V}^\alpha , \\ & & & & \xrightarrow{\mathcal{P}} & +V_0^\alpha . \end{array} \tag{4.6}$$

Thus respective interactions behave as

$$\begin{aligned} \overline{\psi}_2\psi_1\phi^M \quad &\xrightarrow{\mathcal{CP}} \quad -\overline{\psi}_1\psi_2\phi^{\overline{M}}\,, \\ \overline{\psi}_2\gamma^\mu\gamma_5\psi_1 V_\mu^\alpha \quad &\xrightarrow{\mathcal{CP}} \quad \overline{\psi}_1\gamma^\mu\gamma_5\psi_2 V_\mu^{\overline{\alpha}}\,. \end{aligned} \tag{4.7}$$

Here $\overline{M}$ and $\overline{\alpha}$ denote antiparticle, for example $\pi^+ \to \pi^-$ etc.

One immediately finds that $\mathcal{CP}$ invariant combinations are

$$\begin{aligned} H_M^W \quad &\sim \quad (\overline{\psi}_2\psi_1\phi^M - \overline{\psi}_1\psi_2\phi^{\overline{M}})\,, \\ H_V^W \quad &\sim \quad (\overline{\psi}_2\gamma^\mu\gamma_5\psi_1 V_\mu^\alpha + \overline{\psi}_1\gamma^\mu\gamma_5\psi_2 V_\mu^{\overline{\alpha}})\,. \end{aligned} \tag{4.8}$$

Hermiticity of (4.8) Hamiltonians depend on the behaviour of bilinears and of meson fields [4]

$$\begin{aligned} (\overline{\psi}_2\psi_1)^\dagger \quad &= \quad \psi_1^\dagger\gamma_0\psi_2 = \overline{\psi}_1\psi_2\,, \\ (\overline{\psi}_2\gamma^\mu\psi_1)^\dagger \quad &= \quad \overline{\psi}_1\gamma_0(\gamma^\mu)^\dagger\gamma_0\psi_2 = \overline{\psi}_1\gamma^\mu\psi_2\,, \\ (\overline{\psi}_2\gamma^\mu\gamma_5\psi_1)^\dagger \quad &= \quad \overline{\psi}_1\gamma_0\gamma_5\gamma^{\dagger\,\mu}\gamma_0\psi_2 = \overline{\psi}_1\gamma^\mu\gamma_5\psi_2\,, \\ (\phi^M)^\dagger \quad &= \quad \phi^{\overline{M}}\,, \\ (V_\mu^\alpha)^\dagger \quad &= \quad V_\mu^{\overline{\alpha}}\,. \end{aligned} \tag{4.9}$$

Obviously, the Hermitian and CP invariant effective Hamiltonians are

$$\begin{aligned} H_M^W \quad &= \quad \mathrm{i}A(\overline{\psi}_2\psi_1\phi^M - \overline{\psi}_1\psi_2\phi^{\overline{M}})\,, \\ H_V^W \quad &= \quad \epsilon(\overline{\psi}_2\gamma^\mu\gamma_5\psi_1 V_\mu^\alpha + \overline{\psi}_1\gamma^\mu\gamma_5\psi_2 V_\mu^{\overline{\alpha}})\,. \end{aligned} \tag{4.10}$$

If the first term in (4.10) means $\Delta S = 1$ change then the second term means $\Delta S = -1$. Therefore in publications dealing with hypernuclei one usually encounters one (first) of the two terms listed in (4.10).

Combining (4.4), (4.5) and (4.6), one can show that a PC and CP invariant coupling is

$$H_M^W(PC) \sim \overline{\psi}_2\gamma_5\psi_1\phi^M + \overline{\psi}_1\gamma_5\psi_2\phi^{\overline{M}}. \tag{4.11a}$$

The hermicity requires the final form

$$H_M^W(PC) = \mathrm{i}B(\overline{\psi}_2\gamma_5\psi_1\phi^M + \overline{\psi}_1\gamma_5\psi_2\phi^{\overline{M}}). \tag{4.11b}$$

[4] $\gamma^{\dagger\,\mu} = \gamma_0\gamma^\mu\gamma^0$; $\gamma_5^\dagger = \gamma_5$.

Here A and B have the same phase (for example they are both real) as required by the experimental data [11].

5. SU(6)$_W$ symmetry

The SU(6)$_W$ symmetry [12,15-20] has been employed [5,18-20,27] to connected PV $\overline{B}BM$ amplitudes with PV $\overline{B}\gamma_\mu\gamma_5 BV^\mu$ amplitudes. Here M is a pseudoscalar meson while V^μ corresponds to a vector meson. The effective $\Delta S = 1$ weak Hamiltonian, which transforms (almost) as SU(6)$_W$ operators [15] is given by [20]

$$
\begin{aligned}
&H_{PV}^{\Delta S=1} \\
&= a_T(A^{[2}_{(4}\phi^{5]}_{1)} - A^{(2}_{[4}\phi^{5)}_{1]} - A^{[1}_{(3}\phi^{6]}_{2)} + A^{(1}_{[3}\phi^{6)}_{2]} + A^{[2}_{(6}\phi^{3]}_{1)} - A^{(2}_{[6}\phi^{3)}_{1]} - A^{[1}_{(5}\phi^{4]}_{2)} + A^{(1}_{[5}\phi^{4)}_{2]}) \\
&+a_V(A^{[2}_{(3}\phi^{5]}_{2)} + A^{(2}_{[3}\phi^{5)}_{2]} - A^{[1}_{(4}\phi^{6]}_{1)} - A^{(1}_{[4}\phi^{6)}_{1]} + A^{[2}_{(5}\phi^{3]}_{2)} - A^{(2}_{[5}\phi^{3)}_{2]} - A^{[1}_{(6}\phi^{4]}_{1)} + A^{(1}_{[6}\phi^{4)}_{1]}) \\
&+b_T(A^{[2}_{41}\phi^{5]} + A^{25}_{[4}\phi_{1]} - A^{[1}_{32}\phi^{6]} + A^{16}_{[3}\phi_{2]} + A^{[2}_{61}\phi^{3]} - A^{23}_{[6}\phi_{1]} - A^{[1}_{52}\phi^{4]} + A^{14}_{[5}\phi_{2]}) \\
&+b_V(A^{[2}_{32}\phi^{5]} + A^{25}_{[3}\phi_{2]} - A^{[1}_{41}\phi^{6]} - A^{16}_{[4}\phi_{1]} + A^{[2}_{52}\phi^{3]} + A^{23}_{[5}\phi_{2]} - A^{[1}_{61}\phi^{4]} - A^{14}_{[6}\phi_{1]}) \\
&+c_V(A_4\phi^6 - A^6\phi_4 - A_3\phi^5 + A^5\phi_3 + A_6\phi^4 - A^4\phi_6 - A_5\phi^3 + A^3\phi_5)\,,
\end{aligned}
\tag{5.1a}
$$

Here

$$A = \overline{B}B. \tag{5.1b}$$

This can be explicitly written as

$$
\begin{aligned}
&H_{PV}^{\Delta S=1} \\
&= 2a_T[\overline{B}^{ij2}B_{ij1}\overline{\phi}^6_3 - \overline{B}^{ij3}B_{ij6}\overline{\phi}^1_2 - \overline{B}^{ij1}B_{ij2}\overline{\phi}^5_4 + \overline{B}^{ij4}B_{ij5}\overline{\phi}^2_1] \\
&+2a_V[\overline{B}^{ij2}B_{ij5}\overline{\phi}^2_3 - \overline{B}^{ij3}B_{ij2}\overline{\phi}^5_2 - \overline{B}^{ij1}B_{ij6}\overline{\phi}^1_4 + \overline{B}^{ij4}B_{ij1}\overline{\phi}^6_1 - \\
&\qquad - \overline{B}^{i1j}B_{25i}\overline{\phi}^j_4 + \overline{B}^{ij4}B_{i25}\overline{\phi}^j_1 + \overline{B}^{i14}B_{ij5}\overline{\phi}^2_j - \overline{B}^{i14}B_{ij2}\overline{\phi}^5_j \\
&+ b_V[\overline{B}^{ij2}B_{i25}\overline{\phi}^j_3 - \overline{B}^{ij3}B_{i25}\overline{\phi}^j_2 + \overline{B}^{i23}B_{ij5}\overline{\phi}^2_j - \overline{B}^{23i}B_{ij2}\overline{\phi}^5_j - \\
&\qquad - \overline{B}^{1ij}B_{16i}\overline{\phi}^j_4 + \overline{B}^{ij4}B_{16i}\overline{\phi}^j_1 - \overline{B}^{14i}B_{ij6}\overline{\phi}^1_j + \overline{B}^{i14}B_{1ij}\overline{\phi}^6_j] \\
&+ c_V[\overline{B}^{ijk}B_{ij6}\overline{\phi}^k_4 - \overline{B}^{ij4}B_{ijk}\overline{\phi}^6_k - \overline{B}^{ijk}B_{ij5}\overline{\phi}^k_3 + \overline{B}^{ij3}B_{ijk}\overline{\phi}^5_k] - \\
&-2a_T[\overline{B}^{ij1}B_{ij2}\overline{\phi}^6_3 - \overline{B}^{ij6}B_{ij3}\overline{\phi}^2_1 - \overline{B}^{ij1}B_{ij2}\overline{\phi}^5_4 + \overline{B}^{ij4}B_{ij5}\overline{\phi}^2_1] \\
&-2a_V[\overline{B}^{ij5}B_{ij2}\overline{\phi}^3_2 - \overline{B}^{ij3}B_{ij2}\overline{\phi}^5_2 - \overline{B}^{ij1}B_{ij6}\overline{\phi}^1_4 + \overline{B}^{ij4}B_{ij1}\overline{\phi}^6_1] + \ldots
\end{aligned}
\tag{5.2}
$$

The meaning of indices $i = 1, \ldots 6$ is determined by

$$1 \quad u\!\uparrow \overline{u}\uparrow, \quad 2 \quad u\!\downarrow \overline{u}\downarrow, \quad 3 \quad d\!\uparrow \overline{d}\uparrow, \quad 4 \quad d\!\downarrow \overline{d}\downarrow, \quad 5 \quad s\!\uparrow \overline{s}\uparrow, \quad 6 \quad s\!\downarrow \overline{s}\downarrow \tag{5.3}$$

The quark-antiquark product, i.e. meson ϕ_a^b (pseudoscalar and vector) is as denoted in Table 5.1 [20]. The baryon states (octet spin 1/2 and decouplet spin 3/2, the {56} representation) are listed in Table 2 [16,20].

TABLE 5.1a. Mesons in SU(6)$_W$.

$S\,S_z$	Meson	ϕ_1^1	ϕ_2^2	ϕ_3^3	ϕ_4^4	ϕ_5^5	ϕ_6^6
0 0	π^0	$1/2$	$-1/2$	$-1/2$	$1/2$		
1 0	$\rho^0(0)$	$1/2$	$1/2$	$-1/2$	$-1/2$		
1 0	$\eta_1'(0)$	$1/\sqrt{6}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$1/\sqrt{6}$	$1/\sqrt{6}$
0 0	ϕ_1	$1/\sqrt{6}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	$-1/\sqrt{6}$
1 0	$\omega_8(0)$	$1/\sqrt{12}$	$1/\sqrt{12}$	$1/\sqrt{12}$	$1/\sqrt{12}$	$-2/\sqrt{12}$	$-2/\sqrt{12}$
0 0	η_8	$1/\sqrt{12}$	$-1/\sqrt{12}$	$1/\sqrt{12}$	$-1/\sqrt{12}$	$-2/\sqrt{12}$	$2/\sqrt{12}$

TABLE 5.1b. Mesons in SU(6)$_W$ (continued).

$S\,S_z$	Meson	ϕ_1^2	ϕ_3^4	ϕ_5^6	ϕ_1^3	ϕ_2^4
1 1	$\rho^0(\uparrow)$	$1/\sqrt{2}$	$-1/\sqrt{2}$			
1 1	$\omega_8(\uparrow)$	$1/\sqrt{6}$	$1/\sqrt{6}$	$-2/\sqrt{6}$		
1 1	$\eta_1'(\uparrow)$	$1/\sqrt{3}$	$1/\sqrt{3}$	$1/\sqrt{3}$		
0 0	π^+				$1/\sqrt{2}$	$-1/\sqrt{2}$
1 0	$\rho^*(0)$				$1/\sqrt{2}$	$-1/\sqrt{2}$

TABLE 5.1c. Mesons in SU(6)$_W$ (continued).

$S\,S_z$	Meson	ϕ_3^5	ϕ_4^6	ϕ_1^5	ϕ_2^6
0 0	κ^0	$1/\sqrt{2}$	$-1/\sqrt{2}$		
1 0	$K^{*0}(0)$	$1/\sqrt{2}$	$1/\sqrt{2}$		
0 0	K^+			$1/\sqrt{2}$	$-1/\sqrt{2}$
1 0	$K^{*+}(0)$			$1/\sqrt{2}$	$1/\sqrt{2}$

TABLE 5.1d. Mesons in SU(6)$_W$ (continued).

$S\,S_z$	Meson	ϕ_1^4	ϕ_2^3	ϕ_3^6	ϕ_4^5	ϕ_1^6	ϕ_2^5
1 1	$\rho^+(\uparrow)$	1					
1 -1	$\rho^+(\downarrow)$		1				
1 1	$K^{*0}(\uparrow)$			1			
1 -1	$K^{*0}(\downarrow)$				1		
1 1	$K^{*+}(\uparrow)$					1	
1 -1	$K^{*+}(\downarrow)$						1

TABLE 5.2. Baryons in SU(6)$_W$.

$$
\begin{aligned}
p &= \sqrt{2}(B^{114} - B^{123}) & 3B^{114} &= \sqrt{2}p + \Delta^+_{1/2} \\
\Delta^+_{1/2} &= B^{114} + 2B^{123} & 3B^{123} &= -\tfrac{1}{\sqrt{2}}p + \Delta^+_{1/2} \\
n &= \sqrt{2}(-B^{332} + B^{134}) & 3B^{332} &= -\sqrt{2}n + \Delta^0_{1/2} \\
\Delta^0_{1/2} &= B^{332} + 2B^{134} & 3B^{134} &= \tfrac{1}{\sqrt{2}}n + \Delta^0_{1/2} \\
\Lambda^0 &= \sqrt{3}(B^{235} - B^{145}) & 3B^{136} &= \Sigma^0 + \tfrac{1}{\sqrt{2}}Y^{*\,0}_{1/2} \\
\Sigma^0 &= 2B^{136} - B^{235} - B^{145} & 3B^{145} &= -\tfrac{1}{2}\Sigma^0 + \tfrac{1}{\sqrt{2}}Y^{*\,0}_{1/2} - \tfrac{\sqrt{3}}{2}\Lambda \\
Y^{*\,0}_{1/2} &= \sqrt{2}(B^{136} + B^{235} + B^{145}) & 3B^{235} &= -\tfrac{1}{2}\Sigma^0 + \tfrac{1}{\sqrt{2}}Y^{*\,0}_{1/2} + \tfrac{\sqrt{3}}{2}\Lambda \\
\Sigma^+ &= \sqrt{2}(-B^{116} + B^{125}) & 3B^{116} &= -\sqrt{2}\Sigma^+ + Y^{*\,+}_{1/2} \\
Y^{*\,0}_{1/2} &= B^{116} + 2B^{125} & 3B^{125} &= \tfrac{1}{\sqrt{2}}\Sigma^+ + Y^{*\,+}_{1/2} \\
\Sigma^- &= \sqrt{2}(B^{336} - B^{345}) & 3B^{336} &= \sqrt{2}\Sigma^- + Y^{*\,-}_{1/2} \\
Y^{*\,-}_{1/2} &= B^{336} + 2B^{345} & 3B^{345} &= -\tfrac{1}{\sqrt{2}}\Sigma^- + Y^{*\,-}_{1/2} \\
\Xi^0 &= \sqrt{2}(B^{255} - B^{156}) & 3B^{255} &= \sqrt{2}\Xi^0 + \Xi^{*\,0}_{1/2} \\
\Xi^{*\,0}_{1/2} &= B^{255} + 2B^{156} & 3B^{156} &= -\tfrac{1}{\sqrt{2}}\Xi^0 + \Xi^{*\,0}_{1/2} \\
\Xi^- &= \sqrt{2}(B^{455} - B^{356}) & 3B^{455} &= \sqrt{2}\Xi^- + \Xi^{*\,-}_{1/2} \\
\Xi^{*\,-}_{1/2} &= B^{455} + 2B^{356} & 3B^{356} &= -\tfrac{1}{\sqrt{2}}\Xi^- + \Xi^{*\,-}_{1/2}
\end{aligned}
$$

In order to test CP and spatial behaviour one can explicitly write some transitions. (We use $1 =\uparrow$ and $2 =\downarrow$ with spinors.) One has

$$
\begin{aligned}
F(\Lambda \to p\pi^-) &= \tfrac{1}{\sqrt{3}}\left(\tfrac{1}{12}b_T - \tfrac{1}{6}b_V + \tfrac{1}{3}c_V\right) \cdot \textstyle\sum_{s=1,2} \chi_p^{s\,\dagger}\chi_\Lambda^s \phi_{\pi^+}\,, \\
F(p \to \Lambda\pi^+) &= -\tfrac{1}{\sqrt{3}}\left(\tfrac{1}{12}b_T - \tfrac{1}{6}b_V + \tfrac{1}{3}c_V\right) \cdot \textstyle\sum_{s=1,2} \chi_\Lambda^{s\,\dagger}\chi_p^s \phi_{\pi^-}\,, \\
F(\Lambda \to p\rho^-) &= -\sqrt{\tfrac{2}{3}}\left(a_V - \tfrac{1}{12}b_T + \tfrac{1}{3}b_V - \tfrac{1}{2}c_V\right) \cdot \chi_p^{2\dagger}\chi_\Lambda^1 \rho^1_{+1}\,, \\
F(p \to \Lambda\rho^+) &= -\sqrt{\tfrac{2}{3}}\left(a_V - \tfrac{1}{12}b_T + \tfrac{1}{3}b_V - \tfrac{1}{2}c_V\right) \cdot \chi_\Lambda^{1\,\dagger}\chi_p^2 \rho^1_{-1}\,.
\end{aligned} \tag{5.4}
$$

From that one can conclude that baryon densities which multiply meson fields behave as[5]

$$
\begin{aligned}
\chi^\dagger\chi &\to \overline{\psi}\psi \\
\chi^\dagger\sigma_+\chi &\to \overline{\psi}\gamma_{1,2}\gamma_5\psi.
\end{aligned} \tag{5.5}
$$

[5]This are not generators of SU(6)$_W$ which correspond to $(\psi^\dagger\psi)$ and $(\psi^\dagger\gamma_{1,2}\gamma_5\psi)$ [15,17].

Under CP reflection one finds[6]

$$\begin{array}{lcl} \overline{\psi}\psi & \stackrel{\mathcal{CP}}{\longrightarrow} & \overline{\psi}\psi\,, \\ \overline{\psi}\boldsymbol{\gamma}\gamma_5\psi & \stackrel{\mathcal{CP}}{\longrightarrow} & \overline{\psi}\boldsymbol{\gamma}\gamma_5\psi\,, \\ \phi_a^b & \stackrel{\mathcal{CP}}{\longrightarrow} & (-)\phi_a^b\,. \end{array} \tag{5.6}$$

Thus generally

$$\overline{B}^{abc}B_{ijk}\phi_t^r \to (-)\overline{B}^{ijk}B_{abc}\phi_r^t. \tag{5.7}$$

This causes a minus sign appearing in front of the second half of the expression (5.2). According to general arguments, given in Sect. 4. that means that the effective expressions (5.1) and (5.2) are CP invariant. In the relativistic notation the terms (5.4) correspond to generic forms (4.8).

However the expression (5.2) is not Hermitian as one can easily conclude by comparison with generic forms (4.10). In order to deal with the Hermitian effective weak Hamiltonian one has to multiply (5.1) and (5.2) with a factor

$$\mathrm{i}^{1-S_M}. \tag{5.8}$$

Here $S_M = 0, 1$ is the meson spin. This factor has nothing to do with $\mathrm{SU}(6)_W$ symmetry. As a matter of fact, by being meson spin dependent, it breaks that symmetry.

One can also mention that similar considerations and analogous combinations apply to the $\Delta S = 0$ PV effective weak Hamiltonians [19]. There $\mathrm{SU}(6)_W$ symmetry + CP invariance leads to the coupling[7]

$$f_\pi(\overline{p}n\pi_- - \overline{n}p\pi_+) = f_\pi\frac{-\mathrm{i}}{\sqrt{2}}\overline{N}(\boldsymbol{\tau}\times\boldsymbol{\pi})_3 N. \tag{5.9}$$

The form which is used in the literature [19,24,25]

$$\mathrm{i}f_\pi(\overline{p}n\pi_- - \overline{n}p\pi_+) = f_\pi\frac{1}{\sqrt{2}}\overline{N}(\boldsymbol{\tau}\times\boldsymbol{\pi})_3 N \tag{5.10}$$

is Hermitian.

6. *The relative signs of weak* $\overline{B}\,B\,\pi$ *amplitudes*

While an overall sign does not matter, the relative signs of various terms appearing in the Weak Strangeness Violating Potential (WSVP) determine the magnitude of calculated matrix elements [26]. Those relative signs depend on the following

[6] $\psi^C \to C\overline{\psi}^T$, $\overline{\psi}^C \to -\psi^T C^{-1}$, $\psi^P \to \gamma_0\psi$, $C^{-1}\gamma^\mu C = -\gamma^{\mu\,T}$, $C^{-1}\gamma_5 C = -\gamma_5^T = \gamma_5$, $C^{-1}\gamma_5 C = \gamma_5^T = \gamma_5$. Meson states are catalogued in multiplets of $W-$spin [20], see Appendix C.

[7] $\pi_\pm = (\pi_1 \pm i\pi_2)/\sqrt{2}$.

(i) The relative signs of the strong coupling constants, such as $g_{B_1B_2M}$ and $g_{B_1B_2V}$ (here B_i are baryons, M is a pseudoscalar meson and V is a vector meson).

(ii) The relative signs of the weak amplitudes such as A and ϵ_ρ for example [4,5]

(iii) The effective strong and weak Hamiltonians such as for example

$$
\begin{aligned}
\mathcal{H}^W_{\Lambda N\pi} &= \mathrm{i}G_F m_\pi^2 \overline{\psi}(A_\pi + B_\pi\gamma_5)\boldsymbol{\tau}\cdot\boldsymbol{\phi}\begin{pmatrix}0\\1\end{pmatrix}\psi_\Lambda\,,\\
\mathcal{H}^S_{NN\pi} &= \mathrm{i}g_{NN\pi}\overline{\psi}_N\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\phi}\psi_N\,,\\
\mathcal{H}_{\Lambda N\rho} &= G_F m_\pi^2\overline{\psi}_N\left(\alpha_\rho\gamma^\mu + \beta_\rho\frac{\mathrm{i}\sigma^{\mu\nu}\partial_\nu}{2\overline{M}} + \epsilon_\rho\gamma^\mu\gamma_5\right)\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\mu\begin{pmatrix}0\\1\end{pmatrix}\psi_\Lambda\,,\\
\mathcal{H}_{NN\rho} &= \overline{\psi}_N\left(g^V_{NN\rho}\gamma^\mu + \frac{g^T_{NN\rho}}{2M}\sigma^{\mu\nu}\partial_\nu\right)\boldsymbol{\tau}\cdot\boldsymbol{\phi}_\mu\psi_N\,.
\end{aligned}
\tag{6.1}
$$

We will be using matrices and conventions as in Bjorken-Drell book [2,3,7], i.e.

$$
\gamma^0 = \beta, \qquad \boldsymbol{\gamma} = \beta\boldsymbol{\alpha}, \qquad \gamma_5 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \qquad \gamma_\mu\gamma^\mu = \gamma^0\gamma^0 - \boldsymbol{\gamma}\cdot\boldsymbol{\gamma}. \tag{6.2}
$$

The relative signs of the strong coupling constants are those given in Refs. [10,13]. All are in mutual agreement. A collection of the coupling constant values can be found in Table III or Ref. [5]. One finds, for example

$$
\mathrm{Sign}(g_{NN\pi}/g^V_{NN\rho}) = +1\,. \tag{6.3}
$$

(All other signs agree with SU(3) flavour symmetry predicitons.)

The relative signs of the weak amplitudes follow form the $\mathrm{SU}(6)_W$ sum rule [18-20]. Important formulae are reproduced in (64) of Ref. [5]. The connection between notations is for example

$$
\Lambda^0_- \to A(\Lambda^0_-)\,. \tag{6.4}
$$

The experimental values of the amplitudes A and B (6.1) can be found in Ref. [11]. However, when using the results listed in Table I of Ref. [11], reproduced below one has to take into account the following facts: Ref. [11] used $\gamma_5 = -\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, which entered the same form as shown in the formula (6.1). Thus all B amplitudes, but $B(\Sigma^+_0)$ in Table 6.1 should be read with minus sign, i.e.

$$
B(\Lambda^0_-) = (-9.98 \pm 0.24)\cdot(2.21\times 10^{-7}). \tag{6.5}
$$

TABLE 6.1. Numbers given in units of 2.21×10^{-7}.

$M \to m + \mu$	A	B	C_{AB}
$\Lambda^0_- \to p + \pi^-$	1.47 ± 0.01	9.98 ± 0.24	–0.289
$\Lambda^0_0 \to n + \pi^0$	-1.07 ± 0.01	-7.14 ± 0.56	–0.740
$\Sigma^+_+ \to n + \pi^+$	0.06 ± 0.01	19.07 ± 0.07	–0.038
$\Sigma^+_0 \to p + \pi^0$	1.48 ± 0.05	-12.04 ± 0.58	0.982
$\Sigma^-_- \to n + \pi^-$	1.93 ± 0.01	-0.65 ± 0.07	0.003
$\Xi^0_0 \to \Lambda + \pi^0$	1.55 ± 0.03	-5.56 ± 0.33	–0.148
$\Xi^-_- \to \Lambda + \pi^-$	2.04 ± 0.01	-7.49 ± 0.28	0.237

The signs of $A(\Sigma^+_0)$ and $B(\Sigma^+_0)$ depend on the definition of isotriplet components. Usually (and we do the same) one defines

$$\begin{aligned} \Sigma^{\pm} &= \tfrac{1}{\sqrt{2}}(\Sigma_1 \pm \mathrm{i}\Sigma_2)\,, \\ \pi^{\pm} &= \tfrac{1}{\sqrt{2}}(\pi_1 \pm \mathrm{i}\pi_2) \end{aligned} \tag{6.6a}$$

However, in Ref. [11] the spherical isovector components were used

$$\begin{aligned} \Sigma^{\pm} &= \mp\tfrac{1}{\sqrt{2}}(\Sigma_1 \pm \mathrm{i}\Sigma_2)\,, \\ \pi^{\pm} &= \mp\tfrac{1}{\sqrt{2}}(\pi_1 \pm \mathrm{i}\pi_2)\,. \end{aligned} \tag{6.6b}$$

Thus $A(\Sigma^+_+)$ does not change the sign, as Σ^+ and π^+ signs compensate, but $A(\Sigma^+_0)$ amplitude must change the sign. Rule

(i) Change sign of $A(\Sigma^+_0)$

(ii) Change sign of all B amplitudes except of $B(\Sigma^+_0)$.

The signs (3) and the signs in Table 6.1 (with comments) fix relative signs of all *PC potential terms* that can be derived from the Hamiltonians (6.1). With SU(3) symmetry, pole dominance etc. (see Ref. [11]) all relative signs of A_π, B_π, α_ρ and β_ρ can be determined.

The relative signs of A_π and ϵ_ρ, which contribute to *PV potential terms* can be determined by invoking SU(6)$_W$ symmetry [5,12,15-20]. They are expressible in term of parameters a_T, a_V, b_T, b_V and c_V [12-15,21] which were introduced in Sect. 5. There one can find also the derivation of the relative phase terms proportional to A_π and ϵ_ρ appearing in the formula (6.1). Here we list, for the completness sake the sum rules connecting ϵ_ρ with the decay amplitudes which are given in Ref. [5]

$$\begin{aligned} \epsilon &= \tfrac{2}{3}A(\Lambda^0_-) - \tfrac{1}{\sqrt{3}}A(\Sigma^+_0) + \sqrt{3}a_T\,, \\ \epsilon &= A(\Sigma^+_0) - \tfrac{1}{3}a_T \end{aligned} \tag{6.7}$$

The A amplitudes should be "read" from Table 6.1, as explained above [$A(\Sigma^+_0) = -1.48$].

7. Parity-conserving meson exchange weak potential

The weak baryon-baryon-pion vertex is connected with the B amplitude which appears in the matrix element

$$\langle B_f\pi|\, H_W\, |B_i\rangle = \mathrm{i}G_F m_\pi^2 \overline{u}_{B_f}(A - \gamma_5 B)u_{B_i}.$$

The relative sign and phases of A and B in the $\Delta S = 1$ case are experimentally connected as shown in Table 6.1 [11].

The theoretical expression for B contain pole terms (B^P) and separable terms (B^S). A generic pole term, shown in Fig. 7.1, contains the weak amplitude $a_{B'B}$ [W] and the strong $NN\pi$ coupling constants [S]. The relative signs of $a_{B'B}$ (3.7) and the strong coupling constant are determined by theoretical expressions (3.1) and (3.6) and by the experimental data [10,11,13]. As already shown (3.8) the weak B amplitude must have the same phase as A. The theoretical expression, aa for example (3.8) lead to the predictions whose relative signs agree with experimental data [29,30]. The fixing of relative signs is also helped by the famous Goldberger-Treiman relation [6] which is for the nucleon pion case given by (B.11).

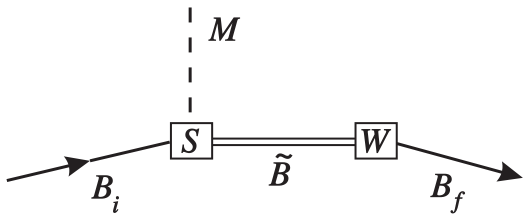

Fig. 7.1. Generic form of a baryon pole term. (The crossed term is not shown!)

Here the relative signs of g_A, f_π and $g_{NN\pi}$ are interconnected.

The separable contribution to B amplitudes can be calculated in the same way as the separable contributions (2.2) (2.6) and (2.9). Obviously one has to exchange $A \to B$ and to insert, or omitt, γ_5's at some places [1].

The pole term contributions to the vector meson exchanges is based on the same type of diagram as shown in Fig. 7.1. Again, $a_{B'B}$ appears in the weak vertex. The relative signs of the strong couplings of pseudoscalara mesons g_{BBM} and the strong couplings of vector mesons g^V_{BBV} are known from the analysis of the experiments [10,13]. So the phase of the PC vector mesons exchange weak potential is fixed[8].

The same reasoning would apply to the contributions of the $3/2^+$ resonances [30] which replace the pole $\widetilde{B}(1/2^+) \to B(3/2^+)$ in Fig. 7.1.

One can also determine relative signs connected with the axial vector meson exchanges. However, as this is a novel inclusion, the discussion is relegated to the following paragraph.

8. *Axial-vector meson exchanges*

The axial vector meson exchange weak potential (AVMWP) contain the separable and the pole terms.

The separable terms appear naturally when in any separable (i.e. current×current) term the axial vector current formfactors are approximated by the axial vector meson pole. For example in expressions (B.3), (B.6) etc. one introduces

$$g_A \to g_A(q^2) = g_A(0)\frac{m^2_{AV}}{m^2_{AV} - q^2}. \tag{8.1}$$

The relative sign of $g_A(0) \equiv g_A$ and of the whole separable contribution (2.3) are experimentally fixed [6] and thus everything is determined. It should be mentioned that PV separable contributions, which contain products of vector and axial vector currents, would contain products

$$g_V \frac{m^2_V}{m^2_V - q^2} g_A \frac{m^2_{AV}}{m^2_{AV} - q^2}. \tag{8.2}$$

But again all relative signs are known experimentally. Besides various g_V's and g_A's can be approximately connected by using SU(3) flavour symmetry [1,6].

In order to calculate pole terms one needs magnitudes of the strong axial vector meson constants g^{AV}_{BBAV} [14]. As long as they are real, what is indicated by general argument (4.4) and (4.6), the $\pm$ sign does not matter.

In order to prove that one has to combine the axial vector field $V^{A\,\alpha}_\mu$ behaviour

$$\begin{aligned} V^{A\,\alpha}_\mu &\xrightarrow{\mathcal{C}} V^{A\,\alpha}_\mu\,, \\ V^{A\,\alpha}_\mu &\xrightarrow{\mathcal{P}} \boldsymbol{V}^{A\,\alpha}\,, \\ &\xrightarrow{\mathcal{P}} -V^{A\,\alpha}_0\,. \end{aligned} \tag{8.3}$$

[8]The overall sign depends on $a_{B'B}$. The strong coupling constant enters the potential quadratically as $(g^V_{BBV})^2$ or $(g_{BBV})^2$, see Fig. 8.1.

The property

$$V_\mu^{A\,\alpha\,\dagger} = V_\mu^{A\,\overline{\alpha}} \tag{8.4}$$

follows from (4.9). All diagrams similar to the one shown in Fig. 8.1 lead to expressions containing $(g_{BBAV}^{AV})^2$. Thus the sign of the pole term contribution is determined by the sign of $a_{B'B}$ which corresponds to the weak vertex.

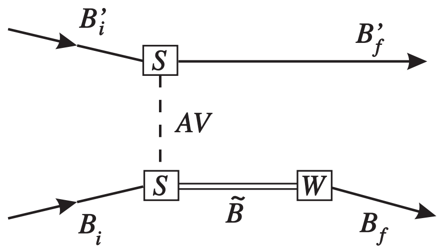

Fig. 8.1. Axial vector meson exchange contribution to $B_i + B_i' \to B_f + B_f'$ process. The same generic diagram applies to π or vector meson exchanges by $AV \to \pi, V$.

9. Weak vector meson vertices, SU(6)$_W$ symmetry, $\Delta I = 1/2$ selection rule and factorization contributions

It is well known [19] that the weak baryon-baryon-meson ($\overline{B}'BM$) couplings correspond to the quark diagram shown in Fig. 9.1

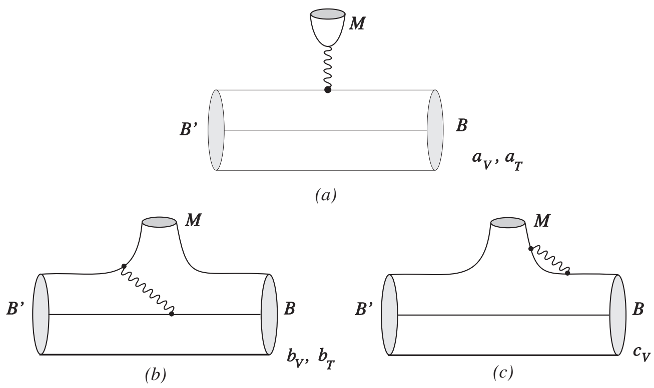

Fig. 9.1. Quark diagrams corrresponding to SU(6)$_W$ parameters.

The indicated $SU(6)_w$ parameters correspond to formula (5.1). The weak vertices, or decay amplitudes are function of those parameters as shown, for example, in (5.4) Some other vertices are [20]:

$$\begin{aligned}
&A(\Lambda^0_-) = \tfrac{1}{\sqrt{3}}\left(-\tfrac{1}{6}b_V + \tfrac{1}{12}b_T + \tfrac{1}{2}c_V\right), \quad A(\Lambda^0_0) = \tfrac{1}{\sqrt{6}}\left(\tfrac{1}{6}b_V - \tfrac{1}{12}b_T - \tfrac{1}{2}c_V\right), \\
&A(\Xi^-_-) = \tfrac{1}{\sqrt{3}}\left(-\tfrac{1}{4}b_V + \tfrac{1}{4}b_T + \tfrac{1}{2}c_V\right), \quad A(\Xi^0_0) = \tfrac{1}{\sqrt{6}}\left(-\tfrac{1}{4}b_V + \tfrac{1}{4}b_T + \tfrac{1}{2}c_V\right), \\
&A(\Sigma^-_-) = -\sqrt{2}\left(\tfrac{1}{9}b_V - \tfrac{5}{36}b_T - \tfrac{1}{6}c_V\right), \quad A(\Sigma^+_+) = -\sqrt{2}\left(\tfrac{1}{36}b_V + \tfrac{1}{36}b_T\right), \\
&A(\Sigma^+_0) = -\left(-\tfrac{5}{36}b_V + \tfrac{1}{9}b_T + \tfrac{1}{6}c_V\right), \quad A(\Sigma^0_0) = -\tfrac{\sqrt{2}}{18}b_V + \tfrac{5}{36\sqrt{2}}b_T + \tfrac{1}{6\sqrt{2}}c_V
\end{aligned} \tag{9.1a}$$

$$\begin{aligned}
&\langle\rho^-(\uparrow)p(\downarrow)|\,H^{(-)}\,|\Lambda(\uparrow)\rangle = \sqrt{\tfrac{2}{3}}\left(-\tfrac{1}{3}b_V + \tfrac{1}{12}b_T + \tfrac{1}{2}c_V - a_V\right), \\
&\langle\rho^0(\uparrow)n(\downarrow)|\,H^{(-)}\,|\Lambda(\uparrow)\rangle = -\tfrac{1}{\sqrt{2}}\left(-\tfrac{1}{6}b_V + \tfrac{1}{4}b_T + \tfrac{1}{2}c_V + a_T\right), \\
&\langle\rho^-(\uparrow)\Lambda(\downarrow)|\,H^{(-)}\,|\Xi^-(\uparrow)\rangle = \sqrt{\tfrac{2}{3}}\left(-\tfrac{1}{12}b_V + \tfrac{1}{12}b_T + \tfrac{1}{6}c_V - \tfrac{1}{3}a_V\right), \\
&\langle\rho^0(\uparrow)\Lambda(\downarrow)|\,H^{(-)}\,|\Xi^0(\uparrow)\rangle = \tfrac{1}{\sqrt{3}}\left(-\tfrac{1}{12}b_V + \tfrac{1}{12}b_T + \tfrac{1}{6}c_V + \tfrac{1}{3}a_T\right), \\
&\langle\rho^-(\uparrow)n(\downarrow)|\,H^{(-)}\,|\Sigma^-(\uparrow)\rangle = \tfrac{1}{9}\left(\tfrac{1}{2}b_V - b_T - c_V + 2a_V\right), \\
&\langle\rho^+(\uparrow)n(\downarrow)|\,H^{(-)}\,|\Sigma^+(\uparrow)\rangle = \tfrac{1}{9}\left(-2b_V + b_T\right), \\
&\langle\rho^0(\uparrow)p(\downarrow)|\,H^{(-)}\,|\Sigma^+(\uparrow)\rangle = \tfrac{1}{9\sqrt{2}}\left(-\tfrac{5}{2}b_V + 2b_T + c_V + 2a_T\right), \\
&\langle\omega^0_8(\uparrow)p(\downarrow)|\,H^{(-)}\,|\Sigma^+(\uparrow)\rangle = \tfrac{1}{9\sqrt{6}}\left(-\tfrac{5}{2}b_V + 2b_T + c_V + 2a_T\right), \\
&\langle\omega^0_8(\uparrow)n(\downarrow)|\,H^{(-)}\,|\Sigma^0(\uparrow)\rangle = \tfrac{1}{18\sqrt{3}}\left(2b_V - \tfrac{5}{2}b_T - c_V - 2a_T\right), \\
&\langle\omega^0_8(\uparrow)n(\downarrow)|\,H^{(-)}\,|\Lambda^0(\uparrow)\rangle = \tfrac{1}{36}\left(4b_V - 5b_T - 6c_V - 12a_T\right), \\
&\langle K^{*0}(\uparrow)p(\downarrow)|\,H^{(-)}\,|p(\uparrow)\rangle = \tfrac{1}{9}\left(-\tfrac{1}{2}b_V + b_T + c_V + 8a_T\right), \\
&\langle K^{*+}(\uparrow)n(\downarrow)|\,H^{(-)}\,|p(\uparrow)\rangle = \tfrac{1}{9}\left(2b_V - b_T - 5c_V + 10a_V\right), \\
&\langle K^{*0}(\uparrow)n(\downarrow)|\,H^{(-)}\,|n(\uparrow)\rangle = \tfrac{1}{9}\left(b_V - \tfrac{1}{2}b_T - 4c_V - 2a_T\right), \\
&\langle K^{*-}(\uparrow)p(\downarrow)|\,H^{(-)}\,|n(\uparrow)\rangle = 0\,.
\end{aligned} \tag{9.1b}$$

Here the longer notation for the vector mesons ρ, ω and K^* is kept. The spin orientations are indicated. However, the spin-spatial dependence on the RHS of (9.1a,b) is omitted. The full reading of, for example, $\langle\rho^- p|\,H\,|\Lambda\rangle$ is

$$\langle\rho^-(\uparrow)p(\downarrow)|\,H^-\,|\Lambda(\uparrow)\rangle = F(\Lambda\to p\rho^-). \tag{9.2a}$$

Here F is given by the formula (5.4).

The $SU(6)_W$ Hamiltonian (5.1) does not satisfy $\Delta I = 1/2$ (or octet dominance) selection rule. The $\Delta I = 1/2$ rule leads, for example to the relations

$$A(\Sigma^+_+) = 0 \quad \text{and} \quad A(\Lambda^0_0) = -\frac{1}{\sqrt{2}} A(\Lambda^0_-)\,. \tag{9.3a}$$

The inspection of the formulae (9.1a,b) shows that this holds if the $SU(6)_w$ parameters satisfy

$$b_V = -b_T. \tag{9.3b}$$

In the vector meson case with the definition

$$\langle \rho^-(\uparrow) p(\downarrow) |\, H^- \,| \Lambda(\uparrow) \rangle = A_\rho(\Lambda^0_-)(\chi^\dagger \boldsymbol{\sigma} \chi \boldsymbol{\rho}) \tag{9.2b}$$

the $\Delta I = 1/2$ rule leads to

$$\begin{aligned} A_\rho(\Lambda^0_0) &= -\tfrac{1}{\sqrt{2}} A_\rho(\Lambda^0_-)\,, \\ A_\rho(\Sigma^+_+) - A_\rho(\Sigma^-_-) &= \sqrt{2} A_\rho(\Sigma^+_0)\,, \\ A_{K^*}(p^+_+) &= A_{K^*}(n^0_-) - A_{K^*}(p^0_0)\,. \end{aligned} \tag{9.4a}$$

With the constraint (9.3b) this can hold only if

$$a_V = -a_T\,. \tag{9.4b}$$

However, this last condition has to be further discussed. As shown in Ref. [19] one can easily calculate a_V and a_T for the real weak Hamiltonian (given, for example, by formula (2.1) of Ref. [1]). For such a Hamiltonian, as it will be discussed below, (9.4) cannot hold.

In a weak Hamiltonian which satisfy $\Delta I = 1/2$ rule, neutral $\Delta S = 1$ currents must appear. In the $\Delta S = 0$ sector, discussed by Ref. [19] one would have neutral currents appearing originally and not only as result of a Fierz rearrangement. Products of currents can be desomposed as

$$\begin{gathered} a_1 b_{-1} + a_{-1} b_1 = 2 \textstyle\sum_T C^{T\,0}_{1\,1\,1\,-1} X^0_T\,, \qquad (T = 0, 2) \\ \tfrac{1}{\sqrt{2}} a_1 \sim -\overline{d} u\,, \qquad \tfrac{1}{\sqrt{2}} b_{-1} \sim \overline{u} d\,, \\ a_0 b_0 = \textstyle\sum_T C^{T\,0}_{1\,0\,1\,0}\, X^0_T\,, \qquad (T = 0, 2) \\ a_0,\ b_0 \sim (\overline{u} u - \overline{d} d)\,. \end{gathered} \tag{9.5a}$$

Here only the flavour content of currents, which are color scalars, is shown. The octet dominance means that only the isospin $T = 0$ is allowed in the sums (9.5a). The factorization means, for example

$$\langle \rho^+ n |\, a_1 b_{-1} \,| p \rangle = \langle \rho^+ |\, a_1 \,| 0 \rangle \langle n |\, b_{-1} \,| p \rangle \qquad\qquad (a_V)\,. \tag{9.6}$$

The combination (9.6) determines a_V while the combination

$$\langle\rho^0 p|\, a_0 b_0\, |p\rangle = \langle\rho^0|\, a_0\, |0\rangle\langle p|\, b_0\, |p\rangle \qquad (a_T) \tag{9.7}$$

would determine a_T.

In a Hamiltonian transforming as an octet one has pieces

$$\begin{aligned} H_8^{(+)} &= (-2)\tfrac{1}{2}\left(C^{0\,0}_{1\,1\,1\,-1}\right)^2 (a_1 b_{-1} + a_{-1} b_1)\,, \\ H_8^{(0)} &= \left(C^{0\,0}_{1\,0\,1\,0}\right)^2 a_0 b_0\,. \end{aligned} \tag{9.8a}$$

This is easily found by using the definition

$$X_T^0 = \sum_m C^{T\,0}_{1\,m\,1\,-m} a_m b_{-m}\,. \tag{9.8b}$$

The consistency of the whole procedure is easily checked by writing (9-5a) in the form

$$\begin{aligned} a_1 b_{-1} + a_{-1} b_1 &\equiv 2\textstyle\sum_{T=0,2}\left(C^{T\,0}_{1\,1\,1\,-1}\right)^2 (a_1 b_{-1} + a_{-1} b_1)\,, \\ a_0 b_0 &\equiv 2\textstyle\sum_{T=0,2}\left(C^{T\,0}_{1\,0\,1\,0}\right)^2 a_0 b_0\,. \end{aligned} \tag{9.5b}$$

By using (9.8) one can calculate

$$\begin{aligned} \langle\rho^+ n|\, H_8^{(+)}\, |p\rangle &= -\left(C^{0\,0}_{1\,1\,1\,-1}\right)^2 \langle\rho^+|\, a_1\, |0\rangle\langle n|\, b_{-1}\, |p\rangle \\ &= -\left(C^{0\,0}_{1\,1\,1\,-1}\right)^2 \cdot \alpha C^{1/2\,-1/2}_{1\,-1\,1/2\,1/2}\beta = \tfrac{1}{3\sqrt{3}}\alpha\beta \\ &\sim a_V\,, \\ \langle\rho^0 p|\, H_8^{(0)}\, |p\rangle &= \left(C^{0\,0}_{1\,0\,1\,0}\right)^2 \langle\rho^0|\, a_0\, |0\rangle\langle p|\, b_0\, |p\rangle \\ &= \left(C^{0\,0}_{1\,0\,1\,0}\right)^2 \cdot \alpha C^{1/2\,1/2}_{1\,0\,1/2\,1/2}\beta = -\tfrac{1}{3\sqrt{3}}\alpha\beta \\ &\sim a_T\,. \end{aligned} \tag{9.9}$$

Thus the relation (9.4) is consistent with $\Delta I = 0$ (i.e. octet dominance) selection rule.

In Ref. [19] a_V and a_T were determined from the weak strangeness conserving Hamiltonian without QCD corrections. For the sake of completeness that calculation is briefly discussed in Appendix D.

Here we will estimate a_V and a_T from $\Delta S = 1$ weak Hamiltonian. First it will be done without QCD corrections. (The QCD corrected Hamiltonian is given in

Ref. [1].) Such "bare" $\Delta S = 1$ weak Hamiltonian is [4,6,8]

$$H^{\Delta S=1} = \frac{G}{\sqrt{2}} \sin\theta_C \cos\theta_C \left[\overline{d}\gamma^\mu(1-\gamma_5)u \cdot \overline{u}\gamma^\mu(1-\gamma_5)s + \ldots\right] . \tag{9.10}$$

In order to determine a_V (a_T) the factorizable (or separable - SEP) contributions of (9.10) $\Delta S = 1$, $\Lambda + p \to n + p$ scattering should be compared with the second order contribution produced by the effective interactions

$$\begin{aligned} H^S_{NN\rho} &= g^V_{NN\rho}\sqrt{2}\ \overline{\psi}_n \gamma^\mu \psi_p \rho^-_\mu \\ &\quad + g^V_{NN\rho}(\overline{\psi}_p \gamma^\mu \psi_p - \overline{\psi}_n \gamma^\mu \psi_n)\rho^0_\mu + \ldots , \\ H^W_{\Lambda N\rho^-} &= -\sqrt{\tfrac{2}{3}} a_V \overline{\psi}_n \gamma^\mu \gamma_5 \psi_\Lambda \rho^+_\mu , \\ H^W_{\Lambda N\rho^0} &= -\tfrac{1}{\sqrt{3}} a_T \overline{\psi}_n \gamma^\mu \gamma_5 \psi_\Lambda \rho^0_\mu . \end{aligned} \tag{9.11}$$

Here H^S corresponds to the standard strong $NN\rho$ coupling [5,7] while H^W are effective weak Hamiltonians, which were read from (9.1). The following equalities are obtained in the factorizable (SEP) approximations

$$\begin{aligned} &\langle pn| -\mathrm{i}H^S_{NN\rho}(\rho - \text{field contracted})H^W_{N\Lambda\rho}(\rho - \text{field contracted})\,|p\Lambda\rangle \\ &\quad = (-\mathrm{i})^2 \left(-\sqrt{\frac{2}{3}}\right) a_V \sqrt{2} g^V_{NN\rho} \overline{u}_n \gamma^\mu u_p \frac{1}{q^2 - m^2_\rho} \overline{u}_p \gamma_\mu \gamma_5 u_\Lambda \\ &\quad = -\tfrac{G}{\sqrt{2}} \sin\theta_C \cos\theta_C \langle n|\, \overline{d}\gamma^\mu u\, |p\rangle \langle p|\, \overline{u}\gamma_\mu\gamma_5 s\, |\Lambda\rangle \\ &\quad = -\frac{G}{\sqrt{2}} \sin\theta_C \cos\theta_C \overline{u}_n \gamma_\mu u_p \frac{m^2_\rho}{m^2_\rho - q^2} \left(-\sqrt{\frac{3}{2}}\right) (F + D/3) \overline{u}_p \gamma_\mu \gamma_5 u_\Lambda . \end{aligned} \tag{9.12a}$$

Here u_i are baryon spinors while the axial vector coupling is given [1,2,4,6] by

$$F + \frac{D}{3} = 0.733 . \tag{9.12b}$$

The vector current formfactor is assumed to be dominated by the ρ meson exchange. That leads to the factor $m^2_\rho/(m^2_\rho - q^2)$ in the last row of the equalities (9.12a). The factor in front of the baryon bilinears $\overline{u}_\alpha \Gamma u_\beta$ should be equal, what gives

$$-\sqrt{\frac{2}{3}} a_V \sqrt{2} g^V_{NN\rho} = \frac{G}{\sqrt{2}} \sin\theta_C \cos\theta_C \sqrt{\frac{3}{2}} (F + D/3) m^2_\rho . \tag{9.12c}$$

With $g^V_{NN\rho} = 3.16$ [5,10,13], $\sin\theta_C \cos\theta_C = 0.22$ and $G = 1.03 \times 10^{-5}/m^2_p$ one immediately finds

$$a_V = -2.7 \times 10^{-7} \tag{9.12d}$$

This seems to be in good agreement with a_V determined for $\Delta S = 0$ transition (see Appendix A) which is $a_V = -2.77 \times 10^{-7}$. It is also in good agreement with the estimate given by Ref. [15] which found

$$a_V = -2.86 \times 10^{-7}. \tag{9.13}$$

However in $\Delta S = 1$ sector one cannot expect the ratio $a_V/a_T = 3$ which was found in the $\Delta S = 0$ sector (see Appendix D). The Fierz rearangement (FR) of Hamiltonian (9.10) leads to

$$-\frac{G}{\sqrt{2}} \sin\theta_C \cos\theta_C \frac{1}{3} \langle n| \overline{d}\gamma^\mu \gamma_5 s \,|\Lambda\rangle \langle p| \overline{u}\gamma_\mu u \,|p\rangle. \tag{9.14a}$$

Here ρ^0 exchange can be associated with the formfactor comming from the last matrix element only.

In $\Delta S = 0$ case, the ρ^0 could have been connected with both corresponding matrix elements [19] (see Appendix D). Starting with (9.14a) and (9.11) one can establish an equality which is analogous to (9.12)

$$-\frac{1}{\sqrt{3}} a_T g^V_{NN\rho} = \frac{G}{\sqrt{2}} \sin\theta_C \cos\theta_C \frac{1}{3} \sqrt{\frac{3}{2}} (F + D/3) \frac{1}{2} m_\rho. \tag{9.14b}$$

Here (see Appendix D) only the isovector piece of the vector current $\langle p| \overline{u}\gamma_\mu u \,|p\rangle$ was extracted as only that piece corresponds to the ρ exchange. The relation is

$$a_T = \frac{1}{3} a_V. \tag{9.15}$$

Obviously the a_V and a_T dependent weak $\Delta S = 1$ potential contains both $\Delta I = 1/2$ (octet dominance) and $\Delta I = 3/2$ pieces. In some applications [5] potential is calculated in the octet dominance approximation. As already stated that means the equality (9.4). In practice the amplitude $A_\rho(\Lambda^0_0)$ is selected and used to determine an effective ϵ_ρ coupling through

$$\begin{array}{c} \langle \rho^0(\uparrow) n(\downarrow)| \,\mathcal{H}^{(-)}\, |\Lambda(\uparrow)\rangle = A_\rho(\Lambda^0_0)\eta \\ \epsilon_\rho = -A_\rho(\Lambda^0_0) = \frac{1}{\sqrt{3}} \left(-\frac{5}{12} b_V + \frac{1}{2} c_V + a_T \right) \end{array} . \tag{9.16}$$

Here η denotes the spin and spatial dependence of the matrix element (9.16).

The parameters b_V and c_V can be expressed through hyperon nonleptonic decay amplitudes [19], i.e.

$$\begin{aligned} b_V &= 6\tfrac{1}{\sqrt{3}} [\overline{A}(\Lambda^0_-) + A(\Sigma^+_0)] \,, \\ c_V &= 3[\sqrt{3} A(\Lambda^0_-) + A(\Sigma^+_0)] \,. \end{aligned} \tag{9.17}$$

With (9.17) one finds

$$\epsilon_\rho = \frac{2}{3}A(\Lambda^0_-) - \frac{1}{\sqrt{3}}A(\Sigma^+_0) + \frac{1}{\sqrt{3}}a_T\,. \tag{9.18}$$

But for the slight missprint[9] this is in the perfect agreement with the formula (64) in Ref. [5]. However one could also start with the expression (9.1) for $A_\rho(\Lambda^0_-)$ which gives

$$\epsilon_\rho = \frac{1}{\sqrt{2}}A_\rho(\Lambda^0_-) = \frac{2}{3}A(\Lambda^0_-) - \frac{1}{\sqrt{3}}A(\Sigma^+_0) - \frac{a_V}{\sqrt{3}}\,. \tag{9.19}$$

As was said, expressions (9.18) and (9.19) could agree if $a_V = -a_T$. Alternatively one can use

$$\begin{aligned} \mathrm{Sign}(a_T) &= -\mathrm{Sign}(a_V)\,, \\ |a_V| &= |3a_T|\,. \end{aligned} \tag{9.20}$$

Inserting that in (9.19) one would produce the expression used by Ref. [5], i.e.[10]

$$\epsilon_\rho = \frac{2}{3}\Lambda^0_- - \frac{1}{\sqrt{3}}\Sigma^+_0 + \sqrt{3}a_T\,. \tag{9.21}$$

One could also check whether that holds for the K* vertices. The corresponding effective PV weak Hamiltonian is

$$C^{PV}_{K^*}(\overline{n}pK^{*\,+} + \overline{n}nK^{*0}) + D^{PV}_{K^*}(\overline{p}p + \overline{n}n)K^{*0}. \tag{9.22}$$

Here the spatial dependence is supressed. Comparing that with (9.1) and using (9.17) one obtains

$$\begin{aligned} C^{PV}_{K^*} &= A_{K^*}(p^+_+) = -\sqrt{3}\Lambda^0_- + \tfrac{1}{3}\Sigma^+_0 + \tfrac{10}{9}a_V\,, \\ D^{PV}_{K^*} &= A_{K^*}(p^+_0) = -\tfrac{2}{3}\Sigma^+_0 + \tfrac{8}{9}a_T\,. \end{aligned} \tag{9.23}$$

Reference [5] has introduced $a_V = 3a_T$ in (9.23). This does not seem quite consistent with (9.19) where $a_V = -3a_T$ was introduced.

When everything is claculated by using the effective Hamiltonian for the $N\Lambda\rho$ coupling

$$H^{W/PV}_{N\Lambda\rho} = \epsilon_\rho\overline{\psi}_N\gamma^\mu\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\begin{pmatrix}0\\1\end{pmatrix}\psi_\Lambda\,, \tag{9.24}$$

[9]Ref. [5] has $\sqrt{3}a_T$ instead of $a_T/\sqrt{3}$.

[10]Bear in mind that the experssion (9.17) correspond to the sign of hyperon nonleptonic decay amplitudes A as used in Ref. [5]. See also Sect. 6. Thus in the following we write $A(B^i_j) \to B^i_j$.

and the expression (9.18) or (9.19) for ϵ_ρ are used, one is implicitly assuming the relation (9.4), i.e.

$$\text{“}a_T\text{”} = -\text{“}a_V\text{”} = \eta\,. \tag{9.25}$$

Here “a_i” symbolizes the effective parameter. However one could use

$$\text{“}a_T\text{”} = a_V = 3a_T\,, \qquad \text{or}\,, \tag{9.26a}$$

$$\text{“}a_T\text{”} = \frac{1}{3}a_V \qquad (\mathit{see}\ (9.13))\,. \tag{9.26b}$$

The Ref. [5] has used (9.26a) for the $N\Lambda\rho$ coupling. In the case of $N\Lambda K^*$ couplings the conventiones are somewhat different. Neither sign nor magnitude are consistent with (9.25) and (9.26a). Here they [5] have used

$$\begin{aligned} \text{“}a_V\text{”} &= a_V\,, && (\mathit{see}\ (9.12))\,,\\ \text{“}a_T\text{”} &= a_T = \tfrac{1}{3}a_V\,, && (\mathit{see}\ (9.12))\,. \end{aligned} \tag{9.27}$$

If one wanted to be consistent with (9.25) one should replace

$$\begin{aligned} a_V &\to -3a_T = -a_V && (\mathit{see}\ (9.12d))\,,\\ a_T &\to \ \ 3a_T = a_V && (\mathit{see}\ (9.12d))\,. \end{aligned} \tag{9.28}$$

Altogether that gives

$$\begin{aligned} \epsilon_\rho &= \tfrac{2}{3}\Lambda^0_- - \tfrac{1}{3}\Sigma^+_0 + \sqrt{3}a_T\,,\\ C^{PV}_{K^*} &= -\sqrt{3}\Lambda^0_- + \tfrac{1}{3}\Sigma^+_0 - \tfrac{1}{3}a_T\,,\\ D^{PV}_{K^*} &= -\tfrac{2}{3}\Sigma^+_0 + \tfrac{8}{9}a_T\,. \end{aligned} \tag{9.29}$$

There is a sign difference in the expression (9.29) for $C^{PV}_{K^*}$ in comparison with Ref. [5].

The replacement (9.28) or the result (9.29) can be justified on the basis of the $\mathrm{SU}(6)_W$ symmetry [19]. From (9.1) and taking into the account (9.4b) one can deduce

$$\begin{aligned} C^{PV}_{K^*} &= \tfrac{1}{9}(3b_V - 5c_V + 10a_V)\,,\\ D^{PV}_{K^*} &= \tfrac{1}{9}\left(-\tfrac{3}{2}b_V + c_V + 8a_T\right)\,,\\ \eta &= \tfrac{1}{9}\left(-\tfrac{3}{2}b_V + c_V + 8a_T\right)\,. \end{aligned} \tag{9.30}$$

Here C corresponds to the coupling $\overline{n}pK^{*+}$, D to the coupling $\overline{p}pK^{*0}$, and η to the coupling $\overline{n}nK^{*0}$. The $\Delta I = 1/2$ rule requires[11]

$$D^{PV}_{K^*} + C^{PV}_{K^*} = \eta \,, \tag{9.31a}$$

i.e.

$$\frac{1}{9}(\frac{3}{2}b_V - 4c_V + 10a_V + 8a_T) = \frac{1}{9}\left(\frac{3}{2}b_V - 4c_V - 2a_T\right) . \tag{9.31b}$$

Both sides are equal only if $a_V = -a_T$, as it was found in comparison between (9.18) and (9.19).

One has every right to ask "Why should one determine ϵ_ρ with the value (9.12) instead of with (9.15)?" It is difficult to make a very learned recomendation. One could use an average and estimate errors. For example: use (9.18) but replace a_T with

$$\text{“}a_T\text{”} = \frac{1}{2}(+a_V + a_T) = \frac{2}{3}a_V \tag{9.32a}$$

and determine an error with respect to the claculated a_V and a_T, i.e.

$$\text{“}a_T\text{”} = -(1.8 \pm 0.9) \times 10^{-5}. \tag{9.32b}$$

A wiser course might be to calculate the $\Delta S = 1$ PV potentials corresponding to a_V and a_T pieces directly in the separable (factorizable) approximation.

Actually the contributions proportional to either a_T or a_V contain both $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces. That can be shown by writing explicitly the flavour dependence of the corresponding effective Hamiltonian (in the following spin-spatial dependence is omitted). The strong interactions are described as in (9.11). The parametrization

$$\mathcal{H}^W_{\Lambda N\rho^-} = \alpha\overline{p}\Lambda\rho^+ + \beta\overline{n}\Lambda\rho^0, \tag{9.33}$$

is used for the weak part. The flavour dependence of the effective PV $\Delta S = 1$ potential is

$$\begin{aligned} V &= \sqrt{2}g^V\alpha(p\Lambda)_1(\overline{n}p)_2 + g^V\beta(\overline{n}\Lambda)_1(\overline{p}p - \overline{n}n)_2 \\ &= \sqrt{2}g^V\alpha \not{c} + g^V\beta(\not{a} - \not{b}) \\ &\quad (g^V \equiv g^V_{N\Lambda\rho}) \end{aligned} \tag{9.34}$$

[11]This is shown in Appendix E.

Here we have introduced the combinations

$$\begin{aligned} a\!\!\!/ &= (\overline{n}\Lambda)(\overline{p}p)\,, & a\!\!\!/ &= \tfrac{1}{2}\beta_1 - \tfrac{1}{6}\beta_\tau + \tfrac{1}{\sqrt{6}}\beta_T\,, \\ b\!\!\!/ &= (\overline{n}\Lambda)(\overline{n}n)\,, & b\!\!\!/ &= \tfrac{1}{2}\beta_1 + \tfrac{1}{6}\beta_\tau - \tfrac{1}{\sqrt{6}}\beta_T\,, \\ c\!\!\!/ &= (\overline{p}\Lambda)(\overline{n}p)\,, & c\!\!\!/ &= \phantom{\tfrac{1}{2}\beta_1 +{}} \tfrac{1}{3}\beta_\tau + \tfrac{1}{\sqrt{6}}\beta_T\,, \end{aligned} \tag{9.35}$$

which were used in Ref. [1]. With the equalities (9.35) one can separate $\Delta I = 1/2$ (β_τ) piece in the effective potential V from the $\Delta I = 3/2$ (β_T) contribution, i.e.

$$V = \left(\sqrt{2}g_V\frac{1}{3}\alpha\right)\beta_\tau + g^V\left(\sqrt{\frac{2}{6}}\alpha + \frac{2}{\sqrt{6}}\beta\right)\beta_T\,. \tag{9.36a}$$

Comparison with (9.11) connects α and β with a_V and a_T

$$\alpha = -\sqrt{\frac{2}{3}}a_V; \qquad \beta = -\sqrt{\frac{1}{3}}a_T\,. \tag{9.36b}$$

The $\Delta I = 1/2$ rule follows from (9.36) only if α and β satisfy the sum rule

$$\begin{gathered} \sqrt{\tfrac{2}{6}}\alpha + \tfrac{2}{\sqrt{6}}\beta = 0\,, \\ a_V = -a_T\,. \end{gathered} \tag{9.37}$$

This agrees with the earlier conclusion (9.4b). However with the realistic values (9.12d) and (9.15) the potential (9.36) must contain both $\Delta I = 1/2$ and $\Delta I = 3/2$ terms. The parameter in front of the $\Delta I = 1/2$ piece (β_τ) is

$$\tilde{\epsilon}_\rho = -\frac{2}{3\sqrt{2}}\,a_V\,. \tag{9.38}$$

This should enter as a "factorizable" contribution in the formulae of Ref. [14,15]. One can easily check that the effective Hamiltonian of Ref. [5] (with all spin-spatial dependence supressed) leads to

$$\begin{gathered} \tilde{\epsilon}_\rho\overline{N}\boldsymbol{\tau}\cdot\boldsymbol{\rho}^{[\mathrm{CT}]}\begin{pmatrix} 0 \\ \Lambda \end{pmatrix} g_V\overline{N}\boldsymbol{\tau}\cdot\boldsymbol{\rho}^{[\mathrm{CT}]}N \\ \to -\tilde{\epsilon}_\rho g^V(\overline{n}\Lambda)_1(\overline{p}p-\overline{n}n)_2 + 2\tilde{\epsilon}_\rho(\overline{p}\Lambda)(\overline{n}p)_2 = \tilde{\epsilon}_\rho g^V\beta_\tau. \end{gathered} \tag{9.39}$$

Here the exponent [CT] denotes contracted fields. With $a_V = 3a_T$ (9.15) one finds an alternative to (9.21)

$$\epsilon_\rho = \frac{2}{3}\Lambda^0_- - \frac{1}{\sqrt{3}}\Sigma^+_0 - \frac{2}{3}a_T. \tag{9.40}$$

However the formula (9.40) was obtained by simply throwing away the $\Delta I = 3/2$ part (β_T) of the potential (9.36). The relative strength of the β_τ and β_T pieces are proportional to

$$\frac{a_V}{a_V + a_T} \cdot \frac{1}{3}\sqrt{\frac{2}{3}}\frac{\sqrt{6 \cdot 3}}{4} = \frac{3a_T}{4a_T}\frac{1}{\sqrt{12}} = \frac{\sqrt{3}}{8} = 0.217\,. \tag{9.41}$$

It does not seem justified to omit the β_T piece. It might be argued that its contribution to the process $\Lambda + N \to N + N$ is to some extent included in (9.21). It is difficult to support that by some explicit mathematical arguments.

10. *Weak PV NNK coupling and sum rules*

The weak nucleon-nucleon pion parity-violating (PV) coupling involves contributions

$$p \to nK^+ \to A(p^+_+)\,, \qquad n \to nK^0 \to A(n^0_0)\,, \qquad p \to pK^0 \to A(p^+_0)\,. \tag{10.1}$$

They can be described by an effective weak vertex which, in the notation of Ref. [1] transforms as

$$(H_W)^2_3\,. \tag{10.2a}$$

One has for example

$$i_1 = \overline{b}^C_3 b^A_C \phi^2_A\,. \tag{10.2b}$$

The complete effective interaction is [1,4,6,8]:

$$\begin{gathered}\mathcal{A}_W(eff) = \delta_1(i_1 - i_1^\dagger + i_7^\dagger - i_7) + \delta_2(i_2 - i_2^\dagger + i_8^\dagger - i_8) + \delta_3(-i_3 + i_3^\dagger - i_4^\dagger + i_4)\,, \\ i_2 = \overline{b}^A_C b^C_3 \phi^2_A\,, \quad i_3 = \overline{b}^A_D b^2_3 \phi^D_A\,, \quad i_4 = \overline{b}^2_3 b^A_D \phi^D_A\,, \quad i_7 = \overline{b}^C_A b^2_C \phi^A_3\,, \quad i_8 = \overline{b}^2_C b^C_A \phi^A_3\,.\end{gathered} \tag{10.2c}$$

The repeated indices are summed over. Only the SU(3) transformation properties are indicated in (10.2). The spatial factor would be

$$\overline{u}u\phi\,, \tag{10.2d}$$

where u corresponds to the hyperon and ϕ stands for the meson field. The physical content of SU(3) terms as b^i_j is given by the attached matrices - see Ref. [1], formula (5.17). One finds for example

$$\overline{b}^3_1 = \overline{p}\,, \qquad \phi^1_3 = K^+\,, \qquad b^3_3 = -\frac{2\Lambda^0}{\sqrt{6}}. \tag{10.2e}$$

The amplitudes (10.1) are connected via the SU(3) sum rules, with the hyperon nonleptonic decay amplitudes

$$\Lambda \to p\pi^- \quad \to A(\Lambda^0_-)\,, \qquad \Sigma \to p\pi^0 \quad \to A(\Sigma^0_+)\,, \qquad \Xi \to \Lambda\pi^- \quad \to A(\Xi^-_-)\,. \tag{10.3}$$

The particle content of various i_k's in (10.2) is shown in Table 10.1. One finds for example

$$\begin{aligned}(i_1)_{\pi^0} &= \overline{b}_3^C b_C^2 \phi_2^2 = (\overline{b}_3^1 b_1^2 + \overline{b}_3^2 b_2^2 + \overline{b}_3^3 b_3^2)\phi_2^2 \\ &= \left[\overline{\Xi}^- \Sigma^- + \overline{\Xi}^0 \left(-\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}}\right) + \left(-\frac{2\overline{\Lambda}^0}{\sqrt{6}}\right) n\right] \left(-\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}}\right),\end{aligned} \tag{10.4}$$

etc. Contributions from invariants (10.2) are listed in Table 10.1.

TABLE 10.1. Contributions to the decay amplitudes.

Amplitude	δ_1	δ_2	δ_3
$A(\Xi^-_-)$	$-\frac{1}{\sqrt{6}}$	$\frac{2}{\sqrt{6}}$	0
$A(\Lambda^0_-)$	$\frac{2}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$	0
$A(\Sigma^+_0)$	0	$\frac{1}{\sqrt{2}}$	0
$A(\Sigma^+_+)$	0	0	1
$A(n^0_0)$	1	-1	1
$A(p^+_0)$	0	-1	0
$A(p^+_+)$	1	0	1

It is easy to reproduce the Lee-Sugawara sum-rule [4,6,8,9]

$$2A(\Xi^-_-)+A(\Lambda^0_-)-\sqrt{3}A(\Sigma^0_0) = -\frac{2}{\sqrt{6}}\delta_1+\frac{4}{\sqrt{6}}\delta_2+\frac{2}{\sqrt{6}}\delta_1-\frac{1}{\sqrt{6}}\delta_2-\sqrt{\frac{3}{2}}\delta_2 = 0. \tag{10.5}$$

One also easily finds

$$A(n^0_0) = \sqrt{\frac{3}{2}}A(\Lambda^0_-) - \frac{1}{\sqrt{3}}A(\Sigma^+_0) + A(\Sigma^+_+), \tag{10.6}$$

and

$$A(p^+_0) = \sqrt{\frac{3}{2}}A(\Lambda^0_-) + \frac{1}{\sqrt{3}}A(\Sigma^+_0) + A(\Sigma^+_+). \tag{10.7}$$

If $A(\Sigma^+_+) = 0$ as it follows in the current algebra (CA) approximation, one obtains the sum rules given in Ref. [5].

It is well known [1] that the current algebra based approximation (including PCAC) produces more constrained connections than the SU(3) symmetry requirements alone. Using CA and PCAC [4,8,9] one obtains for the $A(\Sigma^+_+)$ amplitude

$$\mathrm{i}A(\Sigma^+_+)\overline{u}_n u_\Sigma = \langle \pi^+ n|\, H_W^{PV}\, |\Sigma^+\rangle = (-\mathrm{i})\frac{1}{f_\pi}\left[\langle p|\, H_W^{PC}\, |\Sigma^+\rangle + \sqrt{2}\langle n|\, H_W^{PC}\, |\Sigma^0\rangle\right]. \tag{10.8a}$$

Here PC (PV) means parity-conserving (violating). In deriving (10.8a) one had to calculate the action of a SU(3) generator

$$F_i = \int \mathrm{d}^3x\, V_0^i(x) \tag{10.9a}$$

on a baryon state. Here V_0 is the zero component of the quark vector current

$$V_0^i = \overline{q}\frac{\lambda^i}{2}\gamma_0 q\,. \tag{10.9b}$$

For that purpose one can use formulae from Ref. [9], as for example

$$\begin{aligned} \pi^0|n\rangle &\sim F_3\tfrac{1}{\sqrt{2}}(B_6 + \mathrm{i}B_7) = \tfrac{1}{\sqrt{2}}\mathrm{i}(f_{36k}B_k + if_{37k}B_k) \\ &= -\tfrac{1}{2}\tfrac{1}{\sqrt{2}}(B_6 + \mathrm{i}B_7) = -\tfrac{1}{2}|n\rangle\,, \end{aligned} \tag{10.9c}$$

and

$$\pi^+|n\rangle \sim (F_1 + iF_2)\frac{1}{\sqrt{2}}(B_6 + iB_7) = \frac{1}{\sqrt{2}}(B_4 + iB_5) = |p\rangle\,. \tag{10.9d}$$

If H_W^{PC} transforms as a $\mathrm{SU(3)}_{\mathrm{flavour}}$ octet, i.e. if one has octet dominance, than, on the basis of the Wigner-Eckart theorem [22,23], one finds

$$\langle p|\, H_W^{(8)\,PC}\, |\Sigma^+\rangle = -\sqrt{2}\langle n|\, H_W^{(8)\,PC}\, |\Sigma^0\rangle\,, \qquad A(\Sigma^+_+) = 0\,. \tag{10.8b}$$

Further applications of CA and PCAC [4,8,9] give

$$\begin{aligned} &\mathrm{i}A(\Lambda^0_-)\overline{u}_p u_\Lambda = \langle \pi^+ p|\, H_W^{(8)\,PV}\, |\Lambda^0\rangle = -\mathrm{i}\tfrac{1}{f_\pi}\langle n|\, H_W^{(8)\,PC}\, |\Lambda^0\rangle\,, \\ &\mathrm{i}A(\Sigma^+_0)\overline{u}_p u_\Sigma = \langle \pi^0 p|\, H_W^{(8)\,PV}\, |\Sigma^+\rangle = -\mathrm{i}\tfrac{1}{f_\pi}\left(-\tfrac{1}{\sqrt{2}}\right)\langle p|\, H_W^{(8)\,PC}\, |\Sigma^+\rangle\,, \\ &\mathrm{i}A(n^0_0)\overline{u}_p u_n = \langle \overline{K}^0 n|\, H_W^{(8)\,PV} |n\rangle = -\mathrm{i}\tfrac{1}{f_K}\left[\sqrt{\tfrac{3}{2}}\langle n|\, H_W^{(8)\,PC}|\Lambda^0\rangle - \tfrac{1}{\sqrt{2}}\langle n|\, H_W^{(8)\,PC}|\Sigma^0\rangle\right] \\ &\mathrm{i}A(p^+_+)\overline{u}_n u_p = \langle \overline{K}^+ n|\, H_W^{(8)\,PV}|p\rangle = -\mathrm{i}\tfrac{1}{f_K}\left[\sqrt{\tfrac{3}{2}}\langle n|\, H_W^{(8)\,PC}|\Lambda^0\rangle + \tfrac{1}{\sqrt{2}}\langle n|\, H_W^{(8)\,PC}|\Sigma^0\rangle\right] \end{aligned} \tag{10.10}$$

Using the relation (10.8b) one obtains the sum rules (10.6) and (10.7) with $A(\Sigma^+_+) = 0$. The sum rules

$$\begin{aligned} A(n^0_0) &= \left[\sqrt{\frac{3}{2}}A(\Lambda^0_-) - \frac{1}{\sqrt{2}}A(\Sigma^+_0)\right]\frac{f_\pi}{f_K}\,, \\ A(p^+_0) &= \left[\sqrt{\frac{3}{2}}A(\Lambda^0_-) + \frac{1}{\sqrt{2}}A(\Sigma^+_0)\right]\frac{f_\pi}{f_K}\,, \end{aligned} \qquad (10.11)$$

follow from the octet dominance combined with the soft pion approximation.

If one give up the octet dominance what formally means that the index 8 in $H^{(8)}_W$ is omitted in formulae (10.10), one finds from (10.8a) and (10.10)

$$\frac{1}{f_\pi}\langle n|\, H_W\, |\Sigma^0\rangle = -\left[A(\Sigma^+_0) + \frac{1}{\sqrt{2}}A(\Sigma^+_+)\right]. \qquad (10.12a)$$

The third version of the sum rules for the $A(n^0_0)$ and $A(p^+_+)$ amplitudes is

$$\begin{aligned} A(n^0_0) &= \left[\sqrt{\frac{3}{2}}A(\Lambda^0_-) - \frac{1}{\sqrt{2}}A(\Sigma^+_0) - \frac{1}{2}A(\Sigma^+_+)\right]\frac{f_\pi}{f_K}\,, \\ A(p^+_+) &= \left[\sqrt{\frac{3}{2}}A(\Lambda^0_-) + \frac{1}{\sqrt{2}}A(\Sigma^+_0) + \frac{1}{2}A(\Sigma^+_+)\right]\frac{f_\pi}{f_K}. \end{aligned} \qquad (10.12b)$$

It was obtained by the soft pion theorem [1] using PCAC and CA, without the octet dominance assumption.

Various combinations of Dirac spinors $\overline{u}_{B_f}u_{B_i}$ appearing in the above formulae are ignored, or better to say replaced by a generic term $\overline{u}_B u_B$, i.e.

$$\overline{u}_n u_\Sigma,\quad \overline{u}_p u_\Lambda,\quad \overline{u}_p u_\Sigma,\quad \overline{u}_n u_n,\quad \overline{u}_n u_p,\ \rightarrow\ \overline{u}_B u_B. \qquad (10.13)$$

This is equivalent to assumption that all baryons are mass-degenerate octet members. Thus the expression (10.12) is in a way also octet dependent.

In the nonrelativistic approximation (NRA) where Dirac spinors are replaced by Pauli spinors, this fine distinction does not matter. In NRA all bilinears (10.13) are replaced by a simple product

$$\overline{u}_B u_B \stackrel{\text{NRA}}{\longrightarrow} \chi^\dagger\chi. \qquad (10.14)$$

An additional difference between (10.6), (10.7), (10.11) and (10.12) is in the factor $f_\pi/f_K = 0.83$. The appearence of that factor in (10.11) and (10.12) openly illustrates as already mentioned [1], that the current algebra based approximation differs form that which is based on the SU(3) symmetry with octet dominance. This factor is not included in SU(6)$_W$, and thus SU(3) based sum rules used in Ref. [5]. They have (10.6) and (10.7) with $A(\Sigma^+_+) \equiv 0$.

11. *Nucleon-nucleon-kaon vertices*

The calculation of the weak NNK coupling has been carried out in Ref. [1] where it has been based on the contribution

$$\begin{aligned} A &= A_{\rm cc} + A_{\rm sep} \,, \\ B &= B_8 + B_{\rm sep} \,. \end{aligned} \tag{11.1a}$$

That scheme was analogous to the theoretical description of hyperon nonleptonic decays used by Ref [9]. An alternative theoretical approach, in which decouplet poles were used, has been also developed [30]. Recently it has been adopted for the calculation of the NNK vertices [28]. It contains contributions

$$\begin{aligned} A &= A_{\rm cc} + A_{10} + A_{\Lambda'} \\ B &= B_8 + B_{10} + B_{\Lambda'} \end{aligned} \tag{11.1b}$$

The current algebra contribution $A_{\rm cc}$ and the octet pole contributions B_8 are given by Ref. [1]. Very general expression for decouplet poles, as for instance A_{10} can be found in Refs. [28,30]. By SU(3) relation (adaptation) one finds

$$A_{10}(p_+^+) = -\frac{p-n}{18}\, h_2 g_2 \,\frac{\Sigma^{*\,0}+n}{(\Sigma^{*\,0})^2}\,, \qquad B_{10}(p_0^+) = -\frac{p+n}{18}\, h_3 g_2 \,\frac{\Sigma^{*\,0}-n}{(\Sigma^{*\,0})^2}\,. \tag{11.2a}$$

Here

$$h_2 = -0.2\cdot 10^{-6}, \qquad h_3 = -0.4\cdot 10^{-6}, \qquad g_2 = 15.7\,, \tag{11.2b}$$

while p, $\Sigma^{*\,0}$ etc. are baryon masses.

The $A_{\Lambda'}$ and $B_{\Lambda'}$ amplitudes are small contributions from the $\Lambda(1405) = \Lambda'$ resonance [11,30]. A characteristic Λ' pole term for the p_+^+ amplitude is shown in Fig. 10.1. Proceeding in the standard way [28] one finds

$$A_{\Lambda'}(p_+^+) = \frac{g_{\Lambda' pK}\, a_{n\Lambda'}(p-n)}{(\Lambda-n)(\Lambda'-\Sigma)}, \qquad A_{\Lambda'}(p_0^+) = 0, \qquad A_{\Lambda'}(n_0^0) = 0\,. \tag{11.3}$$

The PC vertices are determined by

$$B_{\Lambda'}(p_+^+) = \frac{g_{\Lambda' pK}\, a_{n\Lambda'}(p+n)}{(\Lambda+n)(\Lambda'-\Sigma)}\,. \tag{11.4}$$

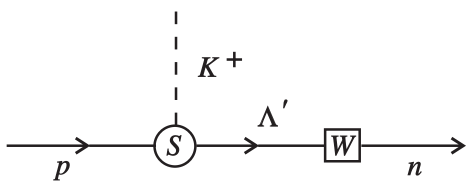

Fig. 11.1 - Λ' pole term contributions to the transition $p \to K^+ n$.

The values of NNK amplitudes obtained using two different theoretical approaches a) Ref. [30] and b) Ref. [1] are summarized in Table 11.1. These amplitudes, together with the amplitudes corresponding to π [1] and η (see Appendix F) exchanges, determine the weak potential. Its form is given by formulae (8.1)-(8.11) in Ref. [1]. The strengths of the weak vertices are summarized in Table 11.2 which is analogous to the Table 9.1 in Ref. [1]. The numerical values appearing in column a) corespond to the case (11.1a). They are identical with the numbers given in Ref. [1].[12]

The column b) corresponds to the combination (11.1b).

TABLE 11.1 - Nucleon-kaon amplitudes are multiplied by 10^6.

Amplitude	(a) [30]	(b) [1]
$A_K(p_0^+)$	0.525	0.408
$B_K(p_0^+)$	-2.35	-2.187
$A_K(p_+^+)$	0.250	0.281
$B_K(p_+^+)$	3.088	4.238
$A_K(n_0^0)$	0.625	0.625
$B_K(n_0^0)$	1.803	1.799

TABLE 11.2. Weak vertices and their connection with the weak nonleptonic amplitudes.

Weak vertices	Analytic expression	Numerical value a)	b)	ΔI
a	$\frac{\sqrt{2}}{3}A(\Lambda_-^0) - \frac{1}{3}A(\Lambda_0^0)$	2.3187	2.3187	1/2
b	$\sqrt{\frac{3}{2}}\left[\frac{\sqrt{2}}{3}A(\Lambda_-^0) + \frac{2}{3}A(\Lambda_0^0)\right]$	-0.051	-0.051	3/2
$\tilde{a}$	$\frac{\sqrt{2}}{3}B(\Lambda_-^0) - \frac{1}{3}B(\Lambda_0^0)$	15.862	15.862	1/2
$\tilde{b}$	$\sqrt{\frac{3}{2}}\left[\frac{\sqrt{2}}{3}B(\Lambda_-^0) + \frac{2}{3}B(\Lambda_0^0)\right]$	0.032	0.032	3/2
c	$\frac{1}{3}\left[A_K(n_0^0) - A_K(p_0^+) + 2A_K(p_+^+)\right]$	0.260	0.200	1/2
d	$\frac{1}{3}\left[A_K(n_0^0) + 2A_K(p_0^+) - A_K(p_+^+)\right]$	0.404	0.475	1/2
e	$\frac{1}{3}\left[-A_K(n_0^0) + A_K(p_0^+) + A_K(p_+^+)\right]$	0.021	0.050	3/2
$\tilde{c}$	$\frac{1}{3}\left[B_K(n_0^0) - B_K(p_0^+) + 2B_K(p_+^+)\right]$	4.154	3.443	1/2
$\tilde{d}$	$\frac{1}{3}\left[B_K(n_0^0) + 2B_K(p_0^+) - B_K(p_+^+)\right]$	-2.271	-1.995	1/2
$\tilde{e}$	$\frac{1}{3}\left[-B_K(n_0^0) + B_K(p_0^+) + B_K(p_+^+)\right]$	0.084	-0.355	3/2
f	$A_{\eta_1}(\Lambda_{\eta_1}^0)$	0.06	0	1/2
$\tilde{f}$	$B_{\eta_1}(\Lambda_{\eta_1}^0)$	27.53	28.16	1/2
g	$A_{\eta_8}(\Lambda_{\eta_8}^0)$	-5.19	-5.63	1/2
$\tilde{g}$	$B_{\eta_8}(\Lambda_{\eta_8}^0)$	22.97	31.10	1/2

[12] Table 9.1 contains some missprints, which are corrected here!

Table 11.2 contains also, for the sake of completeness the η exchange amplitudes. Their derivation is sketched in Appendix F.

One can see that the $\Delta I = 1/2$ pieces of the weak potential in both cases are reasonably close, the discrepancy being within 14% to 30%. However the $\Delta I = 3/2$ pieces differ significantly. For the combination e, the difference is about 130%. The combination $\tilde{e}$ shows the opposite signs with order of magnitude difference in strengths.

While the theoretical predictions for the $\Delta I = 1/2$ potentials might be reliable to within 30%, the $\Delta I = 3/2$ pieces in the potential are rather poorly determined by the present methods[13].

12. Outlook

This review contains two results which will be useful in future applications. Firstly it is shown how the effective weak potential due to the vector meson and axial vector meson exchanges combines with the effective weak potential produced by the exchanges of pseudoscalar mesons. Working within the described theoretical framework, i.e. CA, PCAC, SU(3), SU(6)$_W$, SEP approximations, pole terms etc. one can fix all relative signs within potential, as well as the relative signs with respect to strong interactions and semileptonic processes. That will serve as a good foundation for the derivation of the additional weak potential pieces.

Secondly a novel derivation of NNK and $N\Lambda\eta$ weak vertices is discussed in detail. The changes in the resulting affective weak potentials are presented. When all theoretical derivations are compared, one ends with a theoretical uncertainty which is less then 30% [14] for $\Delta I = 1/2$ potential pieces. Unfortunately the predicted strength of $\Delta I = 3/2$ potential pieces depends very much on the theoretical methods. In the initial analysis of experimental data one cannot rely upon the $\Delta I = 3/2$ potential pieces produced by the pseudoscalar meson exchanges.

Appendix A. Effective weak Hamiltonian and IVB (W)

In the current-current (charged currents are here considered) form the effective weak Hamiltonian $\mathcal{H}_W$ is given by [4]

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} \mathcal{J}_\mu^\dagger(x) \mathcal{J}^\mu(x) \,. \tag{A.1}$$

Fundamentally the $\mathcal{H}_W$ is produced by the current-IVB (W) coupling

$$H(IVB) = g \mathcal{J}_\mu W^{\mu\,\dagger}. \tag{A.2}$$

[13] The consideration of other A, B values (see Table 5 in Ref. [30]) would not change that conclusions.

[14] Sometime less then 14%.

With IVB propagator [2]

$$(-\mathrm{i})\frac{g^{\mu\nu}}{k^2 - M_W^2} + \dots \tag{A.3}$$

one finds in the $k \ll M_W^2$ limit (All integrations $\int \mathrm{d}x$ are omitted in (A.1), (A.2) and in the following)

$$\text{(second order)} \quad (-\mathrm{i})^2 H^2(IVB) = (-\mathrm{i})H_W \quad \text{(first order)} \tag{A.4}$$

$$(-1)H^2(IVB) = \mathrm{i}g^2 \frac{g^{\mu\nu}}{-M_W^2} \mathcal{J}_\mu^\dagger(x)\mathcal{J}_\nu(x) = (-\mathrm{i})\frac{g^2}{M_W^2}\mathcal{J}_\mu^\dagger(x)J^\mu(x).$$

Thus

$$\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{M_W^2} \tag{A.5}$$

as it should be (CKM angles can be assumed, but they are not shown explicitly). The main purpose of this appendix was to test Eq. (A.3). Obviously, (-1) in front of (A.3) is essential.

Appendix B. Pion decay and induced pseudoscalar in semileptonic weak decays

PCAC relation [4] is

$$\begin{aligned} &\partial^\mu A_\mu^i(x) = \frac{f_\pi}{\sqrt{2}} m_\pi^2 \pi^i(x) \qquad (i = 1, 2, 3 \quad \text{(isospin)}\,, \\ &\phi_{\pi^-} = \frac{1}{\sqrt{2}}(\pi^1 + \mathrm{i}\pi^2)\,. \end{aligned} \tag{B.1}$$

The pion decay constant f_π is [4]

$$\langle 0|\, A_\mu^1(x) + \mathrm{i}A_\mu^2(x)\, |\pi^-(k)\rangle = \mathrm{i}f_\pi k_\mu \mathrm{e}^{-\mathrm{i}kx} \qquad (kx = k_0x_0 - \boldsymbol{k}\cdot\boldsymbol{x})\,. \tag{B.2}$$

A general form for the axial vector current matrix element is [1, p.131]

$$\langle p|\, A_\mu^1 + \mathrm{i}A_\mu^2\, |n\rangle = \overline{u}_p(p)[g_A\gamma_\mu\gamma_5 + g_P k_\mu\gamma_5 + \dots]u_n(n), \quad (k_\mu = (p-n)_\mu)\,. \tag{B.3a}$$

Here

$$A_\mu^i(x) = \overline{q}(x)\gamma_\mu\gamma_5 \frac{1}{2}\tau^i q(x). \tag{B.3b}$$

The g_P formfactor is produced by pion exchange diagram shown in Fig. B 1.

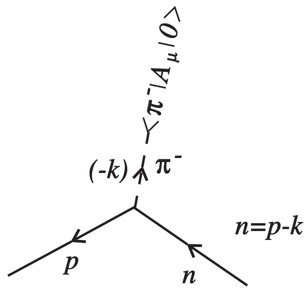

Fig. B 1. Pion exchange contribution to the matrix element (B.3).

The strong interaction [2] of pions is

$$H_S = \mathrm{i}g_\pi \overline{\psi}_N \tau^i \gamma_5 \psi_N \pi^i = \mathrm{i}g_\pi \sqrt{2}\overline{\psi}_p \gamma_5 \psi_n \pi^{(+)} + \dots \tag{B.4}$$

The diagram shown in Fig. B 1 which is the second order perturbation gives

$$\begin{aligned} (IP)_u &= (-\mathrm{i})^2 \overline{u}_p \gamma_5 u_n \frac{\mathrm{i}}{k^2 - m_\pi^2} \mathrm{i}\sqrt{2} g_\pi \mathrm{i} f_\pi k_\mu \\ &= (-\mathrm{i})(-1)\overline{u}_p \gamma_5 u_n \frac{\sqrt{2} g_\pi f_\pi k_\mu}{k^2 - m_\pi^2} \,. \end{aligned} \tag{B.5}$$

This has to be combined with the first order contribution

$$(-\mathrm{i})\overline{u}_p \left[\gamma_\mu \gamma_5 g_A - \frac{\sqrt{2} g_\pi f_\pi k_\mu}{k^2 - m_\pi^2} \gamma_5 \right] u_n g_P \simeq \frac{(-1)\sqrt{2} g_\pi f_\pi}{k^2 - m_\pi^2} \,. \tag{B.6}$$

The PCAC condition can be written also [4] as

$$k_\mu \langle p | \, (A_\mu^1 + \mathrm{i} A_\mu^2) \, | n \rangle \Big|_{m_\pi^2 = 0} = 0 \,. \tag{B.7}$$

Remark We always assume $A_\mu^i = \overline{\psi}\gamma_\mu\gamma_5(\lambda^i/2)\psi$ and net (-) sign as in (4.65b) of Ref. [1]. Thus our $\mathcal{J}_\mu^\pm = V_\mu^1 \pm \mathrm{i}V_\mu^2 - (A_\mu^1 \pm \mathrm{i}A_\mu^2)$. Dirac equation is $(\not{p} - m)u = 0$ $\overline{u}(\not{p} - m) = 0$. Thus $(p-n)^\mu \overline{u}(p)\gamma_\mu\gamma_5 u(n) = (m_p + m_n)\overline{u}(p)\gamma_5 u(n) = 2m_N \overline{u}(p)\gamma_5 u(n)$, and $k = p - n$. Also $\langle \pi^- | \, A_\mu \, |0\rangle = \langle 0| \, A_\mu^\dagger \, |\pi^-\rangle^\dagger = (\mathrm{i}q_\mu f_\pi)^\dagger = -\mathrm{i}q_\mu f_\pi = \mathrm{i}k_\mu f_\pi$, since $(q = -k)$.

The expression (B.3a) gives

$$k_\mu \langle p | \, (A_\mu^1 + \mathrm{i}A_\mu^2) \, | n \rangle = (2m_N g_A + g_P k^2)\overline{u}_p \gamma_5 u_n \,. \tag{B.8}$$

The condition (B.7) holds if

$$g_P = -\frac{2m_N g_A}{q^2} \,. \tag{B.9}$$

From calculation (B.6), one finds with $m_\pi^2 = 0$

$$g_P \to (-\sqrt{2})\frac{g_\pi f_\pi}{q^2}\,. \tag{B.10}$$

The combination of (B.9) and (B.10) produces the Goldberger-Treiman (GT) relation [4]

$$\begin{aligned} 2m_N g_A &= \sqrt{2} f_\pi g_\pi\,, \\ m_N g_A &= \frac{f_\pi}{\sqrt{2}} g_\pi\,. \end{aligned} \tag{B.11}$$

All these confirms the phase “i” appearing in (B.2).

Appendix C. Meson states in SU(6)$_W$ and CP transformation

Meson states ϕ^a_b (Sect. 4) behave as $W-$triplet (vector mesons) and $W-$singlet (pseudoscalar mesons) and also as $B-$spin scalars[15] [15,16,19,20]. The space-time property of a pseudoscalar meson state (SU(3) spins are omitted) is the same as the behaviour of fermion (ψ) density

$$\begin{aligned} \psi^\dagger 1 \psi &= \overline{\psi}\gamma^0\psi\,, \\ \overline{\psi}\gamma^0\psi &\xrightarrow{\mathrm{CP}} (-)\overline{\psi}\gamma^0\psi\,. \end{aligned} \tag{C.1}$$

The result (C.1) is derived in Sect. 3. The CP behaviour of the vector meson states follows the behaviour of densities

$$\psi^\dagger\gamma^0\sigma_1\psi, \qquad \psi^\dagger\gamma^0\sigma_2\psi, \qquad \psi^\dagger\sigma_1\psi\,. \tag{C.2}$$

One can write

$$\psi^\dagger\gamma_0\sigma_i\psi = \overline{\psi}\sigma_i\psi = \overline{\psi}\gamma_0\gamma_i\gamma_5\psi\,, \qquad (i = 1, 2). \tag{C.3}$$

Using the results given in Sects. 3 and 4, one obtains

$$\overline{\psi}\gamma_0\gamma_i\gamma_5\psi \xrightarrow{\mathrm{P}} +\overline{\psi}\gamma_0\gamma_i\gamma_5\psi\,. \tag{C.4}$$

The C transformation means

$$\overline{\psi}\gamma_0\gamma_i\gamma_5\psi \xrightarrow{\mathrm{C}} -\psi^T C^{-1}\gamma_0\gamma_i\gamma_5 C\overline{\psi}^T = -\overline{\psi}\gamma_0\gamma_i\gamma_5\psi\,. \tag{C.5}$$

[15] $B-$spin operators are $B_1 = i\sigma_3\gamma_5/2$, $B_2 = \sigma_3\gamma_0\gamma_5/2$, $B_3 = \gamma_0/2$.

Finally

$$\overline{\psi}\gamma_0\gamma_i\gamma_5\psi \xrightarrow{\mathrm{CP}} -\overline{\psi}\gamma_0\gamma_i\gamma_5\psi\,. \tag{C.6}$$

The formulae (C.1) and (C.6) confirm the meson field behaviour (4.6).

A different behaviour of the third component in (C.2) is compensated by the properties of the corresponding $\overline{B}B$ density. The expression (4.7) is not changed.

Appendix D. Parameters a_T and a_V for $\Delta S = 0$ Hamiltonian

The SU(6)$_W$ parameters a_T and a_V were calculated via the factorization approximation by Ref. [19]. Their relevant formulae, found for the strangeness conserving ($\Delta S = 0$) Hamiltonian are

$$\begin{aligned} \langle\rho^- p|\,H_W(\Delta S=0)\,|n\rangle &= \frac{G}{\sqrt{2}}\cos^2\theta_C\langle\rho^-|\,\overline{d}\gamma_\mu u\,|0\rangle\langle p|\,\overline{u}\gamma^\mu\gamma_5 d\,|n\rangle \\ &= -\frac{G}{\sqrt{2}}\cos^2\theta_C\frac{1}{2}\langle\rho^-|\,V^{-1}_{\mu\,1}\,|0\rangle\langle p|\,A^{\mu\,1}_1\,|n\rangle \\ &\sim \frac{10}{9}a_V\,\mathrm{ctg}\,\theta_C \end{aligned} \tag{D.1}$$

and

$$\begin{aligned} \langle\rho^0 p|\,H_W(\Delta S=0)\,|p\rangle &= -\frac{1}{6}\frac{G}{\sqrt{2}}\cos^2\theta_C\frac{1}{2}\langle\rho^0|\,V^0_{\mu\,1}\,|0\rangle\langle p|\,A^{\mu\,0}_1\,|p\rangle \\ &\sim -\frac{10}{9\sqrt{2}}a_T\,\mathrm{ctg}\,\theta_C\,. \end{aligned} \tag{D.2}$$

Here we have introduced isospin tensors $V_T^{T_Z}$

$$V_1^{-1} = \sqrt{2}\,\overline{d}u = \sqrt{2}\,\overline{q}\tau_+ q\,, \qquad V_1^0 = \overline{q}\tau_3 q\,, \qquad V_1^1 = -\sqrt{2}\,\overline{u}d\,, \tag{D.3}$$

etc. In Ref. [19] one can find

$$V^{1-i2} = \frac{1}{\sqrt{2}}V_1^{-1}\,, \qquad V^3 = \frac{1}{2}V_1^0. \tag{D.4}$$

From (D.1) and (D.2) one obtains

$$\begin{aligned} a_V &= -\frac{9}{20}\frac{1}{\sqrt{3}}G\,sc\,\alpha\beta\,, \\ a_T &= -\frac{3}{20}\frac{1}{\sqrt{3}}G\,sc\,\alpha\beta\,, \\ a_T &= \frac{a_V}{3}\,. \end{aligned} \tag{D.5}$$

Here we have used $s \equiv \sin\theta_C$ and $c \equiv \cos\theta_C$, and

$$\begin{aligned}
\langle \rho^- | \, V_1^{-1} \, |0\rangle \quad &= C^{1\,-1}_{1\,-1\,0\,0} \; \alpha = \alpha \, , \\
\langle \rho^0 | \, V_1^{0} \, |0\rangle \quad &= C^{1\,0}_{1\,0\,0\,0} \; \alpha = \alpha \, , \\
\langle p | \, A_1^{1} \, |n\rangle \quad &= C^{1/2\,1/2}_{1\,1\,1/2\,-1/2} \; \beta = \sqrt{\tfrac{2}{3}}\beta \, , \\
\langle p | \, A_1^{1} \, |n\rangle \quad &= C^{1/2\,1/2}_{1\,0\,1/2\,1/2} \; \beta = -\sqrt{\tfrac{1}{3}}\beta \, .
\end{aligned} \tag{D.6}$$

One can easily sketch the genesis of the product of matrix element which appears in (D.2). That can serve as a model for the $\Delta S = 1$ case.

Using FR one can write in SEP approximation

$$\begin{aligned}
&\langle \rho^0 p | \, \overline{d}_i \gamma_\mu (1-\gamma_5) d_j \overline{u}_j \gamma^\mu (1-\gamma_5) u_i \, |p\rangle \quad \rightarrow \quad L = \\
&-\tfrac{1}{3} \left[\langle \rho^0 | \, \overline{d}_i \gamma_\mu d_i \, |0\rangle \langle p | \, \overline{u}_j \gamma^\mu \gamma_5 u_j \, |p\rangle \; + \; \langle p | \, \overline{d}_i \gamma^\mu \gamma_5 d_i \, |p\rangle \langle \rho^0 | \, \overline{u}_j \gamma_\mu u_j \, |0\rangle \right] .
\end{aligned} \tag{D.7a}$$

Here ρ^0 can be emitted from either $\overline{u}u$ or $\overline{d}d$ combination. Either combination can contribute to the baryon matrix element. In $\Delta S = 1$ case (8.14) there is no such symmetry. With [19]

$$\langle \rho^0 | \, \overline{u} \gamma_\mu u \, |0\rangle = -\langle \rho^0 | \, \overline{d} \gamma_\mu d \, |0\rangle \tag{D.8a}$$

and with

$$V(A)^3 = V(A)^0_1 = \frac{1}{2}(\overline{u}u - \overline{d}d) \, , \tag{D.8b}$$

one can write (D.7a) as

$$L = -\frac{1}{6} \langle \rho^0 | \, V^0_{\mu\,1} \, |0\rangle \langle 0 | \, A_1^{\mu\,0} \, |p\rangle \, . \tag{D.7b}$$

It might be useful to list some isospin (tensorial) relations which were employed in Sect. 8. The isovector triplet containing u and d quarks can be written as

$$\begin{aligned}
\overline{u}d \quad &= -\tfrac{1}{\sqrt{2}} \overline{q} \tau_1^1 q = -T_1^1 \, , \\
\overline{u}u - \overline{d}d \quad &= \tfrac{1}{\sqrt{2}} \overline{q} \tau_1^3 q = T_1^0 \, , \\
\overline{d}u \quad &= \tfrac{1}{\sqrt{2}} \overline{q} \tau_1^{-1} q = T_1^{-1} \, .
\end{aligned} \tag{D.9}$$

One has to compare the matrix elements $\langle n | \, \overline{d}u \, |p\rangle$ and $\langle \overline{p} | \, \overline{u}u \, |p\rangle$. From the second matrix element we need only the isovector

$$\frac{1}{2}(\overline{u}u - \overline{d}d) \tag{D.10}$$

contribution which can be connected with ρ^0 meson. Thus

$$\begin{aligned}
\langle n|\,\overline{d}u\,|p\rangle &= \langle n|\,T_1^{-1}\,|p\rangle = C^{1/2\,-1/2}_{1\,-1\,1/2\,1/2}\alpha = -\sqrt{\tfrac{2}{3}}\alpha\,, \\
\langle p|\,\overline{u}u\,|p\rangle \to \tfrac{1}{2}\langle p|\,(\overline{u}u-\overline{d}d)\,|p\rangle &= \tfrac{1}{\sqrt{2}}\langle p|\,T_0^1\,|p\rangle = \tfrac{1}{\sqrt{2}}C^{1/2\,1/2}_{1\,0\,1/2\,1/2}\alpha \\
&= -\tfrac{1}{\sqrt{2}}\tfrac{1}{\sqrt{3}}\alpha = -\tfrac{1}{2}\sqrt{\tfrac{2}{3}}\alpha\,.
\end{aligned} \tag{D.11}$$

This is the origin of the factor 1/2 which appears on the RHS of (9.14b).

Appendix E. Weak PV vector boson K* exchange and $\Delta I = 1/2$ rule

From (9.1) one can deduce the following weak PV $\overline{N}NK^*$ coupling

$$C\overline{n}pK^{*+} + D\overline{p}pK^{*0} + \eta\overline{n}nK^{*0}. \tag{E.1}$$

Here only hadronic flavours are indicated while the spin-spatial parts are omitted. The strong $\overline{N}\Lambda K^*$ coupling has the flavour dependence

$$g^V(\overline{p}\Lambda\overline{K^{*+}} + \overline{n}\Lambda\overline{K^0}^*)\,. \tag{E.2}$$

When (E.1) and (E.2) are combined via K* exchanges one finds a weak PV potential with the following baryonic content

$$V = g^V C(\overline{p}\Lambda)(\overline{n}p) + g^V D(\overline{n}\Lambda)(\overline{p}p) + g^V\eta(\overline{n}\Lambda)(\overline{n}n)\,. \tag{E.3}$$

Ther isospin content of that V can be found by using formulae (F-13) and (F-14) of Ref. [1], i.e.

$$\begin{aligned}
V &= g^V C\left(\tfrac{1}{3}\beta_\tau + \tfrac{1}{\sqrt{6}}\beta_T\right) \\
&\quad + g^V D\left(\tfrac{1}{2}\beta_1 - \tfrac{1}{6}\beta_\tau + \tfrac{1}{\sqrt{6}}\beta_T\right) \\
&\quad + g^V\eta\left(\tfrac{1}{2}\beta_1 + \tfrac{1}{6}\beta_\tau - \tfrac{1}{\sqrt{6}}\beta_T\right).
\end{aligned} \tag{E.4}$$

Here β_T is the $\Delta I = 3/2$ contribution, which is eliminated if

$$C + D = \eta\,. \tag{E.5}$$

Appendix F. Baryon-baryon-η vertices in hypernuclear decays

Two members of the pseudoscalar meson U(3) nonet are possible mediators of the PV/PC hypernuclear potential in the OME (one meson exchange) picture of the hyperon-hyperon interaction. Their contributions follow the same pattern as was the case with π and K mesons: the PV A-amplitude gets its contribution for separable (SEP), current algebra (CA) and decouplet (10) terms

$$A_\eta(H) = A_\eta^{\mathrm{SEP}}(H) + A_\eta^{\mathrm{CA}}(H) + A_\eta^{10}(H)\,, \tag{F.1}$$

where H is a decaying hyperon. The parity-conserving B-amplitude is then written

$$B_\eta(H) = B_\eta^{\mathrm{SEP}}(H) + B_\eta^{\mathrm{POLE}}(H) + B_\eta^{10}(H)\,, \tag{F.2}$$

where $B_\eta^{\mathrm{POLE}}(H)$ arises from the pole term contribution. Earlier calculations Ref. [1] have used the effective weak four-quark QCD corrected Hamiltonian to obtain the separable contributions to A/B amplitudes. The separable contributions were omitted in the NNK amplitude calculations [30] which relied on decouplet poles. Thus we do not take that into the account when dealing with $NN\eta$ vertices.

The CA contribution to PV amplitude A follows the standard soft-meson procedure [8,9]. A (virtual[16]) decays which are relevant for the hypernuclear potential calculation are

$$\begin{aligned} \Lambda \quad &\to n + \eta_1 \qquad \text{and} \\ \Lambda \quad &\to n + \eta_8\,. \end{aligned} \tag{F.3}$$

Since the U(3) quark structure of the η mesons is given by $\eta \sim (\frac{1}{\sqrt{2}})\overline{q}_i(\lambda^a/2)_{ij}q_j$ with $a = 0, 8$, and $\lambda^0 = \sqrt{2/3}$, the CA contribution comming from η_1 nonet state vanishes whereas the η_8 calculation gives

$$\begin{aligned} \langle n\eta_8|\, H_W^{\Delta S=1}\,|\Lambda\rangle \quad &= -\frac{\sqrt{2}}{f_\eta}\langle n|\,[F_{\eta_8}^{0,A}, H_W^{PV}]\,|\Lambda\rangle = \frac{\sqrt{2}}{f_\eta}\sqrt{\frac{3}{2}}\langle n|\,H_W^{PC}]\,|\Lambda\rangle \\ &= -\sqrt{3}\frac{f_\pi}{f_\eta}A(\Lambda_-^0)\,, \\ \text{or} \qquad A(\Lambda_{\eta_8}^0) \quad &= -\frac{f_\pi}{f_\eta}\sqrt{3}A(\Lambda_-^0)\,. \end{aligned} \tag{F.4}$$

Here $f_\eta = 1.1\, f_\pi$.

The η pole contributions follow again the same procedure except that for the U(3) nonet contribution a generalized SU(3) relation [9] is used

$$\langle B^k|\, \eta^i\, |B^j\rangle = 2g_{\pi NN}[d_{ijk}(1-f)]\,. \tag{F.5a}$$

[16]Recall that the η masses are $m(\eta_1) = 958\,\mathrm{MeV}$ and $m(\eta_8) = 547\,\mathrm{MeV}$.

With $i = 0$ for the nonet (η_1) state the symmetric structure constant is $d_{0mn} = (\sqrt{2/3})\delta_{mn}$ so one obtains the U(3) result

$$\langle \mathbf{8}|\, O\, |\mathbf{8}\rangle = 2g_{\pi NN}\sqrt{\frac{2}{3}}(1-f)\,. \tag{F.6}$$

The resulting total η_1 pole contribution is

$$B^{\rm POLE}(\Lambda_{\eta_1}) = \left[-\frac{1}{\sqrt{2}}A(\Lambda^0_-)\right]\left[g\sqrt{\frac{2}{3}}(1-f)\right]\frac{\Lambda+n}{\Lambda-n}\left(\frac{1}{\Lambda}-\frac{2}{n}\right), \tag{F.7}$$

where the usual subtraction has been made [9]. With $i = 8$ in (F.5a) the η_8 contribution is calculated from the full formula

$$\langle B^k|\, \eta^i\, |B^j\rangle = 2g_{\pi NN}[if_{ijk}\, f + d_{ijk}(1-f)]\,. \tag{F.5b}$$

The strong $\Lambda\Lambda\eta_8$ coupling is calculated to be

$$g_{\Lambda\Lambda\eta_8} = -\frac{2}{\sqrt{3}}g_{\pi NN}(1-f) = -\frac{1}{\sqrt{2}}g_{\Lambda\Lambda\eta_1}\,. \tag{F.8}$$

For $B^k = B^j = n$ the strong coupling is given by

$$g_{nn\eta_8} = -2g_{\pi NN}\frac{\sqrt{3}}{3}(1-2f). \tag{F.9}$$

Since the pole diagrams for η_8 η_1 are the same, the following expresion is obtained

$$B^{\rm POLE}(\Lambda_{\eta_8}) = \left[-\frac{1}{\sqrt{2}}A(\Lambda^0_-)\right]\left[-\frac{\sqrt{3}}{3}g_{\pi NN}\right]\frac{\Lambda+n}{\Lambda-n}\left(\frac{1-f}{\Lambda}-\frac{1+2f}{n}\right). \tag{F.10}$$

Appendix G: Effective weak strangeness violating interaction in coordinate and/or momentum space

The effective one meson exchange (OME) contribution to the weak potential is comming form the second order $S-$matrix term

$$\begin{aligned} H_I &= \langle ac|\, H_W\, |bd\rangle \\ H_W &= \int {\rm d}^4x\, {\rm d}^4y\, \overline{\psi}(x)\Gamma_1^\mu\psi(x)\Delta_{\mu\,\nu}\overline{\psi}(y)\Gamma_2^\nu\psi(y)\,. \end{aligned} \tag{G.1}$$

Here $\Delta_{\mu\nu}(x-y)$ is the free meson propagator [2]. In the case of pesudoscalar mesons ($\pi,\ K,\ \eta$) its $\mu\,\nu$ dependence should be omitted. Γ_i^μ are some suitable spin-isospin operators which are specified in the literature for any particular case.

The fermion operators ψ are associated with fermion fields experiencing some interactions, i.e. being bound by some strong potential. It is convenient to include at least a part of that strong interaction explicitly. Then, instead of expanding ψ's in terms of the free particle solutions

$$\mathrm{e}^{-\mathrm{i}px} \tag{G.2}$$

one should expand using the solutions corresponding to a particular strong (for example shell-model) potential. Such bound state interactions [31] or Furry [32] picture employs

$$\begin{aligned} \psi(x) &= \textstyle\sum_n a_n \psi_n(\boldsymbol{r}) \mathrm{e}^{-\mathrm{i}E_n t} + \text{a.p.} \\ H(V_S)\psi_n(\boldsymbol{r}) &= E_n \psi_n(\boldsymbol{r})\,. \end{aligned} \tag{G.3}$$

Here a.p. stays for antiparticle piece which does not contribute in the present application. The Hamiltonian H has a symbolic and generic meaning. It may represent some relativistic (quasi-relativistic) dynamics. Its solution $\psi_n(\boldsymbol{x})$ can serve as a basis for the later nonrelativistic approximate (NRA) expression.

The particle creation operators $a_n^\dagger$ are indicated in (G.3) For example the operator a_b^+

$$|b\rangle = a_b^+ |0\rangle$$

picks up the appropriate states from the expression (G.3). One is left with the well defined baryon-baryon OME interaction, which is the starting point for the derivation of an effective weak potential. The generic form of such baryon-baryon interaction is

$$H_I = \int \mathrm{d}^3x\, \mathrm{d}^3y\, \overline{\psi}_a(\boldsymbol{x})\Gamma_1\psi_b(\boldsymbol{x})\Delta(\boldsymbol{x}-\boldsymbol{y})\overline{\psi}_c(\boldsymbol{y})\Gamma_2\psi_d(\boldsymbol{y})\,. \tag{G.4}$$

It corresponds to a process

$$d + b \to a + c\,. \tag{G.5}$$

Here it is assumed that the time dependence, such as

$$\psi_b(x) = \psi_b(\boldsymbol{x})\mathrm{e}^{-\mathrm{i}E_b t_x} \tag{G.6}$$

has been integrated out, as shown in Appendix G of Ref.[1].

The transition to the momentum space is achieved by Fourier decomposition, as for example

$$\psi_b(\boldsymbol{r}) = \int \mathrm{d}^3q\, \mathrm{e}^{\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{x}}\psi_b(\boldsymbol{q})\,. \tag{G.7}$$

For a free particle

$$\psi_b(\boldsymbol{q}) = \delta(\boldsymbol{q} - \boldsymbol{p}_b)u_b(\boldsymbol{p})\,. \tag{G.8a}$$

Here $u(p)$ is a Dirac spinor, i.e. the solution of the free particle Dirac equation

$$(\not{p} - m_b)u_b(\boldsymbol{p})\,. \tag{G.8b}$$

Thus in the free particle case one has

$$\psi_b(\boldsymbol{x}) \to u_b(\boldsymbol{p})\mathrm{e}^{\mathrm{i}\boldsymbol{p}_b\boldsymbol{x}}, \tag{G.8c}$$

and $\boldsymbol{p}_b$ is the momentum of that particular particle b.

In the general case, with the propagator given by

$$\Delta(\boldsymbol{x} - \boldsymbol{y}) = \int \mathrm{d}^3q\,\mathrm{e}^{\boldsymbol{q}(\boldsymbol{x}-\boldsymbol{y})}\Delta(q)\,, \tag{G.9}$$

one finds

$$H_I = \int \quad \mathrm{d}^3x\,\mathrm{d}^3y\,\mathrm{d}^3q_a\,\mathrm{d}^3q_b\,\mathrm{d}^3q_c\,\mathrm{d}^3q_d\,\mathrm{d}^3q_f \mathrm{e}^{-\mathrm{i}\boldsymbol{q}_a\boldsymbol{x}}\mathrm{e}^{\mathrm{i}\boldsymbol{q}_b\boldsymbol{x}}\mathrm{e}^{\mathrm{i}\boldsymbol{q}_f(\boldsymbol{x}-\boldsymbol{y})}\mathrm{e}^{-\mathrm{i}\boldsymbol{q}_c\boldsymbol{y}}\mathrm{e}^{\mathrm{i}\boldsymbol{q}_d\boldsymbol{y}}$$
$$\overline{\psi}_a(\boldsymbol{q}_a)\Gamma_1\psi_b(\boldsymbol{q}_b)\Delta(q_f)\overline{\psi}_c(\boldsymbol{q}_c)\Gamma_2\psi_d(\boldsymbol{q}_d)\,. \tag{G.10a}$$

The integration over coordinates can be easily carried out leading to

$$H_I = (2\pi)^6 \int \quad \mathrm{d}^3q_a\,\mathrm{d}^3q_b\,\mathrm{d}^3q_c\,\mathrm{d}^3q_d\,\mathrm{d}^3q_f\,\delta(-\boldsymbol{q}_a + \boldsymbol{q}_b + \boldsymbol{q}_f)\delta(-\boldsymbol{q}_f - \boldsymbol{q}_c + \boldsymbol{q}_d)$$
$$\overline{\psi}_a(\boldsymbol{q}_a)\Gamma_1\psi_b(\boldsymbol{q}_b)\Delta(q_f)\overline{\psi}_c(\boldsymbol{q}_c)\Gamma_2\psi_d(\boldsymbol{q}_d)\,. \tag{G.10b}$$

One can further integrate over, for example d^3q_a and d^3q_d finding

$$H_I = (2\pi)^6 \int \mathrm{d}^3q_a\,\mathrm{d}d^3q_c\,\mathrm{d}^3q_f\,\overline{\psi}_a(\boldsymbol{q}_a)\Gamma_1\psi_b(\boldsymbol{q}_a - \boldsymbol{q}_f)\Delta(\boldsymbol{q}_f)\overline{\psi}_c(\boldsymbol{q}_c)\Gamma_2\psi_d(\boldsymbol{q}_c + \boldsymbol{q}_f)\,. \tag{G.10c}$$

Thus one ends with three integrations instead of two which one had in the coordinate representation (G.4). However, if one deals with the free particle states defined by (G.8) one ends without any integrations. Starting from

$$H_I(\text{free particle}) = \int \quad \mathrm{d}^3x\,\mathrm{d}^3y\,\mathrm{d}^3q\,\mathrm{e}^{-\mathrm{i}\boldsymbol{p}_a\boldsymbol{x}}\mathrm{e}^{\mathrm{i}\boldsymbol{p}_b\boldsymbol{x}}\mathrm{e}^{\mathrm{i}\boldsymbol{q}(\boldsymbol{x}-\boldsymbol{y})}\mathrm{e}^{-\mathrm{i}\boldsymbol{p}_c\boldsymbol{y}}\mathrm{e}^{\mathrm{i}\boldsymbol{p}_d\boldsymbol{y}} \tag{G.11a}$$
$$\overline{u}_a(\boldsymbol{p}_a)\Gamma_1u_b(\boldsymbol{p}_b)\Delta(q_f)\overline{u}_c(\boldsymbol{p}_c)\Gamma_2u_d(\boldsymbol{p}_d)\,,$$

one finds

$$\begin{aligned} H_I(\text{free particle}) \quad &= (2\pi)^6 \int \mathrm{d}^3 q \, \delta(-\boldsymbol{p}_a + \boldsymbol{p}_b + \boldsymbol{q})\delta(-\boldsymbol{p}_c - \boldsymbol{q} + \boldsymbol{p}_d) \\ &\qquad \overline{u}_a(\boldsymbol{q}_a)\Gamma_1 u_b(\boldsymbol{q}_b)\Delta(q_f)\overline{u}_c(\boldsymbol{q}_c)\Gamma_2 u_d(\boldsymbol{q}_d) \qquad \text{(G.11}b\text{)} \\ &= (2\pi)^6 \delta(-\boldsymbol{p}_a + \boldsymbol{p}_b - \boldsymbol{p}_c + \boldsymbol{p}_d)\overline{u}_a(\boldsymbol{p}_a)\Gamma_1 u_b(\boldsymbol{p}_b)\Delta(q_f)\overline{u}_c(\boldsymbol{p}_c)\Gamma_2 u_d(\boldsymbol{p}_d) \end{aligned}$$

with

$$\boldsymbol{q}_f = \boldsymbol{p}_d - \boldsymbol{p}_c = \boldsymbol{p}_a - \boldsymbol{p}_b \qquad \text{(G.11}c\text{)}$$

and

$$\boldsymbol{p}_d + \boldsymbol{p}_b = \boldsymbol{p}_a + \boldsymbol{p}_c \,. \qquad \text{(G.11}d\text{)}$$

The last equality corresponds to the process (G.5) involving *free* particles a, b, c and d. The simple expression (G.11) is sometimes used as "the representation in the momentum space", as the starting formula for NRA. Sometimes that can be of some pedagogical use, as all manipulations look simple and transparent. But one should keep in mind that for particles bound by some potential the more complicated formula (G.10) is the correct expression in the momentum space.

Keeping in mind its complexity it is obviously easier to perform NRA in coordinate space, starting from (G.4) as it was often the case.

If one deals with a simple time dependence (G.3), (G.6), the states $\psi_n(\boldsymbol{x})$ are to be considered as solutions of the stationary Dirac equation with a potential

$$\begin{aligned} E_D \psi(\boldsymbol{r}) \quad &= (\boldsymbol{\alpha} \cdot \boldsymbol{p} + \beta V + \beta M)\psi(\boldsymbol{r}) \,, \\ E_D \quad &\simeq M + E + \ldots \end{aligned} \qquad \text{(G.12}a\text{)}$$

Here the potential

$$V = V(\boldsymbol{r}, \boldsymbol{\alpha}, \boldsymbol{\sigma}, \beta, \ldots) \qquad \text{(G.12}b\text{)}$$

can be, in principle, some general effective description of the strong forces binding nucleons in nuclei.

Formally, to the leading order in M^{-1}, one can approximate the equation (G.12) by a substitution

$$\psi_n(\boldsymbol{r}) \simeq \begin{pmatrix} \phi_n(\boldsymbol{r}) \\ \dfrac{\boldsymbol{\sigma} \cdot \boldsymbol{p}}{2M} \phi_n(\boldsymbol{r}) \end{pmatrix} . \qquad \text{(G.13)}$$

The symbol $\boldsymbol{p}$, appearing in (G.13) is used for traditional reasons. It corresponds to a definite momentum only if one deals with free particle wave functions (G.8).

Generally, $\boldsymbol{p}$ is the spatial derivative acting on the right, for example[17]

$$\boldsymbol{\sigma}\cdot\boldsymbol{p}(A(\boldsymbol{x})B(\boldsymbol{x})) = B(\boldsymbol{x})(-\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}A(\boldsymbol{x})) + A(\boldsymbol{x})(-\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}B(\boldsymbol{x}))\,. \tag{G.14}$$

The approximation (G.13) has its limitations and it has to be improved in various ways if one wants to proceede to higher orders in M^{-n} expansion [33–35].[18]

It is customary to assume that in NRA one ends with a Schrödinger equation describing a nuclear shell model

$$-\frac{1}{2M}\Delta\phi_n(\boldsymbol{r}) + V(\boldsymbol{x},\boldsymbol{\sigma},\ldots)\phi_n(\boldsymbol{x}) = E_n\phi_n(\boldsymbol{x})\,. \tag{G.15}$$

The substitution (G.13) with the nonrelativistic nucleon states described by (G.15) is the foundation of NRA of the expression (G.4). In the following this is illustrated for the $\boldsymbol{x}$ dependent density from (G.4):

$$D_x(\boldsymbol{y}) = \int \mathrm{d}^3x\,\psi_a(\boldsymbol{x})\begin{pmatrix}\Gamma_1^A & \Gamma_1^B\\ \Gamma_1^C & \Gamma_1^D\end{pmatrix}\psi_b(\boldsymbol{x})f(\boldsymbol{x},\boldsymbol{y})\,. \tag{G.16a}$$

Here Γ_1 matrix is subdivided in 2×2 matrices and the meson propagator is f. The indices A, B, ... have a very general meaning. They can contain four-vector labels $\mu,\nu,\ldots = 0,1,2,3$ if one deals with a vector-vector interaction. With

$$\begin{aligned}\overline{\psi} &= \psi^\dagger\beta\,,\\ \beta(\Gamma_1) &= \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}\Gamma_1^A & \Gamma_1^B\\ \Gamma_1^C & \Gamma_1^D\end{pmatrix} = \begin{pmatrix}\Gamma_1^A & \Gamma_1^B\\ -\Gamma_1^C & -\Gamma_1^D\end{pmatrix},\end{aligned} \tag{G.16b}$$

one can write

$$D_x(\boldsymbol{y}) = \int \mathrm{d}^3x\,\left[\phi_a^\dagger,\left(\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M}\phi_a\right)^\dagger\right]\begin{pmatrix}\Gamma_1^A & \Gamma_1^B\\ -\Gamma_1^C & -\Gamma_1^D\end{pmatrix}\begin{pmatrix}\phi_b(\boldsymbol{x})\\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_b}\phi_b(\boldsymbol{x})\end{pmatrix}f(\boldsymbol{x},\boldsymbol{y})\,. \tag{G.16c}$$

By definition (G.13) the operator $\boldsymbol{p}$ in (G.16) acts on the nucleon wave functions.

It is useful to carry out explicitly the matrix multiplication in (G.16c)

$$\begin{aligned}D_x(\boldsymbol{y}) &= \int \mathrm{d}^3x\,\left\{\phi_a\left(\Gamma_1^A\phi_b + \Gamma_1^B\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_b}\phi_b\right)\right.\\ &\qquad \left.-\left(\tfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_a}\phi_a\right)^\dagger\left(\Gamma_1^C\phi_b + \Gamma_1^D\tfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_b}\phi_b\right)\right\}f(\boldsymbol{x},\boldsymbol{y})\,.\end{aligned} \tag{G.16d}$$

[17] With free particles, one has (p_n being a $c-$number): $\boldsymbol{\sigma}pe^{ip_nr} = \boldsymbol{\sigma}p_n e^{\mathrm{i}p_n\cdot r}$.

[18] For example the Foldy-Wouthuysen transformation.

The expression (G.16) can be then evaluated numerically, as it has been done in [26] in order to check the consistency of the calculation. Alternatively one can move the derivative $\boldsymbol{p}$ from the left hand side by partial integration. The generic form of the second term in (G.16d) is

$$\int \mathrm{d}^3x \left(\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_a}\phi\right)^\dagger F(\boldsymbol{x},\boldsymbol{y}) = N(\boldsymbol{y})\,. \tag{G.17a}$$

Taking into account that

$$\left(\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_a}\phi_a\right)^\dagger = \left(\frac{-\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\phi_a}{2M_a}\right)^\dagger = \frac{\mathrm{i}\boldsymbol{\nabla}\phi_a^\dagger\boldsymbol{\sigma}}{2M_a} \tag{G.18}$$

and partially integrating (G.17a), one finds

$$N(\boldsymbol{y}) = -\int \mathrm{d}^3x\,\phi_a^\dagger\frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_a}f(\boldsymbol{x},\boldsymbol{y})\,. \tag{G.17b}$$

Here $\boldsymbol{p} = -\mathrm{i}\boldsymbol{\nabla}$ acts on the right, both on the nucleon wave function ϕ_b and on the meson propagator $f(\boldsymbol{x},\boldsymbol{y})$. It was shown [1] that f has a Yukawa form in the coordinate space.

In some cases the partial integration (G.17b) can lead to an effective potential which does not contain the derivatives (i.e. "speed dependent terms") of the nucleon wave functions. As an example let us apply the above formalism to (G.4) with

$$\Gamma_1 = \gamma_5 \quad \text{and} \qquad \Gamma_2 = 1\,. \tag{G.19}$$

That corresponds to a parity-violating (PV) pseudoscalar meson exchange. Leading terms in the M^{-1} expansion are

$$\begin{gathered} D_x(\boldsymbol{y}) \longrightarrow \textstyle\int \mathrm{d}^3x\,\phi_a\left[\hat{f}\dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_b} - \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{2M_a}\hat{f}\right]\phi_b \\ (\text{with}\quad \Gamma_1^B = 1,\ \Gamma_1^C = 1,\ \Gamma_1^A = 0 = \Gamma_1^D)\,, \end{gathered} \tag{G.20a}$$

and

$$D_y(\boldsymbol{x}) \longrightarrow \int \mathrm{d}^3y\,\left[\phi_c^\dagger(\boldsymbol{y})\phi_d(\boldsymbol{y})\right]\hat{f}(\boldsymbol{x},\boldsymbol{y})\,. \tag{G.20b}$$

However, "hat" on the function $\hat{f}$ means that the same propagator is shared by both terms a and b. By performing the partial integration in (G.20a) and by recombining both terms one finds

$$H_I(\gamma_5,1) = \int \mathrm{d}^3x\,\mathrm{d}^3y\,\left(\phi_a^\dagger\frac{i\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}}{2M_f}f\,\phi_b\right)(\phi_c^\dagger\phi_d)\,. \tag{G.20c}$$

From here one can read the first contribution to the V_3 appearing in the expression (9.6) of Ref.[1].

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SLABI MEZONSKI VRHOVI I HIPERNUKLEONSKI POTENCIJAL

Ovaj je članak nastavak ranijeg članka o hipernuklearnim potencijalima. Predstavljamo nov izvod hipernuklearnog potencijala koji krši stranost a posljedica je izmjene pseudoskalarnih mezona. Usporedba s ranijom metodom pokazuje da je teorijska netočnost manja od 30%. Raspravljaju se podrobno relativni predznaci izmjena pseudoskalarnih mezona i vektorskih (aksijalno vektorskih) izmjena. Daju se dodatne napomene o nerelativističkom približenju.