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UV AND IR ANALYSES OF THE MASS SPECTRUM IN THE SINE-GORDON MODEL

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Dedicated to Professor Kseno Ilakovac on the occasion of his 70$^{\text{th}}$ birthday

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We study the mass spectrum of the sine-Gordon model on a cylinder in the UV and IR regime. This is done by numerical diagonalization of the XXZ spin chain in a transverse field which is a convenient regularization. Our results strongly confirm the conjecture of Klassen and Melzer that sine-Gordon and massive Thirring models are *not* equivalent when defined on a *finite* cylinder. We obtain that the first two breathers have equal scaling dimensions, contrary to the conjecture claimed in literature.

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1. Introduction

It is almost impossible to overestimate the role of the sine-Gordon model (SGM) in modern physics. It emerges in the treatment of two-dimensional low-energy excitations in a wealth of phenomena in different branches, such as tunneling of Cooper pairs in Josephson junction [1] or field-induced gap in antiferromagnetic chain compounds (e.g. Cu benzoate) [2]. We should also mention that SGM solitons are two-dimensional analogues of cosmic strings. The reason for this wide-spread appearance of the SGM is that it gives the simplest non-trivial Lagrangian for single (pseudo)scalar field in 2D which is compactified ("angle variable").

Looking from a purely formal side, SGM is a perfect "laboratory" for testing various ideas and methods[1] for the following reasons. On one side it is a very

[1] Indeed, sine-Gordon and massive Thirring models are certainly the best understood nontrivial massive field theories.

complex quantum field theory (QFT) which posseses: (a) topological excitations (solitons), (b) bound states (breathers) whose number depends on the value of the coupling constant, (c) nontrivial vacuum (condensate) for $\beta^2 > 4\pi$, (d) nondiagonal S-matrix and (e) (possible) phase transition at $\beta^2 = 8\pi$. On the other side, it is integrable, i.e. a complete (infinite) set of conserved charges in involution exists, which enabled nonperturbative results such as: (a) masses of all particles given by Dashen-Hasslacher-Neveu (DHN) formula [3]

$$m_n = 2m \sin \frac{n\pi\beta^2}{2(8\pi - \beta^2)}, \quad n = 1, 2, \ldots < \frac{8\pi}{\beta^2} - 1 \; , \tag{1}$$

where m is the soliton and m_n n^{th} breather mass, (b) complete on-shell solution (S-matrix) [4], (c) S-duality[2] with massive Thirring model (MTM) which establishes equivalence of the models when the respective coupling constants are connected by the Coleman relation [5]

$$1 + \frac{g_0}{\pi} = \frac{4\pi}{\beta^2} \; .$$

and identifies soliton in SGM with elementary fermion in MTM and (d) some exact off-shell amplitudes [6] (complete off-shell solution is still missing).

Motivation for our interest in SGM and MTM are two claims which were put forward recently. First, authors in Ref. [7] calculated mass spectrum of MTM using two different methods and obtained that there is only one breather in the whole attractive regime ($\beta^2 < 4\pi$ in SGM terms) with the mass different from that of the first breather in DHN formula (1). They also claimed to have found wrong assumptions in all previous calculations [3, 4, 8] which gave Eq. (1).

Second claim, which was originally made in Ref. [9], is that SGM and MTM are *not* equivalent when defined on finite space of lenght L, and only become equivalent *on-shell* when $L = \infty$. The authors analysed UV limit and have shown that there are *two* different perturbed conformal field theories (CFT's); a purely bosonic one and a fermionic one, which they identified with SGM and MTM, respectively. Notice that this means that solitons in SGM are *bosons* and are not equivalent to elementary fermions of MTM.

In this paper we continue our investigations (started in Ref. [10][3]) on mass spectra and scaling dimensions of operators creating particle states. Using the conformal perturbation theory [12, 9], it can be shown that the *XXZ* spin chain with an even number of sites and periodic boundary conditions in a transverse magnetic field (σ^x perturbation) is spin chain regularization of the SGM ([9, 10]). It should be mentioned that to our knowledge this is the first analysis of this spin chain (which is believed to be non-integrable). We numerically diagonalize the spin chain Hamiltonian up to 16 sites and extrapolate results to the infinite length continuum limit using the BST extrapolation algorithm [13]. The same method

[2]Applies only in infinite volume, as we shall comment further.
[3]A similar analysis was done for MTM in Ref. [11].

was previously applied to conformal unitary models perturbed by some relevant (usually termal) operator [14]. In this way, we can obtain estimates of mass ratios without further assumptions, particularly those criticized in Ref. [7].

Our results for the mass spectrum are in complete agreement with DHN formula (1) and in disagreement with results of Fujita et al. [7]. Unfortunately, we could only analyse first two breathers because higher ones lie in the continuum part of the spectrum, and this method (contrary to Bethe Ansatz) can be used only for isolated states.

We also confirm conclusion in Ref. [9] that SGM and MTM are not equivalent when defined on a finite space. We calculated scale dimensions of operators creating lowest particle states. For the (anti)soliton we confirmed conjecture from Ref. [9] which means that these particles are *bosons*. Our results also confirmed conjecture for the first breather made in Ref. [9]. But for the second breather, we obtained the same scaling dimension as for the first breather, in contradiction with conjecture made in Ref. [9]. This result is most interesting and completely unexpected because it violates the picture in which n^{th} breather can be described as bound state of n first-breathers [3].

2. *Spin chain regularization of SGM*

The SGM is a $(1+1)$ dimensional field theory of a pseudoscalar field φ, defined classically by the Lagrangian

$$\mathcal{L}_{\text{SG}} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \lambda\cos(\beta\varphi)\ . \tag{2}$$

Here λ is a mass scale (with mass dimension depending on a regularization scheme), β is a dimensionless coupling (which does not renormalize) and one identifies field configurations that differ by a period $2\pi/\beta$ of the potential (because we want to have "ordinary" QFT with a unique vacuum).

In Ref. [9] it was shown that SGM can be viewed as a perturbed CFT when the second term in Eq. (2) is treated as a (massive) perturbation. An unperturbed theory $\lambda = 0$ (approached in UV limit) is the free massless compactified pseudoscalar CFT (known as Gaussian model) which has central charge $c = 1$ and is generated by

$$L_b = \{V_{m,n} | m, n \in \boldsymbol{Z}\}\ . \tag{3}$$

Here $V_{m,n}$ are primary ("vertex") operators whose scaling dimensions and (Lorentz) spins are

$$d_{m,n} = \frac{m^2\beta^2}{4\pi} + \frac{n^2\pi}{\beta^2}, \tag{4}$$

$$s_{m,n} = mn\ . \tag{5}$$

Because of $V^\dagger_{m,n} = V_{-m,-n}$, we can define Hermitian combinations

$$\begin{aligned} V^{(+)}_{m,n} &\equiv \frac{1}{2}\left(V_{m,n} + V_{-m,-n}\right) , \\ V^{(-)}_{m,n} &\equiv \frac{\mathrm{i}}{2}\left(V_{-m,-n} - V_{m,n}\right) \end{aligned}$$

which have equal scaling dimensions and spin. Also, it can be shown that a properly normalized perturbing operator in the SGM (2) is

$$\cos(\beta\varphi) = V^{(+)}_{1,0} \tag{6}$$

which means that λ has mass dimension $y = 2 - d_{1,0} = 2 - \beta^2/4\pi$. From the condition of relevancy of the perturbation, i.e. $y > 0$, we obtain the Coleman's bound $\beta^2 < 8\pi$.

Using the above analysis, it was proposed (Appendix B in Refs. [9] and [10]) that the *XXZ* spin chain with periodic boundary conditions in a transverse magnetic field defined by the Hamiltonian

$$H = -\sum_{n=1}^{N}\left(\sigma^x_n\sigma^x_{n+1} + \sigma^y_n\sigma^y_{n+1} + \Delta\sigma^z_n\sigma^z_{n+1} + h\sigma^x_n\right), \tag{7}$$

where σ^a are Pauli matrices, $N \in 2\boldsymbol{Z}$, $-1 \le \Delta < 1$ (we use the usual parametrization $\Delta = -\cos\gamma$, $0 \le \gamma < \pi$), is a spin chain regularization of the SGM. The argument has two steps; first, one must show that unperturbed theories are equivalent, i.e., that Eq. (7) with $h = 0$ is a spin chain regularization of L_b CFT (3), and second, that in the unperturbed theory ($h = 0$) perturbation operator σ^x_n is a lattice regularization of $V^{(+)}_{1,0}(x)$. For the detailed discusion see Ref. [10].

SGM defined on a cylinder[4] is now obtained from the spin chain (7), where

$$\beta = \sqrt{2(\pi - \gamma)}\ , \tag{8}$$

in the scaling limit $N \to \infty$, $h \to 0$ while keeping fixed scaling parameter $\tilde{\mu}$

$$\tilde{\mu} \equiv hN^{d_\lambda} = hN^{2-\beta^2/4\pi} = hN^{3/2+\gamma/2\pi}. \tag{9}$$

In this limit, the mass gaps of the chain (7) are expected to satisfy a scaling law

$$\tilde{m}_i = h^{1/d_\lambda}\tilde{G}_i(\gamma,\tilde{\mu}) = h^{(2-\beta^2/4\pi)^{-1}}\tilde{G}_i(\gamma,\tilde{\mu}). \tag{10}$$

where γ is connected to β by Eq. (8).

[4]Space is compactified on a circle with circumference L and time is infinite.

It it easy to see [10] that $\tilde{\mu} \propto L^{d_\lambda}$, and the constant of proportionality is not important because we are interested here only in the $L \to \infty$ ($\tilde{\mu} \to \infty$) and $L \to 0$ ($\tilde{\mu} \to 0$) limits.

It is important to observe that although perturbative CFT analysis was used to obtain spin chain regularization, the results obtained using this regularization are in fact *nonperturbative*. One of the reasons is that it is widely believed that for the CFT perturbed by a relevant (i.e. superrenormalizable) operator, a perturbative expansion has a nonvanishing radius of convergence in two dimensions[5]. Taking this into account, it is natural to assume that Hilbert space of the full theory is isomorphic to that of the unperturbed theory. In fact, agreement of our results with SGM and MTM mass-spectra in the $L \to \infty$ limit (as given by (1)) indirectly confirms those assumptions.

3. *Mass spectrum*

Our goal here is to calculate the mass ratios of particles in the SGM in the $L \to \infty$ limit using conection with the spin chain (7). First, we must numerically calculate the mass gaps of the spin chain for finite N and h. This was done for up to 18 sites using the Lanczos algorithm. Then, we must make a continuum (scaling) limit, i.e., take $N \to \infty$ and $h \to 0$, keeping $\tilde{\mu}$ fixed. Finally, we should make $L \to \infty$, i.e., $\tilde{\mu} \to \infty$ limit. In practice, it is preferable to do the following [14]: first take $N \to \infty$ with h fixed and afterwords extrapolate to $h \to 0$. The difference is that in the latter case one does $\tilde{\mu} \to \infty$ before $h \to 0$. These limits are performed using the BST extrapolation method [13].

In Ref. [10], we considered a number of values of coupling $-1 \leq \Delta < 1$ (or $\sqrt{2\pi} \geq \beta > 0$). Starting from $\Delta = -1$, the spectrum contains five clearly isolated states which we name vacuum, soliton, antisoliton, first and second breather. All other levels form "continuum," i.e., they "densely" fill the region between about $2 \times$ (mass of first breather) and some $E_{\max}$. Soliton and antisoliton energies are not degenerate which is a consequence of breaking Z_2 symmetry on the spin chain. Exactly at $\Delta = -1$, we have[6] $\tilde{m}_{B1} = \tilde{m}_S < \tilde{m}_A < \tilde{m}_{B2}$. As we increase Δ $\tilde{m}_S$, $\tilde{m}_A$ and $\tilde{m}_{B2}$ monotonically increase (relative to $\tilde{m}_{B1}$), where $\tilde{m}_S$ and $\tilde{m}_A$ increase faster than $\tilde{m}_{B2}$ and at $\Delta \approx -0.1$ disappear into the "continuum" (i.e., $\tilde{m}_{S,A} > 2\tilde{m}_{B1}$), while $\tilde{m}_{B2}$ asymptotically approach $2\tilde{m}_{B1}$. This was a crude picture visible already from raw data before extrapolation $N \to \infty$ and $h \to 0$, and it is expected from the DHN formula (1). Observe that the exact degeneracy of soliton and first breather masses at $\Delta = -1$ is present in Eq. (1).

As an example, we shall now make continuum analysis for the coupling $\Delta = -0.4$ ($\beta^2 = 3.96$), which was not fully presented in Ref. [10]. In Fig. 1 we present numerical results for the scaled gaps (scaling functions of mass gaps) $\tilde{G}_a$, $a \in \{S, A, B1, B2\}$. This is, of course, a check of the scaling relation (10). BST extra-

[5] With appropriate IR and, if necessary, UV cutoffs

[6] We employ an obvious notation for mass gaps; $\tilde{m}_S$ for soliton, $\tilde{m}_A$ for antisoliton and $\tilde{m}_{Bn}$ for n$^{\text{th}}$ breather, where $n = 1, 2$.

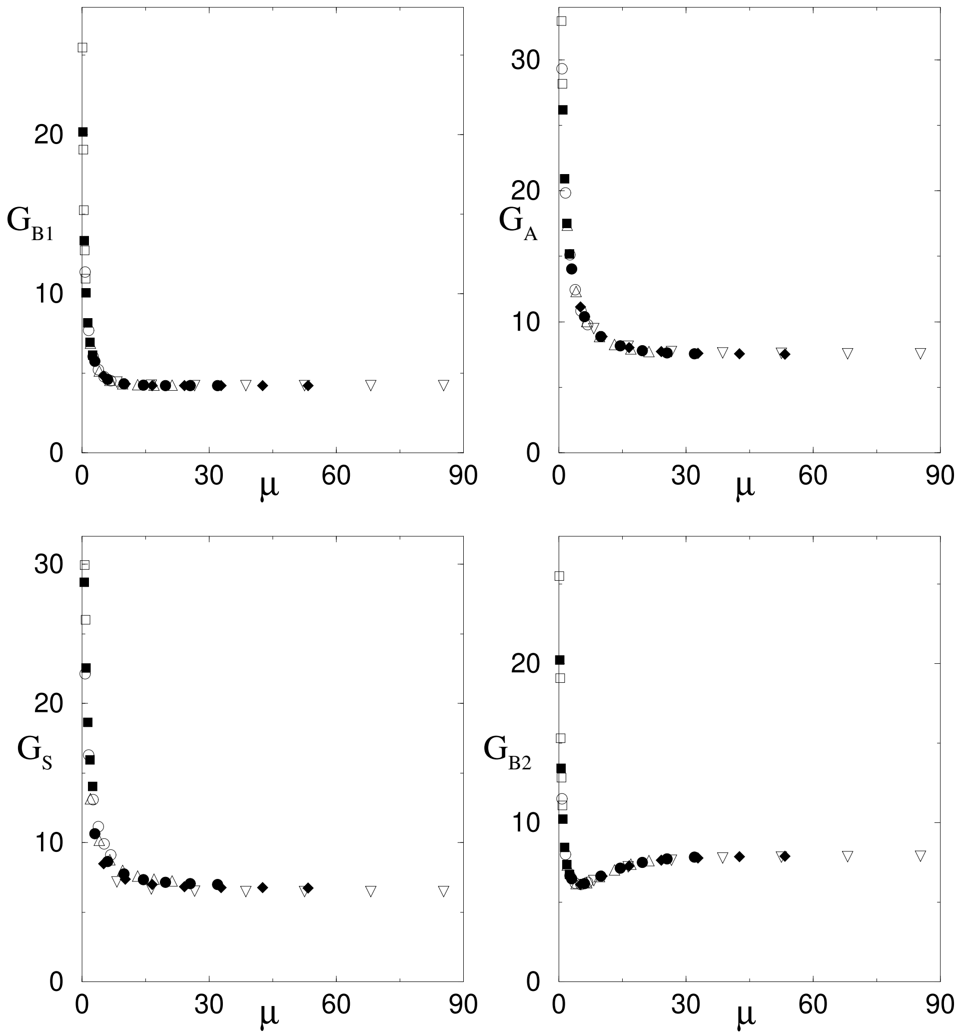

Fig. 1. Scaling functions $\tilde{G}_a(\beta, \mu)$ for the isolated gaps of Hamiltonian (7) at $\Delta = -0.4$ (or $\beta^2 = 3.96$).

polations $N \to \infty$ (with fixed h) of scaled gaps for $h = 0.8, 0.5, 0.3, 0.2$ are given in Table 1. Finally, (partially) extrapolated mass ratios

$$\tilde{r}_a(\Delta, h) = \lim_{\substack{N\to\infty \\ h \text{ fixed}}} \frac{\tilde{m}_a}{\tilde{m}_{B1}} = \lim_{\substack{N\to\infty \\ h \text{ fixed}}} \frac{\tilde{G}_a}{\tilde{G}_{B1}}, \qquad a \in \{S, A, B2\}$$

TABLE 1. Estimates for the scaled gaps $\tilde{G}_a(\beta,\infty)$ as a function of h at $\Delta = -0.4$ ($\beta^2 = 3.96$). The numbers in brackets give the estimated uncertainty in the last given digit.

h	$\tilde{G}_{B1}$	$\tilde{G}_S$	$\tilde{G}_A$	$\tilde{G}_{B2}$
0.8	4.220189 (4)	6.4723 (1)	7.5657 (5)	7.936 (4)
0.5	4.2203 (1)	6.740 (1)	7.509 (4)	7.96 (1)
0.3	4.22820 (5)	6.93 (1)	7.45 (2)	7.97 (2)
0.2	4.233 (1)	7.00 (3)	7.37 (5)	8.06 (7)

are given in Table 2 together with the predictions from DHN formula (1) and Fujita et al. formula [7]. Although we were not able to make final extrapolation $h \to 0$, one can see that our results are in full agreement with DHN and reject the results of Fujita et al.

TABLE 2. Estimates for the mass gap ratios $\tilde{r}_a(\Delta, h)$ as a function of h at $\Delta = -0.4$ ($\beta^2 = 3.96$). We also added predictions obtained from Eq. (1) (DHN) and Fujita et al.

$\tilde{r}_a$	h				DHN	Fujita
	0.8	0.5	0.3	0.2		
S	1.53365 (3)	1.5970 (2)	1.639 (3)	1.654 (8)	1.724	1.367
A	1.7927 (1)	1.779 (1)	1.762 (6)	1.74 (1)	1.724	1.367
B2	1.880 (1)	1.886 (3)	1.885 (5)	1.90 (2)	1.914	

4. *UV limit and scaling dimensions of particle states*

Let us now turn our attention to the UV limit of our results for the spin chain (7). We mentioned in Sect. 2 that it is obtained when $\tilde{\mu} \to 0$. From conformal perturbation theory, we expect the scaling relation

$$\tilde{m}_a = \zeta h^{2\pi/(3\pi+\gamma)} \left[2\pi d_a \tilde{\mu}^{-2\pi/(3\pi+\gamma)} + \tilde{H}_a(\gamma, \tilde{\mu}) \right] \tag{11}$$

where d_a is the scaling dimension of the state a, and ζ is the well-known normalization factor,

$$\zeta = \frac{2\pi \sin\gamma}{\gamma} \, .$$

From Eq. (11) we can obtain the scaling dimensions of the particle states S, A, $B1$ and $B2$ from the condition that $\tilde{H}_a$ should be less singular than $\tilde{G}_a$. In Fig. 2 we show numerical results, again for $\Delta = -0.4$ for reduced scaling functions, where we used values from Table 3 for scaling dimensions. One can see that the scaling relation (11) is very well satisfied.

Let us now make a few comments on the results shown in Table 3. First of all, we see that soliton and antisoliton are spin zero particles, which means they are *not* equivalent to elementary fermion and antifermion of MTM (which are spin-1/2) when L is finite. For the first breather, we obtained expected result because in the UV limit, it is created by

$$V^{(-)}_{1,0} \propto \sin(\beta\varphi) \propto \varphi + O(\varphi^2)\,,$$

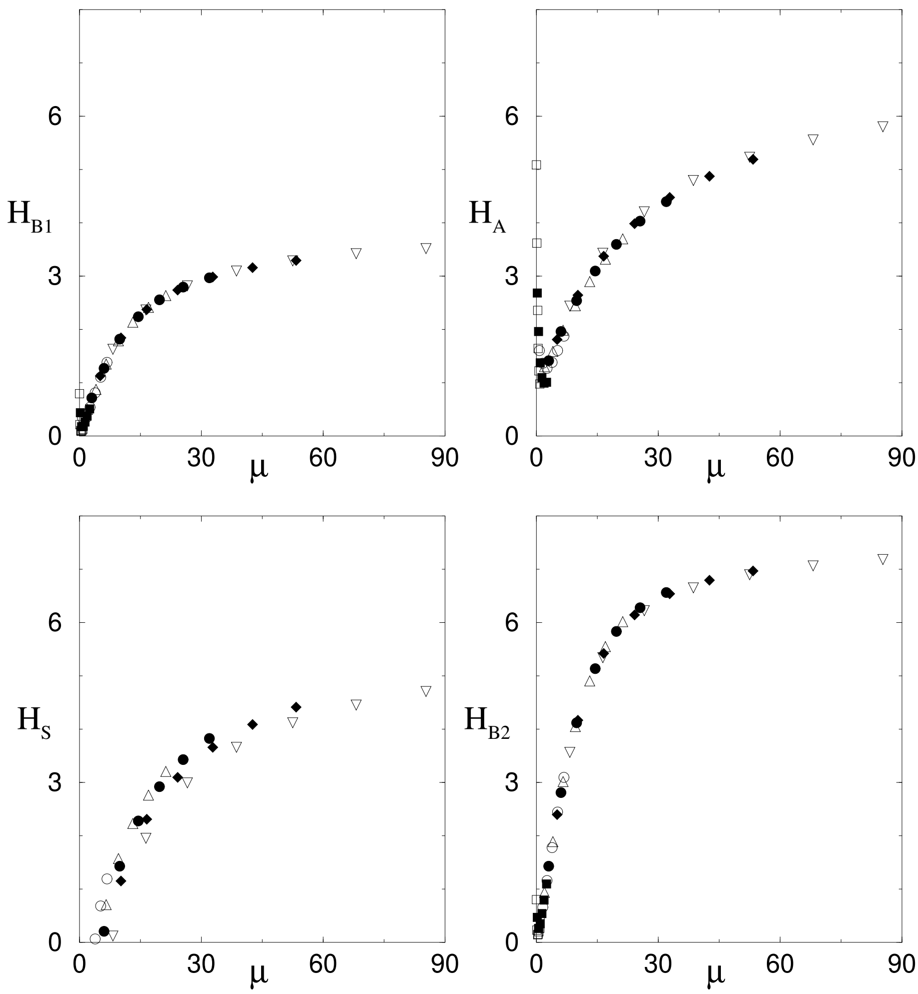

Fig. 2. Reduced scaling functions $\tilde{H}_a(\beta, \mu)$ at $\Delta = -0.4$ (or $\beta^2 = 3.96$).

TABLE 3. Scaling dimensions of particle states in SGM as conjectured from our numerical results.

State	Operator	Scaling dimension
soliton	$V_{0,1}$	$\dfrac{\pi}{\beta^2} = \dfrac{1}{2}\left(1-\dfrac{\gamma}{\pi}\right)^{-1}$
antisoliton	$V_{0,-1}$	$\dfrac{\pi}{\beta^2} = \dfrac{1}{2}\left(1-\dfrac{\gamma}{\pi}\right)^{-1}$
1st breather	$V_{1,0}^{(-)}$	$\dfrac{\beta^2}{4\pi} = \dfrac{1}{2}\left(1-\dfrac{\gamma}{\pi}\right)$
2nd breather	$V_{1,0}^{(+)}$	$\dfrac{\beta^2}{4\pi} = \dfrac{1}{2}\left(1-\dfrac{\gamma}{\pi}\right)$

i.e., by "regularized" elementary SGM field[7]. We should emphasize that these results were first conjectured in Ref. [9] using conformal perturbation theory and were first explicitely calculated in Ref. [10].

In Ref. [9], it was conjectured that the n^{th} breather is created in the UV limit by the operator $V_{n,0}^{((-)^n)}$ with the scaling dimension $n^2\beta^2/4\pi$. The argument goes as follows. Let us take the SGM coupling constant β small and expand DHN mass formula (1)

$$m_n = nm_1\left[1-\frac{1}{6}\left(n^2-1\right)\left(\frac{\beta^2}{16}\right)^2 + O(\beta^6)\right]. \tag{12}$$

It looks like the n^{th} breather could be viewed as loosly bound state of n 1st breathers. Indeed, this can be confirmed using ordinary perturbation theory. We can expand the SGM Lagrangian (2) as

$$\mathcal{L}_{\text{SG}} = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{1}{2}\lambda\beta^2\varphi^2 + \frac{1}{4!}\lambda\beta^4\varphi^4 + O\big((\beta\varphi)^6\big)$$

and see that for small β it reduces to an attractive scalar φ^4 theory with bare mass $\lambda\beta^2$ and coupling constant $\lambda\beta^4$. It is well-known that the non-relativistic limit of this theory gives the potential between elementary bosons $V(x) = -(\beta^2/8)\delta(x)$, which is attractive. Since the interaction is weak, one can compute the binding energy of n bosons solving the non-relativistic n-body Schrödinger equation. Exact result for the ground state gives lowest-order term in Eq. (12). In Ref. [3], this calculation was extended to two-loop order (β^8) in the $n=2$ case (2nd breather), and result agreed with the DHN formula.

Now, we have seen that elementary boson is the first breather, and that operator which creates it in the UV limit is $V_{1,0}^{(-)}$. From the operator product expansion one can see that regularized product of n operators is

$$:(V_{1,0}^{(-)})^n: = V_{n,0}^{((-)^n)}\,. \tag{13}$$

[7]In the UV limit, φ is not well defined operator, i.e., it does not satisfy Wightman axioms.

Using this, the authors in Ref. [9] conjectured that the operator which corresponds to the $n^{\rm th}$ breather is $V_{n,0}^{((-)^n)}$.

Surprisingly, our results show that scaling dimension of the 2[nd] breather is exactly the same as that of the 1[st] breather. This clearly follows from Fig. 3, where we show numerical values for the ratio of the mass gaps $\tilde{m}_{B2}/\tilde{m}_{B1}$, and scaling relation (11) in the UV limit ($\tilde{\mu} \to 0$). Notice that the above conjecture would predict that the ratio in Fig. 3 should converge to 4 when $\tilde{\mu} \to 0$.

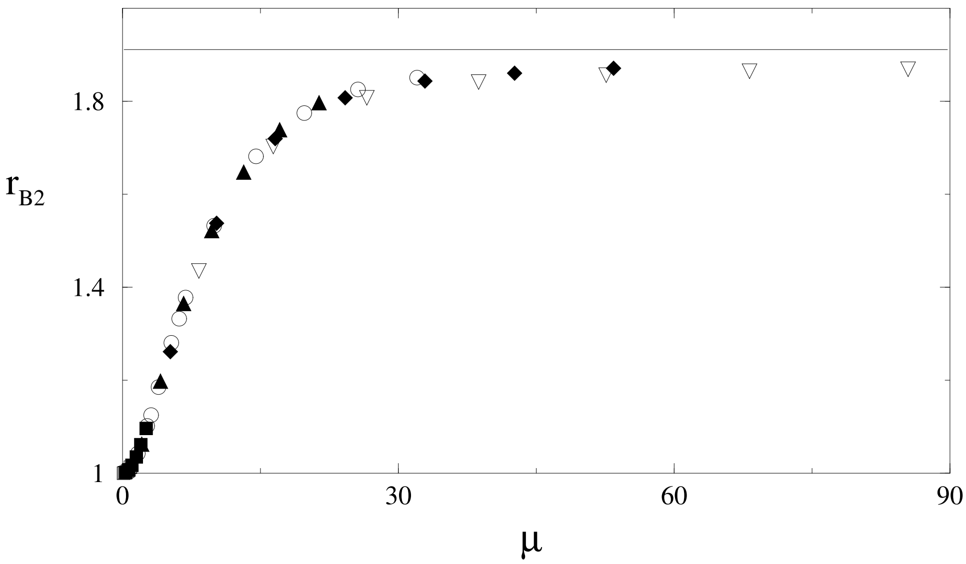

Fig. 3. Ratios of mass gaps $\tilde{m}_{B2}/\tilde{m}_{B1}$ at $\Delta = -0.4$. It is clear that it is equal to 1 in the UV limit $\tilde{\mu} \to 0$, so from Eq. (11) follows that first and second breather have equal scale dimensions.

So, our results predict that the operator connected to the second breather is $V_{1,0}^{(+)}$, because it is the only operator with the same dimension as $V_{1,0}^{(-)}$ which is connected to the first breather. After this result was first obtained in Ref. [10] it was subsequently analytically confirmed in Ref. [15] using extension of the nonlinear integral equation (NLIE) method.

5. Conclusion

In this paper, we have analysed both infrared ($L \to 0$) and ultraviolet ($L \to \infty$) limits of sine-Gordon model defined on a cylinder with the circumference L. We used perturbed CFT aproach, as described in Ref. [9], to find spin chain normalization of the model which was used to numerically calculate the mass-spectrum.

In the IR limit, our results agree with the DHN formula for masses of breathers, and disagree with the recent claims of Fujita et al. Taken together with the results for massive Thirring model [11], our results also confirm Coleman duality relation.

Analysis of the UV (conformal) limit have confirmed the conjecture of Klassen and Melzer [9] that sine-Gordon and massive Thirring models are *not* equivalent when defined on a finite cylinder, and only become equivalent *on-shell* when $L = \infty$. Our results clearly confirm that soliton in the SGM has different scaling dimension than elementary fermion in MTM (the former is *boson* while the latter is *fermion*). We also calculated scaling dimensions of the first two breathers. While for the first breather our results confirmed conjecture made in Ref. [9], for the second breather we obtain a different result, i.e., that it has the same scaling dimension as the first breather. This result is not only interesting, but also surprising because it is in contradiction with intuition gained from perturbative calculations. After it was first obtained in Refs. [10, 11], this result was afterwards confirmed in Ref. [15] using different methods.

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UV I IR ANALIZE MASENOG SPEKTRA U SINUS-GORDONOVOM MODELU

Proučavamo maseni spektar u sinus-Gordonovom modelu u UV i IR uvjetima. To se čini numeričkom dijagonalizacijom spinskog lanca *XXZ* u poprečnom polju koje je pogodna regularizacija. Naši rezultati snažno potvrđuju pretpostavku Klassena i Melzera da sinus-Gordonov model i Thirringov model *nisu* ekvivalentni ako su definirani na *konačnom* valjku. Dobivamo da prva dva disalna stanja imaju jednake skalne dimenzije, suprotno pretpostavki koja se navodi u literaturi.