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REALISTIC REGULARIZATION OF THE QED GREEN FUNCTIONS

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Generalizing the 't Hooft and Veltman method of unitary regulators, we demonstrate for the first time the existence of local, Lorentz-invariant, physically motivated Lagrangians of quantum-electrodynamic phenomena such that: (i) Feynman diagrams are finite and equal the diagrams of QED but with regularized propagators. (ii) n-point Green functions are C-, P- and T-invariant up to a phase factor, Lorentz-invariant and causal. (iii) No auxiliary particles or parameters are introduced.

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1. Introduction

Perturbative predictions about quantum-electrodynamic phenomena implied by a QED Lagrangian can be computed using the Feynman rules, a regularization method to circumvent ultraviolet divergencies and a renormalization scheme. Regularization method results in regularized n-point Green functions; a suitable limiting procedure (a renormalization scheme) then leads to physically sensible predictions that are independent of the particular regularization method used. But no known regularized n-point Green functions can be regarded as being based on physically realistic premises about quantum-electrodynamic phenomena: the derivation of each is formalistic since it disregards some of the basic tenets of conventional physics (e.g., by lacking a Lagrangian, by not being Lorentz-invariant, by introducing particles with wrong metric or statistics...). *So the perturbative predictions of QED presently cannot be directly derived from physically realistic premises*; for a history

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of, and comments on this basic, conceptual inconsistency see, e.g., [1]. Dirac [2] believed that removal of this conceptual inconsistency may lead to an important advance in field theories.

To show that one can remove this inconsistency *already in four-dimensional space-time,* we will introduce a new, physically motivated modification of the QED Lagrangian and consider it within the theoretical framework of 't Hooft and Veltman that presents an alternative to the convential perturbative quantum field theory [3]. They avoid canonical formalism and take diagrams as the basis from which everything must be derived; so they give a perturbative definition of the S-matrix directly in terms of diagrams corresponding to a given Lagrangian as specified by *postulated* Feynman rules. The question is: How do we modify the QED Lagrangian so that the resulting regularized S-matrix is derived from physically realistic premises?

We are using the adjectives formalistic and realistic in the sense of Pauli and Villars [4]. Introducing their formalistic regularization method, they remarked: "It seems very likely that the 'formalistic' viewpoint used in this paper and by other workers can only be a transitional stage of the theory, and that the auxiliary masses will eventually be entirely eliminated, or the 'realistic' standpoint will be so much improved that the theory will not contain any further accidental compensations." Which we intend to do.

Gupta [5] has shown already in 1952 that one can modify the QED Lagrangian so that the new Lagrangian results in the S-matrix of QED, regularized by a certain Pauli-Villars method. And twenty years later, 't Hooft and Veltman [3] introduced the method of unitary regulators (HV-method) that: (i) is a variant of Pauli-Villars methods for regularizing propagators, (ii) requires only an exceedingly simple modification of the initial Lagrangian and (iii) is very suitable for proving the causality of the regularized n-point Green functions and the unitarity of the resulting S-matrix. Unfortunately both methods are formalistic since they introduce also unphysical, auxiliary particles with wrong metric or statistics. To get rid of this serious conceptual deficiency, we will generalize the HV-method to avoid auxiliary particles.

We will demonstrate the utility of the generalized HV-method by showing that there are finite perturbative n-point Green functions of quantum-electrodynamic phenomena derived from a *realistic perturbative theory* (a rp-theory, for short) *such that:*

(i) A rp-theory of quantum-electrodynamic phenomena is specified in a continuous, four-dimensional space-time by a local, Lorentz-invariant, physically motivated modification of a QED Lagrangian.

(ii) The Feynman rules for this modified Lagrangian, *defined* as specified by 't Hooft and Veltman [3], result in regularized Feynman diagrams that equal the diagrams of QED but with regularized propagators that have no additional singularities.

(iii) All constants of a rp-theory are measurable in principle; *there are no auxiliary parameters or particles.*

(iv) For certain values of these constants, the QED propagators are such low-energy approximations to their regularizations as acceptable for renormalization.

(v) The n-point Green functions of a rp-theory, *defined* as specified by 't Hooft and Veltman [3] in terms of regularized Feynman diagrams, are C-, P-, T- and Lorentz-invariant, causal, and charge and total four-momentum conserving.

Such a rp-theory of quantum-electrodynamic phenomena is not yet known; we cannot incorporate a finite-cutoff, Pauli-Villars, dimensional or lattice regularization of QED in a rp-theory.

2. *Lorentz-invariant regularization without singularities*

As in the HV-method to each additional singularity of a regularized Feynman propagator corresponds an additional particle [3], we will first specify Lorentz-invariantly regularized Feynman propagators that have no additional singularities and have the Källén-Lehman representation used in proving causality and unitarity [3,6]. Regarding the metric and other conventions, we follow Refs. [3,6]; in particular, a four-vector $k = (\vec{k}, \mathrm{i}k^0)$, and $k^2 \equiv \vec{k}\cdot\vec{k} - (k^0)^2$.

Consider a Lorentz-invariantly regularized spin 0 Feynman propagator, say, $\Delta_F(x)$ whose space-time Fourier transform

$$(2\pi)^4\mathrm{i}\widetilde{\Delta}_F(k) = \varphi(k^2)(k^2+m^2-\mathrm{i}\epsilon)^{-1}\,, \qquad \varphi(-m^2) = 1\,, \tag{1}$$

where: (a) $\varphi(z)$ is an analytic function of complex variable z with a finite discontinuity somewhere across the segment $z \le z_d < -m^2$ of the negative real axis; (b) $|\varphi(z)| < A|z|^{-r}$ with $r \ge 3/2$ as $|z| \to \infty$; (c) $\varphi(z)$ is real on the positive real axis; (d) $\varphi(z)$ depends on some constant Λ so that for any $\Lambda \ge \Lambda_0 > 0$ it has properties (a) to (c) and

$$\sup_{|z|<z_0} |\varphi^{(n)}(z) - \delta_{n0}| \to 0 \qquad \text{as} \quad \Lambda \to \infty \quad \text{for any} \quad z_0 > 0\,, \quad n = 0,1,2, \tag{2}$$

and

$$\sup_{z\ge 0,\Lambda\ge\Lambda_0} |z^{(n+3)/2}\varphi^{(n)}(z)| < \infty\,, \quad n = 0,1,2.$$

As a consequence, the spin 0 propagator provides a low-energy approximation to its regularization (1) which itself is acceptable for renormalization.

Using Cauchy's integral formula, we can conclude that the Lorentz-invariant regularization (1) of the spin 0 Feynman propagator admits the Källén-Lehman representation

$$(2\pi)^4\mathrm{i}\widetilde{\Delta}_F(k) = \int_0^\infty \frac{\rho(s)}{k^2+s-\mathrm{i}\epsilon}\,\mathrm{d}s \tag{3}$$

with

$$\rho(s) = \delta(s - m^2) + (2\pi\mathrm{i})^{-1}(m^2 - s)^{-1} \lim_{y\to 0}[\varphi(-s - \mathrm{i}y) - \varphi(-s + \mathrm{i}y)]\,, \qquad (4)$$

$s, y > 0$. Note that $\rho(s)$ is real, $\rho(s) = O(s^{-r})$ as $s \to \infty$, and

$$\int_0^\infty s^m \rho(s)\,\mathrm{d}s = 0 \qquad \text{for} \quad m = 0, 1, \ldots < r - 1\,. \qquad (5)$$

So we can decompose the regularized spin 0 propagator $\Delta_F(x)$ into positive and negative energy parts: $\Delta_F(x) = \Theta(x_0)\Delta^+(x) + \Theta(-x_0)\Delta^-(x)$ [3].

The function $-\mathrm{i}(2\pi)^{-4}(\sqrt{\Lambda^2 - m^2} + \Lambda)^n (k^2 + m^2 - \mathrm{i}\epsilon)^{-1}(\sqrt{k^2 + \Lambda^2 - \mathrm{i}\epsilon} + \Lambda)^{-n}$, $\Lambda > m$, $n = 1, 2, \ldots$, is an example of a Lorentz-invariantly regularized spin 0 Feynman propagator that satisfies the above conditions with $r = n/2$. Unfortunately, we cannot use such propagators for a realistic regularization of the QED Green functions since we do not know how to construct the corresponding *local,* Lorentz-invariant Lagrangians.

A propagator that satisfies conditions (a)-(c) is by (3) a generalization of the spin-0 propagator regularized by a Pauli-Villars regulator that has a continuous mass spectrum. Thus, to use such propagators to construct a rp-theory, we have to extend the 't Hooft-Veltman construction of Lagrangians in HV-method [3] to an *infinity* of additional fields. To provide an *example* of how this can be done, we will present a local, Lorentz-invariant Lagrangian whose propagators for *interacting* fields can be taken as spin 1 and spin $\frac{1}{2}$ propagators regularized so that they acquire no additional singularities and have the Källén-Lehman representation.

3. An example of Lagrangian that regularizes QED propagators

Following Veltman [6], we will consider QED with massive photons in unitary gauge. Its Lagrangian reads

$$\mathcal{L}_{\text{QED}} = -\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 - \tfrac{1}{2}\mu^2 A^2 - \bar{\psi}(\gamma^\mu \overleftrightarrow{\partial}_\mu + m)\psi + \mathrm{i}e\bar{\psi}\gamma^\mu\psi A_\mu + A_\mu J_\mu + \bar{J}_e\psi + \bar{\psi}J_e\,, \qquad (6)$$

where $J_\mu(x)$, $\bar{J}_e(x)$ and $J_e(x)$ are four-vector and bispinor source fields, and μ is the non-vanishing photon mass, a physical constant $< 2\times 10^{-16}$ eV [7]. The Feynman propagators for the four-vector field $A_\mu(x)$ and for the bispinor field $\psi(x)$ are

$$-\mathrm{i}(2\pi)^{-4}\frac{\delta_{\mu\nu} + \mu^{-2}k_\mu k_\nu}{k^2 + \mu^2 - \mathrm{i}\epsilon}\,, \qquad -\mathrm{i}(2\pi)^{-4}\frac{-\mathrm{i}\gamma^\mu k_\mu + m}{k^2 + m^2 - \mathrm{i}\epsilon}\,. \qquad (7)$$

We could use $\mathcal{L}_{\text{QED}}$ to define a rp-theory as specified in Sect. 1, were the propagators (7) are decreasing faster when k^2 tends to infinity.

However, one can modify the QED Lagrangian (6) so that the propagators for the fields A_μ and ψ are such regularizations of propagators (7) that have no additional singularities when calculated according to the generalized 't Hooft-Veltman method. Take, for *example,* the following real-valued, local, Lorentz-invariant Lagrangian

$$\mathcal{L}_{\rm TR} = -\mathcal{L}_1 - \mathcal{L}_{1/2} + {\rm ie}\bar{\psi}\gamma^\mu\psi A_\mu + A_\mu J_\mu + \bar{J}_e\psi + \bar{\psi}J_e\,, \tag{8}$$

with

$$\begin{aligned}\mathcal{L}_1 \;\equiv\;& q_1^{-1}\int {\rm d}^4p\,\Psi'_\mu(x,-p)[\Lambda t(p^2) + p^\nu\overleftrightarrow{\partial}_\nu]\Psi^\mu(x,p)\\ &+q_1^{-1}s_1\int {\rm d}^4p\,{\rm d}^4p'\,f({p'}^2)f(p^2)[\Psi'_\mu(x,-p')\Psi'^\mu(x,p)\\ &+p'_\nu p^\nu\Psi_\mu(x,-p')\Psi^\mu(x,p) - p^\mu\Psi_\mu(x,-p'){p'}^\nu\Psi_\nu(x,p)]\,,\end{aligned} \tag{9}$$

$$\begin{aligned}\mathcal{L}_{1/2} \;\equiv\;& q_{1/2}^{-1}\int {\rm d}^4p\,\bar{\Psi}_{1/2}(x,-p)[\Lambda t(p^2) + p^\mu\overleftrightarrow{\partial}_\mu]\Psi_{1/2}(x,p)\\ &-q_{1/2}^{-1}s_{1/2}\int {\rm d}^4p'\,{\rm d}^4p\,f({p'}^2)f(p^2)[\bar{\Psi}_{1/2}(x,-p')\gamma^\mu\Psi_{1/2}(x,p)p_\mu + {\rm c.c.}],\end{aligned} \tag{10}$$

$$A_\mu(x) \equiv \int {\rm d}^4p f(p^2)\Psi_\mu(x,p)\,, \qquad \psi(x) \equiv \int {\rm d}^4p f(p^2)\Psi_{1/2}(x,p)\,, \tag{11}$$

where: $\Psi_\mu(x,p)$ and $\Psi'_\mu(x,p)$ are four-vector-valued functions of two four-vectors x and p, $\Psi_{1/2}(x,p)$ is a bispinor-valued function of x and p, $2a\overleftrightarrow{\partial}_\mu b \equiv a(\partial_\mu b) - (\partial_\mu a)b$; $\bar{\Psi}_{1/2} \equiv \Psi^\dagger_{1/2}\gamma^4$, $t(p^2)$ and $f(p^2)$ are real-valued functions of real p^2, $\int {\rm d}^4p f^2(p^2) = 1$; q_1, s_1, $q_{1/2}$, $s_{1/2}$ and Λ are real constants, not auxiliary parameters.

There are three kinds of reasons for the chosen form (8–11) of the Lagrangian $\mathcal{L}_{\rm TR}$:

(A) It is the purpose of this paper to show that there are Lagrangians that generalize the 't Hooft and Veltman method of unitary regulators [3] to an infinity of additional fields *but do not introduce additional particles.* So we constructed the Lagrangian $\mathcal{L}_{\rm TR}$ modifying $\mathcal{L}_{\rm QED}$ on the analogy of HV-method [3]: (i) We introduced an infinity of four-vector and bispinor fields of x that have a continuous index p, namely $\Psi_\mu(x,p)$, $\Psi'_\mu(x,p)$ and $\Psi_{1/2}(x,p)$. (ii) We replaced the free part of $\mathcal{L}_{\rm QED}$ with the free Lagrangian of these fields, $-\mathcal{L}_1 - \mathcal{L}_{1/2}$, which is of the first order in ∂ and has a nondiagonal mass matrix. (iii) In the interaction and source terms of $\mathcal{L}_{\rm QED}$, we replaced the fields $A_\mu(x)$ and $\psi(x)$ with weighted integrals (11) of $\Psi_\mu(x,p)$ and $\Psi_{1/2}(x,p)$ over the continuous index p.

(B) We tried to simplify the calculations of regularized propagators. We could do without the four-vector function $\Psi'_\mu(x,p)$ which we introduced solely to be able to use the same functions $t(p^2)$ and $f(p^2)$ in $\mathcal{L}_1$ and $\mathcal{L}_{1/2}$. We introduced $\overleftrightarrow{\partial}$ to make $\mathcal{L}_{\rm TR}$ itself real-valued, not only its action real as required.

(C) The physical motivation for the type of Lagrangian we constructed, which we considered in detail in Ref. [8], is twofold: (i) The Euler-Lagrange equations of $\mathcal{L}_{\rm TR}$ resemble the Boltzmann integro-differential transport equation, which can better model rapidly varying, "ultra-high-energy", macroscopic fluid phenomena than the differential equations of motion of fluid dynamics. (To this end, it uses an infinity of fields to take some account of the underlying microscopic behaviour.) So the Euler-Lagrange equations of $\mathcal{L}_{\rm TR}$ may be regarded as classical transport equations of motion for the one-particle distribution of some infinitesimal entities, such as X-ons surmised to underly all physical phenomena by Feynman [9]. (ii) Ever since the EPR gedanken experiment, it is known that interpretations of certain quantum phenomena suggest the existence of causal faster-than-light effects. The Euler-Lagrange equations of $\mathcal{L}_{\rm TR}$ are the first Lorentz-invariant equations of motion that classicaly model such effects, because their retarded solutions have unbounded front velocities [8]. That is a major qualitative advantage of $\mathcal{L}_{\rm TR}$ over $\mathcal{L}_{\rm QED}$.

Using the Euler-Lagrange equations of $\mathcal{L}_{\rm TR}$ with $e = 0$ and proceeding as in Ref. [10], we calculate the causal dependence of $\Psi_\mu(x,p)$ and $\Psi'_\mu(x,p)$ on $J_\mu(x)$, and of $\Psi_{1/2}(x,p)$ on $J_e(x)$. Thereby we can infer that the Feynman propagator for the four-vector field $A_\mu(x)$ defined by (11) equals

$$-\mathrm{i}(2\pi)^{-4}\widetilde{g}_1\,\frac{\delta_{\mu\nu}+\widetilde{\mu}^{-2}k_\mu k_\nu}{k^2+\widetilde{\mu}^2}\,, \tag{12}$$

$$\widetilde{g}_1(k^2)\equiv q_1 s_1^{-2} I_{10} I_{20}^{-2}\,, \qquad \widetilde{\mu}(k^2)\equiv |s_1|^{-1} I_{20}^{-1}\,, \tag{13}$$

where $I_{mn}(k^2)$ is an analytic function of the complex variable k^2 such that

$$I_{mn}(k^2) = 2\pi^2\Lambda^{-m}\int_0^\infty y^{m+n} f^2(y) t^{-m}(y)[\sqrt{1+\Lambda^{-2}k^2 y t^{-2}(y)}+1]^{-m}\mathrm{d}y \tag{14}$$

for $k^2 > 0$; and the Feynman propagator for the bispinor field $\psi(x)$ defined by (11) equals

$$-\mathrm{i}(2\pi)^{-4}\widetilde{g}_{1/2}\,\frac{-\mathrm{i}\gamma^\mu k_\mu+\widetilde{m}}{k^2+\widetilde{m}^2}\,, \tag{15}$$

$$\widetilde{g}_{1/2}(k^2)\equiv q_{1/2} s_{1/2}^{-1} I_{10} I_{20}^{-1}\,, \qquad \widetilde{m}(k^2)\equiv s_{1/2}^{-1}\{1-s_{1/2}^2[I_{10}I_{11}+\tfrac{1}{4}k^2 I_{20}^2]\} I_{20}^{-1}\,; \tag{16}$$

where k^2 has to be replaced everywhere with $k^2-\mathrm{i}\epsilon$, by the Feynman prescription.

If functions $t(p^2)$ and $f(p^2)$ are such that

$$\int_0^\infty f^2(y)t(y)\,|\sqrt{y}/t(y)|^{l+1}\mathrm{d}y = 0 \tag{17}$$

for $l = 0, -1, \ldots, -n$, then for complex values of k as $|k^2| \to \infty$:

$$\left|\widetilde{g}_1 \frac{\delta_{\mu\nu} + \widetilde{\mu}^{-2}k_\mu k_\nu}{k^2 + \widetilde{\mu}^2}\right| = O(|k^2|^{(1-n)/2}), \qquad \left|\widetilde{g}_{1/2}\,\frac{\widetilde{m} - \mathrm{i}\gamma^\mu k_\mu}{k^2 + \widetilde{m}^2}\right| = O(|k^2|^{-n/2}). \tag{18}$$

When the function $y/t^2(y)$ takes only a finite number of real values v_i, $i = 1, 2, \ldots$, we can explicitly evaluate integrals (14); we obtain

$$I_{mn}(k^2) = \Lambda^{-m}\sum_i A_{mni}v_i^m[\sqrt{1 + \Lambda^{-2}v_i k^2} + 1]^{-m}\,, \tag{19}$$

where A_{mni} are real constants. Considering such a case, we can show that for any μ^2, m^2 and integer n, there exist functions $f(p^2)$ and $t(p^2)$, and constants s_1, $s_{1/2}$, q_1, $q_{1/2}$ and $\Lambda_0 > 0$ such that the propagators (12) and (15) with $\Lambda > \Lambda_0$ are regularizations of spin 1 and spin $\frac{1}{2}$ propagators (7) such that: (i) they have properties analogous to those of propagator (1), and (ii) there is a positive constant k_0^2 such that for all $k^2 \geq -\Lambda^2 k_0^2$ the functions $I_{mn}(k^2)$, $\widetilde{g}_{1/2}(k^2)$, $\widetilde{\mu}(k^2)$, $\widetilde{g}_1(k^2)$ and $\widetilde{m}(k^2)$ are real. In such a case: (i) The constants s_1, $s_{1/2}$, q_1 and $q_{1/2}$ are such that

$$\widetilde{\mu}^2(-\mu^2) = \mu^2\,, \qquad \widetilde{m}^2(-m^2) = m^2\,, \tag{20}$$

$$\widetilde{g}_{1/2}(-\mu^2) = 1 + \mathrm{d}\widetilde{\mu}^2(k^2 = -\mu^2)/\mathrm{d}k^2, \quad \widetilde{g}_1(-m^2) = 1 + \mathrm{d}\widetilde{m}^2(k^2 = -m^2)/\mathrm{d}k^2. \tag{21}$$

So the propagators (12) and (15) have poles at $k^2 = -\mu^2$ and $k^2 = -m^2$, where their behaviour is given by the spin 1 and spin $\frac{1}{2}$ propagators (7) with $\epsilon = 0$. (ii) The difference between spin 1 propagator and propagator (12) depends on the value of Λ so that it satisfies relations analogous to (2); and the same goes for spin $\frac{1}{2}$ propagators. (iii) The propagators (12) and (15) are analytic functions of k^2 that (a) are not continuous everywhere across the negative real axis, (b) have no additional singularities to those of spin 1 and spin $\frac{1}{2}$ propagators (7), and (c) satisfy relations (18). For any integer $n \geq 3$, their Källén-Lehman integral representations are superconvergent: in x-space we can decompose the Feynman propagators (12) and (15) into positive and negative energy parts without contact terms [3].

As a consequence of (i) and (ii) above, the classical, inhomogeneous Maxwell equations can be obtained from the Euler-Lagrange equations of $\mathcal{L}_{\mathrm{TR}}$ with $J_\mu = 0$ and $J_e = 0$ and the definitions (11), by limiting $\Lambda \to 0$.

4. Realistic regularization of the QED Green functions

To obtain a perturbative S-matrix of quantum-electrodynamic phenomena based on the Lagrangian $\mathcal{L}_{\rm TR}$, say $S_{\rm TR}$, we use the 't Hooft-Veltman definition of an S-matrix [3]. In view of the results of Sect. 3, there are functions $f(p^2)$ and $t(p^2)$, and constants s_1, $s_{1/2}$, q_1, $q_{1/2}$ and Λ such that the n-point Green functions of $\mathcal{L}_{\rm TR}$ and the corresponding S-matrix $S_{\rm TR}$ have the following properties:

(i) As the Lagrangian $\mathcal{L}_{\rm TR}$ has the same interaction and source terms as the QED Lagrangian $\mathcal{L}_{\rm QED}$, they are expressed in terms of QED diagrams with the spin 1 and spin $\frac{1}{2}$ propagators (7) replaced with their regularizations (12) and (15), whereas the vertices are the same as in QED, i.e., $(2\pi)^4\gamma_\mu$; so all diagrams are finite.

(ii) To any order in the fine structure constant, the n-point Green functions are causal [3], charge and total four-momentum conserving, Lorentz-invariant and C-, P- and T-invariant up to a phase factor [11].

(iii) If not only the propagators (12) and (15) but also the higher-order two-point Green functions of $\mathcal{L}_{\rm TR}$ have no additional singularities, then $S_{\rm TR}$ relates the same particles as the S-matrix of QED with massive photons in unitary gauge: electrons and positrons, each with two possible polarization vectors, and massive photons with three possible polarization vectors; none of them with wrong metric or statistics. As the propagators (12) and (15) admit the Källén-Lehman representation, this scattering matrix $S_{\rm TR}$ is unitary to any order in the fine structure constant [3].

(iv) In the limit $\Lambda \to \infty$, the propagators (12) and (15) behave as sufficies for renormalization.

So, the perturbative n-point Green functions of $\mathcal{L}_{\rm TR}$ are the result of a rp-theory as defined in Sect. 1.

In view of (iv), we can compute by renormalization the renormalized n-point Green functions of QED with massless photons from the n-point Green functions of $\mathcal{L}_{\rm TR}$ by choosing an appropriate dependence of e, s_1, $s_{1/2}$, q_1 and $q_{1/2}$ on Λ, and then limiting $\Lambda \to \infty$ and the renormalized photon mass to zero [6].

5. Comments

Generalizing the 't Hooft and Veltman method of unitary regulators, we have shown, for the first time as far as we know, that one can regularize the QED Green functions in accordance with the basic tenets of theoretical physics by suitably modifying the free part of QED Lagrangian. As we mentioned in Sect. 3, the physical motivation for such modification has been the Feynman surmise about X-ons, the Boltzmann improvement on fluid dynamics by the transport theory based on his equation and interpretations of certain quantum-electrodynamic phenomena that suggest causal faster-than-light effects.

Within the framework of perturbative quantum field theory as defined by 't Hooft and Veltman [3], the Lagrangian $\mathcal{L}_{\mathrm{TR}}$ is related to the physical world solely through the perturbatively defined scattering matrix S_{TR}. We see no physical properties of S_{TR} that require the spectral function (4) and the Hamiltonians corresponding to free Lagrangians $\mathcal{L}_1$ and $\mathcal{L}_{1/2}$ (which are not free-particle Lagrangians) to be positive as they turn out to be within the framework of canonical formalism [1].

The need for a regularization of QED that would result in a realistic physical model was felt very strongly by the founders of QED, Dirac and Heisenberg, already some sixty-five years ago [1]. But neither they nor their contemporaries succeded in getting rid of the ultraviolet divergencies by a physicaly motivated modification of the QED Lagrangian. In the late 1940s, however, Tomonaga, Schwinger and Feynman "solved" the problem of QED ultraviolet divergencies through renormalization—a solution which does not require the preceding regularization to be realistic and removes all parameters characteristic of it. As they obtained spectacularly succesful formulas for quantum-electrodynamic phenomena, the problem of finding a realistic, Lagrangian-based regularization of the QED Green functions was not so urgent any more. As there had been no progress whatsoever towards a solution of this problem, it mainly came to be considered as practically unsolvable [1]; those who hoped otherwise were often considered "irrational", as Isham, Salam and Strathdee [12] complained twenty-five years later. Thus nowadays, as far as we know, no quantum-field theorist, excepting the string theorist, pays much attention to this problem, which many of the preceding generations—e.g., Dirac, Heisenberg, Landau, Pauli and Salam, to mention some—still hoped to be solved somehow someday [1, 4]. But the string theorists abandon one of the basic premises of conventional physics, the four-dimensionality of space-time. We have shown, however, that such drastic steps may be avoided when modifying QED Lagrangian to get rid of ultraviolet divergencies. But the question remains which modification of the type considered is the most appropriate for better describing quantum-electrodynamic phenomena and their faster-than-light effects than the conventional QED, and what is the content of such a perturbative theory.

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REALISTIČNA REGULARIZACIJA GREENOVIH FUNKCIJA QED

Generalizacijom unitarnih regulatora 't Hooft–Veltmanovom metodom pokazujemo po prvi puta postojanje lokalnih, Lorentz-invarijantnih i fizikalno motiviranih Lagrangiana za kvantno mehaničke pojave, za koje su: (i) Feynmanovi diagrami konačni i jednaki diagramima QED, ali s regulariziranim propagatorima. (ii) Greenove funkcije za n-točaka su C-, P-, i T-invariantne do na fazni faktor, Lorentz-invarijantne i kauzalne. (iii) Ne uvode se pomoćne čestice niti parametri.