ISSN 1330–0016
CODEN FIZBE7

HYPERNUCLEAR POTENTIALS AND THE PSEUDOSCALAR MESON EXCHANGE CONTRIBUTION

CESAR BARBERO[a], DUBRAVKO HORVAT[b], FRANJO KRMPOTIĆ[a], ZORAN NARANČIĆ[b] and DUBRAVKO TADIĆ[c]

[a] *Departamento de Física, Facultad de Ciencias, Universidad Nacional de La Plata, C. C. 67, 1900 La Plata, Argentina*

[b] *Department of Physics, Faculty of Electrical Engineering, University of Zagreb, 10000 Zagreb, Croatia*

[c] *Physics Department, University of Zagreb, 10000 Zagreb, Croatia*

E-mail addresses: barbero@venus.fisica.unlp.edu.ar, dubravko.horvat@fer.hr, krmpotic@venus.fisica.unlp.edu.ar, tadic@phy.hr

Received 21 June 2000; revised manuscript received 7 June 2001
Accepted 18 June 2001

The pieces of the hypernuclear strangeness violating potential due to the pseudoscalar meson exchanges are derived using methods which were successfully applied to hyperon nonleptonic decays. The estimates are tested by comparison with measured hyperon nonleptonic decay amplitudes. All isospin changes $\Delta I = 1/2$ and $\Delta I = 3/2$ are included in the derived potential. All computational methods used are reviewed and described in detail.

PACS numbers: 14.20.Jn, 13.30.-a UDC 539.125, 539.126

Keywords: hyperon nonleptonic decays, potential violating hypernuclear strangeness, exchanges of pseudoscalar mesons

1. *Introduction*

Hypernuclei ${}^{A}_{\Lambda}$N, heavier than ${}^{5}_{\Lambda}$He, decay through mesonic and nonmesonic channels. Here we will produce strangeness violating potentials which can be used in the theoretical description of nonmesonic channels. In these channels, the Λ mass excess of 176 MeV is converted into the kinetic energy of the final two-nucleon state. This review is dealing with nuclear potentials induced by the pseudoscalar meson exchanges, such as π, K and η only. The potential pieces due to the vector and/or axial-vector meson exchanges will be studied along the similar lines in the following paper [1]. The same goes for the calculations of the hypernuclear decay widths [2].

In hypernuclear decays, one can study the parity conserving (PC) and parity violating (PV) parts of the weak interaction. In that sense, the nonmesonic decay

$$\begin{array}{lcll} \Lambda + N & \rightarrow & N + N & \text{or} \\ \Lambda + p^+ & \rightarrow & p^+ + n^0 & \text{and} \\ \Lambda + n^0 & \rightarrow & n^0 + n^0 & \end{array} \tag{1.1}$$

is the $\Delta S = 1$ analogy of the weak NN→NN nuclear PV reaction. However, the weak PC ΛN→NN decay is also observable in experiments. Experimental hypernuclear programs at BNL, KEK, CEBAF, DUBNA and DAΦNE promise enough data for exhaustive hypernuclear studies [3–7].

In this paper, we will employ the meson exchange mechanism to induce the effective baryon-baryon (hyperon) potential, as shown in Fig. 1.1.

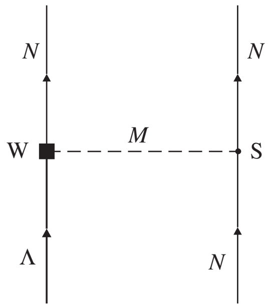

Fig. 1.1. One meson baryon-baryon exchange diagram. N *and* Λ *are* $S = 0$ *and* $S = -1$ *baryons and* M *is a non-strange pseudoscalar meson. S and W are the strong and weak vertices.*

The weak vertex W for the pseudoscalar meson exchange is closely connected with the theory of hyperon nonleptonic decays [8–15]. These decays have been studied and analyzed in detail. Here we will review a particular theoretical approach [8,13,14]. The related methods have already been applied [5] in estimates of the "new" $\Delta S = 1$ vertices, such as NNK which appear in processes (1.1) and which are shown in Fig. 1.1. We intend to include all known and relevant contributions except for the decuplet poles [16].

The strength of a weak vertex W

$$W = \mathrm{i}\bar{u}(p_N)(A - B\gamma_5)u(p_\Lambda) \tag{1.2}$$

can be parameterized by the A and B amplitudes. For certain weak vertices, such as $\Lambda \rightarrow \mathrm{N} + \pi$ for example, the amplitudes A and B can be taken directly from the experiment. In the experimentally measured decay, all particles are on the mass shell. That is not necessarily so in the case of the exchange interaction Fig. 1.1

inside the nuclear matter. However, all possible corrections will be neglected in the following. Theoretical uncertainties, as explained in detail below, are so large that such niceties have no practical importance.

One has to rely entirely on the theory when dealing with the weak vertex corresponding to $\mathrm{N} \to \mathrm{N} + \mathrm{K}$ and $\Lambda \to \mathrm{N} + \eta$ transitions. In that sense, the investigation of the strangeness-violating nuclear interaction is a welcome test for the general theoretical understanding of the hyperon nonleptonic processes. Use will be made of theoretical schemes which were developed for the hyperon nonleptonic decays [13,14,17,18] and for the deduction of the weak parity violating nuclear potential [19,20]. In order to check the theoretical accuracy, the analysis will be carried out for some of directly measurable amplitudes. For the strong meson vertices, the standard form and the flavour SU(3) symmetry is assumed.

The full weak Hamiltonian will be used without any assumptions about octet (i.e. $\Delta I = 1/2$) dominance. Thus, our derived potential (9.11) below, contains both isospin changes: $\Delta I = 1/2$ and $\Delta I = 3/2$.

In the eighties, the study of the hyperon nonleptonic decays had reached certain level of success based on current algebra and pole dominance. Some results [14,21] were elaborated in monographs and reviews [8,11,13]. The theoretically consistent and complete (as far as the mentioned approximations go) results are obtained by combining contributions which were previously used in either one or the other of discussed works. For example, in Ref. [21], the baryon and meson poles were used for describing the $p-$wave (B) amplitudes. On the other hand, the methods employed in Refs. [14,17,18] relied on the baryon poles and separable (factorizable) contributions. It turns out, as discussed in detail below, that one has to combine all three contributions in a well defined way.

Separable contributions associated with operators $\mathcal{O}_1 \ldots, \mathcal{O}_4$ (see Sec. 2 below) contain an axial vector-current matrix element

$$\langle B_f | \, A^a_\mu \, | B_i \rangle \sim \overline{u}_{B_f} [\gamma_\mu \gamma_5 g_A + \mathrm{i} q_\mu \gamma_5 g_P] u_{B_i} \, . \tag{1.3}$$

Here, g_A is the form factor associated with the axial vector meson exchanges [8,22,23] and thus it should be included as “an axial-vector meson-pole contribution”. The term g_P was not included in the earlier estimate [14,21]. It is dominated by the pseudoscalar meson (i.e. kaon) pole, so that its contribution is included in the more general kaon pole contributions. Separable contributions from operators $\mathcal{O}_5$ and $\mathcal{O}_6$ are, as shown below, contained in the meson-pole term.

The parity violating (s–wave) amplitudes were estimated in Ref. [21] using the current-algebra terms (CAT) and the contributions coming from the commutators involving $\mathcal{O}_5$ and $\mathcal{O}_6$ operators. In the case of the p–wave amplitudes, the analogous terms are included in the kaon pole pieces [13,21]. In Refs. [14,17,18], the CAT and the factorizable contributions were used. Obviously, the complete estimate, involving leading poles, should contain everything.

The weak PV (A) amplitude contains *current algebra* and *separable* parts, whereas the PC (B) amplitude gets its contributions from *pole terms* and *separable* parts, i.e.

$$\begin{aligned} A &= A_{\mathrm{CA}} + A_{\mathrm{SEP}} \\ B &= B_{\mathrm{POLE}} + B_{\mathrm{SEP}} \, , \end{aligned} \tag{1.4a}$$

or more precisely

$$\begin{aligned} A &= \sum_{a=\pi,K,\eta} \left[A_{\mathrm{CA}/a} + A_{\mathrm{SEP}/a}\right] \\ B &= \sum_{a=\pi,K,\eta} \left[B_{\mathrm{POLE}/a} + B_{\mathrm{SEP}/a}\right] . \end{aligned} \tag{1.4b}$$

Thus, in comparison with the experimental data, one has to use the complete theoretical expression, as for example

$$A(\exp) = A_{\mathrm{CA}} + A_{\mathrm{SEP}}. \tag{1.5}$$

That determines the values of the matrix elements (5.20) etc. below, as will be explained in detail in Sec. 7.

The following sections will be devoted to specific *pion, kaon* and $\eta-$contributions to PV and PC weak Λ-decay amplitudes. The separable pieces are given in Sec. 3 while the baryon-pole contributions are described in Sec. 4. Sec. 5 is devoted to the so called current-algebra contributions. The connection between separable contributions and the meson poles is discussed in Sec. 6. Sec. 7 compares the calculated weak BBM amplitudes with the measured ones. The theoretically predicted NNK and NNη amplitudes are compared with some other theoretical results. The derivation of the nonrelativistic weak potentials is described in Sec. 8. The $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces are listed separately. The effective $\Delta S = 1$ nonrelativistic potential, which can serve as the input in nuclear calculations, is listed in Sec. 9. Various theoretical details can be found in numerous appendices. Closely related theoretical methods were used in Ref. [5]. Our approach investigates the importance of additional contributions, such as separable terms, kaon poles and so on. Some attention will be paid to the so called “double counting problem” [11,18] by comparing various contributions to B amplitudes.

The weak NNK interactions were investigated using heavy-baryon chiral perturbation theory also [22]. Their results will be compared with results obtained by methods corresponding to the scheme outlined by formulae (1.4). We are hoping that this contribution will illustrate the theoretical uncertainty in the evaluation of NNK and N$\Lambda\eta$ weak vertices which, contrary to N$\Lambda\pi$ vertices, cannot be read directly from the experimental data.

2. *Weak and strong Hamiltonians*

The weak one pion exchange potential can be extracted from the Feynman amplitude shown in Fig. 1.1. This diagram corresponds to a second-order term in the S-matrix expansion. The details of that expansion, performed in an effective field theory, can be found in Sec. 8 below. Here we want to specify the strength of the weak (i.e. A, B) and of the strong vertices (i.e. $g_{B'BM}$). The calculation of the weak strengths A and B, which correspond to the hyperon nonleptonic decay amplitudes [13,14,17,18,21], will be the main topic of the following Sections 3–6.

The weak vertices are determined by the effective weak $\Delta|S| = 1$ Hamiltonian, which in the quark basis is given by [23,24]

$$\mathcal{H}_{\text{eff.}}^{(W)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i \mathcal{O}_i \,. \tag{2.1}$$

This can be conveniently written as

$$\mathcal{H}_{\text{eff}}^{(W)} = -\sqrt{2} G_F \sin\theta_C \cos\theta_C \sum_i C_i \mathcal{O}_i \,. \tag{2.2}$$

Here the C_i's are the QCD Wilson coefficients [23] calculated in the six-quark Standard Model and evaluated at the scale $\mu = 0.5$ GeV. Their values are given in Table 2.1. G_F is Fermi constant and $\sin\theta_C$ and $\cos\theta_C$ (later denoted by s_C and c_C) are the sine and cosine of the Cabibbo angle.

Table 2.1. Wilson coefficients evaluated at the scale $\mu = 0.5$ GeV.

C_1	C_2	C_3	C_4	C_5	C_6
-2.358	0.080	0.082	0.411	-0.080	-0.021

The four-quark $(V - A)$ operators in the effective weak Hamiltonian (2.1) are

$$\begin{aligned}
\mathcal{O}_1 &= :(\bar{d}_L\gamma_\mu s_L)(\bar{u}_L\gamma^\mu u_L) - (\bar{d}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu s_L): && (8,\ 1/2)\\
\mathcal{O}_2 &= :(\bar{d}_L\gamma_\mu s_L)(\bar{u}_L\gamma^\mu u_L) + (\bar{d}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu s_L)\\
&\quad +2(\bar{d}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu d_L) + 2(\bar{d}_L\gamma_\mu s_L)(\bar{s}_L\gamma^\mu s_L): && (8,\ 1/2)\\
\mathcal{O}_3 &= :(\bar{d}_L\gamma_\mu s_L)(\bar{u}_L\gamma^\mu u_L) + (\bar{d}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu s_L)\\
&\quad +2(\bar{d}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu d_L) - 3(\bar{d}_L\gamma_\mu s_L)(\bar{s}_L\gamma^\mu s_L): && (27,\ 1/2)\\
\mathcal{O}_4 &= :(\bar{d}_L\gamma_\mu s_L)(\bar{u}_L\gamma^\mu u_L) + (\bar{d}_L\gamma_\mu u_L)(\bar{u}_L\gamma^\mu s_L)\\
&\quad -(\bar{d}_L\gamma_\mu s_L)(\bar{d}_L\gamma^\mu d_L): && (27,\ 3/2)\\
\mathcal{O}_5 &= :(\bar{d}_L\gamma_\mu\lambda_A s_L)(\bar{u}_R\gamma^\mu\lambda_A u_R) + (\bar{d}_L\gamma_\mu\lambda_A s_L)(\bar{d}_R\gamma^\mu\lambda_A d_R)\\
&\quad +(\bar{d}_L\gamma_\mu\lambda_A s_L)(\bar{s}_R\gamma^\mu\lambda_A s_R): && (8,\ 1/2)\\
\mathcal{O}_6 &= :(\bar{d}_L\gamma_\mu s_L)(\bar{u}_R\gamma^\mu u_R) + (\bar{d}_L\gamma_\mu s_L)(\bar{d}_R\gamma^\mu d_R)\\
&\quad +(\bar{d}_L\gamma_\mu s_L)(\bar{s}_R\gamma^\mu s_R): && (8,\ 1/2)
\end{aligned} \tag{2.3}$$

All operators in (2.3) are normal ordered and their SU(3) (*flavour representation, isospin*) content is also indicated. Moreover, the following notation is used

$$u_L \equiv \frac{1}{2}(1-\gamma_5)u; \quad u_R \equiv \frac{1}{2}(1+\gamma_5)u\,. \tag{2.4a}$$

So, for instance,

$$\begin{aligned}
(\overline{d}_L\gamma_\mu s_L)(\overline{u}_L\gamma^\mu u_L) &\equiv [\overline{d}\tfrac{1}{2}\gamma^\mu(1-\gamma_5)s][\overline{u}\tfrac{1}{2}\gamma_\mu(1-\gamma 5)u] \\
&= \tfrac{1}{4}[\overline{d}\gamma_\mu(1-\gamma_5)s][\overline{u}\gamma^\mu(1-\gamma_5)u] \\
&= \tfrac{1}{4}(\overline{d}s)_{(V-A)}(\overline{u}u)_{(V-A)}, \quad \text{and} \\
(\overline{d}_R\gamma_\mu d_R) &\equiv [\overline{d}\tfrac{1}{2}(1-\gamma_5)\gamma_\mu\tfrac{1}{2}(1+\gamma_5)d] \\
&= \tfrac{1}{2}\overline{d}\gamma_\mu(1+\gamma_5)d\,.
\end{aligned} \tag{2.4b}$$

The effective baryon-baryon-meson strong and weak vertices are given, respectively, by

$$\mathcal{H}^{(S)}_{NN\pi} = \mathrm{i}g_{NN\pi}\overline{\Psi}_N\gamma_5\Psi_N\Phi_\pi \tag{2.5}$$

and

$$\mathcal{H}^{(W)}_{\Lambda N\pi} = \mathrm{i}G_F m_\pi^2\overline{\Psi}_N(A-B\gamma_5)\Psi^S_\Lambda\Phi_\pi\,. \tag{2.6}$$

The value of G_F is

$$G_F m^2_{\pi^+} = 2.21\times 10^{-7} \qquad \text{i.e.} \qquad G_F = 1.16639\times 10^{-5}\,\mathrm{GeV}^{-2}. \tag{2.7}$$

Furthermore, Ψ^S_Λ is the isospin spurion $\begin{pmatrix}0\\1\end{pmatrix}\Psi_\Lambda$, which enforces the $\Delta I = 1/2$ rule. The weak amplitudes A and B can be taken from experiments. But, to check the theoretical methods used in the evaluation of NNK weak vertices, they also have been estimated theoretically (see Table 7.1).

The strong-coupling constants can be inferred from their the SU(3) symmetry, and the following results are obtained (see for instance [25–28]):

- NNπ : $g_{pn\pi^+} = \sqrt{2}g_{NN\pi}$; $g_{nn\pi^0} = -g_{NN\pi}$; $g_{pp\pi^0} = g_{NN\pi}$;
- NΣK : $g_{nK^+\Sigma^-} = \sqrt{2}g_{N\Sigma K}$; $g_{\overline{p}K^0\Sigma^-} = \sqrt{2}g_{N\Sigma K}$; $g_{\overline{p}K^+\Sigma^0} = g_{N\Sigma K}$; $g_{nK^0\Sigma^0} = -g_{N\Sigma K}$;
- $\Xi\Xi\pi$: $g_{\Xi^-\overline{\Xi^0}\pi^+} = -\sqrt{2}g_{\Xi\Xi\pi}$; $g_{\Xi^-\overline{\Xi^-}\pi^0} = -g_{\Xi\Xi\pi}$; $g_{\Xi^0\Xi^0\pi^0} = g_{\Xi\Xi\pi}$;
- NKΛ : $g_{pK^+\Lambda} = g_{\overline{n}K^0\Lambda} = g_{NK\Lambda}$;
- $\Lambda\Sigma\pi$: $g_{\Lambda\Sigma^+\pi^-} = g_{\Lambda\Sigma^-\pi^+} = g_{\Lambda\Sigma^0\pi^0} = g_{\Lambda\Sigma\pi}$.

Sometimes one also uses the parameter $\alpha = 1 - f$. The values of α are, approximately, in the range 0.44-0.6 [25].

Table 2.2. Strong-coupling constants in units of g ($g_{\pi NN} = g = 13.3$, $f = 0.4$, $f + d = 1$.

$M\,B\,B'$	πNN	$\pi\Lambda\Sigma$	$\pi\Sigma\Sigma$	$\pi\Xi\Xi$	KΛN	K$\Xi\Lambda$	KΣN
$g_{MBB'}$	1	$\frac{2(1-f)}{\sqrt{3}}$	$2f$	$(2f-1)$	$\frac{-(1+2f)}{\sqrt{3}}$	$\frac{4f-1}{\sqrt{3}}$	$1-2f$

3. Separable contributions

In this section, we calculate the separable contributions to A and B amplitudes. In the quark basis, they are represented by the diagrams (a) and (b) displayed in Fig. 3.1. Note that the four quarks emerging from the black dot are engendered by the four-quark operators given by (2.1) or (2.2).

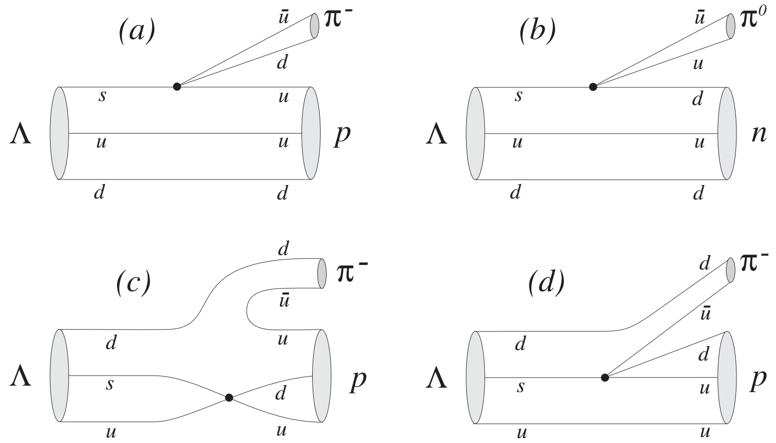

Fig. 3.1. Separable (diagrams (a) and (b)) and non-separable (diagrams (c) and (d)) contributions in Λ decays in a proton (p) and a neutron (n). $\mathcal{H}_{\rm eff}^{(W)}$ acts at the black dot.

In order to account for all possible processes, one has to perform the Fierz transformations (FT) on the operators $\mathcal{O}_i$ and average them over different colours. This averaging procedure brings in a factor of 1/3. The two relations that govern all FT, are

$$\begin{aligned}
[\overline{q}_1\gamma_\mu(1-\gamma_5)q_2]\;[\overline{q}_3\gamma^\mu(1-\gamma_5)q_4] &= [\overline{q}_1\gamma_\mu(1-\gamma_5)q_4]\;[\overline{q}_3\gamma^\mu(1-\gamma_5)q_2] \\
[\overline{q}_1\gamma_\mu(1-\gamma_5)q_2]\;[\overline{q}_3\gamma^\mu(1+\gamma_5)q_4] &= -2[\overline{q}_1(1+\gamma_5)q_4]\;[\overline{q}_3(1-\gamma_5)q_2]\,.
\end{aligned} \tag{3.1}$$

Note that the anticommutativity of fermion fields has been taken into account in (3.1). The Fierz identities for eight Gell-Mann SU(3) λ-matrices, which are necessary for the calculations are

$$\begin{aligned}\lambda^a_b \cdot \lambda^c_d &= \tfrac{16}{9}\delta^a_d\delta^c_b - \tfrac{1}{3}\lambda^a_d\lambda^c_b \\ \lambda^a_d \cdot \lambda^c_b &= 2\delta^a_b\delta^c_d - \tfrac{2}{3}\delta^a_d\delta^c_b \,.\end{aligned} \tag{3.2}$$

The Fierz transformed operators $\mathcal{O}_i^{\rm FT}$ can be expressed in terms of the original $\mathcal{O}_i$ operators:

$$\begin{aligned}\mathcal{O}_1^{\rm FT} &= -\mathcal{O}_1, \quad \text{so that} \quad \mathcal{O}_1 \to \mathcal{O}_1 + \tfrac{1}{3}\mathcal{O}_1^{\rm FT} = \left(1-\tfrac{1}{3}\right)\mathcal{O}_1 \\ \mathcal{O}_2^{\rm FT} &= \mathcal{O}_2, \quad \text{so that} \quad \mathcal{O}_2 \to \mathcal{O}_2 + \tfrac{1}{3}\mathcal{O}_2^{\rm FT} = \left(1+\tfrac{1}{3}\right)\mathcal{O}_2 \\ \mathcal{O}_3^{\rm FT} &= \mathcal{O}_3, \quad \text{so that} \quad \mathcal{O}_3 \to \mathcal{O}_3 + \tfrac{1}{3}\mathcal{O}_3^{\rm FT} = \left(1+\tfrac{1}{3}\right)\mathcal{O}_3 \\ \mathcal{O}_4^{\rm FT} &= \mathcal{O}_4, \quad \text{so that} \quad \mathcal{O}_4 \to \mathcal{O}_4 + \tfrac{1}{3}\mathcal{O}_4^{\rm FT} = \left(1+\tfrac{1}{3}\right)\mathcal{O}_4 \\ \mathcal{O}_6^{\rm FT} &= -2(\overline{d}_L q^i_R)(\overline{q}^i_R s_L), \quad \text{so that} \quad \mathcal{O}_6 \to \mathcal{O}_6 + \tfrac{1}{3}\mathcal{O}_6^{\rm FT} \\ &= \mathcal{O}_6 - \tfrac{2}{3}(\overline{d}_L q^i_R)(\overline{q}^i_R s_L) \\ \mathcal{O}_5^{\rm FT} &= \tfrac{3}{16}\mathcal{O}_6 \quad \text{so that} \quad \mathcal{O}_5 \to \mathcal{O}_5 + \mathcal{O}_5^{\rm FT} = \mathcal{O}_5 + \tfrac{3}{16}\mathcal{O}_6 \,.\end{aligned} \tag{3.3}$$

There is also a simple relation between a matrix element of the operators $\mathcal{O}_5$ and $\mathcal{O}_6$ which is used frequently. It reads:

$$\langle B'|\mathcal{O}_5|B\rangle = \frac{16}{3}\langle B'|\mathcal{O}_6|B\rangle \,. \tag{3.4}$$

The following quark combinations occur in the operators $\mathcal{O}_{5,6}^{\rm FT}$

$$\begin{aligned}(\overline{d}_L s_R)(\overline{d}_R d_L) &\equiv [\overline{d}\tfrac{1}{2}(1+\gamma_5)\tfrac{1}{2}(1+\gamma_5)s][\overline{d}\tfrac{1}{2}(1-\gamma_5)\tfrac{1}{2}(1-\gamma_5)d] \\ &= \tfrac{1}{4}[\overline{d}(1+\gamma_5)s][\overline{d}(1-\gamma_5)d] \,.\end{aligned} \tag{3.5}$$

We also note that from the Dirac equation for quark fields one gets useful relations for the evaluation of the matrix elements of scalar and pseudo-scalar densities, occurring in the operators $\mathcal{O}_5^{\rm FT}$ and $\mathcal{O}_6^{\rm FT}$ (see (3.3) and (3.5)). For instance,

$$\begin{aligned}\overline{d}\gamma_5 u &= (p_d - p'_u)_\mu \overline{d}(p_d)\gamma^\mu\gamma_5 u(p'_u)\ \frac{1}{m_d + m_u} \\ \overline{u}s &= (p_u - p'_s)_\mu \overline{u}(p_u)\gamma^\mu s(p'_s)\ \frac{1}{m_u - m_s} \,.\end{aligned} \tag{3.6}$$

We now briefly outline the procedure that has been followed in the evaluation of the the weak-decay amplitudes, that correspond to the weak vertex in Fig. 1.1. The main assumption is that the invariant decay amplitude for the process (baryon→baryon+meson)

$$B \to B' + M \tag{3.7}$$

is factorizable as

$$\langle M, B'|\, \mathcal{H}_{\rm eff}\, |B\rangle = -\sqrt{2} G_F \sin\theta_C \cos\theta_C \sum_i C_i \langle M|\, (V-A)_i\, |0\rangle \langle B'|\, (V-A)_i\, |B\rangle\,, \tag{3.8}$$

where the currents $(V-A)_i$ account for both: (i) the creation of the meson state $|M\rangle$ from the vacuum $|0\rangle$, and (ii) the transition between two octet baryons B and B'. The character of the meson matrix element $\langle M|\,(V-A)\,|0\rangle$ depends on the meson state. Namely, if M belongs to the SU(3) pseudo-scalar octet, only the A part of the four-quark operator contributes. (When a vector meson is considered only the V part remains.) For the baryon matrix element, both V and A parts occur giving PV or PC contributions. Baryon matrix elements are expressed in terms of vector/axial-vector form factors $v_{BB'}/a_{BB'}$ which occur in the semileptonic baryon decays. These form factors are expressible in terms of F and D constants. The form factors are given in Table 3.1.

Table 3.1. Nucleon vector and axial-vector form factors; $F = 0.461 \pm 0.014$, $D = 0.798 \pm 0.013$.

B B'	SU(3) content	$v_{NN'}$	$a_{NN'}$
n p	$\tilde{\pi}^-$	1	$F+D$
n n	$\tilde{\pi}^0$	$-1/\sqrt{2}$	$-[1/\sqrt{2}](F+D)$
p p	$\tilde{\pi}^0$	$1/\sqrt{2}$	$[1/\sqrt{2}](F+D)$
p p	$\tilde{\eta}_8$	$3/\sqrt{6}$	$[1/\sqrt{6}](3F-D)$
n n	$\tilde{\eta}_8$	$3/\sqrt{6}$	$[1/\sqrt{6}](3F-D)$
n n	$\overline{u}u$	1	$F-D$
n n	$\overline{d}d$	2	$2F$
n n	$\tilde{\eta}_1$	$3/\sqrt{3}$	$[1/\sqrt{3}](3F-D)$
p p	$\tilde{\eta}_1$	$3/\sqrt{3}$	$[1/\sqrt{3}](3F-D)$
Λ p	K^-	$-3/\sqrt{6}$	$-[1/\sqrt{6}](3F+D)$
Λ n	$-\overline{\mathrm{K}^0}$	$-3/\sqrt{6}$	$-[1/\sqrt{6}](3F+D)$

Here we define

$$\langle B'|\, V_a^\mu\, |B\rangle = \overline{u}_{B'}(p')\gamma^\mu v_{BB'}^a u_B(p) \tag{3.9}$$

and

$$\langle B'|\, A_a^\mu\, |B\rangle = \overline{u}_{B'}(p')\gamma^\mu \gamma_5 a_{BB'}^a u_B(p)\,. \tag{3.10}$$

A very important ingredient in the above outlined calculation is the knowledge of the meson matrix element. For the pseudo-scalar (PS) mesons (π, η, K), this matrix element is evaluated from the partially conserved axial-vector current (PCAC) hypothesis [8,9,11]. In the case of vector mesons, it is obtained either from the current-field identity (CFI) [19,29] or from the meson-nucleon $\sigma-$term [30-32].

The PCAC hypothesis is based on the divergence of the axial-vector current

$$A_a^\mu = \overline{\psi}\gamma^\mu\gamma_5\frac{\lambda^a}{2}\psi\,, \tag{3.11}$$

where λ_a is the well known SU(3) matrix [9,26]. The divergence of the operator (3.11) is applied to a (physical) state represented by a *ket* (or *bra*), i.e.

$$\partial_\mu A_a^\mu|0\rangle = \mathrm{i} f_\phi m_\phi^2 \phi^a|0\rangle\,. \tag{3.12}$$

As the form of the meson matrix elements is given by (3.8), the ϕ^a operator has to create a state $|M\rangle$, i.e.

$$\partial_\mu A_a^\mu|0\rangle \sim |M\rangle\,. \tag{3.13}$$

The axial vector current (3.11) transforms as

$$\partial_\mu A_a^\mu \sim |M\rangle \sim \overline{q}\frac{\lambda_a}{\sqrt{2}}q\,. \tag{3.14}$$

That gives

$$\begin{array}{lll} a+\mathrm{i}b & \overline{q}\,\dfrac{\lambda_{a+\mathrm{i}b}}{\sqrt{2}}q & |M\rangle \\ 1+\mathrm{i}2 & \dfrac{1}{\sqrt{2}}\overline{u}d & |\pi^+\rangle \\ 1-\mathrm{i}2 & \dfrac{1}{\sqrt{2}}\overline{d}u & |\pi^-\rangle\,. \end{array} \tag{3.15}$$

Here the combination which transforms as $\overline{u}d$ creates the $|\pi^+\rangle$ states and annihilates the $|\pi^-\rangle$ state. Usually, such field is denoted as ϕ_{π^-}. Proceeding in the same way, one finds

$$\begin{array}{llll} \partial_\mu A_3^\mu|0\rangle & = \frac{\sqrt{2}}{2} f_\pi m_\pi^2|\pi^0\rangle & \partial_\mu A_{1+\mathrm{i}2}^\mu|0\rangle & = f_\pi m_\pi^2|\pi^+\rangle \\ \partial_\mu A_{1-\mathrm{i}2}^\mu|0\rangle & = f_\pi m_\pi^2|\pi^-\rangle & \partial_\mu A_{4+\mathrm{i}5}^\mu|0\rangle & = f_K m_K^2|K^+\rangle \\ \partial_\mu A_{6+\mathrm{i}7}^\mu|0\rangle & = f_K m_K^2|K^0\rangle & \partial_\mu A_8^\mu|0\rangle & = \frac{\sqrt{2}}{2} f_\eta m_\eta^2|\eta\rangle\,. \end{array} \tag{3.16}$$

The above relations enable us to relate the following matrix elements

$$\langle\pi^0|\,\overline{u}\gamma_\mu\gamma_5 u\,|0\rangle = -\langle\pi^0|\,\overline{d}\gamma_\mu\gamma_5 d\,|0\rangle = \frac{\sqrt{2}}{2}\langle\pi^-|\,\overline{d}\gamma_\mu\gamma_5 u\,|0\rangle \tag{3.17}$$

etc. As an example, we list the separable contribution

$$\langle \pi^- p| H_{\rm eff} |\Lambda\rangle = -\sqrt{2} G_F \sin\theta_C \cos\theta_C \sum_i C_i \langle \pi^-| J^i_\mu |0\rangle\langle p| J_i^\mu |\Lambda\rangle \,. \tag{3.18}$$

Particular forms of J_i^μ are to be read from (2.3). For example, the first operator $\mathcal{O}_1$ gives

$$\begin{aligned}\langle \pi^-| J^i_\mu |0\rangle\langle p| J_i^\mu |\Lambda\rangle = & \langle \pi^-| \overline{d}_L\gamma_\mu u_L |0\rangle\langle p| \overline{u}_L\gamma^\mu s_L |\Lambda\rangle - \\ & -\tfrac{1}{3}\langle \pi^-| \overline{d}_L\gamma_\mu u_L |0\rangle\langle p| \overline{u}_L\gamma^\mu s_L |\Lambda\rangle \,.\end{aligned} \tag{3.19}$$

Here the second term was obtained by the Fierz transformation while the first one was directly read from (2.3). The final result is

$$\begin{aligned}\langle \pi^-| J^i_\mu |0\rangle\langle p| J_i^\mu |\Lambda\rangle &= \tfrac{2}{3}\left(-\tfrac{1}{4}\right)\langle \pi^-| \overline{d}\gamma_\mu\gamma_5 u |0\rangle\langle p| \overline{u}\gamma^\mu(1-\gamma_5) s |\Lambda\rangle \\ &= -\mathrm{i}\,\frac{f_\pi}{2\sqrt{6}}(m_\Lambda - m_p)\overline{u}_p(1-\gamma_5)u_\Lambda \,.\end{aligned} \tag{3.20}$$

4. *Baryon pole contributions*

The PC amplitude B gets a part of its contributions from the *baryon pole terms*, which are represented in Fig. 4.1 for the case of the Λ decay.

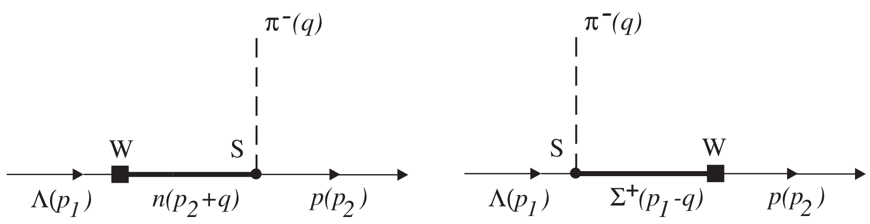

Fig. 4.1. Contributions of baryon-pole terms: (a) s-channel and (b) u-channel.

In order to illustrate the calculation of the baryon pole, we treat now a particular example in full detail. Two possible Λ-decays, which almost fully saturate the Λ width, are

$$\begin{aligned}\Lambda^0_0 : &\qquad \Lambda^0 \to n + \pi^0 \\ \Lambda^0_- : &\qquad \Lambda^0 \to p + \pi^- \,.\end{aligned} \tag{4.1}$$

After assigning the momenta to incoming and outgoing particles, i.e.

$$\Lambda(p_1) \underset{\rm S}{\longrightarrow} \pi^-(q) + \Sigma^+(p_1 - q) \underset{\rm W}{\longrightarrow} p(p_2) \,, \tag{4.2}$$

we calculate the Feynman amplitude relative to the $u-$channel, employing the matrix element (5.26) as follows

$$\overline{u}(p_2)\, a_{\Sigma^+_p} \frac{(\not p_1 - \not q) + m_\Sigma}{(p_1 - q)^2 - m_\Sigma^2} \gamma_5 g_{\Sigma^+\pi^-\Lambda} u(p_1)\,. \tag{4.3}$$

Here $g_{\Sigma\pi\Lambda}$ is the strong coupling constant listed in Table 2.2. Since $p_1 - q = p_2$ and $p_2^2 = m_p^2$, we get

$$\overline{u}(p_2)\, a_{\Sigma\, p} \frac{m_p + m_\Sigma}{(m_p - m_\Sigma)(m_p + m_\Sigma)} \gamma_5 g_{\Sigma\pi\Lambda} u(p_1)\,. \tag{4.4}$$

The result for $B(\Lambda^0_-)$ amplitude, including both the u and s channels, is

$$B^{\rm POLE}(\Lambda^0_-) = g_{\Lambda\Sigma^+\pi^-} \frac{a_{\Sigma^+ p}}{\Sigma^+ - p} + g_{n\pi^- p} \frac{a_{\Lambda n}}{n - \Lambda}\,. \tag{4.5}$$

In the soft pion limit, which is used to determine the a_{ij} amplitude (see Sec. 5 below), the B amplitude has to vanish. This imposes the condition that implies the *subtraction* of the soft amplitude $B^{\rm POLE}(q^2 = 0)$, i.e.

$$B^{\rm POLE}(q^2) - B^{\rm POLE}(0) \equiv B^{\rm POLE}\,(\text{of Eqs. (4.7) and (4.8)})\,. \tag{4.6}$$

Here q^2 is the pion (K, η) four-momentum. In Ref. [3] only $B^{\rm POLE}(q^2)$ was used, which explains some numerical differences that appear in the Tables 7.1, 7.3 and 7.4. We find:

$$B^{\rm POLE}_\pi(\Lambda^0_-) = 2g(\Lambda + p) \left[\frac{a_{\Sigma p}}{\sqrt{3}} \frac{d}{(\Sigma^+ - p)(\Lambda + \Sigma^+)} - \frac{a_{\Lambda n}}{\sqrt{2}} \frac{f + d}{(\Lambda - n)(n + p)} \right] \tag{4.7}$$

$$B^{\rm POLE}_\pi(\Lambda^0_0) = -\frac{1}{\sqrt{2}} B^{\rm POLE}(\Lambda^0_-)\,. \tag{4.8}$$

In the above expression, the following notation is used for the masses: $\Lambda \equiv m_\Lambda$, $p \equiv m_p$, etc. Recall also that $f + d = 1$, so only f's appear in (4.8). (See Table 2.2.) In the same fashion, one can use the diagrams shown in Fig. 4.2 to calculate $B(\Lambda^0_0)$.

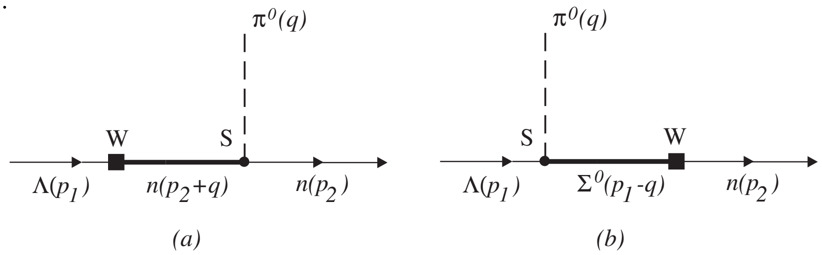

Fig. 4.2. Contributions of baryon-pole terms: (a) s-channel and (b) u-channel, for the π^0 emission.

If one calculates $B_\pi^{\mathrm{POLE}}(\Lambda_-^0)$ using unsubtracted expressions, one obtains $B_\pi^{\mathrm{POLE}}(\Lambda_-^0) = 30.97$. That can be compared with the result obtained using (4.6) which is shown in Table 7.4.

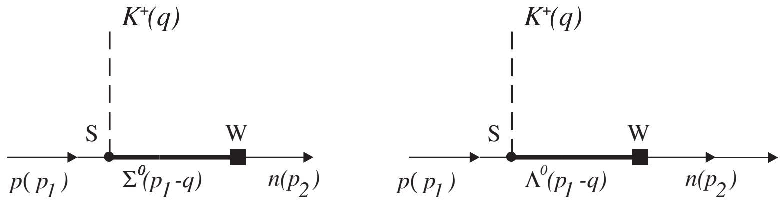

Fig. 4.3. Pole diagrams for the proton nonleptonic decay: $p \to n K^+$.

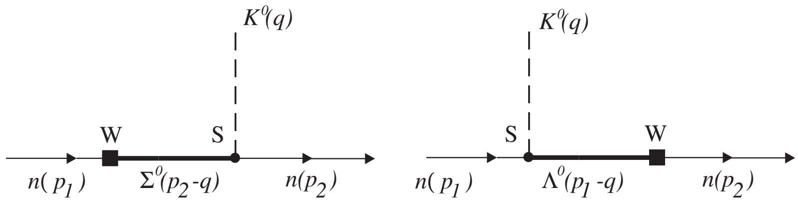

Fig. 4.4. Pole diagrams for the neutron nonleptonic decay: $n \to n K^0$.

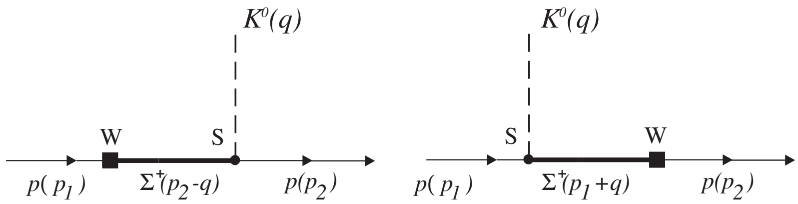

Fig. 4.5. Pole diagrams for the proton nonleptonic decay: $p \to p K^0$.

The diagrams in Figs. 4.3, 4.4 and 4.5 illustrate the weak kaon vertices, and the corresponding analytical expressions read:

$$B_K^{\mathrm{POLE}}(p_+^+) = g(p+n)\Big[(1-2f)\frac{a_{\Sigma^0 n}}{(\Sigma^0 - n)(\Sigma^0 + p)} - \frac{1}{\sqrt{3}}(1+2f)\frac{a_{\Lambda n}}{(\Lambda^0 - n)(\Lambda^0 + p)}\Big] \tag{4.9}$$

$$B_K^{\mathrm{POLE}}(n_0^0) = 2gn\left\{\left[-\frac{1}{\sqrt{3}}(1+2f)\right]\frac{a_{\Lambda n}}{\Lambda^{0^2} - n^2} - (1-2f)\frac{a_{\Sigma^0 n}}{\Sigma^2 - n^2}\right\} \tag{4.10}$$

$$B_K^{\mathrm{POLE}}(p_0^+) = 4gp(1-2f)\sqrt{2}\frac{a_{\Sigma^+ p}}{\Sigma^{0^2} - p^2} \qquad (4.11)\,.$$

Here, for instance, is (see Sec. 5):

$$a_{\Sigma^0 n} = f_\pi \sqrt{2} A(\Sigma^-_-), \quad etc. \tag{4.12}$$

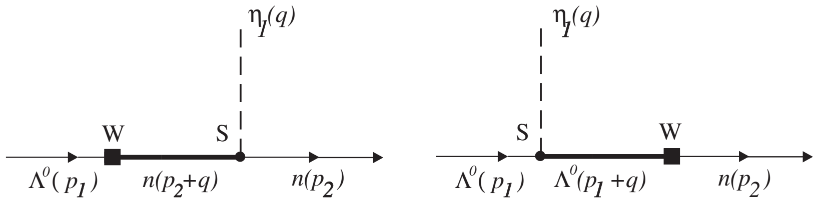

Fig. 4.6. Baryon pole η_1 emission: $s-$channel and $u-$channel contributions.

The weak η vertices are given by: 1) the $u-$channel contribution which contains the Λ^0 as the intermediate hyperon, and 2) the $s-$channel contribution where n is the intermediate baryon. Thus, the strong vertices contain $g_{\Lambda\Lambda\eta}$ and $g_{NN\eta}$ form-factors (see Fig. 4.6). The analytic results are:

$$B^{\rm POLE}_{\eta_8}(\Lambda^0_{\eta_8}) = g\sqrt{\frac{2}{3}}(1-f)\frac{\Lambda+n}{\Lambda-n}\left(\frac{1}{\Lambda}-\frac{2}{n}\right)\left(\frac{1}{\sqrt{2}}\cdot a_{\Lambda n}\right) \tag{4.13}$$

$$B^{\rm POLE}_{\eta_1}(\Lambda^0_{\eta_1}) = \frac{\Lambda+n}{\Lambda-n}\left(\frac{g_{\Lambda\Lambda\eta_1}}{\Lambda}-\frac{g_{NN\eta_1}}{n}\right)\left(\frac{1}{\sqrt{2}}\, a_{\Lambda n}\right). \tag{4.14}$$

The $g_{\Lambda\Lambda\eta_1}$ and $g_{NN\eta_1}$ are strong coupling constants corresponding to the SU(3) singlet η_1.

In the numerical calculations (see Sec. 5 and 7 below), we will be using quantities $\tilde{a}_{if}$ instead of a_{if} introduced above.

5. Current algebra contributions

The calculation of the weak matrix elements by current algebra (CA) methods consists of several approximation procedures which lead eventually to simple relations among transition amplitudes.

The S matrix element is expressed in term of *in* and *out* field operators where [33,34]

$$\begin{aligned} \phi(\mathbf{x},t) &\simeq \phi_{in}(\mathbf{x},t) \qquad \text{for} \quad t\to-\infty \\ \phi(\mathbf{x},t) &\simeq \phi_{out}(\mathbf{x},t) \qquad \text{for} \quad t\to+\infty. \end{aligned} \tag{5.1}$$

(Here we ignore the renormalization constant appearing in front of the *in*/*out*-states.) The initial particle state is of the form

$$|k_1,\ldots,k_N;in\rangle = a^\dagger_{in}(k_N)\cdot\ldots\cdot a^\dagger_{in}(k_1)|0\rangle \tag{5.2}$$

and the S-matrix serves to connect the *in* and *out* states

$$|k_1, \ldots, k_N; in\rangle = S|k_1, \ldots, k_N; out\rangle \,. \tag{5.3}$$

The meson field operator $\phi_A(x)$, for which we are going to use more handy notation, i.e. $\phi_A(x) \to M_A(x)$, could be expressed by the PCAC (3.16) as:

$$M_{a+\mathrm{i}b} = \frac{1}{m_M^2 f_M} \partial_\mu A^\mu_{a+\mathrm{i}b} \,. \tag{5.4}$$

Here, M is a Heisenberg field operator and, therefore, it annihilates a meson state:

$$\langle 0|\, M_{a+\mathrm{i}b}(x)\, |M_{a'+\mathrm{i}b'}(k)\rangle = \frac{1}{(2\pi)^3 2\omega_k} \mathrm{e}^{-\mathrm{i}kx} (\delta_{aa'}\delta_{bb'}) \,. \tag{5.5}$$

Using the time-ordering operator $\mathcal{T}$, one can calculate the matrix element as follows

$$\begin{aligned} &\langle B' M_A(k; out)|\, \mathcal{H}_W(0)\, |B\rangle \\ &\qquad = \mathrm{i} \int \mathrm{d}^4x \frac{\mathrm{e}^{\mathrm{i}kx}}{(2\pi)^3 2\omega_k} (\overrightarrow{\Box_x + m_M^2}) \langle B'|\, \mathcal{T}[M_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)]\, |B\rangle \\ &\qquad = \mathrm{i}\frac{C_M}{f_M}\left(1 - \frac{k^2}{m_M^2}\right) \int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}kx} \langle B'|\, \mathcal{T}[\partial_\mu A^\mu_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)]\, |B\rangle \,. \end{aligned} \tag{5.6}$$

Here C_M depends on the particular PS-meson. The time ordered product can be written as $\mathcal{T}[A(x)B(0)] = \theta(x^0)[A(x), B(0)] + B(0)A(x)$. From

$$\begin{aligned} \mathrm{e}^{\mathrm{i}kx}\theta(x_0)\partial_\mu A^\mu_{a+\mathrm{i}b}(x) \quad &= \partial^\mu \left[\mathrm{e}^{\mathrm{i}kx}\theta(x_0) A^\mu_{a+\mathrm{i}b}(x)\right] - \mathrm{i}k_\mu \theta(x_0)\mathrm{e}^{\mathrm{i}kx} A^\mu_{a+\mathrm{i}b}(x) \\ &- \left[\partial_0\theta(x_0)\right] \mathrm{e}^{\mathrm{i}kx} A^\mu_{a+\mathrm{i}b}(x) \,, \end{aligned} \tag{5.7}$$

and using the derivative of the Heviside function

$$\frac{\partial}{\partial t}\theta(t) = -\frac{\partial}{\partial t}\theta(-t) = \delta(t) \,, \tag{5.8}$$

we find

$$\begin{aligned} &\mathrm{e}^{\mathrm{i}kx}\mathcal{T}[\partial_\mu A^\mu_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)] \\ &\qquad = -\mathrm{i}k_\mu \mathrm{e}^{\mathrm{i}kx}\mathcal{T}[A^\mu_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)] - \delta(x_0)[A^0_{a+\mathrm{i}b}(x), \mathcal{H}_W(0)] \\ &\qquad +\partial_\mu \left\{\mathrm{e}^{\mathrm{i}kx}\mathcal{T}[A^\mu_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)]\right\} . \end{aligned} \tag{5.9}$$

Hence, we can write the above matrix element in the following way

$$\begin{aligned}&\langle B' M_A(k;out)|\, \mathcal{H}_W(0)\,|B\rangle \\ &\quad = \mathrm{i}\frac{C_M}{f_M}\left(1-\frac{k^2}{m_M^2}\right)\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}kx}\{-\mathrm{i}k_\lambda \langle B|\, \mathcal{T}[A^\lambda_{a+\mathrm{i}b}(x)\mathcal{H}_W(0)\,|B\rangle \\ &\quad -\delta(x^0)\langle B'|\,[A^0_{a+\mathrm{i}b}(x), \mathcal{H}_W(0)]\,|B\rangle\}\,.\end{aligned} \tag{5.10}$$

By taking the limit $k \to 0$ (*soft pion limit* for off-shell pions), we get a typical CA relation

$$\mathcal{M}(q\to 0) = -\mathrm{i}\frac{C_M}{f_M}\langle B'|\,[F^5_{a+\mathrm{i}b}(0), \mathcal{H}_W(0)]\,|B\rangle = \mathrm{i}A\,\overline{u}_{B'}\,u_B\,. \tag{5.11}$$

Here the SU(3) (axial) charge is defined by

$$F^5_{a+\mathrm{i}b}(t) = \int \mathrm{d}^3x\, A^0_{a+\mathrm{i}b}(t,\mathbf{x})\,. \tag{5.12}$$

The following CA relations hold [9,13,24]

$$\begin{aligned}[F^5_{a+\mathrm{i}b}, \mathcal{H}_W] &= [F_{a+\mathrm{i}b}, \mathcal{H}_W^{PV}] \\ [F^5_{a+\mathrm{i}b}, \mathcal{H}_W^{PC}] &= [F_{a+\mathrm{i}b}, \mathcal{H}_W^{PV}] \\ [F^5_{a+\mathrm{i}b}, \mathcal{H}_W^{PV}] &= [F_{a+\mathrm{i}b}, \mathcal{H}_W^{PC}]\,.\end{aligned} \tag{5.13}$$

Here $F_{a+\mathrm{i}b}$'s are (vector) charges defined in the similar way as $F^5_{a+\mathrm{i}b}$'s, (5.12) i.e.

$$F_{a+\mathrm{i}b} = \int \mathrm{d}^3x\, V^0_{a+\mathrm{i}b}(t,\mathbf{x})\,. \tag{5.14}$$

In order to evaluate the above commutator, we can use the SU(3) transformation properties of the axial charges and field operators

$$\begin{aligned}M_{a+\mathrm{i}b} &\sim \overline{q}\tfrac{1}{2}(\lambda_a + \mathrm{i}\lambda_b)\gamma_5 q \sim \sqrt{2}F^5_{a+\mathrm{i}b} \quad \text{and} \\ M_A|B\rangle &= \mathrm{i}f_{ABC}|C\rangle\,,\end{aligned} \tag{5.15}$$

where f_{ABC} are the SU(3) structure constants and $q^T = (u,d,s)$ (T stands for *transposed*!). For instance, for the K^0 field we have

$$M_{a+\mathrm{i}b} \to \phi_{K^0} \sim \sqrt{2}F_{\text{"}K^0\text{"}} = \sqrt{2}F_{6-\mathrm{i}7} \sim \overline{q}\frac{1}{2}(\lambda_6 - \mathrm{i}\lambda_7)\gamma_5 q\,. \tag{5.16}$$

The above field anihilates a K^0 in $|\quad\rangle$ or creates a K^0 in $\langle\quad|$. Also, $\phi_{K^+} = \sqrt{2}F_{4-\mathrm{i}5}$ etc. Thus, using the here displayed procedure, we were able to extract the *current algebra* contribution to the matrix element $\mathcal{M}$.

The SU(3) properties can be also expressed using the SU(3) baryon (antibaryon) $\mathcal{B}$ ($\overline{\mathcal{B}}$) and meson $\mathcal{P}$ octet matrices

$$\mathcal{B}^a_b = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}$$

$$\overline{\mathcal{B}^a_b} = \begin{pmatrix} \frac{\overline{\Sigma^0}}{\sqrt{2}} + \frac{\overline{\Lambda^0}}{\sqrt{6}} & \overline{\Sigma^+} & \overline{\Xi^-} \\ \overline{\Sigma^+} & -\frac{\overline{\Sigma^0}}{\sqrt{2}} + \frac{\overline{\Lambda^0}}{\sqrt{6}} & \overline{\Xi^0} \\ \overline{p} & \overline{n} & -\frac{2\overline{\Lambda^0}}{\sqrt{6}} \end{pmatrix} \tag{5.17a}$$

and

$$\mathcal{P}^a_b = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^0}{\sqrt{6}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta^0}{\sqrt{6}} \end{pmatrix}. \tag{5.17b}$$

One can also write

$$\begin{aligned} \Sigma^+ &= \tfrac{1}{\sqrt{2}}(B_1 - \mathrm{i}B_2) & p &= \tfrac{1}{\sqrt{2}}(B_4 - \mathrm{i}B_5) \\ \Sigma^- &= \tfrac{1}{\sqrt{2}}(B_1 + \mathrm{i}B_2) & n &= \tfrac{1}{\sqrt{2}}(B_6 - \mathrm{i}B_7) \\ \Sigma^0 &= B_3 & \Xi^- &= \tfrac{1}{\sqrt{2}}(B_4 + \mathrm{i}B_5) \\ \Lambda^0 &= B_8 & \Xi^0 &= \tfrac{1}{\sqrt{2}}(B_6 + \mathrm{i}B_7)\,. \end{aligned} \tag{5.18a}$$

and

$$\begin{aligned} \pi^+ &= \tfrac{1}{\sqrt{2}}(P_1 - \mathrm{i}P_2) & K^+ &= \tfrac{1}{\sqrt{2}}(P_4 - \mathrm{i}P_5) \\ \pi^- &= \tfrac{1}{\sqrt{2}}(P_1 + \mathrm{i}P_2) & K^0 &= \tfrac{1}{\sqrt{2}}(P_6 - \mathrm{i}P_7) \\ \pi^0 &= P_3 & K^- &= \tfrac{1}{\sqrt{2}}(P_4 + \mathrm{i}P_5) \\ \eta_8 &= P_8 & \overline{K^0} &= \tfrac{1}{\sqrt{2}}(P_6 + \mathrm{i}P_7)\,. \end{aligned} \tag{5.18b}$$

The quark $\mathrm{SU(3)_{flavour}}$ contents of the baryon and meson octet states are:

$$\begin{array}{llllll} \Sigma^+ & \sim & u\,u\,s & p & \sim & u\,u\,d \\ \Sigma^- & \sim & d\,d\,s & n & \sim & u\,d\,d \\ \Sigma^0 & \sim & u\,d\,s & \Xi^- & \sim & d\,s\,s \\ \Lambda^0 & \sim & u\,d\,s & \Xi^0 & \sim & u\,s\,s \end{array} \tag{5.18c}$$

and

$$\begin{array}{llllll} \pi^+ & \sim \overline{d}\,u & & K^+ & \sim \overline{s}\,u \\ \pi^- & \sim \overline{u}\,d & & K^0 & \sim \overline{s}\,d \\ \pi^0 & \sim \frac{1}{\sqrt{2}}(\overline{u}\,u - \overline{d}\,d) & & K^- & \sim \overline{u}\,s \\ \eta_8 & \sim \frac{1}{\sqrt{6}}(\overline{u}\,u + \overline{d}\,d - 2\overline{s}\,s) & & \overline{K^0} & \sim \overline{d}s\,. \end{array} \tag{5.18d}$$

The U(3) isosinglet field η_1 is given in terms of the U(3) λ_0-matrix: $\lambda_0 = \sqrt{\frac{2}{3}}\hat{1}$, i.e. $\eta_1 \sim (\overline{u}\,u + \overline{d}\,d + \overline{s}\,s)/\sqrt{3}$. The quark-antiquark combinations could be inverted and be expressed in terms of corresponding meson states, i.e.

$$\begin{array}{lll} \overline{u}u & = \overline{q}\left(\frac{\sqrt{3}}{6}\lambda^8 + \frac{\sqrt{6}}{6}\lambda^0 + \frac{1}{2}\lambda^3\right) q & \text{corresponding to} \\ \overline{u}u & \to \frac{\sqrt{6}}{6}\eta_8 + \frac{\sqrt{3}}{3}\eta_1 + \frac{\sqrt{2}}{2}\pi^0 & \\ \overline{d}d & = \overline{q}\left(\frac{\sqrt{3}}{6}\lambda^8 + \frac{\sqrt{6}}{6}\lambda^0 - \frac{1}{2}\lambda^3\right) q & \text{corresponding to} \\ \overline{d}d & \to \frac{\sqrt{6}}{6}\eta_8 + \frac{\sqrt{3}}{3}\eta_1 - \frac{\sqrt{2}}{2}\pi^0 & \\ \overline{s}s & = \overline{q}\left(-\frac{\sqrt{3}}{3}\lambda^8 + \frac{\sqrt{6}}{6}\lambda^0\right) q & \text{corresponding to} \\ \overline{s}s & \to -\frac{\sqrt{6}}{3}\eta_8 + \frac{\sqrt{3}}{3}\eta_1\,. & \end{array} \tag{5.18e}$$

The SU(3) meson states can be written in the standard basis [9,35,36] as $|\mathbf{M}\rangle = |\mathbf{8};\ Y, I, I_3\rangle$, with $Y = B + S$. So

$$\begin{array}{lll} |\pi^+\rangle & = \mathcal{P}_1^2|0\rangle & = -|\mathbf{8};\ 0,1,+1\rangle \\ |\pi^-\rangle & = \mathcal{P}_2^1|0\rangle & = \ \ |\mathbf{8};\ 0,1,-1\rangle \\ |\pi^0\rangle & = \frac{1}{\sqrt{2}}(\mathcal{P}_1^1 - \mathcal{P}_2^2)|0\rangle & = \ \ |\mathbf{8};\ 0,1,0\rangle \\ |K^+\rangle & = \mathcal{P}_1^3|0\rangle & = \ \ |\mathbf{8};\ ,1/2,1/2\rangle \\ |K^0\rangle & = \mathcal{P}_2^3|0\rangle & = \ \ |\mathbf{8};1/2,-1/2\rangle \\ |\overline{K}^0\rangle & = \mathcal{P}_3^2|0\rangle & = \ \ |\mathbf{8};\ -1,1/2,1/2\rangle \\ |K^-\rangle & = \mathcal{P}_3^1|0\rangle & = -|\mathbf{8};\ -1,1/2,-1/2\rangle \\ |\eta^0\rangle & = -\frac{3}{\sqrt{6}}\mathcal{P}_3^3|0\rangle & = \ \ |\mathbf{8};\ 0,0,0\rangle\,. \end{array} \tag{5.18f}$$

The corresponding SU(2) (iso)multiplets are

$$
\begin{array}{llll}
K^a = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} & \overline{K}^a = \begin{pmatrix} -\overline{K}^0 \\ K^- \end{pmatrix} & \pi_a^b = \begin{pmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{pmatrix} \\
N^a = \begin{pmatrix} p^+ \\ n^0 \end{pmatrix} & \Xi^a = \begin{pmatrix} -\Xi^0 \\ \Xi^- \end{pmatrix} & \Sigma_a^b = \begin{pmatrix} \Sigma^0/\sqrt{2} & \Sigma^+ \\ \Sigma^- & -\Sigma^0/\sqrt{2} \end{pmatrix} .
\end{array}
\tag{5.18g}
$$

Operators defined in (5.18a,b) act as annihilation operators. The SU(3) charges F_+, F_- and F_3 display the SU(2) group properties:[1]

$$
\begin{array}{llll}
\langle p|F_+ & = \langle n| & \langle n|F_3 & = -\frac{1}{2}\langle n| \\
F_+|\Xi^-\rangle & = -|\Xi^0\rangle & F_3|\Xi^0\rangle & = \frac{1}{2}|\Xi^0\rangle \\
F_+|\Sigma^-\rangle & = \sqrt{2}|\Sigma^0\rangle & F_-|\Sigma^+\rangle & = -\sqrt{2}|\Sigma^0\rangle .
\end{array}
\tag{5.19}
$$

The commutators appearing in the matrix elements $\mathcal{M}$ (5.11) determine the transition amplitudes which are the *current algebra* contributions to A:[2]

$$A(\Lambda_-^0) = -\frac{\sqrt{2}}{f_\pi}\frac{1}{\sqrt{2}}\langle p|\,[F_+, \mathcal{H}_W^{PC}(0)]\,|\Lambda\rangle = -\frac{1}{f_\pi}\langle n|\,\mathcal{H}_W^{PC}(0)\,|\Lambda\rangle = -\frac{1}{f_\pi}a_{\Lambda n} \tag{5.20}$$

$$A(\Lambda_0^0) = -\frac{\sqrt{2}}{f_\pi}\frac{1}{\sqrt{2}}\langle n|\,[F_3, \mathcal{H}_W^{PC}(0)]\,|\Lambda\rangle = \frac{1}{\sqrt{2}f_\pi}a_{\Lambda n} \tag{5.21}$$

$$A(\Xi_-^-) = -\frac{\sqrt{2}}{f_\pi}\frac{1}{\sqrt{2}}\langle \Lambda|\,[F_+, \mathcal{H}_W^{PC}(0)]\,|\Xi^-\rangle = -\frac{1}{f_\pi}\langle \Lambda|\,\mathcal{H}_W^{PC}\,|\Xi^0\rangle = -\frac{1}{f_\pi}a_{\Xi^0\Lambda} \tag{5.22}$$

$$A(\Xi_0^0) = -\frac{1}{\sqrt{2}f_\pi}a_{\Xi^0\Lambda} \tag{5.23}$$

$$A(\Sigma_-^-) = \frac{\sqrt{2}}{f_\pi}a_{\Sigma^0 n} \tag{5.24}$$

$$A(\Sigma_+^+) = -\frac{1}{f_\pi}\left[a_{\Sigma^+ p} + \sqrt{2}a_{\Sigma^0 n}\right] \tag{5.25}$$

$$A(\Sigma_0^+) = \frac{1}{\sqrt{2}f_\pi}a_{\Sigma^+ p} . \tag{5.26}$$

[1] One has to be very careful with the definitions of the initial and final states and with operators acting on these states. The usual definition which is adopted here is that operators *annihilate* the corresponding particle state in *the initial state*.

[2] The above commutators do not contribute to the PC amplitude B. Only the pole diagrams contribute as we have shown in the preceding section. Spinors u are usually omitted.

These connections between decay amplitudes A and the matrix elements a_{if} were used in Ref. [5] to determine a_{if} parameters which appear in the pole contributions, see Sec. 4 above. However, our parametrization is based on the more general expression (1.1). For example, the extraction from the experimental data (see for instance Ref. [16]) is done by using an equation like

$$A(\Lambda^0_-)_{\rm exp} = -\frac{1}{f_\pi}\tilde{a}_{\Lambda n} + A(\Lambda^0_-)_{\rm SEP}. \tag{5.27}$$

All our pole contributions (see Sec. 7 below) are calculated by using the values $\tilde{a}_{if}$ (see also Appendix H).

6. *Pseudoscalar meson poles and separable contributions*

The pseudoscalar meson poles contribute to the form factors which appear in current matrix elements ($g_P(q^2)$ in (6.2) below is one example). Thus, some terms in separable contributions, discussed in Sec. 3, are also included in the meson pole contributions, as illustrated in Fig. 6.1. But, proceeding with some care, the double counting can be avoided. The separable contributions due to operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, and $\mathcal{O}_4$, as calculated in Sec. 3, are not included in the meson contributions. However, the separable contributions from the operators $\mathcal{O}_5$, and $\mathcal{O}_6$ are contained. The pseudoscalar meson pole contributes to the parity conserving B amplitudes only.

The separable PC contributions (3.8) arising from the operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$, and $\mathcal{O}_4$ were evaluated using the product of axial vector currents $A^\mu_a(x)$

$$\langle B_\beta|\, A^\mu_a\, |B_\alpha\rangle \cdot \langle L_a|\, A^a_\mu\, |0\rangle = B({\rm sep})\,. \tag{6.1}$$

Here, in accordance with (3.10),

$$\begin{aligned}\langle B_\beta|\, A^\mu_a(0)\, |B_\alpha\rangle &= \overline{u}_\beta(p')\left[\gamma^\mu\gamma_5 g_A(q^2)\right.\\ &\left.+\frac{1}{M_\alpha+M_\beta}g_P(q^2)q^\mu\gamma_5\right]L_a u_\alpha(p)\,,\end{aligned} \tag{6.2}$$

where $q=p-p'$, $g_A(0)\simeq 1.25$ and L_a stands for a meson[3]. The results presented in Sec. 3 were obtained by assuming that g_P term can be neglected. It is important to stress that the contribution due to the g_P is contained only in the kaon-pole term, as described below. Some useful definitions and conventions are given in Appendix A.

The meson pole contribution to the process in which a meson L is emitted, i.e.

$$B_\alpha \quad\rightarrow\quad B_\beta \quad+\quad L_a\,, \tag{6.3}$$

[3] For $B(\Lambda^0_-)$ $L_a=\pi^-$.

is shown in Fig. 6.1.

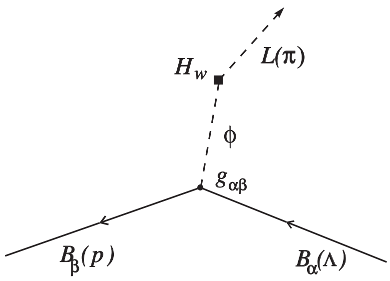

Fig. 6.1. $g_{\alpha\beta}$ stands for the strong vertex, and the weak Hamiltonian H_w contains the current product (6.1). The particles in the process $\Lambda \to \mathrm{p} + \pi$ are indicated in the parentheses.

The amplitude shown in Fig. 6.1 corresponds to ϕ meson pole and reads[4]

$$B^{MP} = \langle L| \, H_w \, |\phi\rangle \frac{\mathrm{i}}{q^2 - m_\phi^2 + \mathrm{i}\epsilon} \overline{u}_\beta \gamma_5 L_a u_\alpha g_{\alpha\beta} \, . \tag{6.4}$$

After approximating

$$\langle L| \, H_w \, |\phi\rangle \simeq \langle L| \, A_\mu^b \, |0\rangle \langle 0| \, A_a^\mu \, |\phi\rangle \, , \tag{6.5}$$

and using both (i) the relation

$$\frac{g_P(q^2)}{M_\alpha + M_\beta} \simeq (-) \frac{g_{\alpha\,\beta} f_\phi(q^2)}{q^2 - m_\phi^2} \, , \tag{6.6a}$$

given in the Appendix A, and (ii) $f_\phi q^\mu = \langle 0| \, A^\mu \, |\phi\rangle$, one can show that

$$B^{MP} \simeq \langle L| \, A_\mu^b \, |0\rangle \; \overline{u}_\beta \frac{1}{M_\alpha + M_\beta} g_P q_\mu \gamma_5 L_a u_\alpha \, . \tag{6.6b}$$

Thus, g_P-contribution to the B(sep) is included in the M(pole) term. This means that in evaluating separable contribution from the operators $\mathcal{O}_1, \ldots, \mathcal{O}_4$, one should include only the g_A form factors, as it has been done in Sec. 3.

The operators $\mathcal{O}_5$ and $\mathcal{O}_6$ contribute with pieces which look like

$$\begin{aligned} \mathcal{O}_x \sim \; & (\overline{\psi}_1 \gamma_5 \psi_2)(\overline{\psi}_3 \gamma_5 \psi_4) \\ & \langle B_\beta| \, \overline{\psi}_1 \gamma_5 \psi_2 \, |B_\alpha\rangle \neq 0 \\ & \langle L| \, \overline{\psi}_3 \gamma_5 \psi_4 \, |0\rangle \neq 0 \, . \end{aligned} \tag{6.7}$$

[4]Here B^{MB} is the contribution arising from the meson pole, while B^{POLE} in Sec. 4 denotes the contribution related with the baryon pole.

Here ψ_i symbolize quark fields appearing in H_w. The corresponding B^{MP} contribution is obtained from (6.4) by the replacement

$$\langle L|\, H_w\, |\phi\rangle \quad \rightarrow \quad \langle L|\, \mathcal{O}_x\, |\phi\rangle, . \tag{6.8}$$

For meson pseudoscalar densities, one uses identities

$$\begin{aligned} \overline{\psi}_i \gamma_5 \psi_j &= \frac{1}{m_i + m_j}(-\mathrm{i})(\partial_\mu \overline{\psi}_i \gamma^\mu \gamma_5 \psi_j) \\ &= \frac{1}{m_i + m_j}(-\mathrm{i})\mathrm{i}(p_i^\mu - p_j^\mu)\overline{\psi}_i \gamma_\mu \gamma_5 \psi_j \, . \end{aligned} \tag{6.9}$$

Thus

$$\begin{aligned} \langle B_\beta|\, \overline{\psi}_1 \gamma_5 \psi_2\, |B_\alpha\rangle &= (-\mathrm{i})\langle B_\beta|\, \partial_\mu A_a^\mu\, |B_\alpha\rangle \frac{1}{(m_1 + m_2)} \\ &= \frac{f_\phi \tilde{q}^2}{q^2 - m_\phi^2} g_{\alpha\beta} \overline{u}_\beta \gamma_5 L_a u_\alpha \cdot \frac{1}{m_1 + m_2} \, . \end{aligned} \tag{6.10}$$

Here the notation $\tilde{q}^2$ was used instead of m_M^2. Thus, the separable contribution corresponding to (3.8) has the generic form

$$B_x(\mathrm{sep}) = \frac{f_\phi \tilde{q}^2}{(m_1 + m_2)(q^2 - m_\phi^2)} g_{\alpha\beta} \overline{u}_\beta \gamma_5 L_a u_\alpha \langle L|\, \overline{\psi}_3 \gamma_5 \psi_4\, |0\rangle \, , \tag{6.11}$$

where the $\phi-$meson pole is openly displayed. So, one expects that it must be included in the B^{MP} contribution. In order to test this assumption one again uses the approximation (6.5) and writes (6.4)

$$B_x^{MP} \simeq \langle L|\, \overline{\psi}_3 \gamma_5 \psi_4\, |0\rangle \langle 0|\, \overline{\psi}_1 \gamma_5 \psi_2\, |\phi\rangle \frac{\mathrm{i}}{q^2 - m_\phi^2} g_{\alpha\beta} \overline{u}_\beta \gamma_5 L_a u_\alpha \, . \tag{6.12a}$$

The first mesonic matrix element is

$$\langle 0|\, \overline{\psi}_1 \gamma_5 \psi_2\, |\phi\rangle = \frac{1}{m_1 + m_2}(-\mathrm{i})\langle 0|\, \partial_\mu A_a^\mu\, |\phi_a\rangle = \frac{1}{m_1 + m_2}(-\mathrm{i}) q^2 f_\phi \, . \tag{6.13}$$

Thus,

$$B_x^{MP} \simeq \frac{f_\phi q^2}{(m_1 + m_2)(q^2 - m_\phi^2)} g_{\alpha\beta} \overline{u}_\beta \gamma_5 L_a u_\alpha \langle L|\, \overline{\psi}_3 \gamma_5 \psi_4\, |0\rangle \, . \tag{6.12b}$$

The approximation (6.12) is exactly equal to $B_x(\mathrm{sep})$ in (6.11) if one puts

$$\tilde{q}^2 \equiv q^2 . \tag{6.14}$$

Alternatively and formally, one can compare the meson pole approximation (6.12a) with the expression (6.11) making use of

$$\frac{f_\phi m_\phi^2}{m_1+m_2} = \frac{\langle 0|\,\partial_\mu A_a^\mu\,|\phi\rangle}{m_1+m_2} = \mathrm{i}\langle 0|\,\overline{\psi}_1\gamma_5\psi_2\,|\phi\rangle \qquad (\tilde{q}^2 = m_\phi^2)\,. \tag{6.15}$$

The insertion of (6.15) in (6.11) converts it into (6.12a). However, in that formal procedure one has neglected the question about the meson ϕ being off-mass-shell. Some additional details can be found in Appendix A.

In the derivation of PCAC constant, one has assumed that the meson ϕ was on the mass shell. As that meson is *not* on the mass shell neither in the diagram in Fig. 6.1, nor in the expression (6.11), the reading (6.14) seems to be amply justified. The separable contributions from operators $\mathcal{O}_5$ and $\mathcal{O}_6$ are included in the meson (for example kaon) pole.

For the process

$$B_\alpha \quad \rightarrow \quad B_\beta \quad + \quad M_a \tag{6.16}$$

one can now formulate *the rule for the calculation of B amplitudes*:

$$B = B(\text{baryon pole}) + B(\text{sep}) + B^{MP}. \tag{6.17}$$

Here B(sep) contains only the contribution from the g_A terms associated with the operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_4$. The g_P terms for these operators and the separable contributions from $\mathcal{O}_5$ and $\mathcal{O}_6$ are included in the B^{MP} piece. The contributions B(baryon pole) $= B^{\text{POLE}}$ were calculated in Sec. 4 while B(sep) can be found in Sec. 3.

7. *Weak baryon-baryon-meson amplitudes and theoretical models*

It is very important to test the quality of the approximate procedure which was used to calculate A and B amplitudes. That can be done by:

(*i*) Comparison with other theoretical calculations,

(*ii*) Comparison with experimental results.

The first task can be easily accomplished by studying the amplitudes corresponding to η and K exchanges given in Tables 7.1–7.4. Parreño et al. [5] did not distinguish η_1 (singlet) from η_8 (octet) exchanges. In their approach, η was supposed to belong to the SU(3) octet. Our results for the octet η are fairly close to that of Ref. [5]. A_{η_8} differs by about 8 %, while the discrepancy for B_{η_8} is 18 %. The origin of those discrepancies is obviously connected with the separable and pole terms which appear in our calculational scheme. As discussed in Sec. 3 and 4, they were not used or they were used in different forms by Ref. [5]. Our theoretical expressions for A and B amplitudes were found using the values $\tilde{a}_{if}$ (5.27). Thus, for example, our theoretical expression for $A_K(p^+_+)$ has the form

$$\begin{aligned} A_K(p^+_+) &= \sqrt{3}\tilde{A}(\Lambda^0_-) + \tfrac{1}{\sqrt{2}}\tilde{A}(\Sigma^+_0) + A_K(p^+_+)_{\rm SEP} = \\ &= -\frac{\sqrt{3}}{f_\pi}\tilde{a}_{\Lambda n} + \frac{1}{2f_\pi}\tilde{a}_{\Sigma p} + A_K(p^+_+)_{\rm SEP}, \end{aligned} \tag{7.1}$$

while the values of Ref. [5] were obtained by using the first term in (7.1) only with a_{if} instead of $\tilde{a}_{if}$. In view of that, the agreement within 18 % suggests that the employed methods are relatively stable and, hopefully, reliable to within about 20 %. That is the accuracy that was hoped for in the theoretical descriptions of the hyperon nonleptonic decays [8–15].

Detailed comparison of our conventions and units with those used in Ref. [5] is given in Appendix I. All numbers, shown in Tables 7.1 - 7.4 are "translated" from Ref. [5]. As discussed in Appendix I, the same goes for Ref. [22], too.

Table 7.1 - Transition amplitudes without the 10^{-7} factor: [BHKNT]≡ present work: (a) a complete amplitude; (b) an amplitude without the separable contribution (this work). The asterisk $^{(*)}$ denotes that the experimental amplitude was used instead.

Amplitude	Exp.	BHKNT(a)	BHKNT(b)	Ref. [5]
$A_\pi(\Lambda^0_-)$	3.23 ± 0.02	$3.25^{(*)}$	2.97	3.28
$B_\pi(\Lambda^0_-)$	22.0 ± 0.5	22.35	19.38	22.25
$A_\pi(\Lambda^0_0)$	-2.37 ± 0.03	$-2.36^{(*)}$	-2.10	-2.32
$B_\pi(\Lambda^0_0)$	-15.9 ± 1.4	-15.93	-13.70	-15.80
$A_\pi(\Sigma^+_0)$	-3.26 ± 0.11	$-3.27^{(*)}$	-3.71	–
$B_\pi(\Sigma^+_0)$	26.7 ± 1.5	18.09	18.04	–
$A_\pi(\Sigma^-_-)$	4.27 ± 0.02	$4.27^{(*)}$	4.60	–
$B_\pi(\Sigma^-_-)$	-1.4 ± 0.2	-7.46	-5.39	–
$A_K(p^+_+)$	–	2.81	2.52	1.68
$B_K(p^+_+)$	–	42.38	34.38	41.77
$A_K(n^0_0)$	–	6.25	6.25	6.30
$B_K(n^0_0)$	–	17.99	21.56	27.12
$A_K(p^+_0)$	–	4.08	5.25	4.62
$B_K(p^+_0)$	–	-21.87	-14.59	-14.65
$A_\eta(\Lambda^0_{\eta_1})$	–	5.60	5.54	–
$B_\eta(\Lambda^0_{\eta_1})$	–	41.10	26.09	–
$A_\eta(\Lambda^0_{\eta_8})$	–	5.19	5.54	3.98
$B_\eta(\Lambda^0_{\eta_8})$	–	38.41	28.82	31.60

Table 7.2. Comparison between transition amplitudes (in 10^{-7} units, w.o. dimension) as given in [5] and this work. A complete amplitude includes a separable contribution. $^{(*)}A_\pi = A_\pi(\Lambda^0_-)$, etc. - see Table 7.1.

Ref. [5]		Total Ampl.		Ampl. without SEP
$A_\pi^{(*)}$:	3.28		3.25	2.97
$B_\pi^{(*)}$:	22.35		22.27	19.38
A_{η_1} :	–		5.60	5.54
A_{η_8} :	3.98		5.19	5.54
B_{η_1} :	–		41.10	26.09
B_{η_8} :	31.60		$38.41(m_\eta^2)$	28.82
C_K^{PV} :	1.68	$A_K(p_+^+)$:	2.81	2.52
C_K^{PC} :	41.77	$B_K(p_+^+)$:	42.38	34.38
D_K^{PV} :	4.62	$A_K(p_0^+)$:	4.08	5.25
D_K^{PC} :	-14.65	$B_K(p_0^+)$:	-21.87	-14.59
$C_K^{PV} + D_K^{PV}$:	6.30	$A_K(n_0^0)$:	6.25	6.25
$C_K^{PC} + D_K^{PC}$:	27.12	$B_K(n_0^0)$:	17.99	21.56

Table 7.3. Nonleptonic amplitudes ($\times 10^7$); † this work, compared with rescaled values of Ref. [22] and [5].

Amplitude	Tot. ampl.†	Ref.[22]	Ref.[5]
$A_K(p_0^+)$	4.08	2.84	4.62
$B_K(p_0^+)$	-21.87	-5.61	-14.65
$A_K(p_+^+)$	2.81	0.76	1.68
$B_K(p_+^+)$	42.38	23.19	41.77
$A_K(n_0^0)$	6.25	3.60	6.30
$B_K(n_0^0)$	17.99	19.86	27.12

The amplitudes appearing with the kaon strangeness violation vertices are compared in Table 7.3. For $A_K(p_0^+)$, $A_K(p_+^+)$ and $A_K(n_0^0)$, the differences are small, below 25 %. They are much larger for the corresponding B amplitudes. Obviously, this is due to our usage of the subtracted pole terms (4.6) instead of the unsubtracted ones which were used in Ref. [5]. The absolute value of $B_K(p_0^+)$ amplitudes, as calculated by [22], is about 3 times smaller than ours. However, the signs are the same in all cases. If $B_K(p_0^+)$ is excluded, the largest discrepancy is about 21 %. Our result for $B_K(p_0^+)$ amplitude differs from Ref. [5] by about 35 % only.

Table 7.4. Nonleptonic $p-$wave (parity conserving) amplitudes ($\times 10^7$); † this work. *Contributions from operators $\mathcal{O}_1 - \mathcal{O}_4$; $^\flat$Contributions from operators $\mathcal{O}_{5,6}$.

Ampl. $\times 10^7$	$B_{\text{exp.}}$	$B^{\text{POLE)}}$ ($\tilde{A}$)	Sep. $(1-4)^*$	Mes. pole	Tot.† amp.	Pole	Sep. $(5-6)^\flat$	Ref. [5]
$B_\pi(\Lambda^0_-)$	22.0	19.38	4.79	-1.9	22.27	23.03	6.98	22.35
$B_\pi(\Lambda^0_0)$	-15.9	-13.70	-1.03	-1.2	-15.93	-16.28	-3.68	-15.80
$B_\pi(\Sigma^+_+)$	42.2	48.73	–	–	48.73	37.68	–	–
$B_\pi(\Sigma^+_0)$	26.7	18.04	-0.45	0.5	18.09	15.85	-1.62	–
$B_\pi(\Sigma^-_-)$	-1.4	-5.39	-2.17	0.1	-7.46	-8.83	-2.79	–
$B_\pi(\Xi^0_0)$	-12.2	-12.67	-0.29	0.3	-12.66	-11.28	-1.05	–
$B_\pi(\Xi^-_-)$	17.5	17.45	-1.40	0.4	16.45	15.97	-1.94	–
$B_K(p^+_+)$	–	34.38	-7.5	15.5	42.38	47.60	-138.6	41.77
$B_K(p^+_0)$	–	-14.59	0.12	-7.4	-21.87	-12.99	-11.76	-14.65
$B_K(n^0_0)$	–	21.56	-10.97	7.4	17.99	24.58	-57.03	27.12
$B_{\eta_1}(\Lambda^0_0)$	–	26.09	1.16	13.85	41.10	28.16	-130.8	–
$B_{\eta_8}(\Lambda^0_0)$	–	28.82	-6.76	16.35	38.41	31.10	762.36	–

Comparison with experimental results is, and can be, performed in a limited sense only. The experimental A_π amplitudes are used to find $\tilde{a}_{B'B}$ matrix elements via a subtraction procedure described in Sec. 5. Those $\tilde{a}_{BB'}$ quantities are then used in baryon pole terms which were defined in Sec. 4. Together with separable contributions and kaon poles, they lead to the theoretical prediction of B_π amplitudes, which are shown in Tables 7.1 and 7.4. Reference [5] has used simple, unsubtracted pole terms as was discussed in Sec. 4. More complicated and, hopefully, better approximation leads to a somewhat better agreement with the experiment in most cases. According to Table 7.4, one finds that discrepancies with experiments for various amplitudes are as follows: $B_\pi(\Lambda^0_-)$, 6%, $B_\pi(\Lambda^0_0)$, 2%, $B_\pi(\Sigma^+_+)$, 14%, $B_\pi(\Sigma^+_0)$, 48%, $B_\pi(\Xi^0_0)$, 4% and $B_\pi(\Xi^-_-)$, 6%. While $B_\pi(\Lambda)$ and $B_\pi(\Xi)$ amplitudes are reproduced within 6%, the theoretical prediction for $B_\pi(\Sigma)$ is poor. Our theoretical $B_\pi(\Sigma^-_-)$ amplitude is off by the factor 5.2. However, the corresponding numbers, which were found using a simplified approximation, analogous to the one applied by Ref. [5] are: $B_\pi(\Lambda^0_-)$ 44%, $B_\pi(\Lambda^0_0)$ 42%, $B_\pi(\Sigma^+_+)$ 35%, $B_\pi(\Sigma^+_0)$ 3 times too small, $B_\pi(\Xi^0_0)$ 1%, $B_\pi(\Xi^-_-)$ 1%, and $B_\pi(\Sigma^-_-)$ disagrees by factor 6.3. Thus, for most amplitudes, simplified approximation gives poorer agreement with the experimental values.

There were always some difficulties associated with the theoretical description of the Σ-hyperon nonleptonic decays. Attempted explanations involved additional $1/2^-(1/2^+)$ baryon resonance poles [37–40], instantons [41], decuplets [37] etc.

Overall, one feels that the first column in Table 7.3 might constitute a better approximation of the real physical results, than given by the last column. No theoretical comparison with Ref. [22] is feasible, as that reference uses a quite different method of evaluation. However, it is encouraging that all methods give similar relative magnitudes and relative sign. The worst disagreement is for $B_K(p_0^+)$. However, $A_K(p_0^+)$ agrees within 14%. If the agreement among various theoretical results is an acceptable indicator, one could say that $A_K(N)$ amplitudes are determined with better accuracy than the $B_K(N)$ ones. A skeptical observer would claim that the results presented in Table 7.3 are the order of magnitude estimates at best.

Our method has produced both $\Delta I = 1/2$ and $\Delta I = 3/2$ pieces in the effective potential (9.6). In the formula (9.11), these pieces are written separately, with isospin operators $\mathbf{1} \cdot \mathbf{1}$ and $\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ for $\Delta I = 1/2$ and $\mathbf{T}_i \tau_j$ for $\Delta I = 3/2$. However, $\Delta I = 3/2$ pieces are small and their magnitude is comparable with the theoretical errors which were discussed above. This can be starkly illustrated if one calculates the $\Delta I = 3/2$ piece, associated with the $\Lambda \to N\pi$ weak amplitudes. One finds

$$B_\pi(\Lambda^0_-)_{\rm exp} + \sqrt{2} B_\pi(\Lambda^0_0)_{\rm exp} = 0.32 \times 10^{-7}. \tag{7.2a}$$

This experimental $\Delta I = 3/2$ piece has a different sign than the predicted one:

$$B_\pi(\Lambda^0_-)_{\rm BHKNT} + \sqrt{2} B_\pi(\Lambda^0_0)_{\rm BHKNT} = -0.26 \times 10^{-7}. \tag{7.2b}$$

The effect is due to the subtraction of large (relatively speaking) numbers. Yet $B_\pi(\Lambda)_{\rm exp}$ agrees within 6% with $B_\pi(\Lambda)_{\rm BHKNT}$. The same degree of agreement is shown by $B(\Xi)$ amplitudes. But in that case $\Delta I = 3/2$ pieces $B(\Xi^-_-) + \sqrt{2} B(\Xi^0_0)$ are 0.013×10^{-7} (exp.) and 0.018×10^{-7} (theor.).

In the deduction of the nuclear potential (9.11), we have used, naturally, the experimental $B_\pi(\Lambda)$ value. Thus the whole discussion, involving the theoretical $B_\pi(\Lambda)$ and $B_\pi(\Xi)$ values, serves as an indication for theoretical uncertainties connected with the predicted $A_K(N)$ and $B_K(N)$ (Table 7.3) values. It is hard to quantify those uncertainties. While the theoretical $\Delta I = 3/2$ piece, connected with $B_\pi(\Lambda)$'s, has wrong sign (but its magnitude is comparable with the experimental value) the $B_\pi(\Xi)$ based $\Delta I = 3/2$ piece agrees with the experimental value within 40%.

The comparison between our results and those of Ref. [5] is discussed in Appendix I.

In a future study one should also consider the contributions of the SU(3) decuplet poles, which were recently employed [37] to describe the hyperon nonleptonic decays.

8. *Effective field theory and the weak* $|\Delta S| = 1$ *potential*

Instead of starting with quarks (u, d, s), we present the formalism in which the baryons $(N,\ \Lambda)$ and the mesons $(\pi,\ K,\ \eta)$ appear. Such an approach facilitates the derivation of the effective potential presented in the next section. We will start with $\pi-$exchange contribution.

The process under consideration is

$$\Lambda + \mathrm{N} \to \mathrm{N} + \mathrm{N}; \qquad N = \begin{pmatrix} p \\ n \end{pmatrix} . \tag{8.1}$$

The typical diagram contains one weak and one strong vertex, as shown in Fig. 1.1. In the perturbation calculation this diagram can be deduced from the effective interaction Hamiltonian (8.2)

$$\begin{aligned} \mathcal{H}_{\text{eff}} &= \mathrm{i} \int \mathrm{d}^4x\, \overline{\psi}_n(x) G_F m_\pi^2 (A - \gamma_5 B) \psi_\Lambda(x) \phi_{\pi^0}(x) \\ &\quad + \mathrm{i} \int \mathrm{d}^4x\, \overline{\psi}_p(x) G_F m_\pi^2 (A - \gamma_5 B) \psi_\Lambda(x) \phi_{\pi^+}(x) \\ &\quad + g_{\pi NN} \mathrm{i} \int \mathrm{d}^4x\, \overline{\psi}_N(x) \gamma_5 \tau_i \psi_N(x) \phi_{\pi_i}(x) \\ &= \mathcal{H}_{W\,\pi^0} + \mathcal{H}_{W\,\pi^+} + \mathcal{H}_S \\ &= \int \mathrm{d}^4x\, (h_0(x) + h_+(x) + h_S(x)) . \end{aligned} \tag{8.2a}$$

The last term in (8.2a) is the strong nucleon-pion interaction, which can be written as

$$\begin{aligned} \mathcal{H}_S &= \mathrm{i} g_{\pi NN} \int \mathrm{d}^4x\, \overline{\psi}_N(x) \gamma_5 \tau_i \psi_N(x) \phi_{\pi_i}(x) \\ &= \mathrm{i} g_{\pi NN} \int \mathrm{d}^4x\, \left\{ \left[\overline{\psi}_p(x) \gamma_5 \psi_p(x) - \overline{\psi}_n(x) \gamma_5 \psi_n(x) \right] \phi_{\pi^0}(x) \right. \\ &\quad \left. + \sqrt{2} \overline{\psi}_p(x) \gamma_5 \psi_n(x) \phi_{\pi^+}(x) + \sqrt{2} \overline{\psi}_n(x) \gamma_5 \psi_p(x) \phi_{\pi^-}(x) \right\} . \end{aligned} \tag{8.2b}$$

Frequently one also writes

$$\begin{aligned} g_{pp\pi^0} &= -\, g_{nn\pi^0} &&= g_{\pi NN} \\ g_{pn\pi^+} &= g_{np\pi^-} &&= \sqrt{2} g_{\pi NN} . \end{aligned} \tag{8.2c}$$

The diagram Fig. 1.1 arises from the quadratic term

$$\begin{aligned} \mathcal{H}_{\text{eff}} \mathcal{H}_{\text{eff}} &= \int \mathrm{d}^4x_1\, \mathrm{d}^4x_2 [h_0(x_1) + h_+(x_1) + h_S(x_1)][h_0(x_2) + h_+(x_2) + h_S(x_2)] \\ &= \int \mathrm{d}^4x_1\, \mathrm{d}^4x_2 [h_0(x_1) h_S(x_2) + h_S(x_1) h_0(x_2) \\ &\quad + h_+(x_1) h_S(x_2) + h_S(x_1) h_+(x_2)] + R . \end{aligned} \tag{8.3}$$

Here only $|\Delta S| = 1$ terms are shown. The part R contains the strong pion exchange such as

$$\int \mathrm{d}^4x_1\, \mathrm{d}^4x_2\, h_S(x_1) h_S(x_2) . \tag{8.4}$$

In the terms shown in (8.3) the pion field contraction can be carried out. This leaves us with an effective baryon-baryon (i.e. Λ+ N$\rightarrow$ N + N) interaction. For instance,

$$\begin{aligned} V_{\mathrm{op}} &= \int \mathrm{d}^4 x_1\, \mathrm{d}^4 x_2 : [\underbracket{\phi_{\pi^0}(x_1)\, \phi_{\pi^0}}(x_2)\overline{\psi}_n(x_1) G_F m_\pi^2 A(\Lambda_0^0)\psi_\Lambda(x_1) \\ &\qquad g_{\pi NN}\overline{\psi}_n(x_2)\gamma_5\psi_n(x_2) : \ + \text{similar terms of the same type.} \end{aligned} \tag{8.5a}$$

One can replace:

$$\int \mathrm{d}^4 x_1\, \mathrm{d}^4 x_2 \underbracket{\phi_{\pi^0}(x_1)\, \phi_{\pi^0}}(x_2) \quad \longrightarrow \quad \int \mathrm{d}^3 x_1\, \mathrm{d}^3 x_2\, \Delta(1,2)\,. \tag{8.5b}$$

Here $\Delta(1,2)$ is the pion propagator. The integration over time components goes into the overall energy conservation. In order to do this, one assumes that all baryon states are stationary, i.e. that baryon operator is of the form

$$\psi(x) = \sum_i a_i\, \mathrm{e}^{-\mathrm{i}E_i t/\hbar}\phi_i(\mathbf{r})\,. \tag{8.5c}$$

More about this issue can be found in Appendix G.

The final result for V_{op} is

$$\begin{aligned} V_{\mathrm{op}} &= \int \mathrm{d}^3 x_1\, \mathrm{d}^3 x_2 \Big\{ \big[-g_{\pi NN} \mathpunct{:}\overline{\psi}_n(x_1) G_F m_\pi^2 [A(\Lambda_0^0) - B(\Lambda_0^0)\gamma_5]\psi_\Lambda(x_1) \\ &\qquad \cdot \overline{\psi}_n(x_2)\gamma_5\psi_n(x_2)\mathpunct{:}\Delta(1,2) + (1 \leftrightarrow 2)\big] \\ &\quad + g_{\pi NN}\mathpunct{:}\overline{\psi}_n(x_1) G_F m_\pi^2 [A(\Lambda_0^0) - B(\Lambda_0^0)\gamma_5]\psi_\Lambda(x_1) \cdot \overline{\psi}_p(x_2)\gamma_5\psi_p(x_2)\mathpunct{:}\Delta(1,2) \\ &\qquad + (1 \leftrightarrow 2) \\ &\quad + g_{\pi NN}\sqrt{2}\mathpunct{:}\overline{\psi}_p(x_1) G_F m_\pi^2 [A(\Lambda_-^0) - B(\Lambda_-^0)\gamma_5]\psi_\Lambda(x_1) \cdot \overline{\psi}_n(x_2)\gamma_5\psi_p(x_2)\mathpunct{:}\Delta(1,2) \\ &\qquad + (1 \leftrightarrow 2)]\Big\}\,. \end{aligned} \tag{8.6}$$

Here the first two terms contribute to $\Lambda + \mathrm{n} \rightarrow \mathrm{n} + \mathrm{n}$, while the rest contributes to $\Lambda + \mathrm{p} \rightarrow \mathrm{n} + \mathrm{p}$. The notation : : indicates the normal ordering.

In the nonrelativistic limit we have:

$$\psi_a \simeq \begin{pmatrix} \chi_a \\ \dfrac{\boldsymbol{\sigma} \cdot \mathbf{p}_a}{2m_a}\chi_a \end{pmatrix}, \tag{8.7}$$

where $\chi_a \equiv \chi_a(\mathbf{r})$ is the single particle wave function. All our conventions regarding spinors, $\gamma-$matrices etc. are those used in Ref. [33]. From the expression (8.6), we can see that there are two contributions to the weak-strong mixing amplitude whose space-time properties are determined by the Lorentz structure of vertices. The structures of the first and second amplitudes, displayed in Fig. 8.1 are, respectively, $[\gamma_5(S)] \otimes [1(W)]$ and $[\gamma_5(S)] \otimes [\gamma_5(W)]$.

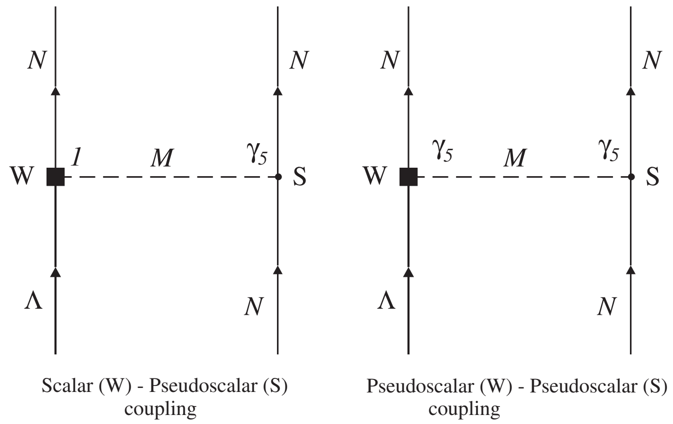

Fig. 8.1. Two different weak (W) - strong (S) combinations for the OME amplitude.

Two different contributions are treated on the same footing. In the nonrelativistic limit, given by (8.7), we can write (to leading terms in $1/m_N$)

$$\overline{\psi}_3\psi_1 \simeq (\chi_3^\dagger\,\chi_1)\,, \text{ scalar} \tag{8.8a}$$

$$\overline{\psi}_3\gamma_5\psi_1 \simeq \left(\frac{1}{2m_N}\right)\chi_3^\dagger(\boldsymbol{\sigma}\cdot\mathbf{q})\chi_1\,, \quad \text{pseudoscalar} \tag{8.8b}$$

$$\overline{\psi}_3\gamma^i\gamma_5\psi_1 \simeq \chi_3^\dagger(\sigma^i)\chi_1\,, \quad \text{space comp. of an axial vector} \tag{8.8c}$$

$$\overline{\psi}_3\gamma^0\gamma_5\psi_1 \simeq \left(\frac{1}{2m_N}\right)\chi_3^\dagger(\boldsymbol{\sigma}\cdot\mathbf{p}_1+\boldsymbol{\sigma}\cdot\mathbf{p}_3)\chi_1\,, \quad \text{time comp. of an axial vector} \tag{8.8d}$$

$$\overline{\psi}_3\gamma^i\psi_1 \simeq \left(\frac{1}{2m_N}\right)\chi_3^\dagger(\sigma^i\boldsymbol{\sigma}\cdot\mathbf{p}_1+\boldsymbol{\sigma}\cdot\mathbf{p}_3\sigma^i)\chi_1\,, \quad \text{space comp. of a vector} \tag{8.8e}$$

$$\overline{\psi}_3\gamma^0\psi_1 \simeq \chi_3^\dagger\chi_1\,, \quad \text{time comp. of a vector}\,, \tag{8.8f}$$

where $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_3$.

From

$$(2\pi)^{-3}(\mathbf{q}^2 + m_M^2)^{-1} = \frac{1}{4\pi}\int \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}}\ \frac{\mathrm{e}^{-m_M r}}{r}\,\mathrm{d}^3\mathbf{r}\,, \tag{8.9a}$$

one gets

$$\Delta(|\mathbf{r}_1 - \mathbf{r}_2|) = V(r) = -\frac{1}{4\pi}\frac{\mathrm{e}^{-m_M r}}{r}\,, \tag{8.9b}$$

i.e. the well known *Yukawa potential*. As discussed in Appendix G the mass dependence can be more complicated than given in (8.9).

For the vertices of the form $[\gamma_5(S)]\otimes[\gamma_5(W)]$, we get a baryon-baryon potential

$$V(r) \sim (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\nabla}_1)(\boldsymbol{\sigma}_2 \cdot \boldsymbol{\nabla}_2)\frac{\mathrm{e}^{-m_M r}}{r}\,, \tag{8.10}$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$. By using some of the identities given in Appendix G , we obtain four different contributions (in the coordinate space) for the effective potential

$$\begin{aligned} V_S &= \frac{1}{3}\left[\frac{m_M^2}{4\pi r}\mathrm{e}^{-m_M r} - \delta(r)\right] \\ V_T &= \frac{1}{3}\frac{m_M^2}{4\pi r}\mathrm{e}^{-m_M r}\left[1 + \frac{3}{m_M r} + \frac{3}{(m_M r)^2}\right] \\ V_{PV} &= \frac{m_M}{4\pi r}\mathrm{e}^{-m_M r}\left(1 + \frac{1}{m_M r}\right). \end{aligned} \tag{8.11}$$

If the weak decay $\Lambda \to \mathrm{N} + \pi$ obeys the isospin selection rule $|\Delta I| = 1/2$, then it is possible to establish the relation

$$\begin{aligned} &\frac{F(\Lambda^0_-)}{F(\Lambda^0_0)} = -\sqrt{2}; \qquad (F = A, B) \\ &\Lambda^0_- \qquad \sim \Lambda^0 \to \mathrm{p} + \pi^- \\ &\Lambda^0_0 \qquad \sim \Lambda^0 \to \mathrm{n} + \pi^0\,. \end{aligned} \tag{8.12}$$

With $f = G_F m_\pi^2 (A - B\gamma_5)$ one can write

$$\begin{aligned} V_{\mathrm{op}} &= \int \mathrm{d}^3x_1\, \mathrm{d}^3x_2 \Big\{ g_{\pi NN}{:}\overline{\psi}_n(x_1) f \psi_\Lambda(x_1) \cdot \big[-\overline{\psi}_n(x_2)\gamma_5\psi_n(x_2) \\ &\qquad + \overline{\psi}_p(x_2)\gamma_5\psi_p(x_2)\big]{:}\Delta(1,2) \\ &\quad -2g_{\pi NN}{:}\overline{\psi}_p(x_1) f \psi_\Lambda(x_1) \cdot \overline{\psi}_n(x_2)\gamma_5\psi_p(x_2){:}\Delta(1,2)\Big\} + (1 \leftrightarrow 2)\,. \end{aligned} \tag{8.13}$$

The bilinear combinations, which contain γ_5 and which are associated with the strong vertex in Fig. 1.1, can be written as

$$\overline{\psi}_N \tau_3 \psi_N = \overline{\psi}_p \psi_p - \overline{\psi}_n \psi_n; \qquad \overline{\psi}_N \tau_+ \psi_N = \overline{\psi}_p \psi_n\,. \tag{8.14}$$

The $|\Delta I| = 1/2$ weak Hamiltonian transforms as the Pauli spinor

$$\chi^{-1/2} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{8.15}$$

Such spinor is sometime known as *spurion*. Thus the bilinear combinations containing Λ field can be written as

$$\begin{aligned} \overline{\psi}_N \tau^3 \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_\Lambda &= (\overline{\psi}_p, \overline{\psi}_n) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_\Lambda \\ &= (-) \overline{\psi}_n \psi_\Lambda . \\ \overline{\psi}_N \tau^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_\Lambda &= (\overline{\psi}_p, \overline{\psi}_n) \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \psi_\Lambda \\ &= \overline{\psi}_p \psi_\Lambda \, . \end{aligned} \tag{8.16a}$$

Formally, one can assume that $\Lambda-$field is a quasi-spinor

$$\psi_{\Lambda\, n} = \begin{pmatrix} 0 \\ \psi_\Lambda \end{pmatrix} . \tag{8.16b}$$

For a future purpose, we also introduce

$$\psi_{\Lambda\, p} = \tau^+ \psi_{\Lambda\, n} = \begin{pmatrix} \psi_\Lambda \\ 0 \end{pmatrix} . \tag{8.16c}$$

Employing the spurion formalism, one can write the first term in (8.13) as

$$\begin{aligned} V_{\rm op}(1/2) \quad &= - \textstyle\sum_i \int {\rm d}^3 x_1 \, {\rm d}^3 x_2 \; g_{\pi NN} : \overline{\psi}_N(x_1) \tau_i \, f \psi_{\Lambda\, n}(x_1) \\ &\qquad \cdot \overline{\psi}_N(x_2) \tau_i \gamma_5 \psi_N(x_2) : , \end{aligned} \tag{8.17}$$

where the relations

$$\begin{aligned} (-) \left(\overline{\psi}_N \tau^3 \psi_{\Lambda\, n} \right) \left(\overline{\psi}_N \tau_3 \psi_N \right) &= + \left(\overline{\psi} \psi_\Lambda \right) \left[\overline{\psi}_p \psi_p - \overline{\psi}_n \psi_n \right] \\ (-) 2 \left(\overline{\psi}_N \tau^+ \psi_{\Lambda\, n} \right) \left(\overline{\psi}_N \tau_- \psi_N \right) &= -2 \left(\overline{\psi} \psi_\Lambda \right) \left(\overline{\psi}_n \psi_p \right) \end{aligned} \tag{8.18}$$

and

$$\sum_i \tau_1^i \tau_2^i = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 = \tau_1^3 \tau_2^3 + 2(\tau_1^+ \tau_1^- + \tau_1^- \tau_2^+) \tag{8.19}$$

have been used. This constitutes the $|\Delta I| = 1/2$ part of the potential $V_{\rm op}$.

One can add the $|\Delta I| = 3/2$ piece (see Appendix C)

$$\begin{aligned} V_{\rm op}(3/2) \quad &= \textstyle\sum_h \int {\rm d}^3 x_1 \, {\rm d}^3 x_2 \; g_{\pi NN} : \overline{\psi}_N(x_1) \\ &\qquad \cdot T_i^m \chi_{1/2}^m \, h \, \psi_\Lambda(x_1) \cdot \overline{\psi}_N(x_2) \tau_i \gamma_5 \psi_N(x_2) : , \end{aligned} \tag{8.20a}$$

where

$$h = G_F m_\pi^2 (\mathcal{A} - \mathcal{B}\gamma_5). \tag{8.20b}$$

The magnitudes of $\mathcal{A}$ and $\mathcal{B}$, as well as their relative signs with respect to A and B (8.2), are calculated theoretically in Sec. 7 and Appendix D. Here

$$\psi_N(x) = \begin{pmatrix} \psi_p(x) \\ \psi_n(x) \end{pmatrix} \tag{8.20b}$$

and

$$\chi_{1/2}^m \quad \rightarrow \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \qquad \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{8.20c}$$

are isospinors, while $\psi_\Lambda(x)$ is an isoscalar.

If one calculates with nuclear wave functions, then the potential can be obtained from

$$V_{\mathrm{op}} = V_{\mathrm{op}}(1/2) + V_{\mathrm{op}}(3/2)\,, \tag{8.21}$$

by omitting the baryon fields ψ_N and ψ_Λ. Thus,

$$V_{\mathrm{op}} \rightarrow V = (-)g_{\pi NN}(f)_1(\gamma_5)_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + g_{\pi NN}(h)_1(\gamma_5)_2 \left(\mathbf{T}_1^m \cdot \chi_{1/2}^m(1)\right) \cdot \boldsymbol{\tau}_2. \tag{8.22a}$$

The notation

$$\mathbf{T}_1^m \cdot \chi_{1/2}^m(1) \tag{8.22b}$$

means that in the actual calculation of this term one has to associate an isospinor $\chi_{1/2}^m(1)$ with the Λ particle, which is also in the position x_1. (Therefore, (8.22) is *not* symmetrized.) In the first term in (8.22a), the Λ particle, which is neutral like neutron, automatically has the spinor $\chi_{1/2}^{-1/2}(1)$. The second term in (8.22a) must be read as

$$N^\dagger \mathbf{T}_1^m \chi_{1/2}^m \boldsymbol{\tau}_2 \rightarrow \begin{cases} (\text{proton :}) & \chi^{1/2\dagger} \mathbf{T}_1^{1/2} \chi^{1/2} \cdot \boldsymbol{\tau}_2 \qquad \text{or} \\ (\text{neutron :}) & \chi^{-1/2\dagger} \mathbf{T}_1^{-1/2} \chi^{-1/2} \cdot \boldsymbol{\tau}_2 \end{cases} \tag{8.23a}$$

with

$$\begin{aligned} \chi^{1/2\dagger} \mathbf{T}_1^{1/2} \chi^{1/2} \cdot \boldsymbol{\tau}_2 &= C^{3/2\,-1/2}_{1\,-1\,1/2\,1/2} \mathbf{t}_1^{-1} \cdot \boldsymbol{\tau}_2 \\ \chi^{-1/2\dagger} \mathbf{T}_1^{-1/2} \chi^{1/2} \cdot \boldsymbol{\tau}_2 &= C^{3/2\,-1/2}_{1\,0\,1/2\,-1/2} \mathbf{t}_1^{0} \cdot \boldsymbol{\tau}_2 \\ \mathbf{t}_1^{-1} \cdot \boldsymbol{\tau}_2 = \tfrac{1}{\sqrt{2}}(1, -\mathrm{i}, 0)(\tau_1, \tau_2, \tau_3) &= \sqrt{2}\tau_- \\ \mathbf{t}_1^0 \cdot \boldsymbol{\tau}_2 &= \tau_2^3\,, \end{aligned} \tag{8.23b}$$

where $C^{3/2\,M}_{1\,r\,1/2\,m_s}$ is the Clebsch-Gordan coefficient.

The notation in equations (8.23) can be understood without the explicit reference to isospinor $\chi^{m}_{1/2}(1)$. (See also Refs. [42,43], formulae (2.29) and (2.43): their $T = T^{\dagger}$(ours).) One can write

$$\overline{\psi}_N(x_1)T_i\psi_{\Lambda}^{-1/2}(x_2)\,. \tag{8.24}$$

In isospin formalism, that means

$$\chi^{m_s\dagger}(1/2)T_i\Delta^M(3/2)\,. \tag{8.25a}$$

Here χ and Δ are $I = 1/2$ and $I = 3/2$ spinors, with the projections m_s and M. The actual values of these projections are

$$\begin{aligned} m_s &= \pm 1/2 && \text{for proton, neutron} \\ M &= -1/2 && \text{for weak transitions}\,. \end{aligned}$$

The transition isospin $\mathbf{T}$ is defined by its matrix elements[5]

$$8.25b\,(T_i)_{m_s\,M} = \sum_r C^{3/2\,M}_{1\,r\,1/2\,m_s} t_i^r\,. \tag{8.25b}$$

Our final results, in the isospin dependent formalism, are displayed in Sec. 9. Some formal details are explained in Appendices C and D.

9. The resulting effective potential

All previous results can be collected in the form of a effective strangeness-violating potential. As already stated, the main aim of this paper was the derivation of such potentials, which will be presented in the form suitable for the usage in nuclear theoretical calculations.

Methods described in Sec. 8. and Appendices A and G allow construction of the strangeness-violating potential which corresponds to the exchange of the pseudoscalar mesons π, K and η. Its radial dependence is described by

$$V_C(M) = \frac{\mathrm{e}^{-m_M r}}{4\pi r} \tag{9.1}$$

$$V_S(M) = \frac{1}{3}\left[\frac{m_M^2}{4\pi r}\,\mathrm{e}^{-m_M r} - \delta(r)\right] \tag{9.2}$$

[5]Other details are given in Appendix C.

$$V_T(M) = \frac{1}{3}\frac{m_M^2}{4\pi r}\,\mathrm{e}^{-m_M r}\left[1 + \frac{3}{m_M r} + \frac{3}{(m_M r)^2}\right] \tag{9.3}$$

$$V_{PV}(M) = \frac{m_M}{4\pi r}\mathrm{e}^{-m_M r}\left(1 + \frac{1}{m_M r}\right) \qquad (M = \{\pi, K, \eta\})\,. \tag{9.4a}$$

According to Appendix G , one can replace m_M by $\epsilon_M = [m_M^2 - (m_\Lambda - m_N)^2/4]^{1/2}$. There is a simple derivative relation between different radial functions, i.e.:

$$\begin{gathered} -\frac{\partial}{\partial r}V_C = V_{PV} \\ \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}V_C\right] = 3V_T = -3\frac{\partial}{\partial r}\left[\frac{1}{r}V_{PV}\right]. \end{gathered} \tag{9.4b}$$

The above radial parts get multiplied by spin-isospin components of the weak-strong vertices and the numerical values of the corresponding amplitudes are given in Sec. 7. Therefore, the generic forms of the potentials will be:

$$V_C \to V_C \cdot 0 \tag{9.5a}$$

$$V_S \to V_S(\{\pi, K, \eta\})\ (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)\ \{B_\pi,\ B_K,\ B_\eta\} \tag{9.5b}$$

$$V_T \to V_T(\{\pi, K, \eta\})\ (\hat{S}_{12})\ \{B_\pi,\ B_K,\ B_\eta\} \tag{9.5c}$$

$$V_{PV} \to V_{PV}(\{\pi, K, \eta\})\ (\hat{\mathrm{r}} \cdot \boldsymbol{\sigma}_2)\ \{A_\pi,\ A_K,\ A_\eta\}\,, \tag{9.5d}$$

with the notation: $\hat{S}_{12} = 3\ (\boldsymbol{\sigma}_1 \cdot \hat{\mathrm{r}}\ \boldsymbol{\sigma}_2 \cdot \hat{\mathrm{r}} - \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2/3)$.

The effective potential can be written in two different (but equivalent) ways: the first version displays the *particle content* of the potential, whereas the second one shows more explicitly its *isospin structure*. The particle content version has to be written in such a way that the particular channel of the Λ−N interaction is specified. On the other hand, the potential that is an operator in the isospace can be applied to both channels.

For the $\Lambda + \mathrm{p} \to \mathrm{p} + \mathrm{n}$ channel, the particle content of the potential is:

$$V(\mathbf{r})_{[\Lambda\ p \to p\, n]} = V_C(r) \cdot 0 + V_1(\mathbf{r}) + V_2(\mathbf{r}) + V_3(\mathbf{r})\,, \tag{9.6}$$

where

$$
\begin{aligned}
V_1(\mathbf{r}) = & (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) \\
& \times\Big\{ V_S(\pi)\,\frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\left(2\tilde{a}+\sqrt{\frac{2}{3}}\tilde{b}\right)\frac{1}{2}[(\overline{n}p)_2(\overline{p}\Lambda)_1+(\overline{n}p)_1(\overline{p}\Lambda)_2] \\
& +V_S(\pi)\frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\left(-\tilde{a}+\sqrt{\frac{2}{3}}\tilde{b}\right)\frac{1}{2}[(\overline{p}p)_2(\overline{n}\Lambda)_1+(\overline{p}p)_1(\overline{n}\Lambda)_2] \\
& +\frac{g_{K\Lambda N}}{2m_N(m_\Lambda+m_N)}\big(V_S(K^+)(\tilde{c}+\tilde{e})\frac{1}{2}[(\overline{n}p)_1(\overline{p}\Lambda)_2+(\overline{n}p)_2(\overline{p}\Lambda)_1] \\
& +V_S(K^0)(\tilde{d}+\tilde{e})\frac{1}{2}[(\overline{p}p)_1(\overline{n}\Lambda)_2+(\overline{p}p)_2(\overline{n}\Lambda)_1]\big) \\
& +\frac{1}{2m_N(m_\Lambda+m_N)}\Big(V_S(\eta_1)\,g_{\eta_1 NN}\tilde{f}\frac{1}{2}[(\overline{p}p)_2(\overline{n}\Lambda)_1+(\overline{p}p)_1(\overline{n}\Lambda)_2] \\
& +V_S(\eta_8)\cdot g_{\eta_8 NN}\tilde{g}\frac{1}{2}[(\overline{p}p)_2(\overline{n}\Lambda)_1+(\overline{p}p)_1(\overline{n}\Lambda)_2]\Big)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
V_2(\mathbf{r}) = & +\hat{S}_{12} \\
& \times\Big\{ V_T(\pi)\frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\left(2\tilde{a}+\sqrt{\frac{2}{3}}\tilde{b}\right)\frac{1}{2}[(\overline{n}p)_2(\overline{p}\Lambda)_1+(\overline{n}p)_1(\overline{p}\Lambda)_2] \\
& +V_T(\pi)\frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\left(-\tilde{a}+\sqrt{\frac{2}{3}}\tilde{b}\right)\frac{1}{2}[(\overline{p}p)_2(\overline{n}\Lambda)_1+(\overline{p}p)_1(\overline{n}\Lambda)_2] \\
& +\frac{g_{K\Lambda N}}{2m_N(m_\Lambda+m_N)}[V_T(K^+)(\tilde{c}+\tilde{e})\frac{1}{2}[(\overline{n}p)_1(\overline{p}\Lambda)_2+(\overline{n}p)_2(\overline{p}\Lambda)_1] \\
& +V_T(K^0)(\tilde{d}+\tilde{e})\frac{1}{2}[(\overline{p}p)_1(\overline{n}\Lambda)_2+(\overline{p}p)_2(\overline{n}\Lambda)_1] \\
& +\frac{1}{2m_N(m_\Lambda+m_N)}\Big([V_T(\eta_1)g_{\eta_1 NN}\tilde{f}\frac{1}{2}[(\overline{p}p)_2(\overline{n}\Lambda)_1+(\overline{p}p)_1(\overline{n}\Lambda)_2] \\
& +V_T(\eta_8)\,g_{\eta_8 NN}\tilde{g}\frac{1}{2}[(\overline{p}p)_2(\overline{n}\Lambda)_1+(\overline{p}p)_1(\overline{n}\Lambda)_2]\Big)\Big\}
\end{aligned}
$$

$$
\begin{aligned}
V_3(\mathbf{r}) = & V_{PV}(\pi)\,\frac{g_{NN\pi}}{2m_N}\left(2a+\sqrt{\frac{2}{3}}b\right)\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{n}p)_2(\overline{p}\Lambda)_1+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{n}p)_1(\overline{p}\Lambda)_2] \\
& +V_{PV}(\pi)\,\frac{g_{NN\pi}}{2m_N}\left(-a+\sqrt{\frac{2}{3}}b\right)\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{p}p)_2(\overline{n}\Lambda)_1+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{p}p)_1(\overline{n}\Lambda)_2] \\
& +\frac{g_{K\Lambda N}}{2m_N}\big(V_{PV}(K^+)(c+e)\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{n}p)_1(\overline{p}\Lambda)_2+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{n}p)_2(\overline{p}\Lambda)_1] \\
& +V_{PV}(K^0)(d+e)\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{p}p)_1(\overline{n}\Lambda)_2+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{p}p)_2(\overline{n}\Lambda)_1]\big) \\
& +\frac{1}{2m_N}\Big(V_{PV}(\eta_1)\,g_{\eta_1 NN}f\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{p}p)_2(\overline{n}\Lambda)_1+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{p}p)_1(\overline{n}\Lambda)_2]\Big) \\
& +V_{PV}(\eta_8)\,g_{\eta_8 NN}g\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{p}p)_2(\overline{n}\Lambda)_1+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{p}p)_1(\overline{n}\Lambda)_2]\Big\}
\end{aligned}
$$

Here the quantities $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ etc. are combinations of the weak nonleptonic amplitudes, and are defined in Table 9.1.

Table 9.1. Weak vertices and their connection with the weak nonleptonic amplitudes.

Weak vertices	Analytic expression	Numerical value	ΔI
a	$\frac{\sqrt{2}}{3}A(\Lambda^0_-)-\frac{1}{3}A(\Lambda^0_0)$	2.3187	1/2
b	$\sqrt{\frac{3}{2}}\left[\frac{\sqrt{2}}{3}A(\Lambda^0_-)+\frac{2}{3}A(\Lambda^0_0)\right]$	-0.0505	3/2
$\tilde{a}$	$\frac{\sqrt{2}}{3}B(\Lambda^0_-)-\frac{1}{3}B(\Lambda^0_0)$	15.862	1/2
$\tilde{b}$	$\sqrt{\frac{3}{2}}\left[\frac{\sqrt{2}}{3}B(\Lambda^0_-)+\frac{2}{3}B(\Lambda^0_0)\right]$	0.032	3/2
c	$\frac{1}{3}\left[A_K(n^0_0)-A_K(p^+_0)+2A_K(p^+_+)\right]$	.597	1/2
d	$\frac{1}{3}\left[A_K(n^0_0)+2A_K(p^+_0)-A_K(p^+_+)\right]$	.367	1/2
e	$\frac{1}{3}\left[-A_K(n^0_0)+A_K(p^+_0)+A_K(p^+_+)\right]$	0.287	3/2
$\tilde{c}$	$\frac{1}{3}\left[B_K(n^0_0)-B_K(p^+_0)+2B_K(p^+_+)\right]$	41.540	1/2
$\tilde{d}$	$\frac{1}{3}\left[B_K(n^0_0)+2B_K(p^+_0)-B_K(p^+_+)\right]$	-22.710	1/2
$\tilde{e}$	$\frac{1}{3}\left[-B_K(n^0_0)+B_K(p^+_0)+B_K(p^+_+)\right]$	0.840	3/2
f	$A_{\eta_1}(\Lambda^0_{\eta_1})$	5.60	1/2
$\tilde{f}$	$B_{\eta_1}(\Lambda^0_{\eta_1})$	27.53	1/2
g	$A_{\eta_8}(\Lambda^0_{\eta_8})$	5.19	1/2
$\tilde{g}$	$B_{\eta_8}(\Lambda^0_{\eta_8})$	22.97	1/2

Similarly, for the $\Lambda + \mathrm{p} \to \mathrm{n} + \mathrm{n}$ channel:

$$V(\mathbf{r})_{[\Lambda\ n\to n\,n]} = V_C(r)\cdot 0 + V_4(\mathbf{r}) + V_5(\mathbf{r}) + V_6(\mathbf{r})\,, \tag{9.7}$$

where

$$\begin{aligned}
V_4(\mathbf{r}) = {}& (\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\
& \times\Big\{\frac{g_{\pi NN}}{2m_N(m_\Lambda+m_N)}\big(V_S(\pi)\left(\tilde{a}-\sqrt{\frac{2}{3}}\tilde{b}\right)\frac{1}{2}[(\overline{n}n)_2(\overline{n}\Lambda)_1+(\overline{n}n)_1(\overline{n}\Lambda)_2]\big) \\
& +\frac{g_{KN\Lambda}}{2m_N(m_\Lambda+m_N)}V_S(K^0)(\tilde{c}+\tilde{d}-\tilde{e})\frac{1}{2}[(\overline{n}n)_2(\overline{n}\Lambda)_1+(\overline{n}n)_1(\overline{n}\Lambda)_2] \\
& +\frac{1}{2m_N(m_\Lambda+m_N)}\Big(g_{\eta_1 NN}V_S(\eta_1)\tilde{f}+g_{\eta_8 NN}V_S(\eta_8)\tilde{g}\Big) \\
& \qquad \tfrac{1}{2}[(\overline{n}n)_2(\overline{n}\Lambda)_1+(\overline{n}n)_1(\overline{n}\Lambda)_2]\Big\}
\end{aligned}$$

$$
\begin{aligned}
V_5(\mathbf{r}) = {} & \hat{S}_{12} \\
& \times\Big\{ \frac{g_{\pi NN}}{2m_N(m_\Lambda+m_N)}\big(V_T(\pi)\left(\tilde{a}-\sqrt{\frac{2}{3}}\tilde{b}\right)\frac{1}{2}[(\overline{n}n)_2(\overline{n}\Lambda)_1+(\overline{n}n)_1(\overline{n}\Lambda)_2]\big) \\
& +\frac{g_{KN\Lambda}}{2m_N(m_\Lambda+m_N)}V_T(K^0)(\tilde{c}+\tilde{d}-\tilde{e})\frac{1}{2}[(\overline{n}n)_2(\overline{n}\Lambda)_1+(\overline{n}n)_1(\overline{n}\Lambda)_2] \\
& +\frac{1}{2m_N(m_\Lambda+m_N)}\Big(g_{\eta_1 NN}V_T(\eta_1)\tilde{f}+g_{\eta_8 NN}V_T(\eta_8)\tilde{g}\Big) \\
& \times\frac{1}{2}[(\overline{n}n)_2(\overline{n}\Lambda)_1+(\overline{n}n)_1(\overline{n}\Lambda)_2]\Big\}
\end{aligned}
$$

$$
\begin{aligned}
V_6(\mathbf{r}) = {} & \frac{g_{\pi NN}}{2m_N}\big(V_{PV}(\pi)\left(a-\sqrt{\frac{2}{3}}b\right)\frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{n}n)_2(\overline{n}\Lambda)_1+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{n}n)_1(\overline{n}\Lambda)_2]\big) \\
& +\frac{g_{KN\Lambda}}{2m_N}V_{PV}(K^0)(c+d-e)\frac{1}{2}[(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{n}n)_2(\overline{n}\Lambda)_1-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{n}n)_1(\overline{n}\Lambda)_2] \\
& +\frac{1}{2m_N}\Big(g_{\eta_1 NN}V_{PV}(\eta_1)f+g_{\eta_8 NN}V_{PV}(\eta_8)g\Big) \\
& \frac{1}{2}[-(\boldsymbol{\sigma}_2\hat{\mathbf{r}})(\overline{n}n)_2(\overline{n}\Lambda)_1+(\boldsymbol{\sigma}_1\hat{\mathbf{r}})(\overline{n}n)_1(\overline{n}\Lambda)_2]\,.
\end{aligned}
$$

The isospin formalism is stated through the bilinear forms (introduced in Sec. 8)

$$
\begin{array}{lll}
\left(\overline{N}\,\mathbf{1}\,\Lambda\begin{pmatrix}0\\1\end{pmatrix}\right)_1\cdot\big(\overline{N}\,\mathbf{1}\,N\big)_2 & \equiv\beta_1 & \Delta I=1/2 \\
\left(\overline{N}\,\boldsymbol{\tau}\Lambda\begin{pmatrix}0\\1\end{pmatrix}\right)_1\cdot\big(\overline{N}\,\boldsymbol{\tau}N\big)_2 & \equiv\beta_\tau; & \Delta I=1/2 \\
\big(\overline{N}\,[\mathbf{T}\,\chi]\,\Lambda\big)_1\cdot\big(\overline{N}\,\boldsymbol{\tau}N\big)_2 & \equiv\beta_T; & \Delta I=3/2
\end{array}
\tag{9.8}
$$

where the change in isospin ΔI has been specified. Their particle content is

$$
\begin{aligned}
\beta_1 &= (\overline{n}\Lambda)_1[(\overline{p}p)_2+(\overline{n}n)_2]\equiv\not{a}+\not{b} \\
\beta_\tau &= (\overline{n}\Lambda)_1[(\overline{n}n)_2-(\overline{p}p)_2]+2(\overline{p}\Lambda)_1(\overline{n}p)_2\equiv\not{b}-\not{a}+2\not{c} \\
\beta_T &= \sqrt{\tfrac{2}{3}}\,\{(\overline{p}\Lambda)_1(\overline{n}p)_2+(\overline{n}\Lambda)_1[(\overline{p}p)_2-(\overline{n}n)_2]\}\equiv\sqrt{\tfrac{2}{3}}\,\{\not{c}+\not{a}-\not{b}\}\,,
\end{aligned}
\tag{9.9}
$$

and the inverse relations read:

$$
\begin{aligned}
\not{a} &= \tfrac{1}{2}\beta_1-\tfrac{1}{6}\beta_\tau+\tfrac{1}{\sqrt{6}}\beta_T \\
\not{b} &= \tfrac{1}{2}\beta_1+\tfrac{1}{6}\beta_\tau-\tfrac{1}{\sqrt{6}}\beta_T \\
\not{c} &= \tfrac{1}{3}\beta_\tau+\tfrac{1}{\sqrt{6}}\beta_T\,.
\end{aligned}
\tag{9.10}
$$

The particle content of the isospin dependent potential can be displayed by using (9.9)[6] The isospin dependent form of the potential is now easily obtained from (9.6) and (9.7) by both: i) rewriting these equations in terms of β_1, β_τ and β_T by means of (9.10), and ii) omitting the baryon fields N and Λ, as already discussed in Sec. 8. Finally, one ends with

$$V(\mathbf{r}) = V_C(r)\cdot 0 + V_7(\mathbf{r}) + V_8(\mathbf{r}) + V_9(\mathbf{r})\,, \tag{9.11}$$

where

$$\begin{aligned}
V_7(r) = &(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\\
&\times\Big\{V_S(\pi)\ \frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\ \tilde{a}\ \frac{1}{2}(\boldsymbol{\tau}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_{2\,\Lambda})\\
&+V_S(\pi)\ \frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\ \tilde{b}\ \frac{1}{2}(\mathbf{T}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\mathbf{T}_{2\,\Lambda})\\
&+\frac{g_{K\Lambda N}}{2m_N(m_\Lambda+m_N)}\Big(V_S(K)\left(\frac{1}{2}\tilde{c}+\tilde{d}\right)\frac{1}{2}[(\mathbf{1}_{1\,\Lambda}\cdot\mathbf{1}_2)+(\mathbf{1}_1\cdot\mathbf{1}_{2\,\Lambda})]\\
&+V_S(K)\tfrac{1}{2}\ \tilde{c}\ \tfrac{1}{2}(\boldsymbol{\tau}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_{2\,\Lambda})\\
&+V_S(K)\ \tilde{e}\ \tfrac{1}{2}(\mathbf{T}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\mathbf{T}_{2\,\Lambda})\Big)\\
&+\frac{1}{2m_N(m_\Lambda+m_N)}\Big(V_S(\eta_1)\ g_{\eta_1 NN}\ \tilde{f}+V_S(\eta_8)\ g_{\eta_8 NN}\ \tilde{g}\Big)\\
&\qquad\qquad\tfrac{1}{2}[(\mathbf{1}_{1\,\Lambda}\cdot\mathbf{1}_2)+(\mathbf{1}_1\cdot\mathbf{1}_{2\,\Lambda})]\Big\}
\end{aligned}$$

$$\begin{aligned}
V_8(\mathbf{r}) = &+\hat{S}_{12}\\
&\times\Big\{V_T(\pi)\ \frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\ \tilde{a}\ \frac{1}{2}(\boldsymbol{\tau}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_{2\,\Lambda})\\
&+V_T(\pi)\ \frac{g_{NN\pi}}{2m_N(m_\Lambda+m_N)}\ \tilde{b}\ \frac{1}{2}(\mathbf{T}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\mathbf{T}_{2\,\Lambda})\\
&+\frac{g_{K\Lambda N}}{2m_N(m_\Lambda+m_N)}\Big(V_T(K)\left(\frac{1}{2}\tilde{c}+\tilde{d}\right)\frac{1}{2}[(\mathbf{1}_{1\,\Lambda}\cdot\mathbf{1}_2)+(\mathbf{1}_1\cdot\mathbf{1}_{2\,\Lambda})]\\
&+V_T(K)\tfrac{1}{2}\ \tilde{c}\ \tfrac{1}{2}(\boldsymbol{\tau}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_{2\,\Lambda})\\
&+V_T(K)\ \tilde{e}\ \tfrac{1}{2}(\mathbf{T}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+\boldsymbol{\tau}_1\cdot\mathbf{T}_{2\,\Lambda})\Big)\\
&+\frac{1}{2m_N(m_\Lambda+m_N)}\Big(V_T(\eta_1)\ g_{\eta_1 NN}\ \tilde{f}\\
&+V_T(\eta_8)\ g_{\eta_8 NN}\ \tilde{g}\Big)\tfrac{1}{2}[(\mathbf{1}_{1\,\Lambda}\cdot\mathbf{1}_2)+(\mathbf{1}_1\cdot\mathbf{1}_{2\,\Lambda})]\Big\}
\end{aligned}$$

[6] For instance, for the PV exchange of the pion, one finds:

$$V_\pi = a\beta_\tau + b\beta_T = \not{a}\left(-a+\sqrt{\frac{2}{3}}b\right)+\not{b}\left(a-\sqrt{\frac{2}{3}}b\right)+\not{c}\left(2a+\sqrt{\frac{2}{3}}b\right).$$

$$
\begin{aligned}
V_9(r) = & V_{PV}(\pi)\ \frac{g_{NN\pi}}{2m_N}\ a\ \frac{1}{2}(-(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{r}})\boldsymbol{\tau}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+(\boldsymbol{\sigma}_1\cdot\hat{\mathrm{r}})\ \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_{2\,\Lambda}) \\
& +V_{PV}(\pi)\ \frac{g_{NN\pi}}{2m_N}\ b\ \frac{1}{2}(-(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{r}})\mathbf{T}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+(\boldsymbol{\sigma}_1\cdot\hat{\mathrm{r}})\boldsymbol{\tau}_1\cdot\mathbf{T}_{2\,\Lambda}) \\
& +\frac{g_{K\Lambda N}}{2m_N}\big(V_{PV}(K)\left(\frac{1}{2}c+d\right)\frac{1}{2}[(\boldsymbol{\sigma}_1\cdot\hat{\mathrm{r}})(\mathbf{1}_{1\,\Lambda}\cdot\mathbf{1}_2)-(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{r}})(\mathbf{1}_1\cdot\mathbf{1}_{2\,\Lambda})] \\
& +V_{PV}(K)\tfrac{1}{2}\ c\ \tfrac{1}{2}((\boldsymbol{\sigma}_1\cdot\hat{\mathrm{r}})\boldsymbol{\tau}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2-(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{r}})\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_{2\,\Lambda}) \\
& +V_{PV}(K)\ e\ \tfrac{1}{2}(-(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{r}})\mathbf{T}_{1\,\Lambda}\cdot\boldsymbol{\tau}_2+(\boldsymbol{\sigma}_1\cdot\hat{\mathrm{r}})\boldsymbol{\tau}_1\cdot\mathbf{T}_{2\,\Lambda}) \\
& +\frac{1}{2m_N}\Big(V_{PV}(\eta_1)\ g_{\eta_1 NN}\ f+V_{PV}(\eta_8)\ g_{\eta_8 NN}\ g\Big) \\
& \times\tfrac{1}{2}[-(\boldsymbol{\sigma}_2\cdot\hat{\mathrm{r}})(\mathbf{1}_{1\,\Lambda}\cdot\mathbf{1}_2)+(\boldsymbol{\sigma}_1\cdot\hat{\mathrm{r}})(\mathbf{1}_1\cdot\mathbf{1}_{2\,\Lambda})]\,.
\end{aligned}
$$

Appendix A: Meson poles and double counting

Baryonic matrix element of a general axial vector current has the form (6.2), i.e.

$$\langle B_\beta|\,A^\mu_a(0)\,|B_\alpha\rangle=\overline{u}_\beta(p')\left[\gamma^\mu\gamma_5 g_A(q^2)+\frac{1}{M_\alpha+M_\beta}g_P(q^2)q^\mu\gamma_5\right]\Lambda_a u_\alpha(p)\,. \quad \text{(A.1)}$$

Here Λ_a is a matrix (for example $\lambda_a/2$) which describes the internal (for example SU(3) flavor) hadron symmetry. Its precise form is not needed here.

The meson matrix element is

$$\langle 0|\,A^\mu_a(0)\,|M_b\rangle=\mathrm{i}\delta_{ab}f_M q^\mu. \quad \text{(A.2)}$$

If the meson is on the mass shell, then

$$q^2=m_M^2\,. \quad \text{(A.3}a\text{)}$$

The PCAC relation is

$$\partial_\mu A^\mu_a=C\Phi_a\,. \quad \text{(A.4)}$$

It leads to

$$
\begin{aligned}
\langle 0|\,\partial_\mu A^\mu_a(x)\,|M_a\rangle & =\partial_\mu\left[\mathrm{e}^{-\mathrm{i}qx}\langle 0|\,A^\mu_a(0)\,|M_a\rangle\right]=(-\mathrm{i})\mathrm{e}^{-\mathrm{i}qx}q_\mu\langle 0|\,A^\mu_a(0)\,|M\rangle \\
& =q^2 f_M(q^2)\mathrm{e}^{-\mathrm{i}qx}=C(q^2)\mathrm{e}^{-\mathrm{i}qx}.
\end{aligned} \quad \text{(A.5)}
$$

If the meson is on the mass shell, one should select

$$C(m_M^2)=m_M^2 f_M\,. \quad \text{(A.3}b\text{)}$$

Also

$$\begin{aligned}
&\partial_\mu \langle B_\beta |\, A^\mu_a(x)\, | B_\alpha\rangle \\
&\quad = \partial_\mu \left[\mathrm{e}^{\mathrm{i}(p'-p)x} \langle B_\beta |\, A^\mu_a(0)\, | B_\alpha\rangle \right] \\
&\quad = \mathrm{i}\mathrm{e}^{\mathrm{i}(p'-p)x} \overline{u}_\beta(p') \left[g_A(\not p - \not p') - \frac{q^2}{M_\alpha + M_\beta} g_P \right] \gamma_5 \Lambda_a u_\alpha(p) \\
&\quad = (-\mathrm{i})\mathrm{e}^{\mathrm{i}(p'-p)x} \overline{u}_\beta(p') \left[(M_\alpha + M_\beta) g_A + \frac{q^2}{M_\alpha + M_\beta} g_P \right] \gamma_5 \Lambda_a u_\alpha(p) \\
&\quad = C \langle B_\beta |\, \Phi_a(x)\, | B_\alpha\rangle \\
&\quad = C \mathrm{e}^{\mathrm{i}(p'-p)x} \langle B_\beta |\, \Phi_a(0)\, | B_\alpha\rangle \\
&\quad = C \mathrm{e}^{-\mathrm{i}qx} \langle B_\beta |\, \Phi_a(0)\, | B_\alpha\rangle\,.
\end{aligned} \tag{A.6}$$

The matrix element $\langle B_\beta|\, \Phi_a \,|B_\alpha\rangle$ can be calculated by using the equation of motion [33]

$$(\Box + m_M^2)\Phi_a(x) = -\mathrm{i} g_{\alpha\beta} \overline{\psi}_\beta(x) \gamma_5 \Lambda_a \psi_\beta(x)\,, \tag{A.7}$$

which leads to

$$(-q^2 + m_M^2) \langle B_\beta |\, \Phi \,| B_\alpha\rangle = -\mathrm{i} g_{\alpha\beta} \overline{u}_\beta \gamma_5 \Lambda_a u_\alpha\,. \tag{A.8}$$

Here $g_A,\ g_P,\ g_{\alpha\beta}$ and C are in general some functions of q^2.

By inserting (A.8) in (A.6) and dropping unimportant factors, one finds a relation

$$(M_\alpha + M_\beta) g_A(q^2) + \frac{q^2}{M_\alpha + M_\beta} g_P(q^2) = m_M^2 f_M \frac{g_{\alpha\beta}(q^2)}{m_M^2 - q^2}\,. \tag{A.9}$$

In the limit $q^2 \to 0$, assuming that

$$\begin{aligned} g_{\alpha\beta}(0) &= g_{\alpha\beta}(m_M^2) = g_{\alpha\beta} \\ g_A(0) &= g_A = 1.25\,, \end{aligned} \tag{A.10}$$

one finds the famous Goldberger-Treiman (GT) relation (generalized):

$$\frac{g_A}{f_M} = \frac{g_{\alpha\beta}}{M_\alpha + M_\beta}\,. \tag{A.11}$$

The induced pseudoscalar formfactor g_P is dominated by the pseudoscalar meson (for example pion) pole. This is described by the diagram shown in Fig. A.1. The diagram in Fig.A.1 corresponds to

$$(\mathrm{Fig.A.1})^\mu = \overline{u}_\beta(p') \gamma_5 \Lambda_a u_\alpha(p) g_{\alpha\beta}(q^2) \frac{\mathrm{i}}{q^2 - m_M^2 + \mathrm{i}\epsilon} \langle M_b |\, A^\mu_b \,|0\rangle\,. \tag{A.12}$$

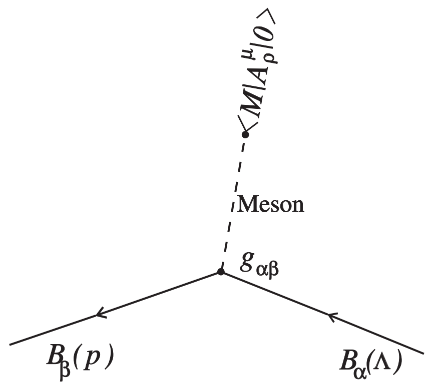

Fig. A.1. Here $g_{\alpha\beta}$ appears in the strong vertex. The axial vector current acts on the intermediate meson M, as indicated.

Inserting (A.2) and compairing the result with (A.1) one finds

$$\frac{g_P(q^2)}{M_\alpha + M_\beta} \simeq (-)\frac{g_{\alpha\beta}(q^2) f_M(q^2)}{q^2 - m_M^2} \,. \tag{A.13a}$$

Inclusion of vertex and other corrections, such as multipion exchanges [6], turns that into

$$\frac{g_P(q^2)}{M_\alpha + M_\beta} = H(q^2) + \frac{g_{\alpha\beta}(q^2) f_M(q^2)}{m_M^2 - q^2} F(q^2) \,. \tag{A.13b}$$

If one neglects H and assumes

$$\begin{aligned} g_{\alpha\beta}(q^2) &\simeq g_{\alpha\beta}(m_M^2) \\ f_M(q^2) &\simeq f_M(m_M^2) = f_M \\ F(q^2) &\simeq 1 \,, \end{aligned} \tag{A.13c}$$

one finds the well-known estimate

$$\frac{g_P(q^2)}{M_\alpha + M_\beta} \simeq \frac{g_{\alpha\beta} f_M}{m_M^2 - q^2} \,. \tag{A.14}$$

For the nucleon (N)-pion (π) system one finds

$$g_P \sim \frac{4M_N^2 g_A}{m_\pi^2 - q^2} \,. \tag{A.15}$$

In the case of the muon capture the effective pseudoscalar constant becomes

$$\frac{g_P}{2M_N}\, m_\mu = \frac{2M_N m_\mu g_A}{m_\pi^2 - q^2(m_\mu)} \frac{2M_N m_\mu g_A}{m_\pi^2 + 0.88 \cdot m_\mu^2} \,. \tag{A.16}$$

Here the muon mass m_μ comes from the lepton current

$$q_\rho \overline{\psi}_\nu \gamma^\rho \gamma_5 \psi_\mu = (-) m_\mu \overline{\psi}_\nu \gamma_5 \psi_\mu \,. \tag{A.17}$$

The expression (A.16) corresponds to the formula (4.37) in [13]. (Their g_P is not equal to our g_P (A.15).)

The GT relation (A.11) and most of the following formulae were derived by using (A.3) convention. Some particular questions that might arise when the meson Φ_a is not no the mass shell will be mentioned in the main text.

Appendix B: Commutators involving bilinear quark field combinations

The canonical anti-commutation relations for Dirac (i.e. quark) fields are [34]

$$\{\psi_\alpha(t,\mathbf{x}), \psi^\dagger_\beta(t,\mathbf{y})\} = \delta_{\alpha\beta}\delta(\mathbf{x}-\mathbf{y}) \,. \tag{B.1}$$

When applying current algebra (CA) on PCAC one encounters the commutators of the bilinear combinations of quark fields such as

$$[\psi^\dagger_\phi(z)\Delta_{\phi\epsilon}\psi_\epsilon(z), \psi^\dagger_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)]\big|_{z_0=x_0} = C \,. \tag{B.2}$$

Here Δ and Γ are some products of Dirac $\gamma-$matrices and the inner group operators corresponding to SU(3) flavor and color. By repeatedly using (B.1) one can write

$$\begin{aligned} \psi^\dagger_\phi \psi_\epsilon \psi^\dagger_\alpha \psi_\beta &= \psi^\dagger_\alpha \psi_\beta \psi^\dagger_\phi \psi_\epsilon + \delta_{\epsilon\alpha} \psi^\dagger_\phi \psi_\beta \delta(\mathbf{x}-\mathbf{z}) \\ &\quad -\delta_{\phi\beta}\psi^\dagger_\alpha \psi_\epsilon \delta(\mathbf{x}-\mathbf{z}) \,. \end{aligned} \tag{B.3}$$

This leads to

$$\begin{aligned} C &= [\psi^+_\phi \Delta_{\phi\epsilon}\Gamma_{\epsilon\beta}\psi_\epsilon - \psi^+_\alpha \Gamma_{\alpha\beta}\Delta_{\beta\epsilon}\psi_\beta]\delta(\mathbf{x}-\mathbf{z}) \\ &= \psi^+(x)[\Delta,\Gamma]\psi(x)\delta(\mathbf{x}-\mathbf{z}) \,. \end{aligned} \tag{B.4}$$

Let us take Δ and Γ of the form

$$\begin{aligned} \Delta &= D\, t_i \\ \Gamma &= G\, t_j; \qquad t_i = \lambda_i/2 \,. \end{aligned} \tag{B.5a}$$

Here λ_i are SU(3)-flavor matrices while D and G are some combinations of Dirac matrices. One can write

$$[t_i\, D, t_j\, G] = \frac{1}{2}[t_i,t_j]\{D,G\} + \frac{1}{2}\{t_i,t_j\}[D,G] \,. \tag{B.5b}$$

It is useful to specify that for some cases:

(1)

$$\begin{aligned} D &= \gamma_5, \qquad G = \gamma_0 \Gamma^\mu_{L,R} \\ \Gamma^\mu_{L,R} &= \gamma_\mu (1 \mp \gamma_5) \,. \end{aligned} \tag{B.5c}$$

This gives

$$\begin{aligned} [D, G] &= \gamma_5 \gamma_0 \Gamma^\mu - \gamma_0 \Gamma^\mu \gamma_5 = 0 \\ \{D, G\} &= 2\gamma_5 \gamma_0 \Gamma^\mu \\ [t_i, t_j] &= \mathrm{i} f_{ijk} t_k \end{aligned} \tag{B.5d}$$

(2)

$$\begin{aligned} D &= 1, \qquad G = \gamma_0 \Gamma^\mu_{L,R} \\ [D, G] &= \gamma_0 \Gamma^\mu - \gamma_0 \Gamma^\mu = 0 \\ \{D, G\} &= 2\gamma_0 \Gamma^\mu \end{aligned} \tag{B.5e}$$

(3)

$$D = \gamma_5, \qquad G_{L,R} = \gamma_0 (1 \mp \gamma_5)$$

$$\begin{aligned} [D, G_{L,R}] &= \gamma_5 \gamma_0 (1 \mp \gamma_5) - \gamma_0 (1 \mp \gamma_5) \\ &= 2\gamma_5 \gamma_0 (1 \mp \gamma_5) \\ \{D, G\} &= \gamma_5 \gamma_0 (1 \mp \gamma_5) + \gamma_0 (1 \mp \gamma_5) \gamma_5 = 0 \,. \end{aligned} \tag{B.6}$$

The relations (B.4) and (B.5) can be used to derive CA [44, 45]. With

$$\begin{aligned} j_i^\mu &= \overline{\psi} \gamma^\mu t_i \psi \\ j_i^{\mu\, 5} &= \overline{\psi} \gamma^\mu \gamma_5 t_i \psi \,, \end{aligned} \tag{B.7a}$$

one finds

$$\begin{aligned} [j_i^0(x), j_j^\mu(y)]_{x_0 = y_0} &= \mathrm{i} f_{ijk} j_k^\mu(x) \delta(\mathbf{x} - \mathbf{y}) \\ [j_i^0(x), j_j^{\mu\, 5}(y)]_{x_0 = y_0} &= \mathrm{i} f_{ijk} j_k^{\mu\, 5}(x) \delta(\mathbf{x} - \mathbf{y}) \\ [j_i^{0\, 5}(x), j_j^\mu(y)]_{x_0 = y_0} &= \mathrm{i} f_{ijk} j_k^{\mu\, 5}(x) \delta(\mathbf{x} - \mathbf{y}) \\ [j_i^{0\, 5}(x), j_j^{\mu\, 5}(y)]_{x_0 = y_0} &= \mathrm{i} f_{ijk} j_k^\mu(x) \delta(\mathbf{x} - \mathbf{y}) \,. \end{aligned} \tag{B.7b}$$

This can be easily transformed into the commutation rules for the SU(3) generators

$$F_i = \int \mathrm{d}^3 x \, j_i^0(x) \tag{B.8a}$$

and the axial vector charges

$$F_i^5 = \int \mathrm{d}^3x\, j_i^{0\,5}(x)\,. \tag{B.8b}$$

Integration over x simply removes delta-functions in (B.7).

General structure of the operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_4$ contains terms as

$$\begin{aligned} j_{\mu\,k}(x)j_\ell^\mu(x) + j_{\mu\,5\,k}(x)j_\ell^{\mu\,5}(x) \;&= b(\mathrm{PC}) \\ j_{\mu\,5\,k}(x)j^{\mu\,\ell}(x) + j_{\mu\,k}(x)j_\ell^{\mu\,5}(x) \;&= a(\mathrm{PV})\,. \end{aligned} \tag{B.9}$$

Quark fields ψ in those terms are not normally ordered. (Some details about that statement can be found in Appendix E.) Thus one can simply apply (B.7) and (B.8) in order to show

$$[F_i^5, \mathcal{O}_A(\mathrm{PV})] = -[F_i, \mathcal{O}_A(\mathrm{PC})] \qquad (A = 1,2,3,4)\,. \tag{B.10a}$$

Operators $\mathcal{O}_5$ and $\mathcal{O}_6$ as shown in Appendix E contain vacuum expectation values (VEV's) when written without normal ordering. Obviously

$$[F_i^5, \tilde{\mathcal{O}}_B(\mathrm{PV})] = -[F_i, \tilde{\mathcal{O}}_B(\mathrm{PC})] \qquad (B = 1,2,3,4) \qquad (\tilde{\mathcal{O}} \text{ is not NOP}) \tag{B.10b}$$

(NOP, normally ordered product). In addition one has to calculate the commutator with the term $\overline{d}(1 \pm \gamma_5)s$. One finds (with $\psi = (u,d,s)$):

$$\begin{aligned} [F_3^5, \overline{d}(1-\gamma_5)s] \;&= -\tfrac{1}{2}\overline{d}(1-\gamma_5)s \\ [F_3^5, \overline{d}(1+\gamma_5)s] \;&= \tfrac{1}{2}\overline{d}(1+\gamma_5)s \\ [F_3, \overline{d}(1-\gamma_5)s] \;&= -\tfrac{1}{2}\overline{d}(1-\gamma_5)s \\ [F_3, \overline{d}(1+\gamma_5)s] \;&= -\tfrac{1}{2}\overline{d}(1+\gamma_5)s\,. \end{aligned} \tag{B.11}$$

Thus, one can write [7]

$$\begin{aligned} [F_3^5, \hat{\mathcal{O}}_6] \;&= [F_3^5, \tilde{\mathcal{O}}_6] - \tfrac{2}{6}\langle 0|\,\overline{d}d\,|0\rangle\overline{d}(1-\gamma_5)s + \tfrac{2}{6}\langle 0|\,\overline{s}s\,|0\rangle\overline{d}(1+\gamma_5)s \\ [F_3, \hat{\mathcal{O}}_6] \;&= [F_3, \tilde{\mathcal{O}}_6] - \tfrac{2}{6}\langle 0|\,\overline{d}d\,|0\rangle\overline{d}(1-\gamma_5)s - \tfrac{2}{6}\langle 0|\,\overline{s}s\,|0\rangle\overline{d}(1+\gamma_5)s\,. \end{aligned} \tag{B.12a}$$

With

$$[F_3^5, \tilde{\mathcal{O}}_6] = -[F_3, \tilde{\mathcal{O}}_6], \quad L_d = \langle 0|\,\overline{d}d\,|0\rangle\overline{d}(1-\gamma_5)s, \quad L_s = \langle 0|\,\overline{s}s\,|0\rangle\overline{d}(1+\gamma_5)s\,, \tag{B.12b}$$

[7] See Appendix E for the definition of $\hat{\mathcal{O}}$.

one can write

$$\begin{aligned}[F_3^5, \hat{\mathcal{O}}_6] = &\ [F_3^5, \tilde{\mathcal{O}}_6] - \tfrac{2}{6} L_d + \tfrac{2}{6} L_s \\ &= -[F_3, \hat{\mathcal{O}}_6] - \tfrac{2}{6} L_d - \tfrac{2}{6} L_s - \tfrac{2}{6} L_d + \tfrac{2}{6} L_s \\ &= -[F_3, \hat{\mathcal{O}}_6] - \tfrac{2}{3} \langle 0 | \, \overline{d} d \, | 0 \rangle \overline{d} (1 - \gamma_5) s \, .\end{aligned} \tag{B.12c}$$

This is the formula (17) in ref. [21] (up to the different sign convention). The operator $\hat{\mathcal{O}}_5$ satisfies the same relation with the replacement

$$L_d \to \frac{16}{3} L_d \, . \tag{B.12d}$$

Everything can be generalized for kaon emmision by selecting appropriate indices in (B.11).

Appendix C: Transition isospin T

This formalism, suitable for isospin $I = 3/2$ (or spin $S = 3/2$) particles is described in Ref. [42] and [43].

An isospin $I = 3/2$ object can be constructed by combining the isovectors[8]

$$\mathbf{t}^{\,1} = \frac{1}{\sqrt{2}} (1, i, 0), \qquad \mathbf{t}^{\,-1} = \frac{1}{\sqrt{2}} (1, -\mathrm{i}, 0), \qquad \mathbf{t}^{\,3} = (0, 0, 1) \, , \tag{C.1}$$

with the isospin function $\chi^m_{1/2}$. That results in

$$Z_i^M = \sum_{r,m} C^{3/2\,M}_{1\,r\,1/2\,m} t_i^r \chi^m_{1/2} \qquad (i = 1, 2, 3) \, . \tag{C.2}$$

A coupling with isospin $I = 1$ field (for example pion π) is now

$$N^{m_s \, \dagger} \pi_i Z_i^M = \chi^{m_s \, \dagger}_{1/2} \sum_{r,m} C^{3/2\,M}_{1\,r\,1/2\,m} \chi^m_{1/2} \mathbf{t}^r \boldsymbol{\pi} \, . \tag{C.3}$$

Here N^{m_s} is either proton ($m_s = 1/2$) or neutron ($m_s = -1/2$), and $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3)$. One can introduce *transition isospin* $\mathbf{T}$ which is defined by its matrix element[9]:

$$\big(\mathbf{T} \big)_{m_s M} = C^{3/2\,M}_{1\,r\,1/2\,m_s} \, \mathbf{t}^r \, . \tag{C.4}$$

---

[8] For spin Ref. [42] uses $\epsilon^1 = -(1, i, 0)/\sqrt{2}$. The + sign here leads to the usual pion field definition $\pi^{\pm} = (\pi_1 \mp \mathrm{i}\pi_2)/\sqrt{2}$.

[9] Here again we have small difference with the Ref. [42] convention: our $\mathbf{T} = \mathbf{T}^*$Ref. [42]. In our case, $I = 3/2$ object will always be on the R.H.S. in bilinear combination (C.3).

Then (C.3) can be written as

$$\begin{aligned} N^{m_s\,\dagger}\,(\mathbf{T}\cdot\boldsymbol{\pi})\,Z^M(3/2) &= F(m_s,M) = \chi_{1/2}^{m_s\,\dagger}\sum\nolimits_{r,m} C_{1\,r\,1/2\,m}^{3/2\,M}\chi_{1/2}^{m}\mathbf{t}^{\,r}\boldsymbol{\pi} \\ &= \sum\nolimits_r C_{1\,r\,1/2\,m_s}^{3/2\,M}\mathbf{t}^{\,r}\cdot\boldsymbol{\pi}\,. \end{aligned} \tag{C.5a}$$

(Here $\chi^{m_s\,\dagger}\chi^{m} = \delta_{m_s\,m}$.) As an example let us take an object with $M = -1/2$. Then for $m_s = -1/2$ one gets

$$\begin{aligned} &\sum\nolimits_r C_{1\,r\,1/2\,-1/2}^{3/2\,-1/2} = C_{1\,0\,1/2\,-1/2}^{3/2\,-1/2} = \sqrt{\tfrac{2}{3}} \\ &F(-1/2,-1/2) = \sqrt{\tfrac{2}{3}}\mathbf{t}^0\cdot\boldsymbol{\pi} = \sqrt{\tfrac{2}{3}}\pi^0 \end{aligned} \tag{C.5b}$$

and

$$\overline{n}\,Z^{-1/2}(3/2)\cdot\pi^0 = \mathcal{F}(-1/2,-1/2)\,. \tag{C.6}$$

The expression (C.6) is the isospin conserving coupling between $I = 1/2$ $\overline{N}(\overline{p},\overline{n})$ field, $I = 3/2$ $\mathcal{E}$ field and π (pion) field.

One also finds for $m_2 = 1/2$:

$$\begin{aligned} \sum\nolimits_r C_{1\,r\,1/2\,1/2}^{3/2\,-1/2} &= C_{1\,-1\,1/2\,-/2}^{3/2\,-1/2} = \tfrac{1}{\sqrt{3}} \\ F(1/2,-1/2) &= \tfrac{1}{\sqrt{3}}\mathbf{t}^{-1}\cdot\boldsymbol{\pi} = \tfrac{1}{\sqrt{3}}\tfrac{1}{\sqrt{2}}(\pi_1 - \mathrm{i}\pi_2) = \tfrac{1}{\sqrt{3}}\pi^+, \end{aligned} \tag{C.7a}$$

and

$$\begin{aligned} \overline{p}Z^{-1/2}(3/2)\cdot\pi^+ &= \mathcal{F}(1/2,-1/2) \\ \mathcal{F}(m_s,M) &= \chi^{m_s\,\dagger}\,Z^M\cdot F(m_s,M) \\ m_s = 1/2 &\quad \text{proton} \\ m_s = -1/2 &\quad \text{neutron}\,. \end{aligned} \tag{C.7b}$$

Remark: the same formalism will be used for weak vertices $\overline{\mathrm{N}}\Lambda\pi$ where Λ will be given the quasi isospin $I = 1/2$ and $I = 3/2$.

Appendix D: $\Delta I = 3/2$ isospin change

The pion exchange contributions contain very small $\Delta I = 3/2$ pieces. This reflects the fact that the experimental $\Lambda\to\mathrm{N}\pi$ decay amplitudes contain very small $\Delta I = 3/2$ contributions. For example, from $B(\Lambda^0_-)_{\mathrm{exp}} = 22.4$ and $B(\Lambda^0_0)_{\mathrm{exp}} = -15.61$ (in units of 10^{-7}) one can deduce from a $\Delta I = 1/2$ based sum rule $-B(\Lambda^0_-)_{\mathrm{exp}}/\sqrt{2} = B(\Lambda^0_0) = -15.84$. With experimental numbers one obtains

$$\left|\frac{B(\Lambda^0_-)/\sqrt{2} + B(\Lambda^0_0)}{B(\Lambda^0_0)}\right|_{\mathrm{exp}} = 0.02$$

instead of zero. Somewhat different picture emerges in the case of weak NNK couplings, whose values are shown in Table D.1. Our results were obtained by including separable contributions (see Table 7.4) which contain $\Delta I = 3/2$ piece. Experimental values were used in pole terms, and they had also $\Delta I = 3/2$ pieces. The relative importance of the $\Delta I = 3/2$ terms can be directly seen from

Table D.1. Nonleptonic amplitudes ($\times 10^7$); †this work, compared with rescaled values of Refs. [22] and [5].

Amplitude	Tot. ampl.†	Ref. [22]	Ref. [5]
$A_K(p_0^+)$	4.08	2.84	4.62
$B_K(p_0^+)$	-21.87	-5.61	-14.65
$A_K(p_+^+)$	2.81	0.76	1.68
$B_K(p_+^+)$	42.38	23.19	41.77
$A_K(n_0^0)$	6.25	3.60	6.30
$B_K(n_0^0)$	17.99	19.86	27.12

the general weak potential (9.11). One can obtain some interesting information from Table D.1 also by checking how well is the $\Delta I = 1/2$ sum rule (D.1) satisfied

$$\begin{aligned} F(p_0^+) + F(p_+^+) &= F(n_0^0) \\ F(p_0^+) + F(p_+^+) &= \Sigma(F) \\ F &= A, B\,. \end{aligned} \tag{D.1}$$

A useful measure of the discrepancy is

$$D(F) = \Big|\frac{\Sigma(F) - F(n_0^0)}{\Sigma(F)}\Big|\,. \tag{D.2}$$

In our case one finds (in 10^{-7} units)

$$\begin{aligned} \Sigma(A) &= 6.87 \\ A(n_0^0) &= 6.25 \\ D(A) &= 0.09 \\ \Sigma(B) &= 20.51 \\ B(n_0^0) &= 17.99 \\ D(B) &= 0.12\,. \end{aligned} \tag{D.3}$$

The amplitudes of Ref. [5] were obtained with the assumption that $\mathcal{H}_W$ contains only $\Delta I = 1/2$ piece. Thus from Table D.1 one finds for their values

$$D(A) = 0; \qquad D(B) = 0\,. \tag{D.4}$$

In the chiral Lagrangian approach of ref. [22] the $\Delta I = 3/2$ contributions are present in B amplitudes

$$D(A) = 0.002, \qquad D(B) = 0.13\,. \tag{D.5}$$

They have also used the $\Delta I = 1/2$ Hamiltonian.

Appendix E: Commutators and normally-ordered operators

The product of fields appearing in the four-quark operators $\mathcal{O}_1,\ \mathcal{O}_2,\ \ldots\ ,\mathcal{O}_6$ comprising the effective weak Hamiltonian are normal-ordered. Thus one has to be careful when calculating the commutators which appear in evaluation of the current algebra contributions (CAC). The current commutators, or more general, the commutators of bilinear quark-field forms, are best used if the operators $\mathcal{O}_i$ are written in the form which is no longer normally ordered. (Some details about commutators can be found in Sec. 5.)

The task of unscrambling the normally ordered product (NOP) can be achieved by using

(a) Wick's theorem for normal-ordered products (WT) [34],

(b) Fierz transformation (FIT) which has been described in Sec.3.

The WT for a product of four quark fields ψ_α (here α denotes all indices: Dirac's components, SU(3) flavour, SU(3) colour, etc.) is:

$$\begin{aligned}\overline{\psi}_\alpha\psi_\beta\,\overline{\psi}_\gamma\psi_\delta =\ & :\overline{\psi}_\alpha\psi_\beta\,\overline{\psi}_\gamma\psi_\delta: + \underbracket{\overline{\psi}_\alpha\psi_\beta}\,:\overline{\psi}_\gamma\psi_\delta: \\ & + :\overline{\psi}_\alpha\psi_\beta:\,\underbracket{\overline{\psi}_\gamma\psi_\delta} + \underbracket{\overline{\psi}_\alpha\psi_\delta}\,\underbracket{\psi_\beta\overline{\psi}_\gamma} + \ldots + \\ & + \underbracket{\overline{\psi}_\alpha\psi_\delta}\,:\psi_\beta\overline{\psi}_\gamma: + \underbracket{\psi_\beta\overline{\psi}_\gamma}\,:\overline{\psi}_\alpha\psi_\delta:\ .\end{aligned} \tag{E.1a}$$

Here $:\mathcal{O}:$ symbolizes the normal-ordering and the lower sign $\underbracket{\qquad}$ the vacuum expectation value (VEV). It means the following

$$\underbracket{\overline{\psi}_\alpha\psi_\beta} = \langle 0|\,\overline{\psi}_\alpha\psi_\beta\,|0\rangle\,. \tag{E.1b}$$

Only the first row is important for the operators $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$ and $\mathcal{O}_4$. For the operators $\mathcal{O}_5$ and $\mathcal{O}_6$ one has to consider the first and the last row only.

This follows from a physical fact that VEV's for non-scalar (be it in the Lorentz-space, or in the "inner"-space, i.e. SU(3) etc.) quantities have to vanish.

This means

$$\begin{aligned} \langle 0|\,\overline{\psi}_\ell \Gamma^\mu_{L,R}\psi_k\,|0\rangle &= 0 \\ \Gamma^\mu_{L,R} &= \gamma^\mu(1\mp\gamma_5) \end{aligned} \tag{E.2a}$$

and

$$\langle 0|\,\overline{\psi}_m \Lambda^m \psi_p\,|0\rangle = 0\,. \tag{E.2b}$$

For a quark combination appearing in the operators $\mathcal{O}_1 - \mathcal{O}_4$ of the form

$$\overline{\psi}_\alpha\,(\Gamma^\mu_L)_{\alpha\beta}\,\psi_\beta, \overline{\psi}_\gamma\,(\Gamma_{\mu\,L})_{\gamma\delta}\,\psi_\delta\,. \tag{E.3a}$$

FIT produces an equality

$$\left(\overline{\psi}_\alpha\,(\Gamma^\mu_L)_{\alpha\beta}\,\psi_\beta\right)\,\left(\overline{\psi}_\gamma\,(\Gamma_{\mu\,L})_{\gamma\delta}\,\psi_\delta\right) = \left(\overline{\psi}_\alpha\,(\Gamma^\mu_L)_{\alpha\delta}\,\psi_\delta\right)\,\left(\overline{\psi}_\gamma\,(\Gamma_{\mu\,L})_{\gamma\beta}\,\psi_\beta\right). \tag{E.3b}$$

For example, one of the terms in (E.1a) can be written as

$$\langle 0|\,\overline{\psi}_\alpha(\Gamma^\mu_L)_{\alpha\delta}\psi_\delta\,|0\rangle\cdot\ :\overline{\psi}_\gamma(\Gamma_{\mu\,L})_{\gamma\beta}\psi_\beta := 0\,. \tag{E.4}$$

The same conclusion can be drawn for all other terms which contain VEV's. When one calculates commutators involving the operators $\mathcal{O}_{1,2,3,4}$, NOP associated complications do not appear.

However FIT $\mathcal{O}_5$ and $\mathcal{O}_6$ contain scalar and pseudoscalar quantities so that their NOP is not trivial. From

$$(\overline{\psi}_\alpha(\Gamma_{\mu\,L})_{\alpha\beta}\psi_\beta)(\overline{\psi}_\gamma(\Gamma^\mu_L)_{\gamma\delta}\psi_\delta) = (-2)(\overline{\psi}_\alpha(1+\gamma_5)_{\alpha\delta}\psi_\delta)(\overline{\psi}_\alpha(1-\gamma_5)_{\gamma\beta}\psi_\beta) \tag{E.5}$$

and from (E.1) one obtains

$$\begin{aligned} :(\overline{\psi}_\alpha(\Gamma_{\mu\,L})_{\alpha\beta}\psi_\beta)(\overline{\psi}_\gamma(\Gamma^\mu_L)_{\gamma\delta}\psi_\delta): \,=\, & (\overline{\psi}_\alpha(\Gamma_{\mu\,L})_{\alpha\beta}\psi_\beta)(\overline{\psi}_\gamma(\Gamma^\mu_L)_{\gamma\delta}\psi_\delta) \\ & +2\ :(\overline{\psi}_\alpha(1+\gamma_5)_{\alpha\delta}\psi_\delta):\langle 0|\,(\overline{\psi}_\gamma\psi_\beta\,|0\rangle \\ & +2\ :(\overline{\psi}_\gamma(1-\gamma_5)_{\gamma\beta}\psi_\beta)\langle 0|\,(\overline{\psi}_\alpha\psi_\delta\,|0\rangle\,. \end{aligned} \tag{E.6}$$

Once this is specified for $\hat{\mathcal{O}}_6$[10] one has to take care of colour indices too. The spinors were actually coupled in colour sectors as follows

$$(\overline{\psi}^i_\alpha\psi_{\beta\,i})(\overline{\psi}^j_\gamma\psi_{\delta\,j})\,. \tag{E.7a}$$

[10]It is defined by $\hat{\mathcal{O}}_6 =: (\overline{d}\Gamma^\mu_L s)[\overline{u}\Gamma_{\mu\,R}u + \overline{d}\Gamma_{\mu\,R}d + \overline{s}\Gamma_{\mu\,R}s] :$.

The rearangement (E.6) turns that into

$$(\overline{\psi}^i_\alpha \psi_{\delta\, j}) \langle 0 |\, \overline{\psi}^j_\gamma \psi_{\beta\, i}\, | 0 \rangle \tag{E.7b}$$

as only the colour singlet can have VEV, one obtains

$$(\overline{\psi}^i_\alpha \psi_{\delta\, j}) \frac{1}{3} \delta_{ij} \langle 0 |\, \overline{\psi}^k_\gamma \psi_{\beta\, k}\, | 0 \rangle \,. \tag{E.7c}$$

Finally this gives

$$\begin{aligned} \hat{\mathcal{O}}_6 = \tilde{\mathcal{O}}_6 \; & + \tfrac{2}{3} \langle 0 |\, \overline{d} d \,| 0 \rangle \overline{d} (1 - \gamma_5) s \\ & + \tfrac{2}{3} \langle 0 |\, \overline{s} s \,| 0 \rangle \overline{d} (1 + \gamma_5) s \,. \end{aligned} \tag{E.8}$$

Here $\tilde{\mathcal{O}}_6$[11] is the $\hat{\mathcal{O}}_6$ operator which is not normal-ordered. This result is in full qualitative agreement with Ref. [21], formula (33). (One has to take into account that $\gamma_5(\text{here}) = -\gamma_5$ [21]). The analogous result can be obtained for $\hat{\mathcal{O}}_5$ which contains SU(3) colour matrices λ_A. They satisfy the equality (3.2)

$$\sum_A (\lambda_A)_{a\, b} (\lambda_A)_{c\, d} = \frac{16}{9} \delta_{ad} \delta_{cb} - \frac{1}{3} (\lambda)_{ad} (\lambda)_{cb} \,. \tag{E.9}$$

FIT of $\hat{\mathcal{O}}_5$ in the Lorentz-space has to be combined with (E.9). It means that in the Lorentz-space $\hat{\mathcal{O}}_5$ must have the same form as $\hat{\mathcal{O}}_6$ (E.7), but one has to introduce (E.9) into the second and third term on the right-hand-side (RHS) of (E.8). The equality (E.2b) means that only the first RHS term in (E.9) contributes. Thus VEV terms in (E.8) are multiplied by 16/3.

$$\begin{aligned} \hat{\mathcal{O}}_5 = \tilde{\mathcal{O}}_5 \; & + \tfrac{32}{9} \langle 0 |\, \overline{d} d \,| 0 \rangle \overline{d} (1 - \gamma_5) s \\ & + \tfrac{32}{9} \langle 0 |\, \overline{s} s \,| 0 \rangle \overline{d} (1 + \gamma_5) s \,. \end{aligned} \tag{E.10}$$

The expression (E.10) agrees fully with the formula (33) of ref. [21].

Some additional formal manipulations are shown in Appendix B.

Appendix F: Isospin and/or baryon decomposition of the weak potential

The isospin decomposition of an effective weak two particle potential acting among baryons depends on the isospin decomposition of the weak vertices. The

[11] Here we are using the definition of $\mathcal{O}_{5,6}$ which differs, with respect to Ref. [11] for instance, by a factor 4, i.e. $\hat{\mathcal{O}}_{5,6}(\text{here}) = 4\ \mathcal{O}_{5,6}$ (Ref.[11]).

amplitudes corresponding to $\Lambda \to B + \pi$ transitions, can be parametrized for $\Delta I = 1/2$ transition as

$$\begin{array}{lll} \Lambda \to & \mathrm{n} + \pi^0; \qquad f(\Lambda_0^0) & \\ & f(\Lambda_0^0) = \frac{1}{\sqrt{3}}\alpha & |\Delta I| = 1/2 \\ \Lambda \to & \mathrm{p} + \pi^-; \qquad f(\Lambda_0^-) & \\ & f(\Lambda_-^0) = -\sqrt{\frac{2}{3}}\alpha\,. & \end{array} \tag{F.1}$$

Here we have displayed only particle content and isospin properties. The same relations (F.1) hold for both $s-$wave and $p-$wave contributions.

The $|\Delta I| = 3/2$ transitions are parametrized as

$$\begin{array}{ll} g(\Lambda_0^0) & = \sqrt{\frac{2}{3}}\beta \\ g(\Lambda_-^0) & = \sqrt{\frac{1}{3}}\beta\,. \end{array} \qquad |\Delta I| = 3/2 \tag{F.2}$$

The weak amplitudes corrresponding to $N \to N + K$ transitions can be parametrized as follows

$$\begin{array}{lll} n \to & \mathrm{n} + \mathrm{K}^0; \qquad f(n_0^0) & \\ & f(n_0^0) = \delta & \\ p \to & \mathrm{p} + \mathrm{K}^0; \qquad f(p_0^+) & |\Delta I| = 3/2 \\ & f(p_0^+) = \frac{1}{2}(\xi + \delta) & \\ p \to & \mathrm{n} + \mathrm{K}^+; \qquad f(p_+^+) & \\ & f(p_+^+) = \frac{1}{2}(-\xi + \delta) & \end{array} \tag{F.3}$$

and

$$\begin{array}{ll} g(n_0^0) & = \frac{1}{2}\epsilon \\ g(p_0^+) & = -\frac{1}{2}\epsilon \\ g(p_+^+) & = -\frac{1}{2}\epsilon\,. \end{array} \tag{F.4}$$

The only weak vertex from which η is emitted corresponds to the proces $\Lambda \to n + \eta$. Here only $|\Delta I| = 1/2$ piece of $\mathcal{H}_W$ can contribute. Thus the weak potential due to the η exchange satisfies the same selection rule.

The potential due to a pion exchange, which corresponds to a diagram analogous to Fig. 1.1, has the following particle (i.e. baryonic) contents:

$$\begin{array}{ll}
(\overline{n}\Lambda)_{1\,w} \; \underbracket{\pi^0\,\overline{\pi}^0}\,(\overline{n}n)_{2\,S}\cdot\frac{1}{\sqrt{3}}\alpha\cdot(-g_{NN\pi}) & \\
(\overline{n}\Lambda)_{1\,w} \; \underbracket{\pi^0\,\overline{\pi}^0}\,(\overline{p}p)_{2\,S}\cdot\frac{1}{\sqrt{3}}\alpha\cdot g_{NN\pi} & |\Delta I| = 1/2 \\
(\overline{p}\Lambda)_{1\,w} \; \underbracket{\pi^-\,\overline{\pi}^-}\,(\overline{n}p)_{2\,S}\cdot(-1)\cdot\sqrt{\frac{2}{3}}\alpha\cdot(\sqrt{2}g_{NN\pi})\,. &
\end{array} \qquad (\mathrm{F}.5)$$

Here $1,2$ denotes the spin and coordinate dependence while w and S mean the weak and the strong vertex respectively. $g_{NN\pi}$ is the strong coupling constant. The combination $\underbracket{M\,\overline{M}}$ symbolizes the meson propagator which appears as a Yukawa function in the weak potential.

The $|\Delta I| = 3/2$ piece is

$$\begin{array}{ll}
(\overline{n}\Lambda)_{1\,w} \; \underbracket{\pi^0\,\overline{\pi}^0}\,(\overline{n}n)_{2\,S}\cdot\sqrt{\frac{2}{3}}\beta\cdot(-g_{NN\pi}) & \\
(\overline{n}\Lambda)_{1\,w} \; \underbracket{\pi^0\,\overline{\pi}^0}\,(\overline{p}p)_{2\,S}\cdot\sqrt{\frac{2}{3}}\beta\cdot g_{NN\pi} & |\Delta I| = 3/2 \\
(\overline{p}\Lambda)_{1\,w} \; \underbracket{\pi^-\,\overline{\pi}^-}\,(\overline{n}p)_{2\,S}\cdot\frac{1}{\sqrt{3}}\beta\cdot(\sqrt{2}g_{NN\pi})\,. &
\end{array} \qquad (\mathrm{F}.6)$$

The kaon exchange results in the combinations

$$\begin{array}{ll}
(\overline{n}\Lambda)_{1\,S} \; \underbracket{K^0\,\overline{K}^0}\,(\overline{n}n)_{2\,w}\cdot\delta\cdot g_{NNK}) & \\
(\overline{n}\Lambda)_{1\,S} \; \underbracket{K^0\,\overline{K}^0}\,(\overline{p}p)_{2\,w}\cdot\frac{1}{2}(\xi+\delta)\cdot g_{NNK} & |\Delta I| = 1/2 \\
(\overline{p}\Lambda)_{1\,S} \; \underbracket{K^-\,\overline{K}^-}\,(\overline{n}p)_{2\,w}\cdot\frac{1}{2}(-\xi+\delta)\cdot g_{NNK}\,. &
\end{array} \qquad (\mathrm{F}.7a)$$

The comparison with Ref. [3] is easier if one introduces the notation

$$\begin{aligned}
\tfrac{1}{2}(\xi+\delta)g_{NNK} &= d \\
\tfrac{1}{2}(-\xi+\delta)g_{NNK} &= d \\
\delta\cdot g_{NNK} &= c+d\,.
\end{aligned} \qquad (\mathrm{F}.7b)$$

In (F.5,6 and 7) and in the following, the Λ baryon is always in the vertex 1, irrespectively whether it is the strong or the weak vertex. (The complete potential, as discussed in Sec. 9, is symmetric in $1 \leftrightarrow 2$.)

The $|\Delta I| = 3/2$ kaon exchange pieces are

$$\begin{array}{ll}
(\overline{n}\Lambda)_{1\,S} \; \underbracket{K^0\,\overline{K}^0}\,(\overline{n}n)_{2\,w}\; e & \\
(\overline{n}\Lambda)_{1\,S} \; \underbracket{K^0\,\overline{K}^0}\,(\overline{p}p)_{2\,w}\; \frac{1}{2}(\xi+\delta)\;(-e) & |\Delta I| = 3/2 \\
(\overline{p}\Lambda)_{1\,S} \; \underbracket{K^-\,\overline{K}^-}\,(\overline{n}p)_{2\,w}\;(-e)\,. &
\end{array} \qquad (\mathrm{F}.8)$$

Here $e \equiv \gamma g_{NNK}/2$.

The expressions (F.5)-(F.8) can be connected with (8.17) and (8.20) by using the following isospin dependent quantities

$$\left.\begin{aligned}
\beta_1 &= \left(\overline{N}\mathbf{1}\begin{pmatrix}0\\1\end{pmatrix}\Lambda\right)_1(\overline{N}\mathbf{1}N)_2 \\
\beta_\tau &= \left(\overline{N}\boldsymbol{\tau}\begin{pmatrix}0\\1\end{pmatrix}\Lambda\right)_1(\overline{N}\boldsymbol{\tau}N)_2
\end{aligned}\right\} \quad \Delta I = 1/2$$
$$\beta_T = \big(\overline{N}(\mathbf{T}\chi)\Lambda\big)_1(\overline{N}\boldsymbol{\tau}N)_2\,. \qquad \Delta I = 3/2 \tag{F.9a}$$

Here

$$\begin{gathered}
\overline{N} = (\overline{p}, \overline{n}) \\
\overline{N}(\mathbf{T}\chi)\boldsymbol{\tau} = \textstyle\sum_{m,r} C^{3/2\,-1/2}_{1\,r\,1\,m}\mathbf{t}^r_1\cdot\boldsymbol{\tau}\,.
\end{gathered} \tag{F.10a}$$

The summation over m goes over two isospin states $m = \pm 1/2$ contained in N. Thus

$$\overline{N}(\mathbf{T}\chi)\boldsymbol{\tau} = \sqrt{\frac{2}{3}}(\tau^- + \tau^3)\,. \tag{F.10b}$$

The symbol Λ carries strangeness and has no isospin dependence. With (F.10) one obtains

$$\begin{aligned}
\beta_1 &= (\overline{n}\Lambda)_1[(\overline{p}p)_2 + (\overline{n}n)_2] \\
\beta_\tau &= (\overline{n}\Lambda)_1[(\overline{n}n)_2 - (\overline{p}p)_2] + 2(\overline{p}\Lambda)_1(\overline{n}p)_2 \\
\beta_T &= \sqrt{\tfrac{2}{3}}\big[(\overline{p}\Lambda)_1(\overline{n}p)_2 + (\overline{n}\Lambda)_1[(\overline{p}p)_2 - (\overline{n}n)_2]\big]\,.
\end{aligned} \tag{F.9b}$$

The isospin dependence of the pion exchange contribution can be written as

$$V_\pi = \tilde{A}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + \tilde{B}\mathbf{T}_1\cdot\boldsymbol{\tau}_2\,. \tag{F.11a}$$

(This is not symmetrized). The factors $\tilde{A}$ and $\tilde{B}$ contain all spatial and spin operators. In theoretical nuclear calculations the operator (F.10a) is to be sandwiched between multiparticle baryon states containing N and Λ.

The correspondence with (F.5) and (F.6) is established if one "reads" (F.11a) as

$$\begin{aligned}
V_\pi \longrightarrow\; & \tilde{A}\beta_\tau + \tilde{B}\beta_T \\
&= (\overline{n}\Lambda)_1(\overline{n}n)_2\left[\tilde{A} - \sqrt{\tfrac{2}{3}}\tilde{B}\right] \\
&+ (\overline{n}\Lambda)_1(\overline{p}p)_2\left[-\tilde{A} + \sqrt{\tfrac{2}{3}}\tilde{B}\right] \\
&+ (\overline{p}\Lambda)_1(\overline{n}p)_2\left[2\tilde{A} + \sqrt{\tfrac{2}{3}}\tilde{B}\right].
\end{aligned} \tag{F.11b}$$

The kaon exchange contribution is

$$\begin{aligned}
V_K &= \tfrac{1}{2}\tilde{C}(1+\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)+\tilde{D}+\tilde{E}\mathbf{T}_1\cdot\boldsymbol{\tau}_2 \\
&\longrightarrow \left(\tfrac{1}{2}\tilde{C}+\tilde{D}\right)\eta_0+\tfrac{1}{2}\tilde{C}\eta_\tau+\tilde{E}\eta_T \\
&= (\overline{n}\Lambda)_1(\overline{n}n)_2(\tilde{C}+\tilde{D}-\tilde{E}) \\
&+(\overline{n}\Lambda)_1(\overline{p}p)_2(\tilde{D}+\tilde{E}) \\
&+(\overline{p}\Lambda)_1(\overline{n}p)_2(\tilde{C}+\tilde{E})\,.
\end{aligned} \tag{F.12}$$

Expressions (F.9b) can be inverted. By introducing the notation

$$\begin{aligned}
\not{a} &= (\overline{n}\,\Lambda)(\overline{p}\,p) \\
\not{b} &= (\overline{n}\,\Lambda)(\overline{n}\,n) \\
\not{c} &= (\overline{p}\,\Lambda)(\overline{n}\,p)\,,
\end{aligned} \tag{F.13}$$

one finds

$$\begin{array}{rclll}
\not{a} &=& \tfrac{1}{2}\beta_1 & -\tfrac{1}{6}\beta_\tau & +\tfrac{1}{\sqrt{6}}\beta_T \\
\not{b} &=& \tfrac{1}{2}\beta_1 & +\tfrac{1}{6}\beta_\tau & -\tfrac{1}{\sqrt{6}}\beta_T \\
\not{c} &=& & +\tfrac{1}{3}\beta_\tau & +\tfrac{1}{\sqrt{6}}\beta_T\,.
\end{array} \tag{F.14}$$

This has been already displayed in (9.10).

Appendix G: Shifted Yukawa function

It is usually stated [44,45] that when a force is transmitted by a particle then the range of that force depends on the mass of the intermediate particle. Ever since Yukawa's seminal work [46], the Yukawa potential

$$V_Y(r) = \frac{\mathrm{e}^{\mu r}}{r} \tag{G.1}$$

was a textbook feature [45], describing the nucleon-nucleon interaction produced by one meson exchange. It turns out that the original statement (G.1) is somewhat modified in hypernuclei [5] where one has nucleons and strange particles.[12] In that case the range of the force depends both on the intermediate meson mass and the baryon masses.

Moreover, in order to connect the more general procedure with the result (G.1) valid for the baryons which all have equal masses, one can alternatively use an interesting generalized definition of the delta-function [47].

[12] For instance the ${}^{12}_{\Lambda}$C nucleus consists of 5 neutrons, 6 protons and the Λ-hyperon.

A transition amplitude $A_{f\,\Lambda}$ corresponding to the one pion (or any meson) exchange has a generic form [45]

$$A_{f\,\Lambda} = \frac{N}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y \int \mathrm{d}^4k \frac{\mathrm{e}^{-\mathrm{i}k(x-y)}}{k^2-\mu^2+\mathrm{i}\epsilon}\cdot\langle f|\,T(S(x)W(y))\,|\Lambda\rangle\,. \tag{G.2a}$$

Here

$$\begin{aligned} k^2 &= k_0^2 - \mathbf{k}^2 \\ kx &= k_0x_0 - \mathbf{k}\cdot\mathbf{x}\,. \end{aligned} \tag{G.2b}$$

Here $S(x)$ and $W(x)$ are some baryon densities (scalar or pseudoscalar) which are sources of the meson (pion) field, which mass is μ. Detailed form of those quantities need not concern us here. If all baryons are nucleons N, then the calculation can be found, for example, in ref. [45], p.213. In a more general case the density $S(x)$ can contain a strange baryon, for example Λ hyperon [5].

The time ordered product in (G.2a) can be written as

$$\begin{aligned} \langle f|\,T(S(x)W(y))\,|\Lambda\rangle &= \theta(x-y)\textstyle\sum_n\langle f|\,S(x)\,|n\rangle\langle n|\,W(y)\,|\Lambda\rangle \\ &+\theta(y-x)\textstyle\sum_s\langle f|\,W(y)\,|s\rangle\langle s|\,S(x)\,|\Lambda\rangle\,. \end{aligned} \tag{G.3}$$

Here the intermediate states

$$|n\rangle; \qquad S=0 \tag{G.4}$$

have no strangeness, while the states

$$|s\rangle; \qquad S=-1 \tag{G.5}$$

must contain a strange baryon.

In (G.2a) one can integrate over times (x_0, y_0). A useful identity is

$$\langle f|\,S(x)\,|n\rangle = \mathrm{e}^{i(E_f-E_n)x_0}\langle f|\,S(\mathbf{x})\,|n\rangle\,. \tag{G.6}$$

The analogous expression hold for other matrix elements. Also

$$\begin{aligned} dx^0\,dy^0 &= 2d\xi\,d\eta; \\ x^0 &= \xi+\eta \\ y^0 &= -\xi+\eta\,. \end{aligned} \tag{G.7}$$

The integration $d\eta$ can be immediately carried out which results in

$$\begin{aligned} A_{f\,\Lambda} &= N\int \mathrm{d}^3x\,\mathrm{d}^3y\,\mathrm{d}\xi\int \mathrm{d}^4k\frac{\mathrm{e}^{-2\mathrm{i}\xi k_0+\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{k^2-\mu^2+\mathrm{i}\epsilon} \\ &\times 2\pi\delta(E_f-E_\Lambda)\big[\theta(\xi)\textstyle\sum_n \mathrm{e}^{\mathrm{i}\Delta_n\xi}\alpha_n(\mathbf{x},\mathbf{y})+\theta(-\xi)\sum_s \mathrm{e}^{-\mathrm{i}\Delta_s\xi}\beta_s(\mathbf{y},\mathbf{x})\big]\,. \end{aligned} \tag{G.8a}$$

Here

$$\begin{aligned}\Delta_i &= E_f + E_\Lambda - 2E_i \\ \alpha_n &= \langle f|\, S(\mathbf{x})\, |n\rangle\langle n|\, W(\mathbf{y})\, |\Lambda\rangle \\ \beta_s &= \langle f|\, W(\mathbf{y})\, |s\rangle\langle s|\, S(\mathbf{x})\, |\Lambda\rangle\,,\end{aligned} \tag{G.8b}$$

and E_k are relativistic energies.

Starting with the expressions (G.8a and b), one first integrates over dξ obtaining

$$\begin{aligned}A_{f\,\Lambda} &= (2\pi \mathrm{i})N\delta(E_f - E_\Lambda)\int \mathrm{d}^3x\,\mathrm{d}^3y\int \mathrm{d}^4k\frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{k^2-\mu^2+\mathrm{i}\epsilon} \\ &\times\left[\sum_n \alpha_n\frac{1}{\Delta_n - 2k_0 + \mathrm{i}\epsilon} + \sum_s \beta_s\frac{1}{\Delta_s + 2k_0 + \mathrm{i}\epsilon}\right].\end{aligned} \tag{G.9}$$

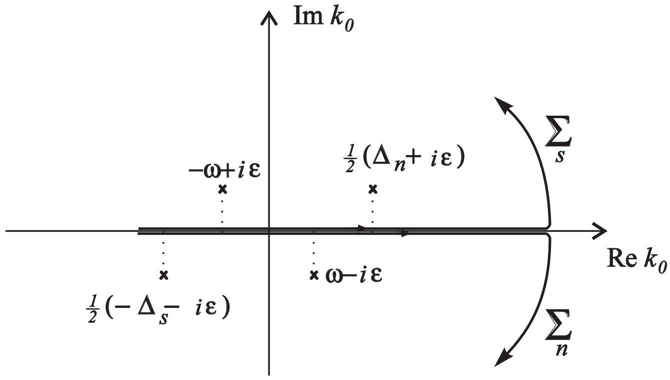

Fig. G.1. The contours in the k_0 plane. Here $\omega^2 = \mathbf{k}^2 + \mu^2$.

Integration in the complex k_0 plane over the contours shown in Fig. G.1 leads to

$$\begin{aligned}A_{f\Lambda} &= 2\pi^2\delta(E_f - E_\Lambda)N\int \mathrm{d}^3x\,\mathrm{d}^3y\int \mathrm{d}^3k\ \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \\ &\times\left[\sum_n \frac{\alpha_n(\mathbf{x},\mathbf{y})}{\omega(\Delta_n - 2\omega)} + \sum_s \frac{\beta_s(\mathbf{y},\mathbf{x})}{\omega(\Delta_s - 2\omega)}\right] \\ &\qquad \omega = \sqrt{\mathbf{k}^2 + \mu^2}\,.\end{aligned} \tag{G.10}$$

In the nonrelativistic limit the energy differences Δ_i can be approximated by the corresponding baryon mass differences. Schematically one can use the following baryonic contents:

$$\begin{array}{ll} |f\rangle & K\text{nucleons} \\ |\Lambda\rangle & 1\Lambda + (K-1)\text{nucleons} \\ |s\rangle & 1\Lambda + (K-1)\text{nucleons} \\ |n\rangle & \text{K nucleons}\,. \end{array} \tag{G.11a}$$

Thus,

$$\begin{array}{lll} E_f & \longrightarrow & K \cdot m_N \\ E_\Lambda & \longrightarrow & (K-1)\cdot m_N + m_\Lambda \\ E_s & \longrightarrow & (K-1)\cdot m_N + m_\Lambda \\ E_n & \longrightarrow & K \cdot m_N \\ \Delta_s & = E_f + E_\Lambda - 2E_s \to -2\delta & \\ \Delta_n & = E_f + E_\Lambda - 2E_n \to 2\delta & \\ \delta & = (m_\Lambda - m_N)/2\ . & \end{array} \tag{G.11b}$$

In such an approximation the factors Δ_i can be taken out of the summations in (G.10). Furthermore,

$$\begin{array}{ll} \sum_n \alpha_n(\mathbf{x},\mathbf{y}) & = S(\mathbf{x})W(\mathbf{y}) \\ \sum_s \beta_n(\mathbf{x},\mathbf{y}) & = W(\mathbf{y})S(\mathbf{x})\,. \end{array} \tag{G.12}$$

In the nonrelativistic approach W and S become operators in the configuration space acting on (Schrödinger) wave functions [45,48]. Thus their order is immaterial. One obtains

$$\begin{aligned} A_{f\Lambda} &= (2\pi^2)N\delta(E_f - E_\Lambda)\textstyle\int \mathrm{d}^3x\,\mathrm{d}^3y \textstyle\int \mathrm{d}^3k\ \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \\ &\times\frac{1}{2}\left[(-1)\frac{1}{\omega(\delta+\omega)} + \frac{1}{\omega(\delta-\omega)}\right]\langle f|\,O(\mathbf{x},\mathbf{y})\,|\Lambda\rangle \\ O(\mathbf{x},\mathbf{y}) &= S(\mathbf{x})W(\mathbf{y})\,. \end{aligned} \tag{G.13}$$

The integration over $\mathbf{k}$ gives the *shifted* Yukawa function

$$\begin{aligned} &\int \mathrm{d}^3k\ \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{2}\left[(-1)\frac{1}{\omega(\delta+\omega)} + \frac{1}{\omega(\delta-\omega)}\right] = \\ &\int \mathrm{d}^3k\ \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{\omega^2-\delta^2} = (-2\pi^2)\frac{\mathrm{e}^{-\epsilon r}}{r} \\ &r = |\mathbf{x}-\mathbf{y}| \\ &\epsilon = \sqrt{\mu^2-\delta^2} = \sqrt{\mu^2 - \tfrac{1}{4}(m_\Lambda - m_N)^2}\,. \end{aligned} \tag{G.14}$$

Eventually one obtains a generic form

$$A_{f\Lambda} = (-4\pi^4)N\int \mathrm{d}^3x\,\mathrm{d}^3y\frac{\mathrm{e}^{-\epsilon r}}{r}\langle f|\,O(\mathbf{x},\mathbf{y})\,|\Lambda\rangle \tag{G.15}$$

in which the Yukawa potential (function) depends on the effective mass

$$\epsilon < \mu \, . \tag{G.16}$$

If one deals with nucleons only, then

$$\begin{aligned} &\delta = 0 \\ &\epsilon \to \mu \, , \end{aligned} \tag{G.17}$$

and the standard textbook [45] form (G.1) is recovered.

This last result can be obtained also by starting from the expression (G.8). In the strong nucleon-nucleon interaction the variable k corresponds to the invariant nucleon momentum transfer [45]

$$\begin{aligned} k^2 &= (p - p'\,)^2 \\ p &= (E_p, \mathbf{p}); \qquad E_p = \sqrt{\mathbf{p}^{\,2} + M_N^2} \; . \end{aligned} \tag{G.18}$$

In the nonrelativistic limit, when $|\mathbf{p}| \ll M_N$ one can approximate

$$\begin{aligned} E_p &\simeq M_N \\ \delta \to E_p - E_p' &\simeq M_N - M_N = 0 \\ k^2 &\simeq -\mathbf{k}^{\,2} \\ \int \mathrm{d}^4 k \frac{\mathrm{e}^{-\mathrm{i}k(x-y)}}{k^2 - \mu^2 + \mathrm{i}\epsilon} &\longrightarrow (-1) \int \mathrm{d}k_0 \; \mathrm{d}^3 k \frac{\mathrm{e}^{-2\mathrm{i}k_0\xi}\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^{\,2} + \mu^2} \\ &= (-1)\pi\delta(\xi) \int \mathrm{d}^3 k \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}}{\mathbf{k}^{\,2} + \mu^2} \\ &= (-1)\delta(\xi) 2\pi^3 \frac{\mathrm{e}^{-\mu r}}{r}. \\ r &= |\mathbf{x} - \mathbf{y}| \, . \end{aligned} \tag{G.19}$$

When this is introduced in (G.8) with $|\Lambda\rangle = |i\rangle$ (nucleon state), one finds

$$\begin{aligned} A_{fi} &= N \int \mathrm{d}^3 x \, \mathrm{d}^3 y \, \mathrm{d}\xi (-1)\delta(\xi) \frac{\mathrm{e}^{-\mu r}}{r} \\ &\times (2\pi)\delta(E_f - E_i) \left[\theta(\xi) \textstyle\sum_n \mathrm{e}^{\mathrm{i}\Delta_n \xi} \alpha_n(\mathbf{x}, \mathbf{y}) + \theta(-\xi) \sum_s \mathrm{e}^{-\mathrm{i}\Delta_s \xi} \beta_s(\mathbf{x}, \mathbf{y})\right] , \end{aligned} \tag{G.20a}$$

with

$$\Delta_i = E_f - E_i \to 0 \, . \tag{G.20b}$$

In order to integrate over ξ one uses the identity

$$\int \mathrm{d}\xi \, \theta(\xi) \, \delta(\xi) \; \mathrm{e}^{\mathrm{i}\alpha\xi} = \frac{1}{2} \, , \tag{G.21}$$

and eventually obtains

$$A_{fi} = A_{f\,\Lambda=i}(\epsilon \to \mu)\,, \tag{G.22}$$

as already mentioned above (G.17). In a sense, that can be considered as an example for the novel delta-function identity derived by ref. [47].

Although interesting as a matter of principle the shifted Yukawa potential does not lead to some startling numerical differences. The range of the potential is somewhat increased, as can be seen by plotting the ratio

$$\frac{\mathrm{e}^{-\epsilon r}}{\mathrm{e}^{-\mu r}} = \mathrm{e}^{(\mu-\epsilon)r} = X(r) \tag{G.23}$$

which is shown in Fig. G.2a. The shifted potential has an increased range. With $\mu = m_\pi = 0.70\,\mathrm{fm}^{-1}$ and $\delta = (m_\Lambda - m_N)/2 = 0.45\,\mathrm{fm}^{-1}$ one finds: $r = 0.5\,\mathrm{fm}$ $X = 1.08$ and $r = 1\,\mathrm{fm}$ $X = 1.17$ for example.

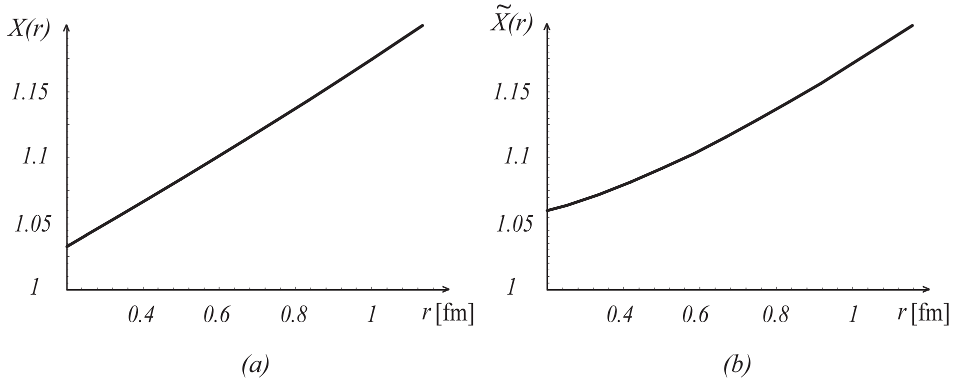

Fig. G.2. The ratio (G.23) (a) and (G.26) (b) is plotted as a function of r.

In the actual calculation with hypernuclei one introduces a monopole form-factor at the each vertex [5]. The denominators in (G.14) and (G.19) are thus replaced by

$$\frac{1}{\mathbf{k}^2+\phi^2} \longrightarrow \frac{(\Lambda^2-\mu^2)}{(\mathbf{k}^2+\phi^2)(\mathbf{k}^2+\Lambda^2)^2} = W(\phi) \qquad (\phi = \epsilon,\ \mu)\,. \tag{G.24}$$

The Fourier transform of that is

$$\mathcal{F}\{W(\phi)\} = 2\pi^2\Big[\frac{\Lambda^2-\mu^2}{\Lambda^2-\phi^2}\left(\frac{\mathrm{e}^{-\phi r}}{r} - \frac{\mathrm{e}^{-\Lambda r}}{r}\right) - \frac{\Lambda^2-\mu^2}{2\Lambda}\,\mathrm{e}^{-\Lambda r}\Big]. \tag{G.25}$$

The ratio

$$\tilde{X}(r) = \frac{\mathcal{F}\{W(\epsilon)\}}{\mathcal{F}\{W(\mu)\}} \tag{G.26}$$

is also plotted in Fig. G.2 (b) for $\Lambda = 1.3\,\mathrm{GeV}$ [5]. One finds $\tilde{X} = 1.09$ at $0.5\,\mathrm{fm}$ for example.

Appendix H: Effective $\widetilde{a}_{BB}$ amplitude and baryon pole terms

Parity violating A amplitudes obtain here the following contributions

$$A = A_{CA} + A_{Sep}\,. \tag{H.1}$$

The first correnspонds to formulae (5.20) to (5.26) while the second one is calculated as described in (3.20). The contributions A (H.1) should be compared with the experimental result A_{exp}, i.e.

$$A_{\mathrm{exp}} \sim A = A_{CA} + A_{Sep}\,. \tag{H.2}$$

The weak vertices in the baryon pole terms (see Fig. 4.1) are determined by the A_{CA} which has a generic form

$$A_{CA} \sim \frac{1}{f_\pi}\langle B'|\, H_W^{PC}\, |B\rangle = \frac{1}{f_\pi}\widetilde{a}_{B'B}\,. \tag{H.3}$$

One actually uses

$$\lim_{q\to 0} A = A_{CA}\,. \tag{H.4}$$

Thus in order to find $\widetilde{a}_{B'B}$ one must use an approximate expression whose a generic form is

$$\frac{1}{f_\pi}\tilde{a}_{B'B} \simeq \left[A_{B'B/exp} - A_{B'B/Sep}\right]. \tag{H.5}$$

Appendix I: Weak amplitudes: conventions and units

Parity violating A and parity conserving B amplitudes for nonleptonic hyperon $|\Delta S = 1|$ decays are given in the usual normalization in [49]. The same normalization is used in the present work. However sometimes a different normalization is used [5,22]. Here we give translational formulae among different notations and normalizations to enable comparisons of theoreticaly calculated amplitudes on the same footing.

In Ref. [5] amplitudes A and B are given in $G_F m_\pi^2$ "units". To relate these amplitudes and those given in [49] one has to multiply in Ref. [5] given amplitudes by the factor $G_F m_\pi^2 = 2.21 \times 10^{-7}$. Also Ref. [5] used an experimentaly extracted amplitude (in their notation) $A_\pi = 1.05$ which is connected to the usual amplitude by the relation

$$A_\pi[5] = \frac{1}{\sqrt{2}}A(\Lambda^0_-)_{\mathrm{exp}}[49] \tag{I.1}$$

(in the obvious notation). Therefore to get the number given in Table 7.1 for instance one has to make the following steps

$$A(\Lambda^0_-)[49] = \sqrt{2}A_\pi[5] = \sqrt{2}\cdot 1.05\cdot 2.21\times 10^{-7}. \tag{I.2}$$

Reference [5] uses somewhat different effective weak Hamiltonian. Instead of our formula (2.6) they have

$$\mathcal{H}^{(W)}_{\Lambda N\pi} = \mathrm{i}G_F m_\pi^2 \overline{\Psi}_N (A + B\gamma_5)\Psi^S_\Lambda \Phi_\pi\,. \tag{I.3}$$

Thus they list all their B amplitudes (see Table II in Ref. [5]) with opposite signs. For example, in our Table 7.1 we list $B_\eta(\Lambda^0_{\eta_8}) = 31.60$, while in Table III of Ref. [5] one finds $B_\eta = -14.3\cdot(2.21\times 10^{-7})$.

In Ref.[22] amplitudes are calculated in the chiral perturbation theory. Transition from their normalization to the usual (this work for instance) is found by comparing the expressions

$$G_F m_\pi^2 \overline{u}_f (A + B\gamma_5)u_i = G_F m_\pi^2 \frac{f_\pi}{f_K}\overline{u}_f \left(\mathcal{A}^{(s)} + 2\frac{k\cdot S_v}{\Lambda_\chi}\mathcal{A}^{(p)}\right)u_i\,. \tag{I.4}$$

This is best done in the rest frame of the initial (heavy) baryon [50]. By simply writing for the baryon spinors

$$u_f = N\begin{pmatrix}\chi \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}_f}{E_f + M_f}\end{pmatrix} \qquad u_i = N\begin{pmatrix}\chi \\ 0\end{pmatrix} \mathbf{p}_f = -\mathbf{k}, \qquad \mathbf{p}_i = 0, \qquad E_i = M_i$$
$$S_v = (1, \boldsymbol{\sigma}/2)\,. \tag{I.5}$$

One obtains

$$\begin{aligned} A &= \frac{f_\pi}{f_K}\mathcal{A}^{(s)} \\ B &= \frac{f_\pi}{f_K}\frac{(M_i + M_f)^2 - m_K^2}{2M_i}\frac{1}{\Lambda_\chi}\mathcal{A}^{(p)} \\ \Lambda_\chi &= 1\,\mathrm{GeV}. \end{aligned} \tag{I.6}$$

Acknowledgements

This research was supported by the Croatian Ministry of Science and Technology grant 119222. The authors also acknowledge the support of ANPCyT (Argentina) under grant BID 1201/OC-AR (PICT 03-04296) and Fundación Antorchas (Argentina) under grant Nr. 13740/01-111. F.K. and C.B. are fellows of the CONICET from Argentina.

References

[1] C. Barbero, D. Horvat, F. Krmpotić, Z. Narančić, and D. Tadić, in preparation.

[2] C. Barbero, D. Horvat, F. Krmpotić, Z. Narančić, and D. Tadić, *On the Weak Decay Puzzle in Hypernuclei*, submitted for publication.

[3] J. Cohen, Prog. Part. Nucl. Phys. **25** (1990) 139.

[4] J. F. Dubach, G. B. Feldman, B. R. Holstein and L. de la Torre, Ann. Phys. **249** (1996) 146.

[5] A. Parreño, A. Ramos, C. Bennhold, Phys. Rev. C **56** (1997) 339.

[6] H. Bando, T. Motoba and J. Zofka, Int. J. Mod. Phys. **A5** (1990) 4021.

[7] C. Bennhold, A. Parreño and A. Ramos, Few-Body Systems Suppl. **9** (1995) 475.

[8] E. D. Commins and P. H. Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cambridge University Press, Cambridge (1983).

[9] R. E. Marshak, Riazuddin and C. P. Ryan, *Theory of Weak Interactions in Particle Phyiscs*, Wiley Interscience, New York (1969).

[10] V. S. Mathur and L. K. Pandit, in *Advances in Particle Physics*, ed. by R. Cool and R. E. Marshak, Wiley Interscience, New York (1968), p. 383.

[11] L. B. Okun, *Leptons and Quarks*, North Holland Publ. Comp., Amsterdam (1982).

[12] F. Palmonari, Riv. Nuovo Cim. **7** (9) (1984) 1.

[13] J. F. Donoghue, E. Golowich and B. Holstein, Phy. Rep. **131** (1986) 319.

[14] D. Tadić and J. Trampetić, Phys. Rev. D **23** (1981) 144.

[15] S. P. Rosen and S. Pakvasa, in *Advances in Particle Physics* ed. by R. Cool and R. E. Marshak, Wiley Interscience, New York (1968), p. 473.

[16] M. D. Scadron and D. Tadić, J. Phys. G: Nucl. Part. Phys. **27** (2001) 163.

[17] D. Horvat and D. Tadić, Z. Phys. C **31** (1986) 311; ibid. **35** (1987) 231.

[18] D. Horvat, Z. Narančić and D. Tadić, Z. Phys. C **38** (1988) 431.

[19] E. Fischbach and D. Tadić, Phys. Rep. C **6** (1973) 125.

[20] B. Desplanques, J. F. Donoghue and B. R. Holstein, Ann. Phys. **124** (1980) 449.

[21] J. F. Donoghue, E. Golowich, W. A. Ponce and B. R. Holstein, Phys. Rev. D **21** (1980) 186.

[22] M. J. Savage and R. P. Springer, Phys. Rev. C **53** (1996) 441; ibid. C **54**(E) (1996) 2786.

[23] M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. **33** (1974) 108; Phys. Rev. D **10** (1974) 897; G. Altarelli and L. Maiani, Phys. Lett. B **52** (1974) 351; A. I. Vainshtein, V. I. Zakharov and M. A. Shifman, Nucl. Phys. B **120** (1977) 316.

[24] J. F. Donoghue, E. Golowich and B. Holstein, *Dynamics of The Standard Model*, Cambridge University Press, New York (1992).

[25] O. Dumbrajs et al., Nucl. Phys. B **216** (1983) 277: H. Kim et all., Nucl. Phys. A **678** (2000) 295.

[26] P. A. Carruthers, *Introduction to Unitary Symmetry*, Interscience Publ., New York (1966).

[27] B. T. Feld, *Models of Elementary Particles*, Blaisdell Publ. Comp., Waltham (1969).

[28] M. Gourdin, *Unitary Symmetries and Their Application to High Energy Physics*, North Holland Publ. Comp., Amsterdam (1967).

[29] J. J. Sakurai, *Currents and Mesons*, Univ. of Chicago Press, Chicago (1969).

[30] J. Gasser and H. Leutwyler, Phys. Rep. **87** (1982) 77.

[31] J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B **253** (1991) 252.

[32] M. E. Sainio, in *Chiral Dynamics: Theory and Experiment* , Proc. Workshop at MIT, Cambridge, July 1994, ed. by A. M. Bernstein and B. R. Holstein, Springer, New York (1995).

[33] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Fields*, McGraw-Hill, New York (1964).

[34] F. Gross, *Relativistic Quantum Mechanics and Field Theory* , John Wiley & Sons, Inc., New York (1993).

[35] J. J. De Swart, Rev. Mod. Phys. **35** (1963) 916.

[36] P. Mc Namee and F. Chilton, Rev. Mod. Phys, **36** (1964) 1005.

[37] M. D. Scadron and M. Visinescu, Phys. Rev. D **28** (1983) 1117; R. P. Springer, Phys. Lett. B **461** (1999) 167.

[38] A. Le Yaouanc, L. L. Oliver, O. Pene and J.-C. Raynal, *Hadron Transitions in The Quark Model*, Gordon and Breach, New York (1988).

[39] D. Palle and D. Tadić, Z. Phys. C **23** (1984) 301.

[40] M. Milošević, D. Tadić and J. Trampetić, Nucl. Phys. B **207** (1982) 461.

[41] D. Horvat, Z. Narančić and D. Tadić, Phys. Rev. D **51** (1995) 6277.

[42] G. E. Brown and B. Weise, Phys. Rep. C **22** (1975) 279.

[43] H. Sugawara and F. von Hippel, Phys. Rev. **172** (1968) 1764.

[44] J. J. Sakurai, *Modern Quantum Mechanics*, Addison-Wesley, Redwood City (1985).

[45] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics*, McGraw-Hill, New York (1964); see p. 211.

[46] H. Yukawa, Proc. Phys.-Math. Soc., Japan, **17** (1935) 48.

[47] D. Zhang, Y. Ding and T. Ma, Am. J. Phys. **57** (1989) 281.

[48] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory*, Row, Peterson and Co., Evanston, Ill. (1961).

[49] Particle Data Group, D. E. Groom et al. Eur. Phys. J. C **15** (2000) 1.

[50] E. Jenkins, Nucl. Phys. B **375** (1992) 561.

HIPERNUKLEARNI POTENCIJALI I DOPRINOS IZMJENE PSEUDOSKALARNIH MEZONA

Izvodimo dijelove hipernuklearnog potencijala koji krši stranost zbog izmjene pseudoskalarnih mezona primjenom metoda koje su uspješno primijenjene za hiperonske neleptonske raspade. Ocjene se provjeravaju usporedbom s mjerenim amplitudama neleptonskih raspada hiperona. Sve su promjene izospina, $\Delta I = 1/2$ i $\Delta I = 3/2$, uključene u izvedeni potencijal. Podrobno opisujemo sve računalne metode.