ISSN 1330–0016
CODEN FIZBE7

ENERGY DISTRIBUTION OF THE BIANCHI TYPE I SOLUTION

IRINA RADINSCHI

"Gh. Asachi" Technical University, Iasi, 6600, Romania
E-mail: iradinsc@phys.tuiasi.ro

Received 7 July 2000; Accepted 11 December 2000

We calculate the energy of an anisotropic model of universe based on the Bianchi type I metric in the Møller prescription. The total energy due to the matter and gravitational field is zero. This result supports the importance of the energy-momentum complexes in the localization of energy.

PACS numbers: 04.20 UDC 530.12

Keywords: matter and gravitational field, total energy, Bianchi type I metric, energy-momentum complexes

1. *Introduction*

The subject of energy-momentum localization has been a problematic issue since the outset of the theory of relativity. Although this subject has problems which still remain unresolved, some interesting results have been found in recent years.

It is possible to evaluate the energy and momentum distribution by using various energy-momentum complexes. According to the opinion of some researchers, the energy-momentum complexes are not useful to get meaningful energy distribution in a given geometry. Virbhadra and his collaborators re-opened the problem of the energy-momentum localization by using the energy-momentum complexes.

Aguirregabiria, Chamorro and Virbhadra [1] showed that several energy-momen-tum complexes coincide for any Kerr-Schild class metric. Also, in his recent paper, Virbhadra [7] emphasized that though the energy-momentum complexes are non-tensors under general coordinate transformations, the local conservation laws with them hold in all coordinate systems. The author calculated the energy distribution of a dilaton dyonic black hole [3] and the energy of a topological black hole [4]. Also, we obtained the energy distribution in a static spherically-symmetric non-singular black hole space-time [5] and the total energy of a Bianchi I type universe [6]. Recently, Virbhadra [7] showed that different energy–momentum complexes give the same and reasonable results for many space–times. By using the Møller energy-momentum complex, I-Ching Yang, Wei-Fui Lin and Rue-Ron Hsu [8] calculated the energy distributions of the GHS dilaton black hole solution and, also, the energy distribution of a dyonic dilaton black-hole solution.

Tryon [9] assumed that the Universe appeared from nowhere about 10^{10} years ago and the conventional laws of physics need not have been violated at the time of the creation of the Universe. According with his Big Bang model, the Universe is homogeneous, isotropic and closed and consists of matter and anti-matter equally. Also, the total energy of the Universe may be equal to zero. In the Einstein prescription, Rosen [10] computed the energy of a closed homogeneous isotropic universe described by a Friedmann-Robertson-Walker (FRW) metric. The total energy is zero. Johri [11], by using the Landau and Lifshitz energy-momentum complex, found that the total energy of a FRW spatially-closed universe is zero at all times, irrespective of equations of state of the cosmic fluid. Also, the total energy enclosed within any finite volume of the spatially flat FRW universe is zero at all times. Banerjee and Sen [12] calculated in the Einstein prescription the total energy density of a model of universe based on the Bianchi type I solutions. It is known that the Bianchi type I solutions, under a special case, reduce to the spatially flat FRW solutions. The total energy density is found to be zero everywhere. Using the symmetric energy-momentum complexes of Landau and Lifshitz, Papapetrou, and Weinberg, Xulu [13] obtained the energy of the Universe in anisotropic Bianchi type I cosmological models.

The purpose of this work is to compute the energy of a special anisotropic model of universe based on the Bianchi type I metric by using the Møller prescription. We use the geometrized units $(G = 1, c = 1)$ and follow the convention that Latin indices run from 0 to 3.

2. *Energy in the Møller prescription*

We consider the line element [14] which describes a special anisotropic model of universe based on the Bianchi type I metric

$$\mathrm{d}s^2 = \mathrm{d}t^2 - A^2\,(t)\,\mathrm{d}x^2 - B^2\,(t)\,\mathrm{d}y^2 - C^2\,(t)\,\mathrm{d}z^2, \tag{1}$$

where

$$\begin{aligned} A\,(t) &= (m_1 s_1 t)^{1/m_1}\,, \\ B\,(t) &= (m_2 s_2 t)^{1/m_2}\,, \\ C\,(t) &= (m_3 s_3 t)^{1/m_3}\,. \end{aligned} \tag{2}$$

In Eqs. (2), m_i and s_i $\left(i = \overline{1,3}\right)$ are positive constants. We exclude the case $m_i = 0$.

The metric given by (1) reduces to the spatially flat Friedmann-Robertson-Walker metric in a special case when $A\,(t) = B\,(t) = C\,(t)$. We define $R\,(t) =$

$(mst)^{1/m}$ and transform the line element (1) according to

$$\begin{cases} x = r\sin\theta\cos\varphi, \\ y = r\sin\theta\sin\varphi, \\ z = r\cos\theta. \end{cases} \tag{3}$$

We obtain the line element

$$\mathrm{d}s^2 = \mathrm{d}t^2 - R^2\left(t\right)\left[\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\ \mathrm{d}\varphi^2\right)\right], \tag{4}$$

which describes the spatially flat Friedmann-Robertson-Walker space-time.

For the solution (1), the non-zero components of the energy-momentum tensor due to the matter are

$$\begin{aligned} T_1^1 &= \left(\frac{m_2-1}{m_2^2} + \frac{m_3-1}{m_3^2} - \frac{1}{m_2m_3}\right)t^{-2}, \\ T_2^2 &= \left(\frac{m_1-1}{m_1^2} + \frac{m_3-1}{m_3^2} - \frac{1}{m_1m_3}\right)t^{-2}, \\ T_3^3 &= \left(\frac{m_1-1}{m_1^2} + \frac{m_2-1}{m_2^2} - \frac{1}{m_1m_2}\right)t^{-2}, \\ T_0^0 &= \left(\frac{m_1+m_2+m_3}{m_1m_2m_3}\right)t^{-2}. \end{aligned} \tag{5}$$

The Møller energy-momentum complex [15] is given by

$$\Theta_i^{\ k} = \frac{1}{8\pi}\frac{\partial\Gamma_i^{\ kl}}{\partial x^l}, \tag{6}$$

where

$$\Gamma_i^{\ kl} = \sqrt{-g}\left(\frac{\partial g_{im}}{\partial x^n} - \frac{\partial g_{in}}{\partial x^m}\right)g^{kn}g^{lm}. \tag{7}$$

The energy component E is

$$E = \int\!\!\int\!\!\int \Theta_0^{\ 0}\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3 = \frac{1}{8\pi}\int\!\!\int\!\!\int \frac{\partial\Gamma_0^{\ 0l}}{\partial x^l}\mathrm{d}x^1\mathrm{d}x^2\mathrm{d}x^3. \tag{8}$$

The nonzero Γ_i^{k0} components are

$$
\begin{aligned}
\Gamma_1^{10} = -\Gamma_1^{01} &= -2\frac{(m_1 s_1 t)^{1/m_1} (m_2 s_2 t)^{1/m_2} (m_3 s_3 t)^{1/m_3}}{m_1 t}, \\
\Gamma_2^{20} = -\Gamma_2^{02} &= -2\frac{(m_1 s_1 t)^{1/m_1} (m_2 s_2 t)^{1/m_2} (m_3 s_3 t)^{1/m_3}}{m_2 t}, \\
\Gamma_3^{30} = -\Gamma_3^{30} &= -2\frac{(m_1 s_1 t)^{1/m_1} (m_2 s_2 t)^{1/m_2} (m_3 s_3 t)^{1/m_3}}{m_3 t}.
\end{aligned}
\tag{9}
$$

From Eqs. (6) and (9), we obtain

$$\Theta_0^0 = 0. \tag{10}$$

The total energy density (due to the matter plus field) vanishes everywhere.

3. Discussion

The main purpose of the present paper is to show that it is possible to "solve" the problem of the localization of energy in relativity by using the energy-momentum complexes.

The energy-momentum localization subject has been associated with much debate. Bondi [16] gave his opinion that a nonlocalizable form of energy is not admissible in relativity so its location can in principle be found. The energy-momentum complexes are not tensorial objects and one is compelled to use "Cartesian coordinates". Due to this reason, this subject was not taken seriously for a long time and was re-opened by the results obtained by Virbhadra and his collaborators. Some interesting results, which have been found recently [1–7,10–13], show that several energy-momentum complexes can give the same and acceptable result for a given space-time. Also, in his recent paper, Virbhadra [7] emphasized that though the energy-momentum complexes are non-tensors under general coordinate transformations, the local conservation laws with them hold in all coordinate systems. Chang, Nester and Chen [17] showed that the energy-momentum complexes are actually quasilocal and legitimate expressions for the energy-momentum.

We used a special case of the Bianchi type I metric and obtained, in the Møller prescription, a result for the energy distribution which is in agreement with the viewpoint of Rosen [10] and Johri [11]. The total energy density vanishes everywhere because the energy contributions from the matter and gravitational field inside an arbitrary two-dimensional surface cancel each other in the case of the special anisotropic model of universe based on the Bianchi type I metric. We completed the investigation of Banerjee and Sen [12], Xulu [13] and ours [6] with one more energy-momentum complex. The results in this paper sustain the importance of the energy-momentum complexes in the localization of the energy.

References

[1] J. M. Aguirregabiria, A. Chamorro and K. S. Virbhadra, Gen. Rel. Grav. **28** (1996) 1393.

[2] A. Chamorro and K. S. Virbhadra, Int. J. Mod. Phys. D **5** (1995) 251.

[3] I. Radinschi, Acta Physica Slovaca **49**, (5) (1999) 1.

[4] I. Radinschi, Fizika B **9** (2000) 1.

[5] I. Radinschi, Mod. Phys. Letters **A15** 11&12 (2000) 803.

[6] I. Radinschi, Acta Physica Hungarica, in press.

[7] K. S. Virbhadra, Phys. Rev. D **60** (1999) 104041.

[8] I-Ching Yang, Wei-Fui and Rue-Ron Hsu, Chin. J. Phys. (Taipei) **37** (1999) 113.

[9] E. P. Tryon, Nature **246** (1973) 396.

[10] N. Rosen, Gen. Rel. Grav. **26** (1994) 319.

[11] V. B. Johri, D. Kalligos, G. P. Singh and C. W. F. Everitt, Gen. Rel. Grav. **27** (1995) 313.

[12] N. Banerjee and S. Sen, Pramana - J. Phys. **49** (1997) 609.

[13] S. S. Xulu, gr-qc/9910015.

[14] M. S. Berman, Gen. Rel. Grav. **8** (1988) 841.

[15] C. Møller, Ann. Phys. (NY) **4** (1958) 347.

[16] H. Bondi, Proc. Royal Soc. London A **427** (1990) 249.

[17] Chia-Chen Chang, J. M. Nester and Chiang-Mei Chen, Phys. Rev. Lett. **83** (1999) 1897.

RASPODJELA ENERGIJE ZA BIANCHIJEVO RJEŠENJE TIPA I

Izračunali smo energiju anizotropnog modela Svemira zasnovanog na Bianchijevoj metrici tipa I prema Møllerovim pravilima. Ukupna energija mase i gravitacijskog polja jednaka je nuli. Taj ishod podržava važnost kompleksa energije–impulsa za lokalizaciju energije.