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LETTER TO THE EDITOR

THE DEPENDENCE OF THE ASYMPTOTIC BEHAVIOUR OF THE $\gamma^\star\gamma$ TO π^0 TRANSITION ON THE DRESSED QUARK-PHOTON VERTICES

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It is discussed how various *Ansätze* for the dressed quark-photon ($qq\gamma$) vertices $\Gamma^\mu(q,q')$ influence the asymptotics of the $\gamma^\star\gamma \to \pi^0$ transition form factor. In this regard, we rectify certain misconceptions present in the literature.

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It has been shown in Ref. [1] that, in contrast to the older constituent quark models with constant quark mass parameters, its modern version provided by the Schwinger-Dyson (SD) approach, leads to the $\gamma^\star(k)\gamma(k') \to \pi^0(p)$ transition form factor $T_{\pi^0}(k^2, k'^2)$ which (for large spacelike $k^2 = -Q^2 < 0$) has the asymptotic momentum dependence

$$T_{\pi^0}(-Q^2, 0) = \frac{\mathcal{K}}{Q^2} \qquad (\mathcal{K} \to \text{const as } Q^2 \to \infty), \tag{1}$$

consistent with the data at the presently largest accessible Q^2 [2] and in agreement (up to the precise value of the coefficient $\mathcal{K}$) with perturbative QCD (pQCD) where $\mathcal{K} = 2f_\pi$ [3], operator product expansion (OPE) where $\mathcal{K} = 4f_\pi/3$ [4], and QCD sum rules where $\mathcal{K} \approx 1.6 f_\pi$ [5].

The SD approach, where the quark propagators $S(q) = [A(q^2)\not{q} - B(q^2)]^{-1}$ are dynamically dressed, requires consistently dressed quark-photon ($qq\gamma$) vertices

$\Gamma^\mu(q,q')$ in order to satisfy the vector Ward-Takahashi identity (WTI). [Using for the pseudoscalar vertex the quark-antiquark pion Bethe-Salpeter (BS) bound-state vertex $\Gamma_{\pi^0}(q,p)$ and the consistently dynamically dressed quark propagator $S(q)$, as well as a $qq\gamma$-vertex $\Gamma^\mu(q,q')$ dressed so as to satisfy the WTI, defines an impulse approximation sometimes called the *generalized impulse approximation* (GIA).] Nevertheless, even just approximately adequate SD solutions for $\Gamma^\mu(q,q')$ are not yet available, and in practice the more or less realistic WTI-satisfying *Ansätze* still must be used.

The topic of this paper is the dependence of the values (in the SD approach) of the asymptotic coefficient $\mathcal{K}$, on the choice of the dressed $qq\gamma$ vertex $\Gamma^\mu(q,q')$. This topic needs clarification, since Ref. [6] recently expressed an implicit disagreement with one detail in what we found earlier [1] in this regard.

The simplest partial solution of the vector WTI is the Ball–Chiu (BC) [7] vertex *Ansatz*

$$\Gamma^\mu_{BC}(q',q) = A_+(q'^2,q^2)\frac{\gamma^\mu}{2} + \frac{(q'+q)^\mu}{(q'^2-q^2)}\{A_-(q'^2,q^2)\frac{(\not q'+\not q)}{2} - B_-(q'^2,q^2)\}\ , \quad (2)$$

where $H_\pm(q'^2,q^2) \equiv [H(q'^2) \pm H(q^2)]$, for $H = A$ or B. This particular solution of the vector WTI reduces to the bare vertex in the free-field limit as must be in perturbation theory, has the same transformation properties under Lorentz transformations and charge conjugation as the bare vertex and has no kinematic singularities. Note that it does not introduce any new parameters as it is completely determined by the dressed quark propagator $S(q)$. In phenomenological calculations in the SD-BS approach, this minimal WTI-satisfying *Ansatz* (2) is still the most widely used $qq\gamma$ vertex (*e.g.*, Refs. [8–12]). A general WTI-satisfying vertex can be written [7] as $\Gamma^\mu = \Gamma^\mu_{BC} + \Delta\Gamma^\mu$, where the addition $\Delta\Gamma^\mu$ does not contribute to WTI, since it is transverse, $(q'-q)_\mu\Delta\Gamma^\mu(q',q) = 0$. That is, $\Delta\Gamma^\mu(q',q)$ entirely lies in the hyperplane spanned by the eight vectors $\mathcal{T}^\mu_i(q',q)$ $(i = 1,...,8)$ transverse to the photon momentum $k = q'-q$. Curtis and Pennington (CP) [13] advocated a transverse *Ansatz* for $\Delta\Gamma^\mu(q',q)$ exclusively along the basis vector usually labeled by $i = 6$:

$$\Delta\Gamma^\mu(q',q) = \mathcal{T}^\mu_6(q',q)\frac{A_-(q'^2,q^2)}{2d(q',q)}\ ; \qquad \mathcal{T}^\mu_6(q',q) \equiv \gamma^\mu(q'^2-q^2) - (q'+q)^\mu(\not q' - \not q)\ . \quad (3)$$

Then, the coefficient multiplying $\mathcal{T}^\mu_6(q',q)$ can be suitably chosen to ensure multiplicative renormalizability in the context of solving fermion SD equations beyond the ladder approximation in QED$_4$ [13]. To this end, $d(q',q)$ should be a symmetric, singularity free function of q' and q, with the limiting behaviour $\lim_{q^2>>q'^2} d(q',q) = q^2$; for example,

$$d_\pm(q',q) = \frac{1}{q'^2+q^2}\left\{(q'^2 \pm q^2)^2 + \Big[M^2(q'^2) + M^2(q^2)\Big]^2\right\}\ , \quad (4)$$

where $M(q^2) \equiv B(q^2)/A(q^2)$ is the DχSB-generated dynamical mass function, which in our case has the large-q^2 dependence [14] in agreement with pQCD.

The choice $d = d_-$ corresponds to the CP vertex *Ansatz* Γ^μ_{CP} introduced in Ref. [13], and $d = d_+$ to the *modified* CP (mCP) vertex Γ^μ_{mCP}, suggested in Ref. [1]. It should also be noted that Γ^μ_{mCP} is essentially equal[1] to the high-q^2 or q'^2 leading part of the vertex of Cudell *et al.* [15].

We used both the CP and the mCP vertex (in addition to the BC one) in analytic calculations of $T_{\pi^0}(-Q^2, 0)$, which are possible not only for $Q^2 = 0$ (yielding the famous Abelian axial anomaly amplitude), but also for $Q^2 \to \infty$, yielding the presently interesting asymptotic behaviour of the transition form factor. The mCP vertex was introduced because it is more suitable for our [1] numerical computations of $T_{\pi^0}(-Q^2, 0)$ (unavoidable for the finite values of Q^2), while retaining the important merits of the original CP vertex. In contrast to the BC one, the mCP vertex is consistent with multiplicative renormalizability by the same token as the CP one.

In the present context, the important *qualitative* difference between the BC vertex on one side, and the CP as well as the modified mCP vertex on the other side, is that $\Gamma^\mu_{BC}(q', q) \to \gamma^\mu$ when *both* $q'^2, q^2 \to \pm\infty$, whereas $\Gamma^\mu_{CP}(q', q) \to \gamma^\mu$ and $\Gamma^\mu_{mCP}(q', q) \to \gamma^\mu$ as soon as *one* of the squared momenta tends to infinity.

Reference [1] showed analytically that the SD approach predicts the asymptotic coefficient $\mathcal{K} = 4f_\pi/3$ (exactly the same as OPE [4]) for *all* $qq\gamma$ vertices $\Gamma^\mu(q', q)$ which go into the bare one (γ^μ) even if just one of the squared momenta q^2 or q'^2 becomes infinite. This was illustrated by the examples of the CP and mCP vertices. [For the latter, $T_{\pi^0}(-Q^2, 0)$ was numerically calculated also for finite values of Q^2.] Both the CP and mCP vertices are multiplicatively renormalizable, so that our result [1] on the asymptotic behaviour of $T_{\pi^0}(-Q^2, 0)$ subsequently received further support from Ref. [16]. In this reference, the derivation of Ref. [1] was generalized by taking into account renormalization explicitly, showing that the asymptotics of Ref. [1] with $\mathcal{K} = 4f_\pi/3$ must be precisely reproduced for *all* $qq\gamma$ vertices which are consistent with multiplicative renormalizability.

Nevertheless, Ref. [1] also showed that the usage of the "minimal" WTI-satisfying $qq\gamma$ vertex Γ^μ_{BC}, namely the Ball-Chiu (BC) one, leads to the asymptotic coefficient $\mathcal{K} = 4\widetilde{f}_\pi/3$, where $\widetilde{f}_\pi$ is the quantity given by the same Mandelstam-formalism expression as the pion decay constant f_π, except that its integrand is modified by the factor $[1 + A(q^2)]^2/4$. (In the case of our solutions [14], this gives $\widetilde{f}_\pi = 1.334 f_\pi = 124$ MeV.) As pointed out in Ref. [17], the arguments of Ref. [16] do not apply to the BC vertex since this vertex is *not* consistent with multiplicative renormalizability [18, 13], so that this enhancement of the asymptotic coefficient $\mathcal{K}$ is possible. The modification of $\mathcal{K}$ is caused by the different asymptotic behaviour of the BC vertex, which tends to the bare vertex, $\Gamma^\mu_{BC}(q', q) \to \gamma^\mu$, only when *both* $q'^2, q^2 \to \pm\infty$, *i.e.*, when the squared momenta in *both* fermion legs tend to infinity. The origin of the factor $(1/2)^2[1 + A([q + p/2]^2)][1 + A([q - p/2]^2)] \approx [1 + A(q^2)]^2/4$ modifying the integrand, when $\Gamma^\mu(q', q) = \Gamma^\mu_{BC}(q', q)$, is then clear: the transition form factor $T_{\pi^0}(k^2, k'^2)$ is extracted from the tensor amplitude $T^{\mu\nu}_{\pi^0}(k, k')$ for the

[1] Up to the inclusion of the mass functions $M(q^2)$, and for the simplest choice of their [15] η-function: $\eta(q', q) \equiv 1$, *i.e.*, the choice $n = 0$ for the exponent in their definition of the η-function.

GIA triangle diagram (Fig. 1 for $P = \pi^0$),

$$T^{\mu\nu}_{\pi^0}(k,k') \propto \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \mathrm{Tr}\{\Gamma^\mu(q-\frac{p}{2}, k+q-\frac{p}{2}) S(k+q-\frac{p}{2})$$

$$\times \Gamma^\nu(k+q-\frac{p}{2}, q+\frac{p}{2}) S(q+\frac{p}{2}) \Gamma_{\pi^0}(q,p) S(q-\frac{p}{2})\} + (k \leftrightarrow k', \mu \leftrightarrow \nu)\,, \quad (5)$$

and since all quark loop momenta q contribute, the small values of $(q \pm p/2)^2 \approx q^2$ in one quark leg will prevent the BC vertex $\Gamma^\mu_{BC}(q',q)$ from reducing to the bare γ^μ-vertex, even when a hard virtual photon momentum $k^2 = -Q^2$ makes "bare" the other fermion leg in $\Gamma^\mu_{BC}(q',q)$.

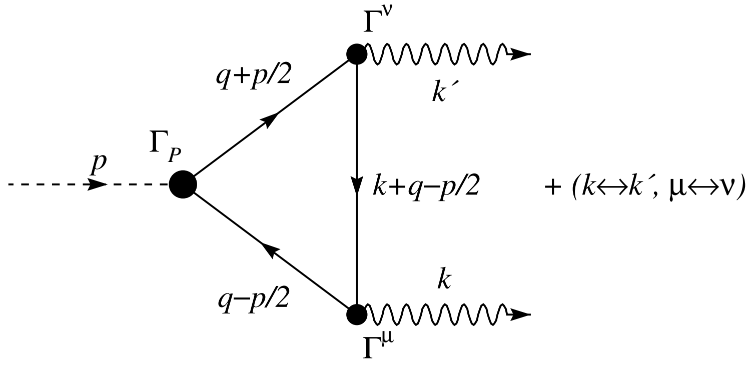

Fig. 1. The pseudoscalar–vector–vector (PVV) diagram for the transitions $P \to \gamma\gamma$ of neutral unflavoured pseudoscalar mesons ($P = \pi^0, \eta, \eta', \eta_c, \eta_b$) to two photons. Within the scheme of the generalized impulse approximation, the propagators and vertices are dressed.

However, this result on the asymptotics of $T_{\pi^0}(-Q^2, 0)$ when using the BC vertex, caused some controversy since Ref. [6] claimed that even for the BC vertex, the asymptotic coefficient should be $\mathcal{K} = 4f_\pi/3$, *i.e.*, that no modification occurs for the BC vertex due to one soft quark leg. The argument of Ref. [6] (see its Sect. *4.*) is that there are in fact no soft legs in the $qq\gamma$ vertices when Q^2 becomes very large. The on-shell condition for the pion and one photon

$$k'^2 = 0 = (p-k)^2 = M_\pi^2 - 2p\cdot k - Q^2 \approx -2p\cdot k - Q^2\,, \quad (6)$$

and $k^2 = -Q^2$ are used to argue that the pion momentum p has components which must scale like k and thus like Q. Then, $A([q \pm p/2]^2) = A(q^2 \pm q\cdot p + M_\pi^2/4)$ would tend to 1 as $Q^2 \to \infty$ even for very soft loop momenta q, just because of $p \sim Q$, causing $\Gamma^\mu_{BC} \to \gamma^\mu$.

We will now demonstrate that this argument does not hold. The very fact that the size of the *components* is invoked makes the argument suspect, because it is a frame-dependent statement. The argument of Ref. [6] relies on working in a Lorentz frame such as the one where

$$k = (0, 0, 0, \sqrt{Q^2}) \,, \tag{7a}$$

$$k' = (E_\pi, 0, 0, -E_\pi) \,, \tag{7b}$$

$$p = (E_\pi, 0, 0, \sqrt{Q^2} - E_\pi) \,, \tag{7c}$$

and where

$$E_\pi = \frac{Q^2 + M_\pi^2}{2\sqrt{Q^2}} \,. \tag{8}$$

Even if one sticks just to that choice in one's calculation, one can expect persistent soft contributions because of those soft loop momenta q which are also perpendicular to p so that $p \cdot q = 0$. However, the shortest and clearest demonstration that, at least in this application, $q \cdot p$ cannot be hard if q is soft, is noting that one can make a Lorentz transformation to the pion rest frame. In this case, it is the boost transformation along the z-axis and with the parameter

$$\beta = \frac{Q^2 - M_\pi^2}{Q^2 + M_\pi^2} \,. \tag{9}$$

In that frame,

$$k = (M_\pi - E_\gamma, 0, 0, E_\gamma) \,, \tag{10a}$$

$$k' = (E_\gamma, 0, 0, -E_\gamma) \,, \tag{10b}$$

with

$$E_\gamma = \frac{Q^2 + M_\pi^2}{2M_\pi} \,, \tag{11}$$

whereas $p = (M_\pi, 0, 0, 0)$, making it clear that for the light pion, $A([q \pm p/2]^2)$ is approximated well by $A(q^2)$ and not by $A(\pm q \cdot p)$ which allegedly [6] would be 1.

We want to make clear that we of course give precedence to the value $\mathcal{K} = 4f_\pi/3$ for the asymptotic coefficient as the one having the more fundamental meaning, resulting from the $qq\gamma$ vertices such as the CP or mCP ones, which have properties closer to the true vertex solution, such as being renormalizable. Also indicative is the asymptotics (12) found by Ref. [1] for the case when both photons are off-shell, $k'^2 = -Q'^2 \leq 0$, while the magnitude of $k^2 = -Q^2 < 0$ is mauch larger than any other relevant scale:

$$T_{\pi^0}(-Q^2, -Q'^2) = \frac{4}{3} \, \frac{f_\pi}{Q^2 + Q'^2} \,. \tag{12}$$

This is found for the $qq\gamma$ vertices which reduce to the bare γ^μ as soon as just one of the quark legs is hard, while the usage of the BC vertex (2) again modifies this result by the substitution $f_\pi \to \widetilde{f}_\pi$. Equation (12) agrees with the leading term of the OPE result derived by Novikov *et al*. [19] for the special case $Q^2 = Q'^2$. The distribution-amplitude dependence of the pQCD approach cancels out for that symmetric case, so that $T_{\pi^0}(-Q^2, -Q'^2)$ in this approach (*e.g.*, see Ref. [20]), in the limit $Q^2 = {Q'}^2 \to \infty$, exactly agrees with both our Eq. (12) and Ref. [19]. Therefore, for that symmetric case, we should have even the precise agreement of the coefficients irrespective of the description of the pion internal structure encoded in the distribution amplitude. Obviously, this favours those $qq\gamma$ vertices which reduce to the bare γ^μ as soon as just one of the quark legs is hard, over the BC vertex, and $\mathcal{K} = 4f_\pi/3$ over $\mathcal{K} = 4\widetilde{f}_\pi/3$. However, the BC vertex, which is the simplest WTI-preserving vertex and has been the one most widely used in phenomenological applications, may anyway be the one which is more phenomenologically successful not only for the presently accessible Q^2, but also for much larger values before starting to fail. For that reason, it is important to understand the asymptotic behaviour to which the BC vertex leads.

In addition to this, note that even if one favours some other vertex, one can use the simple BC *Ansatz* for checking reliability of the numerics, but only if one knows what the correct asymptotic coefficient for the BC *Ansatz* should be. The $\gamma^*\gamma \to \pi^0$ transition form factor predictions for *finite* Q^2, where they have to be calculated numerically, differ in Ref. [6] drastically from those in our Refs. [1] and [17], although they all use the BC vertex. (Compare Fig. 3 in Ref. [6] with Fig. 2 in Ref. [1] and Fig. 1 in Ref. [17].) Such very different behaviours cannot be explained by the usage of somewhat different propagator functions $A(q^2)$ and $B(q^2)$. Such discrepancy, therefore, puts in doubt the validity of the employed numerical approaches. Since the numerical results must tend to the *correct* asymptotic limit, the presented clarification of the $Q^2 \to \infty$ asymptotics for the case of the BC vertex, resolves the doubt in favour of Refs. [1] and [17], where, as Q^2 grows, the numerical results clearly tend to the analytically found asymptotics.

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OVISNOST ASIMPTOTSKOG PONAŠANJA PRIJELAZA $\gamma^\star\gamma$ U π^0 O OBUČENIM VRHOVIMA KVARK-FOTON

Raspravljamo kako razne postavke za obučene vrhove kvark-foton ($qq\gamma$) $\Gamma^\mu(q,q')$ utječu na asimptotsko ponašanje formfaktora prijelaza $\gamma^\star\gamma \to \pi^0$. Ispravljamo neka pogrešna shvaćanja koja se nalaze u literaturi.