

ON DISSIPATION MECHANISM IN THE INTRINSIC JOSEPHSON EFFECT
IN LAYERED SUPERCONDUCTORS WITH COHERENT INTERLAYER
TUNNELLING

S. N. ARTEMENKO

*Institute for Radioengineering and Electronics of the Russian Academy of Sciences,
Moscow 103907, Russia*

Dedicated to Professor Boran Leontić on the occasion of his 70th birthday

Received 22 November 1999; Accepted 21 February 2000

The mechanism of conductivity in the regime of the intrinsic Josephson effect is studied in layered superconductors with singlet d-wave pairing. The cases of coherent and incoherent interlayer tunneling of electrons are considered. The theory with coherent tunnelling describes main qualitative features of the effect observed in highly anisotropic high- T_c superconductors at voltages smaller than the amplitude of the superconducting gap, the mechanism of resistivity being related to the excitations of quasiparticles via the d-wave gap. Interaction of the Josephson junctions formed by the superconducting layers due to the charging effects is shown to be small.

PACS numbers: 74.50.+r, 74.25.Fy, 74.80.Dm

UDC 538.945

Keywords: highly anisotropic high- T_c layered superconductors, intrinsic Josephson effect, singlet d-wave pairing, coherent and incoherent interlayer tunneling

1. Introduction

Theoretical understanding of the intrinsic Josephson effect (IJE) in layered high- T_c superconductors is limited by insufficient knowledge of transport properties of both the superconducting and normal states. Studies of the energy structure [1,2] show that the electronic spectral density in directions $(0, \pi)$ is affected by a strong interaction with spin fluctuations [3]. At the same time, at low energies, in the directions (π, π) , corresponding to zeros of the order parameter Δ , the electronic structure can be described in terms of the Fermi liquid. This gives chances to

describe the IJE at low temperatures and small voltages in terms of the standard Fermi liquid approach. The aim of this work is to study the resistive properties of layered high- T_c superconductors and to understand to which extent they can be described in the frame of the BCS model with intralayer singlet d-wave pairing.

The intrinsic Josephson effect is expected to be qualitatively different in the cases of coherent and incoherent interlayer transfer. If the interlayer tunneling is coherent, a layered superconductor is a strongly anisotropic 3D crystal, whose normal-state resistivity is induced by scattering. On the other hand, a superconductor with incoherent tunneling can be considered as a stack of Josephson tunnel junctions.

If the tunneling is incoherent, the in-layer momentum is not conserved in interlayer transitions. This results in finite tunneling resistivity along the c -axis. If the tunnel junctions are formed by conventional s-wave superconductors, the product of critical current and normal state resistivity $V_c = I_c R_N$ is of the order of the gap value which corresponds to the experimental data for high- T_c superconductors. However, for the d-wave order parameter, independent averaging over directions of the electron momentum in neighbouring layers in the case of incoherent tunneling would result in the zero critical current along the c -axis, and in order to explain the experimentally observed large values of the critical current along the c -axis, one must assume a special d-wave-like angle dependence for the probability of random interlayer hopping. The fraction of the s-component in the order parameter in BSCCO was found in the recent tunneling measurements [4] to be below 10^{-3} . Without the assumption of the special momentum dependence of the interlayer transfer integral for the incoherent interlayer tunneling, such symmetry would result in a negligibly small critical current I_c , and in V_c about three orders of magnitude smaller than the experimentally observed values [5]. Furthermore, with such small values of j_c , the regime of branching in which some junctions are in the superconducting state, while others are in a resistive state, would be impossible in the range of voltages $V \sim \Delta_0/e$ per one resistive junction. Such branching is the most typical manifestation of the IJE. Thus, it is difficult to understand main features of the IJE in the frame of incoherent interlayer tunneling and d-wave pairing.

Since a layered superconductor with coherent tunneling is an anisotropic 3D crystal, in this case one must not assume a special angle dependence for the interlayer transfer integral in order to explain large experimental values of j_c . But the question whether IJE can be observed in a clean quasi two-dimensional crystal is still not clear. The finite normal-state resistivity ρ_c in crystals is induced by scattering, $\rho_c \sim 1/\tau$, in the clean superconductor the scattering rate being small, $\hbar/\tau \ll \Delta_0$. Typical voltages in experimental studies of IJE are a few times smaller than the maximum energy gap Δ_0/e [5], i. e., much larger than $\hbar/e\tau$. At such voltages (and frequencies of Josephson oscillations), resistivity is expected to decrease as voltage and frequency increase because of the Drude-like regime of scattering [6]. However, such decaying branches are not observed. The region of finite resistivity may become more pronounced if the intralayer scattering by impurities is resonant. Then the bandwidth of the gapless states formed by the scattering is relatively

large $\gamma \approx \sqrt{\Delta_0 \hbar / e\tau}$ [7]. But in a clean superconductor, this gives the region of linear resistivity still at voltages much smaller, than Δ_0 . Thus, typical voltage per one junction in the resistive state, $eV \sim \Delta_0$, can be explained only provided an additional mechanism for the resistivity is present at frequencies $\omega > \hbar/\tau$. Such a mechanism was considered recently in the study of bandwidth of the Josephson plasma resonance at low temperatures [8]. It is related to the dissipation induced by the electron excitation via the d-wave gap. However, this mechanism dies out at voltages larger than the amplitude of the d-wave gap, and to get a finite resistivity at high voltages, one must take into account some other mechanisms of scattering. Finite resistivity may be caused by some contribution of incoherent interlayer tunneling at large voltages, or inelastic scattering, e.g. on spin-wave excitations, may become important at large energies. We do not study these processes here.

In this paper we study, first, the IJE in a perfect highly-anisotropic superconducting crystal with coherent interlayer charge transfer. We derive equations for the charge and current densities and calculate the I - V curves at voltages and frequencies of the order of the gap, paying attention to the charging effects. Then, for comparison, we calculate the I - V curves for the case of incoherent interlayer tunneling. We found that the I - V curve calculated for the case of incoherent tunneling looks rather similar to the experimental data at large voltages.

2. Main equations

Having in mind to study large frequencies and voltages, we calculate expressions for the current and charge densities using collisionless transport equations in the Keldysh diagram technique for Green's functions at coinciding times $\widehat{G}(t, t)$ derived by Volkov and Kogan [9]. We generalize these equations to the case of layered superconductors, assuming the interlayer interaction described by the tight-binding approximation, and the superconductivity described by a BCS type the Hamiltonian leading to the singlet d-wave pairing. This approach to interlayer transitions describes the case of a layered single crystal. It is contrary to the case of random interlayer hopping considered in Ref. [10]. Our results have also been checked using quasiclassical transport equations for Green's functions integrated over momenta [11] generalized for layered superconductors with coherent interlayer tunneling [12,13]. For the case when the time dependence of the scalar potential can be neglected, the results can be calculated using a standard expression for the conductivity in terms of the spectral densities in neighbouring layers. Using this approach, we calculate I - V curves for the case of incoherent interlayer tunneling.

We consider homogeneous interlayer currents along the c-axis. This case is realized in narrow samples with the width in the ab-plane smaller than the Josephson length. We solve equations for diagonal and off-diagonal with respect to spin-index

components $g_{nm}(t)$ and $f_{nm}(t)$ of Keldysh propagator,

$$\begin{aligned}
& i\hbar \frac{\partial}{\partial t} g_{nm} + \Delta_n f_{nm}^* + f_{nm} \Delta_m + (\mu_n - \mu_m) g_{nm} \\
& = t_{\perp} \sum_{i=\pm 1} (A_{n,n+i} g_{n+i,m} - g_{n,m+i} A_{m+i,m}), \\
& i\hbar \frac{\partial}{\partial t} f_{nm} - 2\xi f_{nm} - \Delta_n g_{nm}^* + g_{nm} \Delta_m + (\mu_n + \mu_m) f_{nm} \\
& = t_{\perp} \sum_{i=\pm 1} (A_{n,n+i} f_{n+i,m} - f_{n,m+i} A_{m+i,m}^*).
\end{aligned} \tag{1}$$

Functions $g_{nm}(\xi, \phi, t)$, $f_{nm}(\xi, \phi, t)$ are matrices in layer indices. Here $\xi_p = \epsilon(\mathbf{p}) - \epsilon_F$, where $\epsilon(\mathbf{p})$ is the single-electron energy in the normal state, ϵ_F is the Fermi energy and \mathbf{p} the electron momentum in the ab-plane, ϕ is the angle of the in-plane electron momentum and t_{\perp} is the interlayer transfer integral. Furthermore, $\Delta_n = \Delta(\phi)_n$ and χ_n are the anisotropic superconducting order parameter and its phase in layer n , $\mu_n = e\Phi_n + (\hbar/2)(d\chi_n/dt)$ is the gauge-invariant scalar potential, Φ_n is the electric potential, $A_{n,n+1} = \exp \varphi_n$, where $\varphi_n = \chi_{n+1} - \chi_n - (2\pi s/\Phi_0)A_z$ is the gauge-invariant phase difference between the layers, and A_z is the component of the vector potential in the direction perpendicular to the layers. Electric field between the layers is expressed via the gauge-invariant potential as

$$eE_n s = \mu_n - \mu_{n+1} + \frac{\hbar}{2} \frac{d\varphi_n}{dt}. \tag{2}$$

The scalar potential μ is related to branch imbalance [14]. It is responsible for the charging effects in the Josephson plasma oscillations [15] and in IJE [6,16].

The current density between the layers n and $n+1$ and charge density in the layer n can be calculated as

$$j_{n,n+1} = \frac{et_{\perp}}{2s} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} (A_{n+1,n} g_{n,n+1} - g_{n+1,n} A_{n,n+1}), \tag{3}$$

$$\rho_n = -\frac{e}{4is} \int \frac{d\mathbf{p}}{(2\pi\hbar)^2} (g_{nn} + g_{nn}^*), \tag{4}$$

where s is the crystalline period in the c-direction.

3. Charge and current densities

We solve Eqs. (1) perturbatively with respect to t_{\perp} and consider the case of low temperatures $T \ll \Delta_0$. The contributions of interlayer transitions to the charge density are quadratic in t_{\perp} . Therefore, in the main approximation, we can take

into account only intralayer contribution which in the Fourier representation has a form

$$g_{nn} + g_{nn}^* = \left[\frac{2\xi}{\varepsilon} - \frac{8\Delta^2\mu_\omega}{\varepsilon(4\varepsilon^2 - \omega^2)} \right] \tanh \frac{\varepsilon}{2T}, \quad (5)$$

where $\varepsilon = \sqrt{\xi^2 + \Delta^2}$. Inserting this expression into (4), we get in the time representation

$$\rho_n = -\frac{\kappa^2}{8\pi} \int_0^\infty dt_1 F(t_1) \mu_n(t - t_1), \quad (6)$$

where κ is the inverse Thomas-Fermi screening radius,

$$F(t) = \left\langle \int_{-\infty}^\infty d\xi \frac{2\Delta^2 \sin 2\varepsilon t}{\varepsilon^2} \tanh \varepsilon/2T \right\rangle, \quad (7)$$

and $\langle \dots \rangle$ means averaging over ϕ .

The integral in Eq. (7) describes the non-exponential relaxation of μ with a relaxation time of the order of \hbar/Δ_0 . We will need equation (6) in the case of slow variations of the scalar potential when $F(t) \rightarrow 2\delta(t)$ so that the integral in Eq. (6) reduces to $2\mu(t)$. Then, inserting the expression for the charge density into the Poisson's equation, with electric field determined by Eq.(2), we express the difference of the scalar potentials between neighbouring layers, $\delta\mu_n = \mu_{n+1} - \mu_n$, in terms of the time derivative of the phase differences

$$\delta\mu_n = \frac{a}{16\sqrt{1+a}} \sum_m (\dot{\varphi}_{n+m+1} + \dot{\varphi}_{n+m-1} - 2\dot{\varphi}_{n+m}) \left(\frac{\sqrt{1+a}-1}{\sqrt{1+a}+1} \right)^{|m|}, \quad (8)$$

with $a = 4\varepsilon_\perp/(\kappa s)^2$, where ε_\perp is the high frequency dielectric constant in the perpendicular direction.

To calculate the current density, we must solve Eqs. (1) in the linear approximation on t_\perp . The equations are still difficult to solve for an arbitrary $\mu_n(t)$ and $\varphi_n(t)$. Therefore, we calculate the expressions for the current density in two limiting cases. First, we find the solutions in the linear approximation with respect to the potential μ which describes the charging effects. The conditions for small effects of the approximation will be discussed later.

$$g_{n,n+i} = \frac{4t_\perp\Delta^2}{\varepsilon} \tanh \frac{\varepsilon}{2T} \left\{ \frac{C_\omega}{\varepsilon(4\varepsilon^2 - \omega^2)} + \int_{-\infty}^\infty \frac{d\omega_1}{2\pi} \left[\frac{(2\omega_1 - \omega)C_{\omega-\omega_1}}{4\varepsilon^2 - (\omega - \omega_1)^2} - \frac{i\omega S_{\omega-\omega_1}}{4\varepsilon^2 - \omega^2} \right] \frac{\mu_{\omega_1}}{4\varepsilon^2 - \omega_1^2} \right\}. \quad (9)$$

In this limit, the current density between the layers n and $n + 1$ consists of a component determined by the phase difference φ_n between the same layers only, and of an additional component which depends on the difference of the scalar potentials in these layers,

$$j_{n,n+1}(t) \equiv j^\varphi(t) + j^\mu(t).$$

Assuming that t_\perp does not depend on the momentum, we get

$$j^\varphi(t) = j_c \int_0^\infty dt_1 F(t_1) \cos \frac{\varphi_n(t-t_1)}{2} \sin \frac{\varphi_n(t)}{2}, \quad (10)$$

$$\begin{aligned} j^\mu(t) = & j_c \int_0^\infty dt_1 \int_0^\infty dt_2 F(t_2) \left\{ \left[\cos \frac{\varphi_n(t-t_1)}{2} \delta\mu_n(t-t_1-t_2) \right. \right. \\ & - \left. \left. \cos \frac{\varphi_n(t-t_1-t_2)}{2} \delta\mu_n(t-t_1) \right] \cos \frac{\varphi_n(t)}{2} \right. \\ & \left. + \left. \sin \frac{\varphi_n(t-t_1)}{2} \delta\mu_n(t-t_1-t_2) \sin \frac{\varphi_n(t)}{2} \right\}. \end{aligned} \quad (11)$$

Since according to Eq. (8) $\delta\mu_n(t)$ depends on the phase differences between different layers, the component (12) of the current describes the interaction between the "Josephson junctions" due to the charging effects. Interaction due to the charging effects was studied in the phenomenological model by Koyama and Tachiki [17], however, the results are different.

For $s = 1.5$ nm, $1/\kappa = 0.2$ nm and $\epsilon_\perp = 12$, we get $a \approx 0.85$, and the factor in front of the sum in Eq. (8) is about 0.04. Thus, $\delta\mu_n$ is small compared to the time derivatives of the phase differences. Therefore, the charging effects and the contribution of $\delta\mu_n$ to the electric field between the layers must be small as well (cf. Eq. (2)).

Equations (10) and (12) can be simplified in the limiting cases. At $T \ll \hbar\omega$ and $eV \ll \Delta_0$

$$j^\varphi(t) = j_c \sin \varphi_n + j_c \frac{\pi}{2\Delta_0} \frac{d\varphi_n}{dt} (1 - \cos \varphi_n), \quad (12)$$

$$j^\mu(t) = 2j_c \int_0^\infty dt_1 \sin \frac{\varphi_n(t-t_1)}{2} \delta\mu_n(t-t_1) \sin \frac{\varphi_n(t)}{2}. \quad (13)$$

So, the current can be considered to consist of the superconducting, normal and interference components, and of the quasiparticle contribution related to the difference of the scalar potential. In the limit of a linear response, the dissipative

term in Eq. (12) vanishes because in the spatially uniform case excitations of the quasiparticles via the superconducting gap are forbidden. In the nonlinear regime, the related dissipation mechanism is effective because in the presence of a current along the c -axis, the phase depends on the layer index. This makes the system non-uniform and the excitations via the gap become allowed [8]. In the regime of the linear response, this dissipation mechanism becomes possible due to the scattering. Taking into account the scattering in a similar way as in Ref. [8], we get

$$\frac{j(t)}{j_c} = \varphi_n + \frac{2}{3\tau\Delta_0^2} \frac{d\varphi_n}{dt} - \frac{\pi}{2\Delta_0} \delta\mu_n. \quad (14)$$

The expression for the current density simplifies also at large frequencies and voltages,

$$\omega, V \gg \omega_p, \quad (15)$$

where ω_p is the Josephson plasma frequency. In the most anisotropic materials, like Bi- and Tl-based cuprates, $\Delta_0 \gg \omega_p$. Under the condition (15), the electronic AC current is shunted by the displacement current,

$$v_{\text{AC}} \sim v_{\text{DC}} \left(\frac{\omega_p^2}{\omega^2} \right) \ll v_{\text{DC}},$$

and time dependence of the phase differences become simple,

$$\varphi_n \approx 2\omega_n t + \varphi_\omega, \quad \varphi_\omega \ll 1.$$

When all layers are in the resistive state, then $\delta\mu = 0$, and the I - V curve has the form

$$j = \left\{ \theta(2\Delta_0 - V) \left[\frac{2\Delta_0}{V} \mathbf{K} \left(\frac{V}{2\Delta_0} \right) - \mathbf{E} \left(\frac{V}{2\Delta_0} \right) \right] + \theta(V - 2\Delta_0) \left[\mathbf{K} \left(\frac{2\Delta_0}{V} \right) - \mathbf{E} \left(\frac{2\Delta_0}{V} \right) \right] \right\} \tanh \frac{V}{4T}, \quad (16)$$

where, again, V is the voltage per one junction. This expression is also valid in the limit $a \rightarrow 0$ in which $\delta\mu = 0$.

At low voltages the I - V curve (16) is described by the linear quasiparticle conductivity $\sigma_q = \pi e s j_c / \Delta_0 = e^2 / (8\pi \lambda^2 \Delta_0)$. This value agrees with the experimental data of Latyshev et al. [7] and differs from the linear conductivity calculated for the case of the resonant scattering [7] by the factor $8/\pi^2 \approx 1$. At larger voltages, the shape of the I - V curve is different from that observed experimentally. It has a logarithmic anomaly at $V = 2\Delta_0$ and a decaying branch at $V > 2\Delta_0$. This demonstrates that our approach is not valid at large V when contributions of electrons with energy of order Δ_0 are important. Such electrons are strongly scattered

by spin fluctuations which we did not take into account. Any additional mechanism of scattering, in particular, inelastic processes or presence of the component with the incoherent charge transfer in the interlayer tunneling, will smear the logarithmic anomaly at $V = 2\Delta_0$ and add an Ohmic contribution to I - V curves at large voltages.

Now we consider the regime of branching. At the current values $j < j_c$, a regime is possible in which some "junctions" are in the superconducting state while others are in the resistive state. In this case the DC current through the superconducting junctions is transported as the superconducting current, and the AC current is transported mainly as the displacement current. The total voltage is formed by the sum of the voltages across resistive junctions, and the I - V curves consist of branches which differ by the number of the junctions in the resistive state. The number of branches is equal to the total number of the layers in the sample. In the limit of a small a , the n -th branch is described by the first term in Eq. (16) with V substituted by V/n . Under the condition (15), the current density was calculated also for arbitrary relation between $\delta\mu_n$ and $\dot{\varphi}_n$. At $\dot{\varphi}_n \gg T$,

$$j_{n,n+1} = \sqrt{\frac{\dot{\varphi}_n + \delta\mu_n}{(\dot{\varphi}_n - \delta\mu_n)^3}} \left[\mathbf{K} \left(\frac{\sqrt{\dot{\varphi}_n^2 - \delta\mu_n^2}}{2\Delta_0} \right) - \mathbf{E} \left(\frac{\sqrt{\dot{\varphi}_n^2 - \delta\mu_n^2}}{2\Delta_0} \right) \right]. \quad (17)$$

Note that $\dot{\varphi}_n$ differs from the voltage across the resistive junction by $\delta\mu_n$ (cf. Eq. (2)). In the limit $a \rightarrow 0$, when $\delta\mu_n = 0$, the current as function of the voltage V per one resistive junction is identical for all branches. At finite a , according to (8), the shape of the branches depends on the neighbouring junctions, whether they are in the resistive state or not. However, for reasonable values of the parameter a , the difference between the shapes of the branches presented as functions of the total voltage divided by the number of the "junctions" in the resistive state is small, less than 3–4%. Already at $a = 0.2$, the branches almost coincide, which corresponds to the experimental results [4,5].

4. Conclusion

The model with the BCS-type d-wave pairing and coherent interlayer transport describes qualitatively such features of the IJE like the branching with typical voltages per one junction $V \sim \Delta_0$ at low temperatures. The charging effects are found to be small for reasonable values of the parameters. The damping in the system and, hence, the resistivity at such voltages is induced by the transitions of the quasiparticles via the d-wave gap. However, this model does not describe the experimental curves at voltages of the order Δ_0 and higher because in the regime when all layers are in the resistive state, it results in the logarithmic singularity at $V = \Delta_0$ and a decaying I - V curve at larger voltages. This discrepancy may be related either to the inapplicability of the simple BSC-type model with d-wave pairing, or to some other mechanisms of scattering becoming effective at larger

energies, like the inelastic scattering, e.g., on spin wave excitations, or other effects which smear the spectral density. We do not address these processes here.

Note that at elevated temperatures and smaller voltages of the order of $1/\tau$, especially near T_c , one may expect a different regime in which resistivity is related to the quasiparticle scattering. This mechanism dies out at large frequencies and voltages $\omega, V \gg 1/\tau$.

References

- 1) M. R. Norman et al., Phys. Rev. Lett. **79** (1997) 3506;
- 2) Y. F. Fong et al., Nature **398** (1999) 588;
- 3) L. B. Ioffe and A. J. Millis, Phys. Rev. B **58** (1998) 11631;
- 4) M. Möhle and R. Kleiner, Phys. Rev. B **59** (1999) 4486;
- 5) K. Schlenga, R. Kleiner et al., Phys. Rev. B **57** (1998) 14518;
- 6) S. N. Artemenko and A. G. Kobelkov, Phys. Rev. Lett. **78** (1997) 3551;
- 7) Yu. I. Latyshev, T. Yamashita, L. N. Bulaevskii et al., Phys. Rev. Lett. **83** (1999) 2336;
- 8) S. N. Artemenko, L. N. Bulaevskii, M. P. Maley and V. M. Vinokur, Phys. Rev. B **59** 11, 587 (1999); ISPITATI
- 9) A. F. Volkov and Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **65** (1973) 2038;
- 10) A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. **73** (1977) 299; [Sov. Phys. JETP **46** (1977) 155];
- 11) S. N. Artemenko, Zh. Eksp. Teor. Fiz. **79** (1980) 162; [Sov. Phys. JETP **52** (1980) 81];
- 12) S. N. Artemenko and A. G. Kobelkov, Phys. Rev. B **55** (1997) 9094;
- 13) M. J. Graf, M. Palumbo, D. Rainer and J. A. Sauls, Phys. Rev. B **52** (1995) 10588;
- 14) M. Tinkham and J. Clarke, Phys. Rev. Lett. **28** (1972) 1366;
- 15) S. N. Artemenko and A. G. Kobel'kov, JETP Letters **58** (1993) 445; Physica C **253** (1995) 373;
- 16) D. Ryndyk, Phys. Rev. Lett. **80** (1998) 3376;
- 17) S. Koyama and M. Tachiki, Phys. Rev. B **45** (1996) 16183.

DISIPATIVNI MEHANIZMI INTRINSIČNOG JOSEPHSONOVOG EFEKTA U SLOJEVITIM SUPRAVODIČIMA S MEĐUSLOJNIM TUNELIRANJEM

Proučavaju se mehanizmi vodljivosti u uvjetima intrinzičnog Josephsonovog efekta u slojevitim supravodičima sa singletnim d-valnim sparivanjem. Razmatraju se slučajevi koherentnog i nekoherentnog međuslojnog tuneliranja elektrona. Teorija s koherentnim tuneliranjem opisuje glavne kvalitativne značajke efekta koji se opaža u jako anizotropnim visokotemperaturnim supravodičima, na naponima manjim od supravodljivog procijepa, za koji se otpornost svodi na uzбудu kvazičestica putem d-valnog procijepa. Pokazuje se da je međudjelovanje Josephsonovih spojeva među supravodljivim slojevima, koje je posljedica učinaka nabijanja, malo.