ISSN 1330–0008
CODEN FIZAE4

IONISATION OF HYDROGENIC IONS BY ANTIPROTON IMPACT

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Received 18 August 1998; revised manuscript received 25 November 1998
Accepted 18 January 1999

Ionisation of hydrogenic ions by impact of antiprotons have been studied at collision energies varying from 2 to 2000 keV/amu. Total and double-differential cross-sections are calculated employing a final state wave function which incorporates distortion due to Coulomb fields of both the projectile and the target nucleus. The present calculated values are compared with the existing theoretical predictions.

PACS number: 34.50.Fa UDC 535.353, 539.125.46

Keywords: hydrogenic ions, ionisation by impact of antiprotons, calculations for collision energies 2 to 2000 keV/amu

1. Introduction

Ionisation of charged particles is a process of great importance in view of its significant role in understanding large number of processes in atomic physics, plasma physics and astrophysics. For many applications in these areas, it is useful to measure the ionisation cross-sections to a given transition of atomic-ionic system as a result of charged particle impact. Although collision processes have been studied for the last several decades, it is still a challenge to understand in detail even the simplest of ionisation process. This is due to the complex dynamics of the system which consists of more than two particles that interact via Coulomb force.

The process of electron emission resulting from ionisation of atoms/ions by fast charged-particle impact is quite complicated. Ionisation can be probed directly by the use of projectiles of different masses and of opposite charges. In the Bethe-Born theory, ionisation cross-section depends on the square of projectile charge and is independent of its mass. Nevertheless, evidence for the charge-dependence dynamical effect, such as Barkas' effect, in stopping-power measurements has been known for some time.

Antiproton collision with atomic hydrogen and hydrogenic ions provide a unique and fundamental testing ground for the development of non-perturbative quantal scattering technique. For proton impact, the final state is a superposition of elastic scattering, excitation, ionisation and charge transfer, whereas for antiproton impact only the first three processes can occur. This essential difference is obviously a result of change in the charge sign of the projectile which is always available to the theoretical physicist for assessment to antimatter. From the theoretical point of view, the main difficulty is the representation of many-body Coulomb state. Due to the long-range nature of the Coulomb potential, the plane-wave function describing the continuum state remains correlated even to a macroscopic distance.

The first measurements of antiproton-atom collisions were performed by Andersen et al. [1], in which single- and double-differential ionisation cross-sections of He were measured by antiproton impact. A year later, Fainstein et al. [2] applied a theoretical model to calculate the total ionisation cross-section of He by antiproton impact. Recently, Knudsen et al. [3] accurately measured the total ionisation cross-section of hydrogen atom taking antiproton as projectile.

In the present work, we have calculated the total ionisation cross-sections of hydrogenic ions (He^+ to O^{7+}) and double-differential cross-sections of He^+ by antiproton impact. The results of total ionisation cross-section for He^+ are compared with the values obtained by Shultz et al. [4].

In Section 2, we describe briefly the theoretical method adopted here and in Section 3 we present our results with discussion.

2. Theory

We consider the ionisation of hydrogenic ions by collision with antiproton of charge Z_P. We use the perturbative approach [5] which has already been successfully applied to the proton-hydrogen system. A brief discussion of the approach may be useful to study the present case. We adopt the impact parameter formalism, where the motion of the electron is subjected to the quantum mechanical laws while the internuclear motion is classically treated as $\vec{R} = \vec{\rho} + \vec{v}t$, where $\vec{R}$ is the internuclear distance, t the time, $\vec{\rho}$ the impact parameter and $\vec{v}$ the relative velocity of the nuclei. Let the positions of the target and the projectile be at T^+ and P^-, respectively, as shown in Fig. 1. The midpoint of the line joining the points T^+ and P^- is taken to be at rest and is chosen as the origin. Hence, P^- and T^+ move with equal and opposite velocities $\vec{v}/2$ and $-\vec{v}/2$, respectively. The perpendicular (with respect to $\vec{v}$) distance between T^+ and P^- is the impact parameter, $\vec{p}$. Time is measured from the instance when the two nuclei are closest to each other.

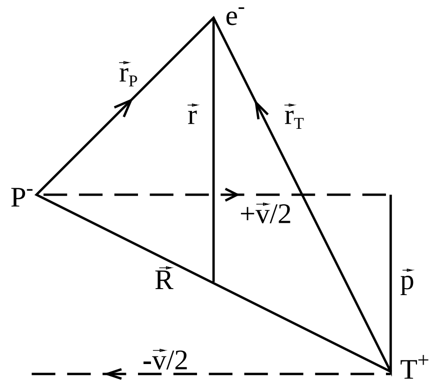

Fig. 1. The collision diagram for antiproton - hydrogen system.

Let $\vec{r}_{\mathrm{P}}$, $\vec{r}_{\mathrm{T}}$ and $\vec{r}$ denote the position vectors of the electron relative to the projectile, target and origin, respectively. The Hamiltonian H_e may be written (atomic units are considered throughout) as

$$H_e = -\frac{1}{2}\nabla_r^2 - \frac{Z_{\mathrm{P}}}{r_{\mathrm{P}}} - \frac{Z_{\mathrm{T}}}{r_{\mathrm{T}}}, \tag{1}$$

where $Z_{\mathrm{P}} = -1$ and Z_{T} varies from 2 to 7.

The nucleus-nucleus interaction term is omitted here as this interaction can be removed by a canonical transformation and as such it should not affect any transition probability for a particular impact parameter.

The development in time of the transition amplitude for ionisation with the ejection of electron having momentum $\vec{k}$ can be written as

$$\frac{\mathrm{d}a_k}{\mathrm{d}t} = \int \left[\Psi_{kc}^{-*}\left(H_e - \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\right)\Psi_i\right]\mathrm{d}\vec{r}, \tag{2}$$

with initial condition that at $t = -\infty$, $a_k = 0$. The ionisation probability corresponds to $|a_k(t=+\infty)|^2$.

The continuum-state wave function in the present work is represented by the product of two Coulomb wave functions,

$$\Psi_{kc}^{-} = N_1 N_2 \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{r}}\,{}_1\mathrm{F}_1(\mathrm{i}\alpha_{\mathrm{P}}, 1; -\mathrm{i}(k_P r_{\mathrm{P}} + \vec{k}_{\mathrm{P}}\cdot\vec{r}_{\mathrm{P}}))\,{}_1\mathrm{F}_1(\mathrm{i}\alpha_{\mathrm{T}}, 1; -\mathrm{i}(k_T r_{\mathrm{T}} + \vec{k}_{\mathrm{T}}\cdot\vec{r}_{\mathrm{T}}))\mathrm{e}^{-\mathrm{i}k^2/2}, \tag{3}$$

where $\alpha_{\mathrm{P}} = 1/k_{\mathrm{P}e}$ and $\alpha_{\mathrm{T}} = -Z_{\mathrm{T}}/k_{\mathrm{T}e}$, and $k_{\mathrm{P}e}$ and $k_{\mathrm{T}e}$ are the velocities of electron in the projectile and the target frame of reference and

$$N_1 = \exp\left(-\frac{\pi\alpha_{\mathrm{P}}}{2}\right)\Gamma(1+\mathrm{i}\alpha_{\mathrm{P}}) \qquad \text{and} \qquad N_2 = \exp\left(-\frac{\pi\alpha_{\mathrm{T}}}{2}\right)\Gamma(1+\mathrm{i}\alpha_{\mathrm{T}}).$$

The above wave function Ψ^-_{kc} asymptotically satisfies the Schrödinger equation

$$\left(-\frac{1}{2}\nabla^2_r - \frac{Z_\mathrm{P}}{r_\mathrm{P}} - \frac{Z_\mathrm{T}}{r_\mathrm{T}} - \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\right)\Psi^-_{kc} = 0. \tag{4}$$

Considering Eqs. (3) and (4), the ground-state wave function of the hydrogen atom around the target nucleus and the contour representation [6] for the confluent hypergeometric function, we may write Eq. (2) as

$$a_k(p) = C_k \oint \mathrm{d}t_1 \mathrm{d}t_2\, t_1^{-1-\mathrm{i}\alpha_\mathrm{P}} (t_1-1)^{\mathrm{i}\alpha_\mathrm{P}}\, t_2^{-1-\mathrm{i}\alpha_\mathrm{T}} (t_2-1)^{\mathrm{i}\alpha_\mathrm{T}} \int \mathrm{e}^{\mathrm{i}Et}\mathrm{d}t \tag{5}$$

$$\times \int \mathrm{e}^{-\vec{K}\cdot\vec{r}}\, \frac{1}{r_\mathrm{T}}\, \mathrm{e}^{-\lambda r_\mathrm{T}}\, \mathrm{e}^{\mathrm{i}(t_1 k_\mathrm{P} r_\mathrm{P} + t_2 k_\mathrm{T} r_\mathrm{T})}\, \mathrm{e}^{\mathrm{i}(t_1 \vec{k}_\mathrm{P}\cdot\vec{r}_\mathrm{P} + t_2 \vec{k}_\mathrm{T}\cdot\vec{r}_\mathrm{T})}\, \mathrm{d}\vec{r},$$

where $\vec{K} = \vec{k} + \vec{v}/2,\ E = k^2/2 - v^2/2 - \varepsilon_0$ and $C_k = (\lambda^{3/2} N_1^* N_2^*)/(4\pi^{5/2})$. The detailed steps of evaluation of Eq. (5) has been described in the previous paper [5].

The double-differential cross-section and the total cross-section are calculated by performing the following integrals

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}E_e \mathrm{d}\Omega_e} = k \int \mathrm{d}^2\vec{p}\ |a_k(p)|^2$$

and

$$\sigma_{total} = 2\pi \int_0^\infty \frac{\mathrm{d}^2\sigma}{\mathrm{d}E_e \mathrm{d}\Omega_e} \sin\theta_e \mathrm{d}\theta_e \mathrm{d}E_e$$

3. Results and discussion

3.1. Total ionisation cross-section (TICS)

In Fig. 2, we compare the present calculated results for total ionisation cross-section as function of collision energy with the values of Shultz et al. [4] for antiproton impact on He^+. The present results for total ionisation cross-section show good agreement with the values calculated by applying continuum distorted-wave eikonal initial-state (CDW-EIS) calculation [4], throughout the energy region considered. Below 150 keV, the present results, however, fail to agree with the computed results of Ref. 4 obtained from the time-dependent Schrödinger equation (TDSE), classical trajectory Monte Carlo (CTMC) and hidden crossing (HC) calculations. The present approximation is essentially a high-energy approximation and does not predict accurate cross-sections in the low-energy region.

Figure 3 displays the values of total ionisation cross-sections as a function of collision energy for antiproton impact on Li^{2+}, Be^{3+}, B^{4+}, C^{5+}, N^{6+} and O^{7+}. The magnitude of the cross-section is multiplied by a factor 10 and compared with the results for He^+ [4]. It is evident from the figure that the values of the cross-

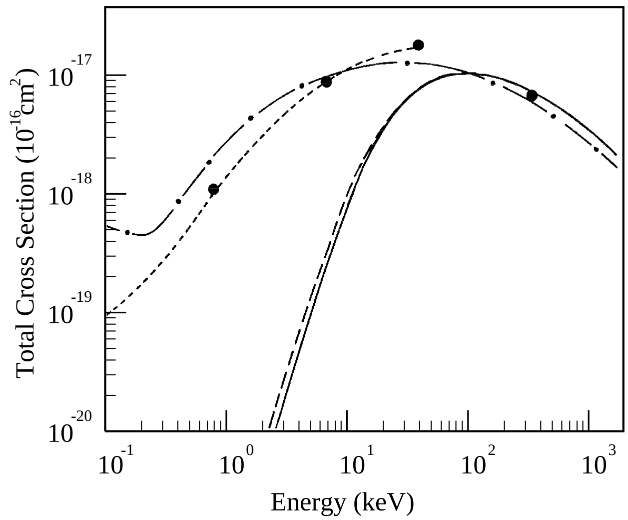

Fig. 2. Total ionisation cross-sections for ionisation of He^+ *by antiproton impact: present results: (——-) , CDW-EIS: (– – – –), HC: (- - - - -), CTMC: (— · — · —) and TDSE: (• • • •).*

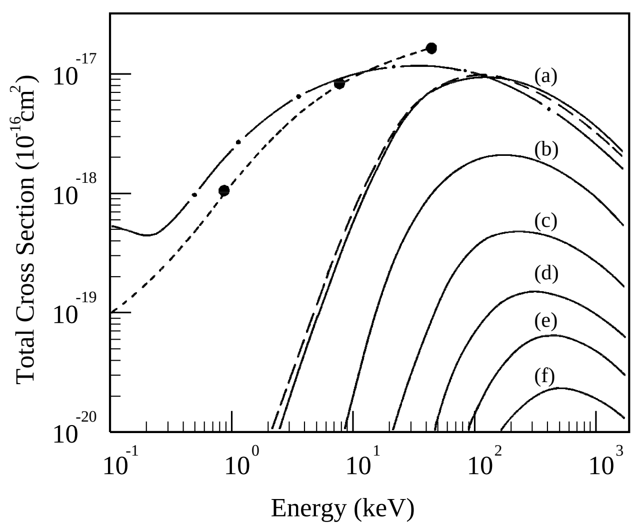

Fig. 3. Total ionisation cross-sections for ionisation of (a) Li^{2+}*, (b)* Be^{3+}*, (c)* B^{4+}*, (d)* C^{5+}*, (e)* N^{6+}*, (f)* O^{7+}*, Present results (———) for (a), (b) , (c), (d), (e) and (f) and total ionisation cross-sections of* He^+ *(Ref. 4): CDW-EIS: (– – – –), HC: (- - - - -), CTMC: (– · – · –) and TDSE: (• • • •).*

sections for different hydrogenic ions show the same trend as that for He^+, though the magnitude in each case differs from one another.

3.2. Double-differential cross-section (DDCS)

In Fig. 4, we display the present calculated results of double-differential cross-sections for 300-keV antiproton impact on He^+ for fixed emission angles of 0° and 10°. The double-differential cross-sections display prominent features in the ejected spectra (a sharp exponential dip in the region of the peak due to electron capture to continuum (ECC) in the forward direction (Fig. 4a) and remains an important feature even at 10° (Fig. 4b)). The exponential dip arises from ejected

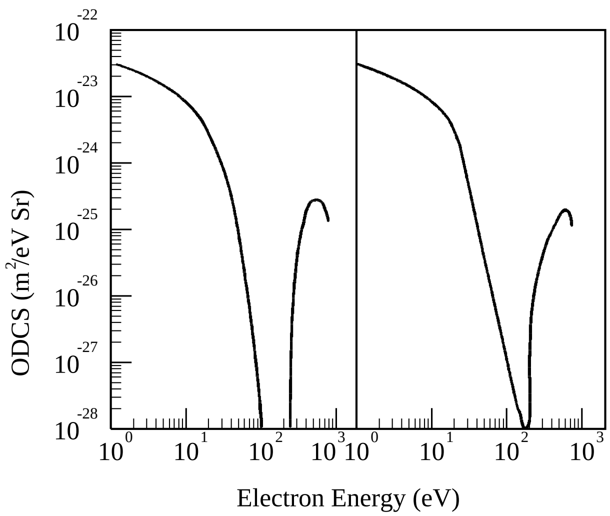

Fig. 4. Double-differential cross-sections of He^+ *as a function of ejection energy by impact of antiproton of incident energy 300* keV *for a fixed emission angle of (a)* 0° *and (b)* 10°*. Present results (———).*

electrons travelling with nearly the same velocity as that of the projectile. The dip in ejected spectra can be considered as a two-center electron-emission effect (TCEE), i.e., as the related effects on the electron-emission spectra due to the motion of ejected electron under the influence of a two-centre potential (Coulomb fields of the projectile and the residual target) in the final continuum state. In the present case, the normalization factor N_1 of the final continuum state wave function (Eq. (3)) appears in the cross-section as

$$|N_1|^2 = \frac{2\pi\alpha_P}{\exp(2\pi\alpha_P) - 1} \, .$$

It is responsible for producing different structure in DDCS, as for a negatively charged projectile ($Z_P < 0$),

$$\lim_{k_{Pe}\to 0} |N_1|^2 \to 2\pi\alpha_P \exp(-2\pi\alpha_P) \to 0$$

gives rise to a singular structure which leads to a pronounced dip in DDCS. Fainstein et al. [7] extensively studied the emission spectra for both positive and negative ion impact on He atom. They reported that at forward angles, instead of maximum in DDCS due to increased attraction by heavy ion, one obtains minimum due to the repulsion between the electron and the negatively-charged projectile, and as the angle increases the dip disappears.

4. Conclusion

In this work, the single-electron ionisation of hydrogenic ions by impact of antiprotons have been calculated. The results for the total ionisation cross-section for He^{+} are found to be in a reasonably good agreement with CDW-EIS values and also interpret well the values of other theoretical calculations, especially in the high-energy region. We also present the total cross-section values for other hydrogenic ions (Li^{2+} to O^{7+}) and compare them with the results for He^{+} [4]. The double-differential cross-section for He^{+} is calculated and the ejected spectra are discussed to some extent.

The present theoretical approach is comparatively simple and easier to tackle numerically. Fairly good results can be predicted for the cross-sections in the intermediate and high-energy region.

Acknowledgements

The authors would like to express their thanks to the International Atomic Energy Agency (IAEA), Vienna, for encouragement under the Research Agreement No. 8003/CF.

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IONIZACIJA VODIKU-SLIČNIH IONA UDAROM ANTIPROTONA

Proučava se ionizacija vodiku-sličnih iona pri sudarima s antiprotonima na energijama 2 do 2000 keV/ajm. Ukupni i dvostruko-diferencijalni presjeci se računaju upotrebom valne funkcije konačnog stanja koja sadrži izobličenje Coulombovim poljem projektila i jezgri mete. Dobiveni se ishodi računa uspoređuju s ishodima drugih teorijskih predviđanja.