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ELEMENTARY EXCITATION INTERACTIONS IN BOSE FLUIDS IN THE BOGOLIUBOV APPROXIMATION

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Dedicated to Professor Mladen Paić on the occasion of his 90$^{\text{th}}$ birthday

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The effective interaction energy of quasiparticles in the Bose fluid, which is described by "ladder state diagrams" in the Bogoliubov approximation, was studied for a model potential. It is shown that in the Bogoliubov approximation, the interaction of excitations in liquid ^{4}He can not be described even qualitatively. It means that a model of the weakly interacting dilute Bose gas fails in describing the interaction between excitation in Bose-condensed liquid, although this model qualitatively incorporates superfluidity in itself.

1. Introduction

The ground state and phonon–roton spectrum of liquid ^{4}He may be regarded as one of the best solved problems in condensed matter physics. Recently, surprising second-order light scattering results have aroused interest for this system. Experiments by Ohbayashi *et al.* [1] have shown that the negative shift in the energy of the two-roton peak in liquid ^{4}He, interpreted as the signal of a two-roton bound state, decreases in magnitude with increasing pressure until the energy shift becomes positive above about 2 atm. A complete understanding of the two-roton state requires an exploration of the effective interaction between a pair of rotons [2]. In this paper, we focus on the question of interaction between excitations in, Bogoliubov boson systems as the first step in the examination of this problem [3]. At the same time, this is an extension of the Bogoliubov theory to the interaction between excitations. This also provides an opportunity to apply a powerful many-body perturbation method which was first developed by Feenberg [4]. Ljolje [5] showed that the Feenberg perturbation theory has an additional advantage, compared to the Brillouin-Wigner (BW) perturbation theory. Namely, it shows the correct behaviour on the number of particles, N, through second order, and thus is applicable to many-body systems through that order. Ljolje rearranged the Feenberg formula to obtain properly behaving expressions to the fifth order. This method was applied to the ground state and excited state energies and the Bogoliubov's well known results were obtained. Recently, on the same basis, the interaction between excitations was studied by C. Campbell [6].

In this paper the expression for the interaction between a pair of quasiparticles was further examinated. The next logical step was performed. After including the matrix elements, which produce the Bogoliubov approximation for the phonon-roton spectrum in the Feenberg perturbation theory [5,6], it was analysed for an appropriate model potential. In Sec. 2, the Feenberg perturbation energy expression was outlined and a rearrangement of it which shows correct behaviour on the number of particles through fifth order and includes two elementary excitations [6]. The interaction between elementary excitations, described approximately by ladder state diagrams with Feenberg denominators, and a corresponding Bethe-Goldstone-Salpeter (BGS) equation, is presented here, too. In Sec. 3 this equation was studied in the Bogoliubov approximation. Some details of the calculation, for a model boson system with short range repulsion and long range attraction, and discussion are given in Sec. 4.

2. Survey of basic relations

For the hamiltonian of a homogenous system of N particles in the volume Ω

$$H = T + V, \tag{1}$$

the Feenberg perturbation energy expression [4] reads

$$\begin{aligned} E_l &= e_l + V_{ll} + \sum_{n \neq l} \frac{V_{ln} V_{nl}}{E_l - E_n^F} + \\ &+ \sum_{\substack{n \neq l \\ n' \neq ln}} \frac{V_{ln} V_{nn'} V_{n'l}}{(E_l - E_n^F)(E_l - E_{nn'}^F)} + \dots, \end{aligned} \tag{2}$$

where l is an arbitrary state index, e_l the eigenvalue of T and the denominators contain the renormalized Feenberg energies:

$$
\begin{aligned}
E_n^F &= e_n + V_{nn} + \sum_{n' \neq ln} \frac{V_{nn'}V_{n'n}}{E_l - E_{nn'}^F} + \\
&+ \sum_{\substack{n' \neq ln \\ n'' \neq lnn'}} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(E_l - E_{nn'}^F)(E_l - E_{nn'n''}^F)} + \dots \qquad (3)\\
E_{nn'}^F &= e_{n'} + V_{n'n'} + \sum_{n'' \neq lnn'} \frac{V_{n'n''}V_{n''n'}}{E_l - E_{n'n''}^F} + \\
&+ \sum_{\substack{n'' \neq ln' \\ n''' \neq ln'n''}} \frac{V_{n'n''}V_{n''n'''}V_{n'''n'}}{(E_l - E_{nn'n''}^F)(E_l - E_{nn'n''n'''}^F)} + \dots \qquad (4)\\
&\vdots
\end{aligned}
$$

The injunctions under the summation signs do not permit any two indices to be equal (there is no repetition of matrix elements).

In the second quantized form the hamiltonian is given by

$$T = \sum_k e_k a_k^+ a_k \qquad (5)$$

$$V = \frac{1}{2\Omega} \sum_{k_1 k_2 q} V_q a_{k_1+q}^+ a_{k_2-q}^+ a_{k_2} a_{k_1}, \qquad (6)$$

where

$$e_k = \frac{\hbar^2 k^2}{2m} \qquad (7)$$

$$V_q = \int V(r) e^{-i\vec{q}\vec{r}} \mathrm{d}\vec{r} \qquad (8)$$

a_k^+ and a_k are the bare particle creation and annihilation operators which satisfy the commutation rules:

$$[a_k a^+_{k'}] = \delta_{kk'} \quad , \quad [a_{k'} a_k] = [a^+_{k'} a^+_k] = 0\,. \tag{9}$$

A procedure for regrouping the terms in the Feenberg relation (2) has been already developed: Ljolje [5] has done this for the non-interacting N-body Bose ground state

$$|l> = |N, 0>,$$

and for the single excited states

$$|l> \;\; = \;\; |N-1, 1_k> \;\; .$$

Campbell [6] has rearranged it for the doubly excited state

$$|l> \;\; = \;\; |N-2, 1_k 1_{k'}> \;\; \equiv \;\; |2>$$

and it was shown that successive approximation of the fifth order has the correct behaviour on N when one takes the unperturbed state to have two particles (out of the condensate) with momenta $\hbar\vec{k}$ and $\hbar\vec{k}'$; it takes the form

$$\begin{aligned} E_2^{(5)} \quad &= \quad E_2^{(1)} + \sum_n{}^{\odot} \frac{V_{2n}V_{n2}}{E_2^{(3)} - \in_n^{(3)}} + \sum_{n,n'}{}^{\odot} \frac{V_{2n}V_{nn'}V_{n'2}}{(E_2^{(2)} - \in_n^{(2)})(E_2^{(2)} - \in_{n'}^{(2)})} \\ &+ \quad \sum_{nn'n''}{}^{\odot} \frac{V_{2n}V_{nn'}V_{n'n''}V_{n'2}}{(E_2^{()} - \in_n^{(1)})(E_2^{(1)} - \in_{n'}^{(1)})(E_2^{(1)} - \in_{n''}^{(1)})} \\ &+ \quad \sum_{\substack{nn' \\ n''n'''}}{}^{\odot} \frac{V_{2n}V_{nn'}V_{n'n''}V_{n''n''}V_{n'''2}}{(E_2^{(0)} - \in_n^{(0)})(E_2^{(0)} - \in_{n'}^{(0)})(E_2^{(0)} - \in_{n''}^{(0)})(E_2^{(0)} - \in_{n'''}^{(0)})}\,, \end{aligned} \tag{10}$$

where the subscripts $n, n', \ldots$ denote the general states different from the state $|2>$ defined above, the mark $\odot$ meanning that all indices are different, and the quantities are defined by:

$$E_2^{(0)} = e_2 = \frac{\hbar^2}{2m}(k^2 + k'^2)$$

$$E_n^{(0)} = e_n \equiv \in_n^{(0)}$$

$$E_2^{(1)} = e_2 + V_{22}$$

$$E_n^{(1)} = e_n + V_{nn} \equiv \in_n^{(1)}$$

$$E_2^{(2)} = E_2^{(1)} + \sum_n^{\odot} \frac{V_{2n}V_{n2}}{E_2^{(0)} - \in_n^{(0)}}$$

$$E_n^{(2)} = \in_n^{(2)} + D_n^{(2)}$$

$$\in_n^{(2)} = \in_n^{(1)} + \sum_{n'(\neq a)}^{\odot} \frac{V_{nn'}V_{n'n}}{E_2^{(0)} - \in_{n'}^{(0)}} + \sum_{n'(=a)}^{\odot} \frac{V_{nn'}V_{n'n}}{\in_n^{(0)} - \in_{n'}^{(0)}}$$

$$D_n^{(2)} = \sum_{n'(=a)}^{\odot} \frac{V_{nn'}V_{n'n}}{E_2^{(0)} - \in_{n'}^{(0)}} - \sum_{n'(=a)}^{\odot} \frac{V_{nn'}V_{n'n}}{\in_n^{(0)} - \in_{n'}^{(0)}}$$

$$E_2^{(3)} = E_2^{(1)} + \sum_n^{\odot} \frac{V_{2n}V_{n2}}{E_2^{(1)} - \in_{n'}^{(1)}} + \sum_{nn'}^{\odot} \frac{V_{2n}V_{nn'}V_{n'}}{(E_2^{(0)} - \in_n^{(0)})(E_2^{(0)} - \in_{n'}^{(0)})}$$

$$E_n^{(3)} = \in_n^{(3)} + D_n^{(3)}$$

$$\in_n^{(3)} = \in_n^{(1)} + \sum_{n'(\neq a)}^{\odot} \frac{V_{nn'}V_{n'n}}{E_2^{(1)} - \in_{n'}^{(1)}} + \sum_{n'(=a)}^{\odot} \frac{V_{nn'}V_{n'n}}{\in_n^{(1)} - \in_{n'}^{(1)}} +$$

$$\sum_{\substack{n'n'' \\ (\neq a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(E_2^{(0)} - \in_{n'}^{(0)})(E_2^{(0)} - \in_{n''}^{(0)})} + \sum_{\substack{n'n'' \\ (=a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(\in_n^{(0)} - \in_{n'}^{(0)})(\in_n^{(0)} - \in_{n''}^{(0)})}$$

$$D_n^{(3)} = \sum_{n'(=a)}^{\odot} \frac{V_{nn'}V_{n'n}}{E_2^{(1)} - \in_{n'}^{(1)}} - \sum_{n'(=a)}^{\odot} \frac{V_{nn'}V_{n'n}}{\in_n^{(1)} - \in_{n'}^{(1)}}$$

$$+ \sum_{\substack{n'n'' \\ (=a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(E_2^{(0)} - \in_{n'}^{(0)})(E_2^{(0)} - \in_{n''}^{(0)})} - \sum_{\substack{n'n'' \\ (=a)}}^{\odot} \frac{V_{nn'}V_{n'n''}V_{n''n}}{(\in_n^{(0)} - \in_{n'}^{(0)})(\in_n^{(0)} - \in_{n''}^{(0)})}.$$

The letter $\underline{a}$ denotes the states which lead to a total term with N as a factor. In the expression (10) the sums contain only states which lead to an $N-$dependence of order N or less.

Campbell [6] has explicitly recognized all interaction "ladder state diagrams" up to the fifth order. Summing them up resulted with an interaction energy between excitations:

$$E_{int} = < 1_k 1_{k'} | M | 1_k 1_{k'} >$$

$$= \quad < 1_k 1_{k'} |V| 1_k 1_{k'} > \; + \; \frac{1}{2} \sum_p \; < 1_k 1_{k'} |V| 1_{k+p} 1_{k'-p} > \; \cdot$$

$$\cdot \quad \frac{< 1_{k+p} 1_{k'-p} |M| 1_k 1_{k'} >}{E_2 - E_p^F} \; , \qquad (11)$$

where

$$< 1_{k+p} 1_{k'-p} |M| 1_k 1_{k'} > \; = \; < 1_{k+p} 1_{k'-p} |V| 1_k 1_{k'} >$$

$$+ \; \frac{1}{2} \sum_t{}' \; \frac{< 1_{k+p} 1_{k'-p} |V| 1_{k+t} 1_{k'-t} >< 1_{k+t} 1_{k'-t} |M| 1_k 1_{k'} >}{E_2 - E_t^F} \quad . \qquad (12)$$

This expression has the form of the BGS equation. The difference from the ordinary theory is only in the denominator.

3. Analysis of the interaction energy in the Bogoliubov approximation

Ljolje [5] has shown that the Bogoliubov theory of boson fluids is obtained from the Feenberg perturbation theory under the condition that there are no "mixed" terms in the second order term of the Feenberg expression. This means that in Eq. (11) we keep only the sum over momentum $\vec{p}$ and that all matrix elements must depend on $\vec{k}$, $\vec{k}'$ and $\vec{p}$. Both energies E_2 and E_p^F, which appear in the denominator of Eq. (11), were obtained in the Bogoliubov approximation by Campbell [6], and they read

$$E_2 \; = \; 2 \quad [\quad \sqrt{e_k^2 + 2\rho e_k V_k} \; + \; \sqrt{e_{k'}^2 + 2\rho e_{k'} V_{k'}} \;] \; + \; \sqrt{e_p^2 + 2\rho e_p V_p}$$

$$- \quad [e_k + e_{k'} + e_p + \rho(V_k + V_{k'} + V_p)]. \qquad (13)$$

$$E_p^F \; = \; \frac{3}{2} \; E_2 - \frac{1}{2} a - \alpha_p - \alpha_{k+p} - \alpha_{k'-p} \; - \; \frac{1}{2} \; [(E_2 - a - 2\,\alpha_p)^2 \; - \; 4\gamma_p^2]^{\frac{1}{2}} \; -$$

$$\frac{1}{2} \; [(E_2 - a - 2\,\alpha_{k+p})^2 \; - \; 8\gamma_{k+p}^2]^{\frac{1}{2}} \; - \; \frac{1}{2} \; [(E_2 - a - 2\,\alpha_{k'-p})^2 \; - \; 8\gamma_{k'-p}^2]^{\frac{1}{2}}, \quad (14)$$

where

$$
\begin{aligned}
a &= e_{k+p} + e_{k'-p} + \rho(V_{k+p} + V_{k'-p}), \\
\alpha_p &= e_p + \rho V_p, \\
\gamma_p^2 &= \rho^2 V_p^2.
\end{aligned} \tag{15}
$$

The interaction energy in the Bogoliubov approximation is then

$$
E_{int} = < 1_k 1_{k'} |V| 1_k 1_{k'} > + \frac{1}{2} \sum_p{}' \frac{| < 1_k 1_{k'} |V| 1_{k+p} 1_{k'-p} > |^2}{E_2 - E_p^F}, \tag{16}
$$

Introducing matrix elements, derived in Refs. 5 and 6, into (16), and transforming it in an integral form, one finds

$$
E_{int} = \frac{1}{\Omega} V_{k-k'} + \frac{\Omega}{(2\pi)^3} \frac{1}{2\Omega^2} \int \mathrm{d}\vec{p} \; \frac{(V_p + V_{k-k'+p})^2}{E_2 - E_p^F} \tag{17}
$$

where

$$
\begin{aligned}
E_2 - E_p^F &= \frac{1}{2} [\, f_{k,k'} + 3\,(e_p + \rho\, V_p + f_{k+p,k'-q}\,) - F_{kk'p}] \\
&+ \frac{1}{2} \left\{ [F_{kk'p} - (f_{k,k'} + f_{k+p,k'-p} + 3(e_p + \rho V_p))]^2 - 4\rho^2 V_p^2 \right\}^{\frac{1}{2}} \\
&+ \frac{1}{2} \left\{ [F_{kk'p} - (f_{k,k'} + f_{p,k+p} + 3(e_{k'-p} + \rho V_{k'-p}))]^2 - 8\rho^2 V_{k'-p}^2 \right\}^{\frac{1}{2}} \\
&+ \frac{1}{2} \left\{ [F_{kk'p} - (f_{k,k'} + f_{p,k'-p} + 3(e_{k+p} + \rho V_{k+p}))]^2 - 8\rho^2 V_{k+p}^2 \right\}^{\frac{1}{2}}
\end{aligned} \tag{18}
$$

and

$$
\begin{aligned}
f_{m,n} &= e_m + e_n + \rho(V_m + V_n) \\
F_{l,m,n} &= 2(\eta_l + \eta_m) + \eta_n \\
\eta_t &= \sqrt{e_t^2 + 2\rho e_t V_t} \,.
\end{aligned}
$$

Although the Bogoliubov approximation is not valid for liquid helium, we can analyse expression (17) for a model potential which has a qualitative behaviour of a short repulsion and an attraction at longer range. Thus, in view of the magnitude of the excited momenta $\hbar|\vec{k}|$ and $\hbar|\vec{k}'|$, one can talk about phonon-phonon, maxon-maxon, roton-roton and so on interactions. Using relation (17), the dependence of the interaction on the angle between momenta can be investigated as well.

4. *Numerical calculation and discussion*

The integration in the interaction energy (Eq. (17)) depends on the shape of V_q. For quantum Bose liquids it can be obtained qualitatively by the Feynman formula

$$V_q = \frac{e_q^2}{2\rho}(S_q^{-2} - 1),$$

where S_q is the liquid structure factor. For liquid helium, it becomes equal one beyond a finite value of q. Thus we may suppose that the model interaction in the k-space is zero beyond a "distance" g. Regarding the form of the numerator in the integrand, it is useful to split the integration in to two parts: first with the numerator V_p^2 and second with the numerator $(V_{k-k'+p}^2 + 2\ V_p V_{k-k'+p})$. As $\vec{k},\ \vec{k}'$ are the parameters they can be placed in the yz plane. The first integration can be performed simply by placing k' in the z axis of the spherical coordinates and taking p as the spherical distance. The second integration is more complicated. For this case all vectors and angles are shown in Fig. 1. For each set of the parameters k, k' and Θ there is a vector $\vec{a} = \vec{k} - \vec{k}'$. The following relations are used:

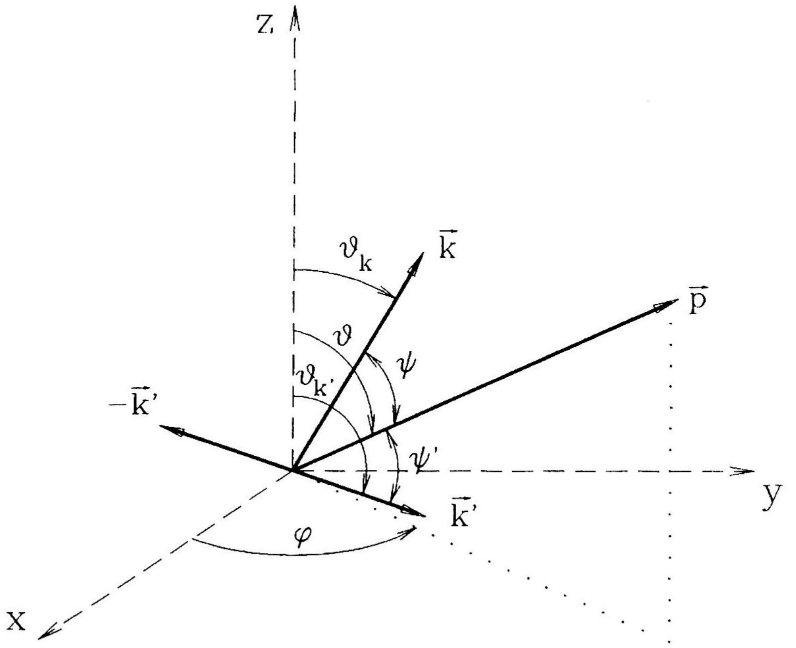

Fig. 1. Graphical representation of the coordinates and parameters used in the numerical integration.

$$\left.\begin{array}{rcl} |\vec{k}+\vec{p}| &=& \{k^2+p^2+2kp\ cos\psi\}^{1/2} \\ |\vec{k}'-\vec{p}| &=& \{k'^2+p^2-2k'p\ cos\psi'\}^{1/2} \\ |\vec{k}-\vec{k}'+\vec{p}| &=& \{a^2+p^2+2ap\ cos\vartheta\}^{1/2} \\ |\vec{k}-\vec{k}'| &=& \{k^2+k'^2-2kk'\ cos\Theta\}^{1/2} \ = \ a \\ \cos\psi &=& cos\vartheta\ cos\vartheta_k + sin\vartheta\ sin\vartheta_k\ sin\varphi \\ cos\psi' &=& cos\vartheta\ cos\vartheta_{k'} + sin\vartheta\ sin\vartheta_{k'}\ sin\varphi \\ cos\vartheta_k &=& \frac{k-k'\ cos\Theta}{a} \\ \vartheta_{k'} &=& \vartheta_k\ +\ \Theta. \end{array}\right\} \tag{19}$$

It is convenient to introduce the dimensionless quantities, whose order is defined with respect to the ^{4}He parameters: $m = m_0 \times 10^{-27}$ kg, $\rho = \rho_0 \times 10^{28}$ m^{-3}, $(m/\hbar^2) = m_0 C_h \times 10^{41}$ kgJ^{-2}s^{-2}, $C_h = 0.89915$, $V_q = V_q^0 \times 10^{-49}$ Jm^3, $p = p_0 \times 10^{10}$ m^{-1}, and all other wave vectors in the same way. In the dimensionless units, the expression (17) becomes (the index zero being droped in all wave vectors!)

$$N\cdot E_{int}10^{21} \ = \ \rho_0\cdot V^0_{k-k'} \ + \ \frac{12.5\ C_1}{\pi^3}\{\int\limits_0^g \mathrm{d}p\ p^2 \int\limits_0^\pi \mathrm{d}\vartheta \sin\vartheta \int\limits_0^{2\pi} \mathrm{d}\varphi\ \frac{V_p^{02}}{D_1^0} \ + \ I_2^0\}, \tag{20}$$

where

$$I_2^0 \ = \{\int\limits_0^{g-a} \mathrm{d}p\ p^2 \int\limits_0^\pi \mathrm{d}\vartheta\sin\vartheta \int\limits_0^{2\pi}\mathrm{d}\varphi \ + \ \int\limits_{g-a}^{g+a} \mathrm{d}p\ p^2 \int\limits_{\vartheta_1}^\pi \mathrm{d}\vartheta\sin\vartheta \int\limits_0^{2\pi}\mathrm{d}\varphi\ \} \frac{2V_p^0V^0_{k-k'+p} \ + \ V^{02}_{k-k'+p}}{D_2^0}, \qquad for\ a\le g \tag{21}$$

and

$$I_2^0 \ = \ \int\limits_{a-g}^{a+g} \mathrm{d}p\ p^2 \int\limits_{\vartheta_1}^\pi \mathrm{d}\vartheta\sin\vartheta \int\limits_0^{2\pi}\mathrm{d}\varphi\ \frac{2V_p^0V^0_{k-k'+p} \ + \ V^{02}_{k-k'+p}}{D_2^0}, \qquad for\ a\ge g. \tag{22}$$

where
$C_1 = 2\rho_0 m_0 C_h, \quad \vartheta_1 = \arccos\frac{g^2-p^2-a^2}{2ap}$

$$\begin{array}{rcl} D_1^0 &=& [\ f^0_{k,k'} \ + \ 3\ (p^2 \ + C_1V_p^0 \ + \ f^0_{k+p,k'-q}\ ) \ - \ F^0_{kk'p}] \\ &+& \left\{\left[F^0_{kk'p} - (f^0_{k,k'} + f^0_{k+p,k'-p} + 3(p^2+C_1V_p^0))\right]^2 - 4(C_1V_p^0)^2\right\}^{\frac{1}{2}} \\ &+& \left\{\left[F^0_{kk'p} - (f^0_{k,k'} + f^0_{p,k+p} + 3(|\vec{k}'-\vec{p}|^2 + C_1V^0_{k'-p}))\right]^2 - 8(C_1V^0_{k'-p})^2\right\}^{\frac{1}{2}} \end{array}$$

$$+ \quad \left\{ \left[F^0_{kk'p} - (f^0_{k,k'} + f_{p,k'-p} + 3(|\vec{k}+\vec{p}|^2 + V^0_{k+p})) \right]^2 - 8(C_1 V_{k+p})^2 \right\}^{\frac{1}{2}} \quad (23)$$

and

$$\begin{aligned} f^0_{m,n} &= m^2 + n^2 + C_1(V^0_m + V^0_n), \\ F^0_{l,m,n} &= 2(\eta^0_l + \eta^0_m) + \eta^0_n, \\ \eta^0_t &= \sqrt{t^2 + 2C_1 V^0_t} \; . \end{aligned}$$

$$\begin{aligned} |\vec{k}+\vec{p}| &= \sqrt{k^2 + p^2 + 2kp\cos\psi}, \\ |\vec{k}'-\vec{p}| &= \sqrt{k'^2 + p^2 - 2k'p\cos\vartheta}, \\ \cos\psi &= \cos\vartheta\cos\Theta + \sin\vartheta\sin\Theta\sin\varphi \; . \end{aligned}$$

The quantity D^0_2 has the same form as D^0_1, but expressions taken from (19) should be used.

A model interaction which has a simple form and the desired qualitative behaviour was chosen. Let the potential in the momentum space read

$$V_q = \begin{cases} V \quad (-q^2 + g^2) \;, \; V > 0 & q \le g \\ 0, & q > g \end{cases}$$

and in r–space

$$V(r) = \frac{1}{\pi^2} \, V g^3 \, \frac{1}{r^2} \, j_2(gr) \; ,$$

where V and g are constants and j_2 is the spherical Bessel function. The parameters g, V, density ρ, atomic mass and the "roton" wave vector are related to the corresponding helium parameters. To obtain g and V, the Bruch–McGee potential for helium atoms [7] was used. After fitting, one finds $V = 2.87 \times 10^{-51}$ Jm^{-5} and $g = 2.39 \times 10^{10}$ m^{-1}.

Regarding the Ohbayashi results, the interaction between two excitations with antiparalel momenta was considered; it means with the values of k and k' near the helium roton wave vector ($k \approx 2 \times 10^8$ cm).

In order to integrate the relation (20), the Gauss quadrature was used. Contrary to the anticipated results [6], it was found that the interaction energy is positive for the value of the density lower one milion times than for liquid ^{4}He density at svp. Furthermore, for higher densities the interaction energy is imaginary. Thus one may conclude that Bogoliubov approximation is not appropriate to describe even qualitatively the interaction between elementary excitations. Having in mind the fact that the Bogoliubov model incorporates superfluidity, this is in a way an unexpected result.

Furthermore, if the denominator in Eq. (17) is approximated up to the first order in density (one term more beyond the Rayleigh-Schrödinger term), one finds the results derived by Campbell [6] which are qualitatively different from the results presented in this paper. Since in the Bogoliubov approximation an infinite number of terms is collected, it shows that the Bogolibov approximation does not have the low density limit previously established [6].

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INTERAKCIJA MEĐU ELEMENTARNIM POBUĐENJIMA BOZONSKOG FLUIDA U BOGOLJUBOVLJEVOJ APROKSIMACIJI

Pokazano je da u Bogoljubovljevoj aproksimaciji interakcija elementarnih pobuđenja u Bozonskim fluidima ne može biti opisana čak ni kvalitativno. U računu je korišten rearanžirani Feenbergov račun smetnje. Ovaj je rezultat neočekivan glede činjenice da opis pojedinačnih elementarnih pobuđenja u modelu "slabo međudjelujućeg bozonskog plina", kvalitativno sadrži suprafluidni spektar.