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CHARGE SIGNATURE ON IONIZATION–EXCITATION OF He BY FAST PROJECTILES

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An analytical expression for the differential cross–section for ionization–excitation of He to He^+(np) states by high energy charged particles is formulated. The ratio of the differential cross–section by a negative projectile and of the differential cross–section by an equivelocity positive projectile is found to be greater than one when the energy of the ejected electron is less than half the ground state energy of He atom.

1. *Introduction*

Many experimental and theoretical results on total ionization–excitation (IE) of He by charged particles are known, whereas differential cross–sections (DCS) with respect to the energy and/or solid angle of the final particles have very seldom been reported. The present paper is devoted to the comparative study of the DCS with respect to the energy of the ejected electron for the IE of He by positive and negative projectiles using a field theoretical technique.

The total IE of He by charged particles and bare ions were experimentally investigated by Fuelling et al. [1] and Pedersen and Folkmann [2]. A relation was

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found between the total double ionization (DI) and total IE of He. In both DI and EI, the total cross–section by negatively charged projectiles was found to be a factor of two larger than that by an equivelocity positively charged projectile [3]. Several theories [4-6] are known to explain successfully the DI by charged particles, but there is no quantum calculation for IE as yet. McGuire [7] and Ford and Reading [8] have suggested that IE, being a two–electron process like DI, can be treated by the same model calculations as were made for DI. Before taking up the present problem, let us discuss some of these models for DI.

According to McGuire [4], the interaction mechanisms responsible for DI are the shake–off (SO) process and the two step (TS) process. Electron–electron correlation is necessary for non–zero contribution from SO. The TS process, on the other hand, arises out of (i) dynamic correlation between the ejected electron and the bound electron and is termed as two-step-one (TS1) process, and (ii) double collision between the projectile and the two bound electrons in He and is termed as two-step-two (TS2) process. The amplitudes from the SO and TS1 processes are proportional to the projectile charge Z_p, and the amplitude due to the TS2 process is proportional to Z_p^2. The interference term of the scattering amplitudes proportional to Z_p^3 is responsible for the difference of the cross-section for DI between the negative and the positive projectiles.

However, Reading and Ford [5] forward an argument that dispute the McGuire's model for DI since the shake–off and the two–step processes cannot interfere because of the different final states of the outgoing electrons. Using forced impulse method the DI cross–section was expressed by them in a Born series expansion in the charge of the projectile. The diffference of the cross–section due to the negative and positive projectiles occurs because of Z_p^3 term in the series that arises due to the non–dipole nature of the interaction between the projectile and the target electrons.

The classical trajectory Monte Carlo calculation of Olson [6], and Vegh's polarization model [9] are other two existing models to compute DI.

In the present paper we are guided by the McGuire's model to compute IE, but use the QED technique to calculate the amplitudes. The technique was earlier applied to study the phenomena of charge transfer, transfer ionization [10], single and double capture [11,12] and various other processes [13]. In most of the cases the results are in fairly good agreement with experiments. There is no conclusive theory yet on IE of a two–electron target. Eventually, we address the question of IE from the field theoretical point of view.

The ratio of the differential cross–sections, thus obtained, for IE of He to He^+(np) states by negative projectile to that by equivelocity positive projectile is found to be greater than one, provided the energy of the ejected electron is less than $I_2/2$. I_2 (=79 eV) is the threshold for double ionization of He ($1s^2$). With the increase of the ejected electron energy above $I_2/2$, the ratio becomes less then one. Further, the ratio is found to be independent of the projectile energy between 0.5 to 2.5 MeV/amu.

2. *Theoretical formalism*

In QED, composite system of bound particles are represented by a string of field operators operating on particle vacuum and multiplied by unperturbed solution of the Schrödinger equation [14]. The creation and the annihilation operators of the particles, bound to the Coulomb field of the nucleus, obey equal time free field commutation relations. There will be no loss of generality to represent the bound electrons by the Feynman directed lines. Feynman diagrams of appropriate order are computed to study IE by antiproton and proton.

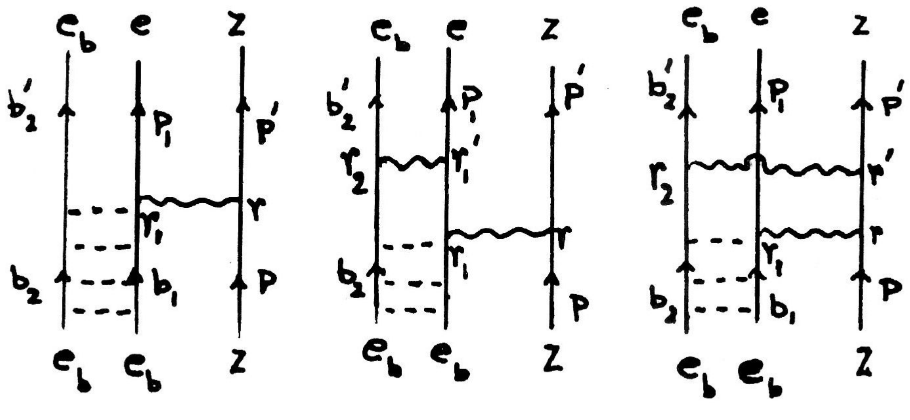

Fig. 1. Feynman diagram for ionization excitation of He *by projectiles of charge* Z*.* e_b *and* e *represent bound and free electrons, respectively.*
(a) Shake-up mechanism.
(b) Two-step-one mechanism.
(c) Two-step-two mechanism.

We use the terminologies of McGuire and draw the Feynman diagrams corresponding to the shake–up (SU) process (Fig. 1a) and the two–step processes TS1 (Fig. 1b) and TS2 (Fig. 1c). The high energy charged projectile interacts with the electron cloud of the He atom ejecting one of the electrons and leaving behind He^+ in an excited state. In SU process, ejection of electron occurs due to the current–current interaction between the projectile and one of the bound electrons, while the other electron is excited due to the change of the Coulomb field. The correlated wave function [15] of the helium atom takes care of the Coulomb effects. In TS1 process (Fig. 1b) virtual photon exchange between the projectile and one of the bound electrons causes ejection of that electron. The resulting free electron in turn exchanges a photon with the other bound electron, raising it to an excited state. In TS2 process the projectile exchanges two virtual photons with the target electrons causing simultaneous ejection and excitation (Fig. 1c). Similarity and differences that may arise between McGuire's approach and the present approach are discussed as follows.

According to McGuire, the amplitudes from TS1 and TS2 terms of IE are expected to behave, essentially, as the products of single ionization and excitation amplitudes, whereas the SU amplitude is expected to behave as the product of one single ionization amplitude and a term which depends on the electron–electron correlation.

The SU amplitude in QED approach is obtained from the second order S–matrix and is represented by second–order Feynman diagram (Fig. 1a). With one photon exchange, the SU term can be identified with the first order Born amplitude as given by McGuire. The TS1 and TS2 amplitudes, however, are obtained from fourth–order S–matrices, and are represented by two fourth–order Feynman diagrams, Fig. 1b and Fig. 1c, respectively. Although these amplitudes can be identified with the second–order Born terms in matter–radiation field coupling, yet, unlike McGuire's TS amplitudes, they cannot be taken as the products of S–matrices for single ionization and excitation. Both the TS1 and TS2 amplitudes contain two photon propagators and one particle propagator. The particle propagator in the TS1 process is an electron propagator necessary for dynamic correlation between the outgoing electron and the bound electron which is excited. In TS2 process the particle propagator is the projectile propagator, essential for double collision causing simultaneous ionization and excitation. Because of the anticommutative and covariant nature of the algebra, contributions from the TS1 and TS2 terms and the term due to interference between SU and TS will be at variance with that obtained from usual quantum mechanical method.

2.1. Amplitude for IE

The reaction under consideration is

$$\mathrm{He} + \mathrm{p}^{+}(\mathrm{p}^{-}) \rightarrow \mathrm{He}^{+}(\mathrm{np}) + \mathrm{e}^{-} + \mathrm{p}^{+}(\mathrm{p}^{-}) \tag{1}$$

where the positive projectile p^{+} (negative projectile p^{-}) collides with He atom with velocity much greater than the electron orbital velocity. Collision of the fast projectile with He atom results in the excitation of one of the electron in a higher orbit and the ejection of the other. In the present paper we calculate the differential cross–sections for ionization–excitation of He to $\mathrm{He}^{+}(\mathrm{np})$ states with respect to the energies of the emitted electrons integrated over all angles. The amplitude for the reaction is the combined sum of the amplitudes due to the SU mechanism, the TS1 and the TS2 mechanisms. To write the S–matrix in a covariant way we define the following terms.

2.2. Definitions

Let $p(p_0, \vec{p})$ and $p^{'}(p_0^{'}, \vec{p}^{\,'})$ be the four–momenta of the projectile before and after the interaction, respectively, $b_1(b_{10}, 0)$ and $b_2(b_{20}, 0)$ the four–momenta of the two target electrons and $p_1(p_{10}, \vec{p}_1)$ and $b_2(b^{'}_{20}, 0)$ the four–momenta of the ionized electron and the excited electron, respectively. The relativistic energies are

$$E^R_p = p_0 = E + M$$

$$E^R_{p'} = p^{'}_0 = E^{'} + M$$

$$E^R_{p_1} = p_{10} = E_1 + m$$

where E, $E^{'}$ and E_1 are the respective kinetic energies and M and m are the projectile and electron masses, respectively.

Let $U(p)$ and a_p be the Dirac spinor and the annihilation operator, respectively, for the charged projectile. Corresponding four–current at r in Fig. 1a is

$$[J_\nu(p, p^{'})]_r = \sqrt{\frac{M^2}{E^R_p E^R_{p'}}} (2\pi)^{-3} \mathrm{e}^{\mathrm{i}(p^{'}-p)r} \sum_\lambda \left[\overline{U}_\lambda(p^{'}) \gamma_\nu U_\lambda(p) \right] a^+_{p'} a_p. \qquad (2)$$

Let c_p and $u(p_1)$ be the annihilation operator and Dirac spinor, respectively, for an electron with momentum $p_1(p_{10}, \vec{p}_1)$. The Dirac spinor for bound electron [14] is given by

$$V(b_i, r_i) = \left[1 - \frac{1}{2} \alpha \zeta(\gamma, \vec{n}) \right] (2\pi)^{-3/2} u(b_i) \mathrm{e}^{-\mathrm{i} b_i r_i} \qquad (3)$$

where $\vec{n} = \vec{r}_i/|r_i|$, $\alpha = 1/137$, $\zeta =$ target nuclear charge, and $u(b_i)$ is the Dirac spinor for a free electron with momentum b_i.

The four–current at r_1 with the initial bound–electron line and the final free electron line (Fig. 1a) [16] is given by

$$[L_\mu(p_1, b_1)]_{r_1} = \sqrt{\frac{m}{E^R_{p_1}}} (2\pi)^{-3/2} \mathrm{e}^{\mathrm{i} p_1 r_1} \sum_\lambda \left[\overline{u}_\lambda(p_1) \gamma_\mu V_\lambda(b_1, r_1) \right] c^+_{p_1} c_{b_1}. \qquad (4)$$

The virtual photon propagator between the projectile and one of the target electrons (Fig. 1a) is given by

$$D(r - r_1) = (2\pi)^{-4} \int \frac{\mathrm{e}^{\mathrm{i} q_1 (r_1 - r)}}{q_1^2 + \mathrm{i}\epsilon} \mathrm{d}^4 q_1. \qquad (5)$$

The electron propagator in the Feynman diagram (Fig. 1b) for the TS1 mechanism is

$$S_e(r_1^{'}-r_1)=(2\pi)^{-4}\int\frac{\mathrm{e}^{\mathrm{i}s_e(r_1^{'}-r_1)}}{s_e\gamma-m+\mathrm{i}\epsilon}\mathrm{d}^4s_e. \tag{6}$$

The projectile propagator in the TS2 mechanism (Fig. 1c) is given by

$$S_z(r^{'}-r)=(2\pi)^{-4}\int\frac{\mathrm{e}^{\mathrm{i}s_z(r^{'}-r)}}{s_z\gamma-M+\mathrm{i}\epsilon}\mathrm{d}^4s_z, \tag{7}$$

where m and M are the masses of electron and the projectile, respectively.

2.3. *Interaction terms*

From the second–order Feynman diagram (Fig. 1a) the current–current interaction for SU mechanism is given by

$$S_{SU}=Ze^2\int[J_\nu(p^{'},p)]_rD(r-r_1)[L_\mu(p_1,b_1)]_{r_1}\mathrm{d}^4r\ \mathrm{d}^4r_1. \tag{8}$$

The fourth–order Feynman diagram corresponding to the TS1 mechanism (Fig. 1b) contains two propagator, $D(r-r_1)$ and $D(r_2-r_1^{'})$, and one electron propagator $S_e(r_1^{'}-r_1)$. The interaction term for the TS1 mechanism is described by

$$S_{\mathrm{TS1}}=Ze^4\int[J_\mu(p^{'},p)]_r\sqrt{\frac{m}{E^R_{p_1}}}(2\pi)^{-3/2}c^+_{p_1}\mathrm{e}^{\mathrm{i}p_1r_1^{'}}$$

$$\overline{u}(p_1)\gamma_\mu S_e(r_1^{'}-r_1)\gamma_\nu c_{b_1}V(b_1,r_1)[L_\nu(b_2^{'},b_2)]_{r_2}D(r_2-r_1^{'})D(r-r_1)\mathrm{d}^4r_1\mathrm{d}^4r_2\mathrm{d}^4r_1^{'}\mathrm{d}^4r. \tag{9}$$

The fourth–order Feynman diagram for TS2 mechanism (Fig. 1c) contains the projectile propagator $S_Z(r^{'}-r)$ and two photon propagators, $D(r-r_1)$ and $D(r^{'}-r_1)$. The interaction term for the TS2 mechanism is given by

$$S_{\mathrm{TS2}}=Z^2e^4\int\sqrt{\frac{M^2}{E^R_{p^{'}}E^R_p}}(2\pi)^{-3/2}a^+_{p^{'}}\mathrm{e}^{\mathrm{i}p^{'}r^{'}}$$

$$\overline{U}(p^{'})\gamma_\mu S_Z(r'-r)\gamma_\nu a_pU(p)\mathrm{e}^{-\mathrm{i}pr}[L_\nu(p_1,b_1)]_{r_1}[L_\mu(b_2^{'},b_2)]_{r_2}$$

$$D(r-r_1)D(r^{'}-r_2)\mathrm{d}^4r_1\mathrm{d}^4r_2\mathrm{d}^4r^{'}\mathrm{d}^4r. \tag{10}$$

2.4. State vectors of the interacting system

Let $\vec{r}_1$, $\vec{r}_2$ be the space coordinates of the two bound electrons and $\vec{R}$ the coordinate of the centre–of–mass (CM) of the He atom, from an arbitrary origin. In CM and relative coordinates, the unperturbed solution of the Schrödinger equation for the He atom is

$$\Phi_i(X,Y,R) = (2\pi)^{-3/2} \mathrm{e}^{\mathrm{i}\vec{R}\vec{L}} \phi_{1s^2}(X,Y), \tag{11}$$

where

$$\vec{X} = \vec{r}_1 - \vec{R}$$

$$\vec{Y} = \vec{r}_2 - \vec{R} \tag{12}$$

and $\vec{L}$ is the momentum of the CM. $\phi_{1s^2}(X,Y)$ is the correlated wave function [15] of the He atom in the ground state.

For fast collision with high energy projectile, the momentum transfer is usually small and the projectile trajectory hardly deviates from the projectile beam direction. The IE is dominated by low energy ejected electrons. The low energy electrons are represented by Coulomb distorted plane wave.

After interaction, the final wave function for $\mathrm{He}^+(n,l)$ and the ionized electron is described by

$$\Phi_f(X,Y,R) = (2\pi)^{-3/2} \mathrm{e}^{\mathrm{i}\vec{R}\vec{L}'} F_c(X) \phi_{nl}(Y). \tag{13}$$

$F_c(X)$ is the Coulomb distorted plane wave for the ionized electron, $\phi_{nl}(Y)$ is the wave function of the bound electron in the $\mathrm{He}^+(nl)$ state and L' is the CM momentum.

In QED, the state vector for a system of interacting particles is represented by a string of field operators operating on particle vacuum and multiplied by the solution of the Schrödinger equation. Since $\Phi_i(X,Y,R)$ and $\Phi_f(X,Y,R)$ represent the solutions in the initial state and in the final state, respectively, the state vectors of the respective system become

$$|\Psi_i >= (2\pi)^{-1/2} \mathrm{e}^{-\mathrm{i}R_0 L_0} \Phi_i(x,y,R) c^+_{b_1} c^+_{b_2} a^+_p |0> \tag{14}$$

$$|\Psi_f >= (2\pi)^{-1/2} \mathrm{e}^{-\mathrm{i}R_0 L'_0} \Phi_f(x,y,R) c^+_{p_1} c^+_{b'_2} a^+_{p'} |0> . \tag{15}$$

L_0 and L'_0 represent the energy components of the four–momenta $L(L_0, \vec{L})$ and $L'(L'_0, \vec{L}')$, respectively. R_0 is the time component of the four–coordinate $R(R_0, \vec{R})$ of the CM.

2.5. The S–matrix elements

The S–matrix elements for the different interaction mechanisms are computed between the initial (14) and the final (15) states as follows.

(a) *Shake–up (SU) mechanism*

The S–matrix element for shake–up mechanism is obtained using Eqs. (8), (14) and (15)

$$M^Z_{\rm SU} =< \Psi_f|S_{\rm SU}|\Psi_i > . \tag{16}$$

After some lengthy computation we get

$$M^Z_{\rm SU} = \underline{C} Ze^2 I\delta^4(p_i - p_f)K_1 \tag{17}$$

where $\underline{C}$ is a constant and

$$I = \int \phi^{'*}(x, y)\phi_{1s^2}(x, y){\rm e}^{{\rm i}\vec{x}(\vec{p}_1+\vec{p}^{\,'}-\vec{p})}{\rm d}^3x{\rm d}^3y \tag{18}$$

$$K_1 = \frac{T_3T_1}{(p^{'} - p)^2} \tag{19}$$

$$T_1 = \overline{u}(p_1)\gamma_\mu u(b_1) \tag{20}$$

$$T_3 = \overline{U}(p^{'})\gamma_\mu U(p). \tag{21}$$

(b) *Two-step-one (TS1) mechanism*

The amplitude $M^Z_{\rm TS1}$ for the TS mechanism is obtained using Eq. (9) and taking the matrix element between Ψ_i and Ψ_f:

$$M^Z_{\rm TS1} =< \Psi_f|S_{\rm TS1}|\Psi_i >= \underline{C} Ze^4 I\delta^4(p_i - p_f)K_2, \tag{22}$$

where p_i and p_f are the initial and the final four–momenta of the interacting particles, respectively, and

$$K_2 = \frac{2\pi T_3T^{'}_1T_2}{(p^{'} - p)^2(b^{'}_2 - b_2)^2(d^2 - m^2)} \tag{23}$$

$$d = p - p^{'} + b_1$$

$$T_1^{'} = \overline{u}(p_1)\gamma_\mu(\gamma d + m)\gamma_\nu u(b_1) \tag{24}$$

$$T_2 = \overline{u}(b_2^{'})\gamma_\mu u(b_2). \tag{25}$$

(c) *Two-step-two (TS2) mechanism*

The amplitude M_{TS2}^Z for TS2 mechanism is obtained by taking the matrix element of S_{TS2} in Eq. (10) between Ψ_i and Ψ_f:

$$M_{\mathrm{TS2}}^Z = < \Psi_f|S_{\mathrm{TS2}}|\Psi_i > = \underline{C} Z^2 e^4 I \delta(E_i - E_f)\delta^3(\vec{p}_i - \vec{p}_f)K_3 \tag{26}$$

where

$$K_3 = \frac{2\pi T_1 T_3^{'} T_2}{(p_1 - b_1)^2 (b_2^{'} - b_2)^2 [(p - p_1 + b_1)^2 - M^2]} \tag{27}$$

$$T_3^{'} = \overline{U}(p^{'})\gamma_\mu(\gamma d^{'} + M)\gamma_\nu U(p) \tag{28}$$

and $d^{'} = p - p_1 + b_1$.

3. Cross–section

The amplitude for IE, M_{fi}^Z, by a projectile of charge Z, is the sum of the amplitudes due to SU, TS1 and TS2 mechanisms

$$M_{fi}^Z = M_{SU}^Z + M_{TS1}^Z + M_{TS2}^Z = \underline{C}\ \delta^4(p_i - p_f) I W^Z$$

where

$$W^Z = e^2 Z K_1 + e^4 Z K_2 + e^4 Z^2 K_3. \tag{29}$$

The cross–section is given by

$$\sigma^Z = \frac{M}{|\vec{p}|}\int |M_{fi}^Z|^2 \frac{\mathrm{d}^3 p^{'} \mathrm{d}^3 p_1}{(2\pi)^6}.$$

Integrating $\mathrm{d}^3 p^{'}$ over momentum δ–function one obtains

$$\sigma^Z = \frac{\underline{C}^2\ I^2 m M}{(2\pi)^6}\frac{2\pi}{|\vec{p}\,|}\int |\vec{p}_1\ |\ |W^Z|^2 \sin\vartheta \mathrm{d}\theta \mathrm{d}E_1. \tag{30}$$

ϑ is the angle between $\vec{p}$ and $\vec{p}_1$. $\sin\vartheta$ can be expressed in terms of E, E_1 and M. On using the following momentum and energy conservation relations

$$\vec{p}^{\,'} + \vec{p}_1 = \vec{p}$$

and

$$E - E^{'} = E_1 + \nabla_{nl}$$

where

$$\nabla_{nl} = \epsilon_{1s^2} - \epsilon_{nl} \tag{31}$$

ϵ_{1s^2} and ϵ_{nl} are the electron orbital energies of He$(1s^2)$ and He$^+(nl)$, respectively, one obtains

$$\cos\vartheta = \frac{\vec{p}_1^{\;2} + \vec{p}^{\;2} - \vec{p}^{\;'2}}{2|\vec{p}_1||\vec{p}|} = \frac{mE_1 + M(E_1 + \nabla_{nl})}{\sqrt{mMEE_1}} = f(E, E_1, M).$$

$$\sin\vartheta = \sqrt{1 - f^2(E, E_1, M)} = F(E, E_1, M). \tag{32}$$

The DCS for ionization–excitation of He with recpect to the energy E_1 of the emitted electron finally becomes

$$\frac{\mathrm{d}\sigma^Z}{\mathrm{d}E_1} = \frac{2\pi^2 \underline{C}^2 \; I^2 mM}{(2\pi)^6} \frac{|\vec{p}_1|}{|\vec{p}\,|} |W^Z|^2 F(E, E_1, M). \tag{33}$$

3.1. Ratio R of the DCS by the negative and that by the equivelocity positive projectiles of equal masses

Let W^- and W^+ are the values of W^Z for the negative and the positive projectiles. From Eq. (33) the ratio for IE to nl states by antiproton (p$^-$) and proton (p$^+$) is

$$R_{nl} = \frac{|W^-|^2}{|W^+|^2} = \frac{|K_1 + e^2K_2 - e^2K_3|^2}{|K_1 + e^2K_2 + e^2K_3|^2}. \tag{34}$$

Neglecting the terms containing e^4, we get

$$|W^{\mp}| = |K_1 + e^2K_2 \mp e^2K_3|^2 = K_1K_1^* + e^2(K_1^*K_2 + K_1K_2^*) \mp e^2(K_1^*K_3 + K_1K_3^*). \tag{35}$$

Substituting for K_1 from Eq. (19) one obtains

$$K_1^* K_1 = (T_3 T_1)^* (T_3 T_1) X_1^{-4} = (T_3^* T_3)(T_1^* T_1) X_1^{-4} \tag{36}$$

where

$$X_1^2 = (p - p^{'})^2 = (p_0 - p_0^{'})^2 - (\vec{p} - \vec{p}^{\,'})^2$$

$$= (E - E^{'})^2 - \vec{p}_1^{\,2} = (E_1 + \nabla_{nl})^2 - 2mE_1 \tag{37}$$

from Eq. (31). Since the projectile energy is much less than the projectile rest mass, the trace part of the S–matrices gives a mass–dependent factor such that

$$(T_3^* T_3)(T_1^* T_1) = \frac{1}{2}\left(1 + \frac{m}{M}\right). \tag{38}$$

The contributions from the interference term in $|W^{\mp}|^2$ are $(K_1^* K_2 + K_1 K_2^*)$ and $(K_1^* K_3 + K_1 K_3^*)$.

Substituting from Eqs. (19) and (23)

$$K_1^* K_2 + K_1 K_2^* = 2\pi (X_1^4 X_2^2 y_1)^{-1} \Big[(T_3 T_1)^* T_3 T_1^{'} T_2 + \text{Complex conjugate term} \Big] \tag{39}$$

where

$$X_2^2 = (b_2^{'} - b_2)^2 = (\epsilon_{1s} - \epsilon_{nl})^2 \tag{40}$$

$$y_1 = (d^2 - m^2) = [(p - p^{'} + b_1)^2 - m^2]^2 =$$

$$= 2m\nabla_{nl} - 2\epsilon_{1s}(E_1 + \nabla_{nl}) + (E_1 + \nabla_{nl})^2. \tag{40a}$$

Using Eqs. (20), (21), (24) and (25), after a lengthy computation, one obtains the dominant contribution from the trace part in Eq. (39)

$$(T_3 T_1)^* (T_3 T_1^{'} T_2) + (T_3 T_1)(T_3 T_1^{'} T_2)^* = 2|T_3^* T_3||T_1^* T_1^{'}||T_2| = 3m. \tag{41}$$

In deriving Eq. (41) we have used the Gordon decomposition of current [17]

$$T_2 = \overline{u}(b_2^{'}) \gamma_\mu u(b_2) =$$

$$= \overline{u}(b_2^{'}) \frac{(b_2^{'} - b_2)^\mu}{2m} u(b_2) + \overline{u}(b_2^{'}) \sigma_{\mu\nu} \frac{b_2^{'} - b_2}{2m} u(b_2) \tag{42}$$

where $\sigma_{\mu\nu} = (1/2)\{\gamma_\mu, \gamma_\nu\}$.

The remaining interference term in Eq. (35) is computed in a similar way

$$K_1^* K_3 + K_1 K_3^* = 2\pi (X_1^2 X_2^2 X_3^2 Y_2)^{-1} \left[(T_3 T_1)^* (T_3^{'} T_1 T_2) + (T_3 T_1)(T_3^{'} T_1 T_2)^* \right]. \quad (43)$$

X_1 and X_2 are given by Eqs. (37) and (40), respectively, and

$$X_3^2 = (p_1 - b_1)^2 = 2mE_1 \left(\frac{\epsilon_{1s}}{E_1} - 1 \right) - 2E_1 \epsilon_{1s} \quad (44)$$

$$Y_2 = (d_2^{'} - M^2) = (p - p_1 + b_1)^2 - M^2 = 2MF_{nl} \quad (45)$$

where

$$F_{nl} = (\nabla_{nl} - \epsilon_{1s}) - \frac{E}{M} \left(1 - \frac{E_1 + \epsilon_{1s}}{2E} \right) (E_1 + \epsilon_{1s}). \quad (46)$$

The trace part is

$$2|T_3^* T_1^{'}||T_1^* T_1||T_2| = \frac{3}{2}(M + E). \quad (47)$$

For the projectile energy range 0.5 to 2.5 MeV/amu, we obtain an analytical expression for $|W^{\mp}|^2$. Neglecting terms like E/M, E_1/m and ϵ_{1s}/m, one obtains

$$|W^{\mp}|^2 = (8E_1^2 m^2 X_2^2)^{-1} \left[X_2^2 + 12\pi \frac{e^2}{\nabla_{nl}} \pm 6\pi e^2 \frac{1}{(\epsilon_{1s}/E_1 - 1)F_{nl}} \right] \quad (48)$$

where $2\epsilon_{1s} = \epsilon_{1s^2} = 79$ eV.

Finally, the ratio R of the differential cross–sections by equivelocity negative and positive projectiles, using Eqs. (34) and (48), becomes

$$R_{nl} = \frac{G_{nl}^-}{G_{nl}^+} \quad (49)$$

where

$$G_{nl}^{\mp} = X_2^2 + \frac{12\pi e^2}{\nabla_{nl}} \pm 6\pi e^2 \frac{1}{(\epsilon_{1s}/E_1 - 1)F_{nl}}. \quad (50)$$

4. Results and disscusions

In the ionization–excitation collisions under consideration the momentum and energy transfers are usually very small. Hence energy E_1 of the ejected electron should also be small. From Eq. (48) the ratio is found to be very sensitive to the value of E_1 relative to ϵ_{1s}, $R > 1$ for $E_1 < \epsilon_{1s}$ and $R < 1$ for $E_1 > \epsilon_{1s}$. Dependence of R on projectile energy enters through F_{nl}. However, from Eq. (46), for $E \ll M$ we find

$$F_{nl} = \nabla_{nl} - \epsilon_{1s}.$$

For the projectile energy range between 0.5 to 2.5 MeV/amu, at which experiments [1,2] were conducted, the ratio becomes independent of the projectile energy and projectile mass. Knowing that

$$\epsilon_{1s} = 39.5 \text{ eV}, \quad \nabla_{2p} = 65.4 \text{ eV}, \quad \nabla_{3p} = 72.96 \text{ eV}$$

and $e^2/4\pi = 1/137$, we get for IE to the $2p$ and $3p$ states the ratios R_{2p} and R_{3p}, where,

$$R_{2p} = \frac{2.344 + 1.997/t}{2.344 - 1.997/t} \tag{51}$$

and

$$R_{3p} = \frac{2.802 + 1.1429/t}{2.802 - 1.1429/t}, \tag{52}$$

$$t = \frac{\epsilon_{1s}}{E_1} - 1.$$

We find that the ratios become equal to 2 when E takes the values 11 eV and 18 eV, respectively, for IE to $2p$ and $3p$ state.

TABLE 1.

Ratio of the differential cross-sections for ionization–excitation of He by antiproton to that by equivelocity proton. R_{2p} is the ratio for excitation to the $2p$ state. E_1 (in eV) is the energy of the emitted electron.

E_1 (in eV)	7	8	9	10	11	12	13	14	15
R_{2p}	1.4	1.6	1.7	1.9	2	2.2	2.5	2.8	3.2

TABLE 2.
Same as in Table 1 for excitation to 3p states.

E_1 (in eV)	12	13	14	15	16	17	18	19	20	21	22	23
R_{3p}	1.4	1.5	1.6	1.7	1.8	1.9	2	2.2	2.4	3	3.1	3.6

4.1. Ratio of the DCS with $(\mathrm{e}^-, \mathrm{e}^+)$ and $(\mathrm{e}^-, \mathrm{p}^+)$ pairs

When the projectiles are electrons or positrons, exchange diagrams corresponding to Figs. 1a, 1b and 1c are to be considered. However, when the energy and momentum transfer are low, the contribution from exchange diagrams are negligible [17]. The ratio of the DCS by $(\mathrm{e}^-, \mathrm{e}^+)$ pair will then be the same as in Eq. (34). Taking into account the contribution from the trace part given by Eq. (38), $G^{\mp}_{nl}$ in Eq. (49) becomes

$$G^{\mp}_{nl} = 2X_2^2 + \frac{12\pi e^2}{\nabla_{nl}} \pm \frac{6\pi e^2}{[(E_1/\epsilon_{1s}) - 1]F_{nl}}. \tag{53}$$

It is to be noted that the first term in Eq. (50) is multiplied by a factor 2 in Eq. (53), while the other terms remain unaltered.

For IE to $2p$ state, using Eq. (53), we get

$$R_{2p} = \frac{G'^{-}_{2p}}{G'^{+}_{2p}} = \frac{3.25142 + 1.9974/t}{3.25142 - 1.9974/t}. \tag{54}$$

When $E_1 = 14$ eV, the ratio becomes 2. For excitation to the $3p$ states it follows

$$R_{3p} = \frac{4.31564 + 1.1429/t}{4.31564 - 1.1429/t}. \tag{55}$$

R_{3p} becomes 2 for $E_1 = 22$ eV.

When the projectile pair is $(\mathrm{e}^{-1}, \mathrm{p}^+)$, the masses are unequal. With equivelocity projectiles and the same incident flux from Eq. (33), the ratio of the differential cross–sections becomes

$$R_{nl} = \frac{|W^-|^2 F(E, E_1, m)}{|W^+|^2 F(E, E_1, M)} = \frac{\sigma^-_{nl} F(E, E_1, m)}{\sigma^+_{nl} F(E, E_1, M)}. \tag{56}$$

Since $F(E, E_1, M)$ is weakly dependent on mass (32), the factor $F(E, E_1, m)/F(E, E_1, M)$ can be taken as 1, and for the ratios at $2p$ and $3p$ states we arrive at

$$R_{2p} = \frac{G'^{-}_{2p}}{G'^{+}_{2p}} = \frac{3.25142 + 1.9974/t}{2.344 - 1.9974/t}, \tag{57}$$

$$R_{3p} = \frac{4.31564 + 1.1429/t}{2.802 - 1.1429/t}. \tag{58}$$

R_{2p} and R_{3p} become 2 for E_1 equal to 7.6 eV and 10.79 eV, respectively.

5. *Conclusions*

We have given an analytical expression of DCS for ionization–excitation of He to different *nl* states. The ratio of the differential cross–section by negative and by equivelocity positive projectiles are derived for $(\mathrm{p}^-, \mathrm{p}^+)$, $(\mathrm{e}^-, \mathrm{e}^+)$ and $(\mathrm{e}^-, \mathrm{p}^+)$ projectile pairs. The ratio is found to depend strongly on the value of the energy E_1 of the ejected electron. According to the theoretical predictions [4] and experimental results [1,2], the ratio is greater than one. The present ratio becomes greater than one only when E_1 is less than half the threshold for double ionization from the ground state of He (79 eV). DCS by negative projectile becomes a factor of 2 larger than that by positive projectiles when E_1 takes values less than 39.5 eV, as shown in the previous section.

Thus we find, when the projectile energy is less than its rest mass and momentum transfer is low, the differential cross–section by negative projectile is greater than that by equivelocity positive projectiles. The difference in the cross–sections arises because of the interference of the two–step–two amplitude (M^Z_{TS2}) with the shake–up amplitude (M^Z_{SU}) and two–step–one amplitude (M^Z_{TS1}).

In conclusion we like to say that the present field theoretic result agrees in principle with the existing theoretical predictions by Ford and McGuire. However there is as yet no experimental or other theoretical results to compare with.

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NABOJSKA OVISNOST IONIZACIJE–EKSCITACIJE HELIJA BRZIM ČESTICAMA

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Formuliran je analitički izraz za diferencijalni udarni presjek ionizacije–ekscitacije He u He^+(np) stanja visokoenergijskim nabijenim česticama. Nađeno je da je omjer diferencijalnog udarnog presjeka negativnih i pozitivnih projektila jednakovaljane brzine veći od jedan ako je energija izbačenog elektrona manja od polovine energije osnovnog stanja helijeva atoma.