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A SIMPLE ANALYSIS OF THE BURSTEIN-MOSS SHIFT IN ULTRATHIN FILMS OF BISMUTH IN THE PRESENCE OF CROSSED ELECTRIC AND QUANTIZING MAGNETIC FIELDS

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We present a simple theoretical analysis of the Burstein-Moss shift in ultrathin films of bismuth in presence of crossed electric and quantizing magnetic fields in the presence of spin and broadening of Landau levels. The numerical results are presented for McClure and Choi, hybrid, Cohen, Lax and ellipsoidal parabolic energy band models. It was found that the shift increases for thinner films and in weaker magnetic field. In addition, the shift increases with increasing electron concentration, and the quantum oscillations, in accordance with the Mc Clure and Choi model, show up much more significantly than with other models of bismuth.

1. Introduction

Bismuth has been the subject of a large number of experimental and theoretical investigations at low temperatures due to the fact that it is easy to observe various phenomena when subjected to different physical conditions [1-20]. Bismuth is considered to be a semimetal because its electronic properties are between those of a metal and semiconductor. The $E-\vec{k}$ dispersion relations of carriers in bismuth differ considerably from simpler spherical constant-energy surfaces of the degenerate electron gas. Initial work demonstrated that carriers in bismuth could be described by the ellipsoidal parabolic model or one-band model [2-3]. Several workers [4-5] have observed that many nonlinear effects in the high-frequency region or in strong magnetic fields in bismuth can be described by the Lax ellipsoidal nonparabolic model [6], whereas Dinger and Lawson [7] indicated that the Cohen model [8] is in better agreement with the experimental results on cyclotron resonance. In a work on magnetic surface resonance, Takoaka et al. [9] concluded that neither the Lax model nor the Cohen model are adequate. They proposed a hybrid band model of bismuth and showed that the Lax and Cohen models are simplified limiting cases of their hybrid dispersion relation [9]. In 1977, McClure and Choi [10] presented a new model of bismuth that was more general than the previous models. They showed that their model fit the data for a large number of magneto-oscillatory and resonance experiments. Though considerable work has already been done, there still remain scopes to be investigated while the interest for further research of the different other electronic properties of Bi is becoming increasingly important. One such important feature is the Burstein-Moss shift (hereafter referred to as BMS) that has been relatively less investigated under the conditions of quantum confinement [21]. It appears that the same shift in ultrathin films of Bi has yet to be studied for the more interesting case which occurs from the presence of crossed electric and quantizing magnetic field. The crossed-field configuration is of fundamental importance for classical and quantum transport phenomena in crystalline solids at low temperatures [22]. Optical investigations of solid in crossed electric and magnetic fields started with the work of Hensel and Peter [23], who indicated that the influence of an electric field on the Landau levels should lead to observable effects in cyclotron resonance experiments.

In ultrathin films, when the film dimension is comparable with the de-Broglie wavelength of the carriers, the restriction of the motion of the carriers in the direction normal to the film (say, the y-direction) may be viewed as carrier confinement in an infinitely deep one-dimensional square potential well, leading to the quantization (known as the quantum size effect, QSE) of the wave vector of the carriers along y-axis, which produces a discrete energy spectrum. The other criteria to be met in order to observe QSE are that the film thickness be extremely uniform and the lifetime of the electronic particle be long enough that the new QSE properties are not broadened beyond recognition [24]. The QSE in ultrathin films of bismuth under the above conditions has been experimentally observed [24].

In Section 2.1. we shall derive the BMS in ultrathin films of bismuth in the presence of crossed electric and magnetic fields by using the McClure and Choi model. We shall then derive an expression for the appropriate electron concentration by using the same model for the purpose of investigating the dependence of BMS on doping. The results for the corresponding BMS by using a hybrid model are given in Section 2.2, from which the simplified limiting cases of the Cohen, Lax and parabolic models can easily be derived. The doping, film thickness and magnetic field dependences of the BMS in various band models have also been investigated.

2. Theoretical background

2.1. Derivation of the BMS in ultrathin films of bismuth using the McClure-Choi model under crossed-field configuration

Following McClure and Choi [10], the dispersion relation of the carriers in bulk specimens of bismuth [11] can be expressed as

$$E(1+\alpha E) = \frac{p_x^2}{2m_{1c,v}} + \frac{p_y^2}{2m_{2c,v}} + \frac{p_z^2}{2m_{3c,v}} + \frac{p_y^2}{2m_{2c,v}}\alpha E\left(1 - \frac{m_{2c,v}}{m'_{2c,v}}\right) + \frac{\alpha p_y^4}{4m_{2c,v}m'_{2c,v}} - \frac{\alpha p_x^2 p_y^2}{4m_{1c,v}m_{2c,v}} - \frac{\alpha p_y^2 p_z^2}{4m_{2c,v}m_{3c,v}}, \tag{1}$$

where E is the carrier energy in the absence of any quantization, $\alpha = 1/E_g$, E_g is the band gap, $\vec{p} = \hbar\vec{k}$, $\vec{p}$ is the momentum vector of the carriers, $\hbar = h/2\pi$, h is the Planck's constant, $\vec{k}$ is the wave vector, $p^2 = p_x^2 + p_y^2 + p_z^2$, p_x, p_y and p_z are the components of the momentum along x, y, and z directions, respectively, $m_{ic,v}$ ($i = 1, 2$ and 3) are the effective masses at the band edge along p_x, p_y and p_z directions for electron or holes, respectively, m'_{2c} is the longitudinal effective mass of the holes for the valence bands in the case of electrons and m'_{2v} is the longitudinal effective mass of the electron for the conduction band in the case of holes. In the presence of an electric field E_0 along the trigonal axis and a quantizing magnetic field B along the bisectrix axis, the modified electron and hole energy spectra can be expressed by extending the method of Zawadski and Lax [25] as follows

$$E(1+\alpha E) = \left(n + \frac{1}{2}\right)\hbar\omega_{0c} + \frac{p_y^2}{2m_{2c}} - \frac{E_0}{B}(1+2\alpha E)p_x - \frac{1}{2}m_{1c}(E_0/B)^2(1+2\alpha E)^2 + \frac{p_y^2}{2m_{2c}}\alpha E\left(1 - \frac{m_{2c}}{m'_{2c}}\right) + \frac{\alpha p_y^4}{4m_{2c}m'_{2c}} - \frac{p_y^2}{2m_{2c}}\left(n + \frac{1}{2}\right)\hbar\omega_{0c} \pm \frac{1}{2}g_c\mu B - \frac{\alpha p_x^2 m_{1c}E_0^2}{4m_{2c}B^2}(1+2\alpha E)^2 \tag{2}$$

and

$$E(1+\alpha E) = \left(n' + \frac{1}{2}\right)\hbar\omega_{0v} + \frac{p_y^2}{2m_{2v}} - \frac{E_0}{B}(1+2\alpha E)p_x - \frac{1}{2}m_{1v}(E_0/B)^2(1+2\alpha E)^2 +$$

$$\frac{p_y^2}{2m_{2v}}\alpha E\left(1-\frac{m_{2v}}{m_{2v}^{'}}\right)+\frac{\alpha p_y^4}{4m_{2v}m_{2v}^{'}}-\frac{\alpha p_y^2}{2m_{2v}}\left(n^{'}+\frac{1}{2}\right)\hbar\omega_{0v}$$

$$\pm\frac{1}{2}g_v\mu B-\frac{\alpha p_x^2 m_{1v}E_0^2}{4m_{2v}B^2}(1+2\alpha E)^2, \tag{3}$$

where n and $n^{'}$ are the Landau quantum numbers for electrons and holes, respectively, e is the carrier charge, $\omega_{0c,v}=eB(m_{1c,v}\ m_{3c,v})^{-1/2}$ and $g_{c,v}$ are the band edge g-factors for electron and holes, respectively. In an ultrathin film, the carriers are assumed to be confined in a one-dimensional potential well of a width d_0, leading to the quantization of the wave vector in the direction normal to the film (here in the y-direction). Therefore, in the presence of size quantization, the modified carrier energy spectra in ultrathin films of bismuth under crossed-field configuration can be written, as follows

$$E(1+\alpha E)=\left(n+\frac{1}{2}\right)\hbar\omega_{0c}+\frac{\hbar^2}{2m_{2c}}\left(\frac{l\pi}{d_0}\right)^2-\frac{E_0}{B}(1+2\alpha E)p_x-\frac{1}{2}m_{1c}(E_0/B)^2(1+2\alpha E)^2$$

$$+\frac{\hbar^2}{2m_{2c}}\left(\frac{l\pi}{d_0}\right)^2\alpha E\left(1-\frac{m_{2c}}{m_{2c}^{'}}\right)+\frac{\alpha\hbar^4}{4m_{2c}m_{2c}^{'}}\left(\frac{l\pi}{d_0}\right)^4\pm\frac{1}{2}g_c\mu B$$

$$-\hbar^2\alpha\left(\frac{l\pi}{d_0}\right)^2\left[(2m_{2c})^{-1}\left(n+\frac{1}{2}\right)\hbar\omega_{0c}+(4B^2m_{2c})^{-1}m_{1c}E_0^2(1+2\alpha E)^2\right] \tag{4}$$

and

$$E(1+\alpha E)=\left(n^{'}+\frac{1}{2}\right)\hbar\omega_{0v}+\frac{\hbar^2}{2m_{2v}}\left(\frac{l^{'}\pi}{d_0}\right)^2-\frac{E_0}{B}(1+2\alpha E)p_x-\frac{1}{2}m_{1v}(E_0/B)^2(1+2\alpha E)^2$$

$$+\frac{\hbar^2}{2m_{2v}}\left(\frac{l^{'}\pi}{d_0}\right)^2\alpha E\left(1-\frac{m_{2v}}{m_{2v}^{'}}\right)+\frac{\alpha\hbar^4}{4m_{2v}m_{2v}^{'}}\left(\frac{l^{'}\pi}{d_0}\right)^4\pm\frac{1}{2}g_v\mu B$$

$$-\hbar^2\alpha\left(\frac{l^{'}\pi}{d_0}\right)^2\left[(2m_{2v})^{-1}\left(n^{'}+\frac{1}{2}\right)\hbar\omega_{0v}+(4B^2m_{2v})^{-1}m_{1v}E_0^2(1+2\alpha E)^2\right], \tag{5}$$

where l and $l^{'}$ are the size quantum numbers for electrons and holes, respectively.

Thus, the BMS can be written as

$$\Delta=E_F+E_g+E_1, \tag{6}$$

where E_F is the Fermi energy and E_1 can be expressed from (4) and (5) as

$$E_1 = (2p_1)^{-1}\left[-Q_1 + \sqrt{Q_1^2 + 4p_1R_1}\right] \tag{7}$$

in which

$$p_1 = \left[\alpha - \frac{1}{2}m_{1v}(E_0/B)^2 4\alpha^2 - \alpha\hbar^2\left(\frac{\pi}{d_0}\right)^2 \frac{m_{1v}\alpha^2E_0^2}{B^2m_{2v}}\right],$$

$$Q_1 = 1 - 2\alpha m_{1v}(E_0/B)^2 + \frac{\alpha\hbar^2}{2m_{2v}}\left(\frac{\pi}{d_0}\right)^2\left(1 - \frac{m_{2v}}{m_{2v}^{'}}\right) -$$

$$\alpha^2\hbar^2\left(\frac{\pi}{d_0}\right)^2\frac{m_{1v}E_0^2}{m_{2v}B^2} - 2\alpha\Phi,$$

$$\Phi = E_F(1+\alpha E_F)(1+2\alpha E_F)^{-1} - \frac{1}{2}\hbar\omega_{0c}(1+2\alpha E_F)^{-1} - \frac{\hbar^2}{2m_{2c}}\left(\frac{\pi}{d_0}\right)^2(1+2\alpha E_F)^{-1}$$

$$+\frac{1}{2}m_{1c}(E_0/B)^2(1+2\alpha E_F) - \frac{\hbar^2}{2m_{2c}}\left(\frac{\pi}{d_0}\right)^2\alpha E_F(1+2\alpha E_F)^{-1}\left(1 - \frac{m_{2c}}{m_{2c}^{'}}\right)$$

$$-\frac{\alpha\hbar^4}{4m_{2c}m_{2c}^{'}}\left(\frac{\pi}{d_0}\right)^4(1+2\alpha E_F)^{-1} + \alpha\hbar^2\left(\frac{\pi}{d_0}\right)^2(1+2\alpha E_F)^{-1}\times$$

$$\left((2m_{2c})^{-1}\frac{1}{2}\hbar\omega_{0c} + (4B^2m_{2c})^{-1}m_{1c}E_0^2(1+2\alpha E_F)^2\right) -$$

$$(1+2\alpha E_F)^{-1}\frac{1}{2}g_c\mu B$$

and

$$R_1 = \Phi - \left[\frac{\hbar^2}{2m_{2v}}\left(\frac{\pi}{d_0}\right)^2 - \frac{1}{2}\hbar\omega_{0v} - \frac{1}{2}m_{1v}(E_0/B)^2 + \alpha\hbar^4(4m_{2v}m_{2v}^{'})^{-1}\left(\frac{\pi}{d_0}\right)^4\right.$$

$$\left.-\alpha\hbar^2\left(\frac{\pi}{d_0}\right)^2(2m_{2v})^{-1} + \frac{1}{2}\hbar\omega_{0v} - \alpha\hbar^2\left(\frac{\pi}{d_0}\right)^2 m_{1v}E_0^2(4B^2m_{2v})^{-1} - \frac{1}{2}\mu Bg_v\right].$$

It appears that the evaluation of the BMS as a function of carrier concentration requires an expression of electron statistics which can be expressed, including spin and broadening, as

$$n_0 = \frac{g_v eB}{h} \sum_{n=0}^{n_{max}} \sum_{l=1}^{l_{max}} \frac{1 + A_0 \cos l_0}{1 + A_0^2 + 2A_0 \cos l_0} \tag{8}$$

where g_v is valley degeneracy,

$$A_0 = \exp(-\eta_{nl}), \quad \eta_{nl} = \frac{E_F - E_{nl}}{k_B T},$$

E_{nl} is obtained by putting $p_x = 0$ in (4), k_B is the Boltzmann constant, T is the temperature, $l_0 = \Gamma / k_B T$ and Γ is the broadening parameter [26].

2.2. *The BMS in hybrid model and in several limiting cases*

The hybrid model of bismuth can be written [9] as follows

$$E(1 + \alpha E) = \frac{\beta(E) p_y^2}{2M_{2v,c}} + \frac{\alpha \gamma p_y^4}{4M_{2v,c}^2} + \frac{p_x^2}{2m_{1c,v}} + \frac{p_z^2}{2m_{3c,v}} \tag{9}$$

where

$$\beta(E) = 1 + \alpha(E)(1 - \gamma) + \delta,$$

$\gamma = M_{2v,c}/m'_{2v,c}$, $M_{2v,c}$ is the transverse effective mass of the holes (electrons) and $\delta = M_{2v,c}/m_{2v,c}$. The modified carrier energy spectra in ultrathin films of bismuth under crossed-field configuration can, according to the hybrid model, be expressed as

$$E(1 + \alpha E) = \left(n + \frac{1}{2}\right)\hbar\omega_{0c} - (E_0/B)p_x(1 + 2\alpha E) - \frac{m_{1c}E_0^2}{2B^2}(1 + 2\alpha E)^2 +$$

$$\beta(E)\frac{\hbar^2}{2M_{2v}}\left(\frac{l\pi}{d_0}\right)^2 + \frac{\alpha\gamma\hbar^4}{4M_{2v}^2}\left(\frac{l\pi}{d_0}\right)^4 \pm \frac{1}{2}\mu B g_c \tag{10}$$

and

$$E(1 + \alpha E) = \left(n' + \frac{1}{2}\right)\hbar\omega_{0v} - (E_0/B)p_x(1 + 2\alpha E) - \frac{m_{1v}E_0^2}{2B^2}(1 + 2\alpha E)^2 +$$

$$\beta(E)\frac{\hbar^2}{2M_{2v}}\left(\frac{l'\pi}{d_0}\right)^2 + \frac{\alpha\gamma\hbar^4}{4M_{2v}^2}\left(\frac{l'\pi}{d_0}\right)^4 \pm \frac{1}{2}\mu B g_v. \tag{11}$$

The general forms of the BMS and the electron statistics of bismuth for the hybrid model are given by (6) and (8), respectively, where E_1 and E_{nl} are expressed by

$$E_1(1+\alpha E_1) = \frac{1}{2}\hbar\omega_{0v} + T_1(1+2\alpha E_1) - \frac{m_{1v}E_0^2}{2B^2}(1+2\alpha E_1)^2 - \frac{1}{2}\mu B g_v +$$
$$\frac{\beta(E_1)\hbar^2}{2M_{2v}}\left(\frac{\pi}{d_0}\right)^2 + \frac{\alpha\gamma\hbar^4}{4M_{2v}^2}\left(\frac{\pi}{d_0}\right)^4 \tag{12}$$

and

$$E_{nl}(1+\alpha E_{nl}) = \left(n+\frac{1}{2}\right)\hbar\omega_{0v} - \frac{m_{1c}E_0^2}{2B^2}(1+2\alpha E_{nl})^2 \pm \frac{1}{2}\mu B g_c +$$
$$+\frac{\beta(E_{nl})\hbar^2}{2M_{2v}}\left(\frac{l\pi}{d_0}\right)^2 + \frac{\alpha\gamma\hbar^4}{4M_{2v}^2}\left(\frac{l\pi}{d_0}\right)^4 \tag{13}$$

in which

$$T_1 = \Bigg[E_F(1+\alpha E_F)(1+2\alpha E_F)^{-1} + \frac{m_{1c}E_0^2}{2B^2}(1+2\alpha E_F) - \frac{1}{2}\hbar\omega_{0c}(1+2\alpha E_F)^{-1}$$
$$-\beta(E_F)\hbar^2\left(\frac{\pi}{d_0}\right)^2\left[2M_{2v}(1+2\alpha E_F)\right]^{-1} - \alpha\gamma\hbar^4\left(\frac{\pi}{d_0}\right)^4\left[4M_{2v}^2(1+2\alpha E_F)\right]^{-1}\Bigg].$$

For $\gamma = 0$ and $\delta = 0$, (9) is simplified to the Lax model, whereas for $\delta = 0$ the same equation leads to the Cohen model as stated in the literature [9]. Thus, simply by changing E_1 and E_{nl}, we can obtain the BMS and the electron statistics in ultrathin films of bismuth under crossed-field configuration for either the Lax or the Cohen model.

If $\alpha \to 0$ (as for the parabolic energy bands), (1) gets simplified as

$$E = \frac{p_x^2}{2m_{1c,v}} + \frac{p_y^2}{2m_{2c,v}} + \frac{p_z^2}{2m_{3c,v}}, \tag{14}$$

which is the ellipsoidal-parabolic model of bismuth. In addition, under the conditions $\alpha = 0$ and $\gamma \gg 1$, the hybrid model as given by (9) is simplified to (14). Therefore, under the above conditions, the dispersion relations of the carriers in ultrathin films of bismuth under the crossed-field configurations can be expressed, as in the parabolic model, by

$$E = \left(n + \frac{1}{2}\right)\hbar\omega_{0c} - \frac{E_0}{B}p_x - \frac{1}{2}m_{1c}\left(\frac{E_0}{B}\right)^2 + \frac{\hbar^2}{2m_{2c}}\left(\frac{l\pi}{d_0}\right)^2 \pm \frac{1}{2}\mu B g_c \tag{15}$$

and

$$E = \left(n' + \frac{1}{2}\right)\hbar\omega_{0v} - \frac{E_0}{B}p_x - \frac{1}{2}m_{1v}\left(\frac{E_0}{B}\right)^2 + \frac{\hbar^2}{2m_{2v}}\left(\frac{l'\pi}{d_0}\right)^2 \pm \frac{1}{2}\mu B g_v. \tag{16}$$

Even for parabolic energy bands the general forms of BMS and n_0 are given by (6) and (8), respectively, where

$$E_1 = \frac{1}{2}\hbar\omega_{0v} + E_F - \frac{1}{2}\hbar\omega_{0c} + \frac{1}{2}m_{1c}\left(\frac{E_0}{B}\right)^2 - \frac{\hbar^2}{2m_{2c}}\left(\frac{\pi}{d_0}\right)^2$$
$$+\frac{1}{2}\mu B g_c - \frac{1}{2}m_{1v}\left(\frac{E_0}{B}\right)^2 + \frac{\hbar^2}{2m_{2v}}\left(\frac{\pi}{d_0}\right)^2 - \frac{1}{2}\mu B g_v \tag{17}$$

and

$$E_{nl} = \left(n + \frac{1}{2}\right)\hbar\omega_{0c} - \frac{1}{2}m_{1c}\left(\frac{E_0}{B}\right)^2 + \frac{\hbar^2}{2m_{2c}}\left(\frac{l\pi}{d_0}\right)^2 \pm \frac{1}{2}\mu B g_c. \tag{18}$$

Finally, under the conditions $E_0 = 0$, $\alpha = 0$, $m_1 = m_2 = m_3 = m^*$ and neglecting broadening, the expression of electron concentration assumes the form [27]

$$n_0 = \frac{g_v e B}{h}\sum_{n=0}^{n_{max}}\sum_{l=1}^{l_{max}} F_{-1}(\eta_{nl}) \tag{19}$$

where

$$\eta_{nl} = (k_B T)^{-1}\left[E_F - \left\{\left(n + \frac{1}{2}\right)\hbar\omega_{0v} - \frac{1}{2}m^*\left(\frac{E_0}{B}\right)^2 + \frac{\hbar^2}{2m^*}\left(\frac{l\pi}{d_0}\right)^2 \pm \frac{1}{2}\mu B g_c\right\}\right]$$

$$\omega_0 = \frac{eB}{m^*},$$

and $F_j(\eta_{nl})$ is the Fermi-Dirac integral of order j [28].

3. *Results and discussion*

Using Eqs (6), (8), (12), (13), (17) and (18) and taking the following values of the parameters [9-16] $g_v = 3$, $m_{1c} = m_0/172$, $m_{2c} = m_0/0.8 = m'_{2c}$, $m'_{3c} = m_0/88.5$, $E_g = 0.0153$ eV, $M_{2v} = 1.28m_0$, $M_{2c} = 1.2m_0$, $d_0 = 40$ nm, $g_c = 55$, $g_v = 20$, $\Gamma = 10^{-4}$ eV, $B = 2.3$ T, $m_{1v} = m_{2v} = m_0/14.9 = m_{2v'}$, $m_{3v} = m_0/1.41$, and $T = 4.2$ K we have calculated the BMS in ultrathin films of bismuth under the

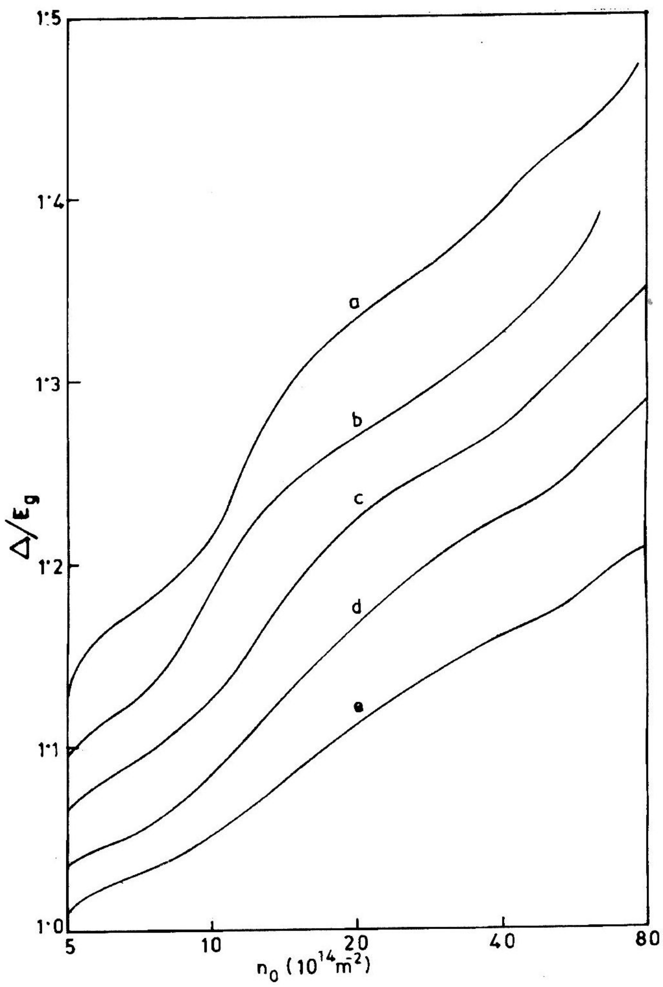

Fig. 1. Plot of the BMS at 4.2 K as a function of the electron concentration in ultrathin films of bismuth in the presence of crossed electric and magnetic fields for $d_0 = 40$ nm and $B = 1$ T. The curves show the results of calculations based on the (a) McClure and Choi model, (b) hybrid model, (c) Lax model, (d) Cohen model and (e) the anisotropic parabolic model.

crossed-field configuration as a function of electron concentration. The results based on the McClure and Choi, hybrid, Cohen, Lax and parabolic-ellipsoidal band models are shown in Fig. 1. Using the same parameters, we present in Figs. 2 and 3 the computed BMS as a function of film thickness and of magnetic field, respectively, for the aforementioned band models of bismuth. Fig. 1 shows that the BMS is an oscillatory function of the electron concentration. The oscillatory dependence of the BMS on the electron concentration is most prominent in the McClure and Choi model. The dependence of the BMS on the surface electron concentration is determined by the particular band structure because of its direct relevance to the Fermi energy.

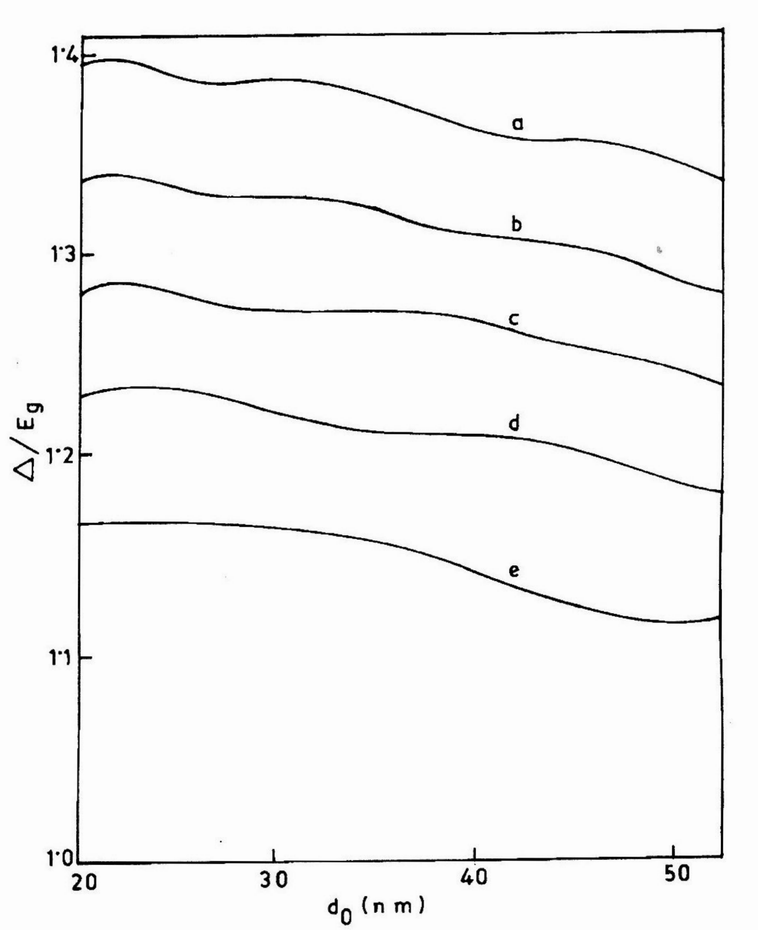

Fig. 2. Plot of the BMS at 4.2 K *as a function of the film thickness in ultrathin films of bismuth in the presence of crossed electric and magnetic fields for* $n_0 = 2 \times 10^{14}$ m^{-2} *and* $B = 1$ T. *The curves show the results of calculations based on the (a) McClure and Choi model, (b) hybrid model, (c) Lax model, (d) Cohen model and (e) the anisotropic parabolic model.*

From Fig. 2 we note that the BMS is strongly dependent on the thickness of the film. The influence of the size quantization is immediately apparent from Fig. 2, since the BMS is strongly dependent on the thickness of the ultrathin films under crossed-field configuration in contract with bulk specimens of bismuth. The appearance of the humps in Figs. 1 and 2 is due to the redistribution of electrons among the quantized energy levels.

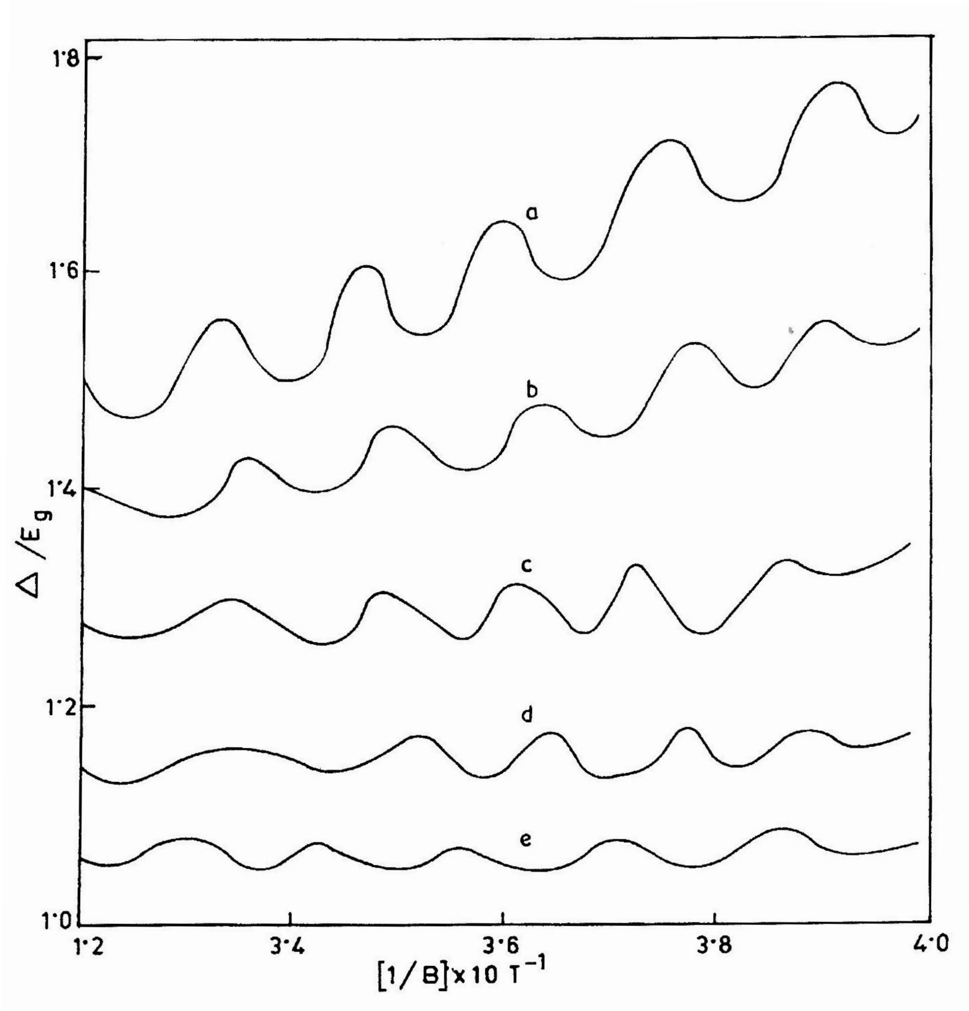

Fig. 3. Plot of the BMS at 4.2 K *as a function of* $1/B$ *in the presence of crossed electric and magnetic fields for* $n_0 = 2 \times 10^{14}$ m^{-2} *and* $d_0 = 40$ nm, respectively. *The curves show the results of calculations based on the (a) McClure and Choi model, (b) hybrid model, (c) Lax model, (d) Cohen model and (e) the anisotropic parabolic model.*

In Fig. 3 we observe that the BMS oscillates with the reciprocal magnetic field. The results of calculations of the quantum oscillations of the BMS in bismuth, as shown in the figures, are considerably larger for the McClure and Choi model than for other models.

4. Conclusions

We have formulated the BMS in ultrathin films of bismuth in a crossed-field configuration by deriving the respective expression of the carrier statistics for the McClure and Choi, the hybrid, the Cohen, the Lax and the ellipsoidal-parabolic models. If $\alpha = 1/E_g \to 0$, the result based on the McClure and Choi model, as given by (1) reduces to (14), that is, the equation for anisotropic parabolic energy bands. Also, under the limiting conditions $\alpha \to 0$ and $\delta \gg 1$, the result for the hybrid model as given by (9) reduces to (14). The Cohen and Lax models, which are the special cases of the hybrid models, also reduce to (14) under the condition $\alpha \to 0$. Therefore, under these limiting conditions, the results for n_0 and BMS for all the four models get simplified to the recpective expressions for the anisotropic parabolic model. The Cohen model is used to describe the dispersion relation of the carriers of lead chalcogenide materials [28]. The Lax model under the condition of effective isotropic carrier effective mass at the edge (i.e. $m_1 = m_2 = m_3 = m^*$) reduces to the two-band Kane model, which is often used in studies of the physical properties of III-V compound semiconductors, excluding n-InAs [28]. Besides, under the condition $E_g \to \infty$, together with the aforementioned equality, the two-band Kane model gets simplified to the well-known form for isotropic parabolic energy bands: $E = \hbar^2 k^2/2m^*$, which is used often for investigating electronic properties of wide-gap materials. Thus, the analysis of our paper is valid, not only for semimetals like bismuth, but also for other types of semiconductors having various band structures.

The variations of the BMS are entirely band structure dependent. The BMS in ultrathin films of semimetals under crossed-field configuration can be assessed from our present work. We have not considered other types of semimetals or other physical variables. With different sets of energy band parameters we shall get different numerical values of BMS, though the nature of variations will be unaltered.

Our result will be changed for arbitrary crossed-field configuration. It may be noted that, although the effects of electron-electron interactions should properly be considered, this simplified analysis exhibits the basic qualitative aspects of the BMS in ultrathin films of bismuth under crossed-field configuration. It may finally be remarked that the basic aim of the present paper was not to investigate only the BMS in ultrathin films of bismuth under crossed-field configuration, but also to formulate the electron concentration in accordance with the various band models, by including spin and broadening, since the various transport and other phenomena in semimetals and the derivation of the expressions of many important physical parameters are based on the temperature-dependent electron statistics in such materials.

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JEDNOSTAVNA ANALIZA BURSTEIN-MOSSOVOG POMAKA U VEOMA TANKIM SLOJEVIMA BIZMUTA U UKRŠTENOM ELEKTRIČNOM I KVANTIZIRAJUĆEM MAGNETSKOM POLJU

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Učinjen je pokušaj jednostavne teorijske analize Burstein-Mossovog pomaka u veoma tankim slojevima bizmuta u ukrštenom električnom i kvantizirajućem magnetskom polju. U razmatranje je uključen i spin te širenje Landauovih nivoa. Numerički rezultati prikazani su za modele McClurea i Choia, Cohenov, Laxov, te za model eliptičnih paraboličnih energetskih vrpci. Postavljene su relacije za svaki model. Nađeno je da pomak raste smanjenjem debljine sloja i porastom jakosti magnetskog polja. Također, pomak raste s povećanjem koncentracije elektrona. Kvantne oscilacije pokazuju se značajnim, što je u skladu s modelom McClurea i Choia, za razliku od predskazanja drugih modela.