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ON THE BAND-TAIL SPREADING ENERGY IN HEAVILY DOPED LASER DIODES UNDER DIFFERENT PHYSICAL CONDITIONS

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In this paper we have studied the band-tail spreading energy in heavily doped laser diodes in bulk specimens, in the presence of magnetic quantization and also under cross-field configurations. We have also computed the same energy in quantum wire lasers having degenerate electron concentration forming band tails. The investigations are based on a newly formulated electron dispersion law of laser materials having highly degenerate carrier concentration. It is found, taking InSb junction laser as an example of a heavily doped laser diode, that the band tail spreading energy oscillates as a function of the inverse magnetic field, increases with increasing carrier density and increasing electric field in the respective cases. Under 2D and 1D quantizations, the same energy decreases with increasing film thickness in an oscillatory way and the heavy doping affects significantly the values of the band tail spreading in all types of quantum confinement. In addition, we have suggested the experimental procedure of determining the band tail spreading energy for laser materials having arbitrary band structure.

1. Introduction

The band-tails [1-3] associated with random impurity distribution in semiconductors contribute to many important effects [4]. One of those is the emission of radiation from semiconductor lasers. The spreading energy characterizing the band tails is thus an important physical feature in studies of stimulated emission from laser diodes. Besides, with the advent of MBE, FLL and other experimental techniques, quantum well lasers and quantum wire lasers have uncovered various new physical bases of optical science. Investigations on the band-tail spreading energy have been published assuming Gaussian band tails [5] and also using the nonphysical assumption of isotropic parabolic energy band models of carriers for III-V laser materials [6].

In laser diodes the radiative recombination process takes place in the p-region in close proximity to the chemical p-n junction. Electrons are injected into the donor band states in the compensated p-region and recombine with holes in the acceptor band. Near a temperature of 0 K, the electrons fill all the energy states in the partially compensated p-region up to a point E_n throught the active region assumed to be more or less uniform. Though the electron states in energies above E_n are mostly empty, the density of states per unit energy at E_n is large enough so that a population inversion sufficient to overcome losses is established.

In this paper an attempt is made to study the BTSE in heavily doped laser diodes having non-parabolic energy bands on the basis of a newly derived electron dispersion law. We shall also study the BTSE in the presence of magnetic quantization and also under cross-field configuration since the mentioned types of quantization of band states provide useful informations regarding various physical properties of heavily doped laser diodes. Besides, the BTSE has further been studied in quantum well and quantum wires laser diodes, respectively, by formulating the respective carrier statistics. We shall also suggest an experimental method of determining BTSE for laser materials. We have investigated the dependence of the spreading energy in heavily doped laser diodes, taking InSb laser diodes as an example, on the doping, magnetic field, electric field and thickness.

2. Theoretical background

2.1. Formulation of BTSE in heavily doped bulk laser diodes

The dispersion relation for the conduction electrons in unperturbed laser materials having narrow-band gap can be expressed as [7]

$$E = a_0 k^2 - b_0 k^4 \tag{1}$$

where E is the total electron energy as measured from the edge of the unperturbed conduction band, $a_0 = \hbar^2/2m^*$, $\hbar = h/2\pi$, h is the Planck's constant, m^* is the effective electron mass at the edge of the unperturbed conduction band, $b_0 = (1 - - m^* m_0^{-1})^2 \cdot (a_0)^2 [3E_g^2 + 4E_g C + 2C^2][E_g(E_g + C)(2C + 3E_g)]^{-1}$, m_0 is the free electron mass, E_g is the unperturbed band gap and C is the spin-orbit splitting parameter. The modified electron energy spectrum due to heavy doping can be expressed, as a result of first and second order perturbation, as [8]

$$\hbar^2 k^2/2m^* = \gamma(E) \tag{2}$$

where $\gamma(E) = [G - \sqrt{H - IE}]$, $G = a_0 A/2B_0$, $A = (a_0 - \alpha D_0 + \beta D_0)$, $D_0 = (4A_s^2 \pi N_i m^*/N\Omega a^3 \hbar^2)$, $\alpha = 2b_0/a$, $\beta = 4/a^2$, $B_0 = [b_0 - D\alpha\beta]$, $D = C_0 + D_0$, $C_0 = \dfrac{4\pi N_i A_s}{N\Omega a^2} - \dfrac{N_i(4\pi A_s)^2 \pi b_0}{4\pi^2 N\Omega(aa_0^3)}$, $H = H_0 a_0^2$, $H_0 = [(A^2/4B_0^2) - (D/B_0)]$, $I = I_0 a_0^2$, $D_0 = B_0^{-1}$, N_i is the number of impurity atoms per N atoms of the crystal, Ω is the volume of the unit-cell, $A = e^2/4\pi\varepsilon_s$, e is the electron charge, ε_s is the semiconductor permitivity and

$$a = (\pi\varepsilon_s)^{1/2}(\pi/3)^{1/6}((N^{-1/3})(\hbar^2/m_0 e^2)^{1/2}).$$

The use of Eq. (2) leads to the following expression of electron concentration as [8]

$$n_0 = (3\pi^2)^{-1}[A_1(E_F) + A_2(E_F)] \tag{3}$$

where $A_1(E_F) = [2m^*\gamma(E_F)/\hbar^2]^{3/2}$, E_F is the Fermi energy, $A_2(E_F) = \sum_{r=1}^{S} \Theta_r[A_1(E_F)]$, $\Theta_r = 2(k_B T)^{2r}(1 - 2^{1-2r})\zeta(2r)\dfrac{\mathrm{d}^2 r}{\mathrm{d}E_F^2 r}$, k_B is the Boltzmann constant, T is temperature, r is the set of real positive integers whose upper limit is S and $\zeta(2r)$ is the zeta function of order $2r$ [9].

The BTSE can, in general, be expressed [6] as

$$f^2 = (e^2/\varepsilon_s)^2 n_0 L_D \tag{4a}$$

where L_D is the screening length. Since we do not consider low-dimensional lasers, L_D can be expressed [10] as

$$L_D^{-2} = (e^2/\varepsilon_s)\left(\frac{\mathrm{d}n_0}{\mathrm{d}E_F}\right), \tag{4b}$$

and we can combine Eqs. (4a) and (4b) to obtain an expression of BTSE as

$$f^2 = (e^3/\varepsilon_s^{3/2}) n_0 \left(\frac{\mathrm{d}E_F}{\mathrm{d}n_0}\right)^{1/2}. \tag{5}$$

Since the carrier statistics is given by Eq. (3), we can combine Eqs. (3) and (5) to obtain an expression of BTSE in this unperturbed parabolic energy band as [11]

$$n_0 = N_C F_{\frac{1}{2}}(\eta) \tag{6a}$$

where $N_C = 2(2\pi m^* k_B T/h^2)^{3/2}$, $\eta = E_F/k_B T$ and $F_j(\eta)$ is the one-parameter Fermi-Dirac integral of order j as defined by Blakemore [11].

2.2. Formulation of BTSE under magnetic quantizing in heavily doped laser diodes

In the presence of a quantization magnetic field B along the z-direction, the electron energy spectrum assumes the form

$$(\hbar^2 k_z^2/2m^*) + \left(n + \frac{1}{2}\right)\hbar\omega_0 = \gamma(E) \tag{6b}$$

where $n (= 0, 1, 2, \ldots)$ is the Landau quantum number and $\omega_0 = eB/m^*$. The electron concentration can be expressed as

$$n_0 (eB\sqrt{2m^*}/\pi^2\hbar^2) \sum_{n=0}^{n_{\max}} [A_3(E_F) + A_4(E_F)] \tag{7}$$

where $A_3(E_F) =$ Real part of $\left[\gamma(E_0) - \left(n + \frac{1}{2}\right)\hbar\omega_0\right]^{1/2}$, $E_0 = E_F + i\Gamma$, $i = \sqrt{-1}$, $\Gamma (= \pi k_B T_D)$ is the broadening parameter, T_D is the Dingle temperature [12] and

$$A_4(E_F) = \sum_{r=1}^{S} \Theta_r [A_3(E_F)].$$

Thus combining Eqs. (7) and (5) we can obtain an expression of BTSE. Further, under the conditions $E_g \to \infty$, $N_i \to 0$ and $T_D \to 0$, Eq. (7) assumes the well-known form as

$$n_0 = N_C \Theta \sum_{n=0}^{n_{\max}} F_{-\frac{1}{2}}(\eta_1) \tag{8}$$

where $\Theta = \hbar\omega_0/k_B T$ and $\eta_1 = (k_B T)^{-1}\left[E_F - \left(n + \frac{1}{2}\right)\hbar\omega_0\right]$.

2.3. Formulation of BTSE under crossed-field configuration in heavily doped laser diodes

In the presence of an electric field ϵ_0 along the x-directions and quantizing magnetic field B along the z-direction the modified electron energy spectrum can be expressed as

$$\gamma(E) = \left(n + \frac{1}{2}\right)\hbar\omega_0 + (\hbar^2 k_z^2/2m^*) - \frac{\epsilon_0}{B}\hbar k_y L_1(E) - \frac{m^* \epsilon_0^2 L_1^2(E)}{2B^2} \tag{9}$$

where $L_1(E) = I/2(H - IE)^{1/2}$.

The expression for electron concentration can be written as

$$n_0 = \frac{2\sqrt{2m^*}B}{2\epsilon_0\hbar^2\pi^2 L_x}\sum_{n=0}^{n_{\max}}[A_5(E_F)+A_6(E_F)] \tag{10}$$

where L_x is the sample length along the x-direction, $A_5(E_F)$ = Real part of

$$\left[\frac{1}{L_I(E_0)}\left[\left\{\gamma(E_0)-\left(n+\frac{1}{2}\right)\hbar\omega_0+e\epsilon_0 L_x L_1(E_0)-\frac{m^*\epsilon_0^2 L_1^2(E_0)}{2B^2}\right\}^{3/2}-\right.\right.$$
$$\left.\left.-\left\{\gamma(E_0)-\left(n+\frac{1}{2}\right)\hbar\omega_0-\frac{m^*\epsilon_0^2 L_1^2(E_0)}{2B^2}\right\}^{3/2}\right]\right]$$

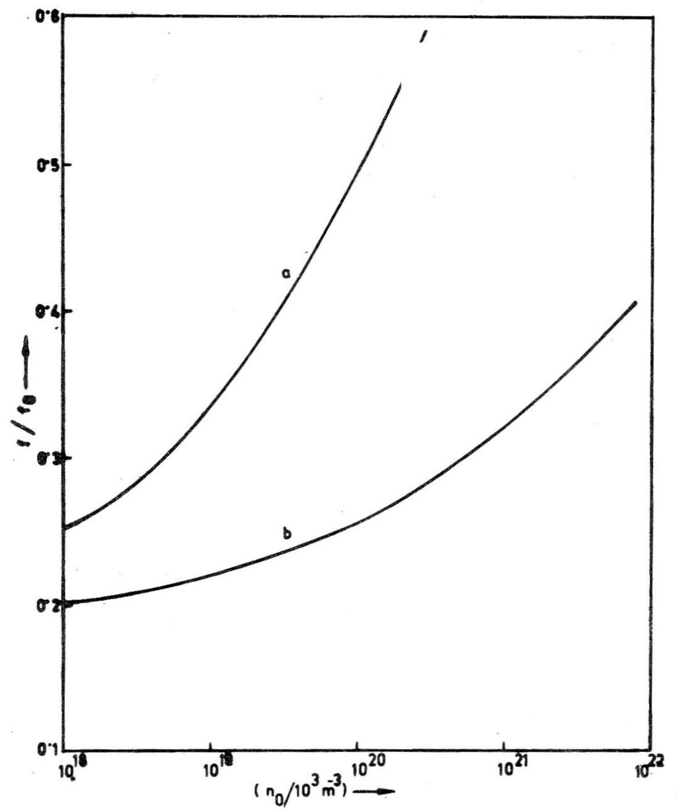

Fig. 1. Plot of normalized BTSE versus n_0 in bulk specimens of heavily doped InSb *laser diode in accordance with (a) our proposed dispersion law, (b) $N_i = 0$.* $T = 4.2$ K.

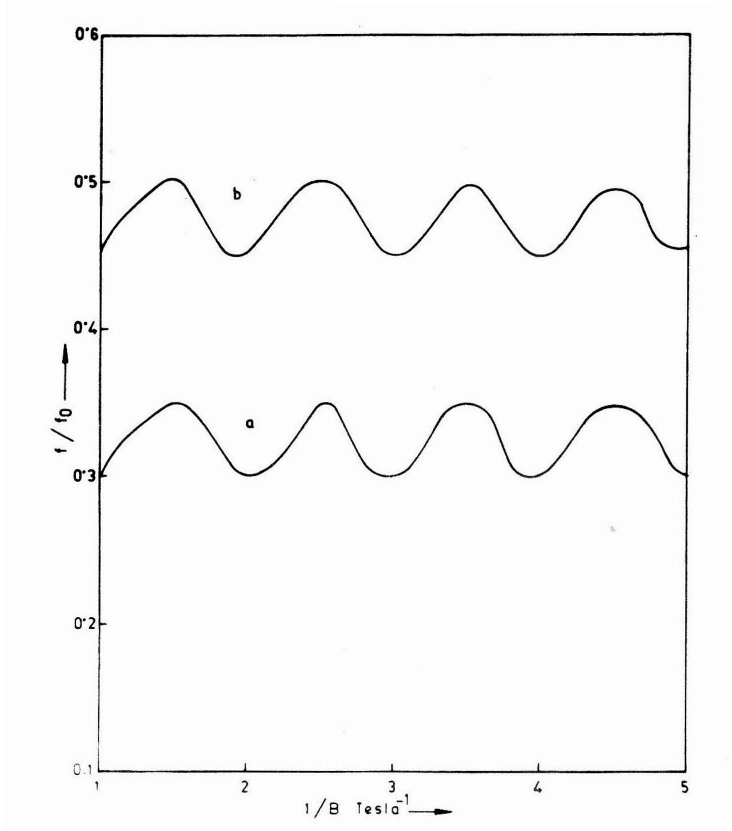

Fig. 2. Plot of normalized BTSE versus $1/B$ under magnetic quantization for the cases of Fig. 1 ($n_0 = 10^{22}\,\mathrm{m}^{-3}$ and $T_D = 7.2$ K). $T = 4.2$ K.

and $A_6(E_F) = \sum_{r=1}^{S} \Theta_r[A_5(E_F)]$. Thus combining Eqs. (5) and (10) we can get an expression of BTSE in this case. For $N_i \to 0$, $E_g \to \infty$ and $T_D \to 0$, the electron concentration can be expressed as

$$n_0 = C_5 \sum_{n=0}^{n_{\max}} \left[F_{\frac{1}{2}}(\eta_2) - F_{\frac{1}{2}}(\eta_3) \right] \tag{11}$$

where $\eta_2 = (k_B T)^{-1}(E_F - E_2)$, $E_2 = \left[\left(n + \frac{1}{2} \right) \hbar\omega_0 - e\epsilon_0 L_x + \frac{m^* \epsilon_0^2}{2B} \right]$, $\eta_3 = (k_B T)^{-1}(E_F - E_3)$, $E_3 = E_2 + e\epsilon_0 L_x$ and $C_5 = \left[2B(2m^*\pi)^{1/2}(k_B T)^{3/2}/L_x h^2 \epsilon_0 \right]$. For $\epsilon_0 \to 0$, Eq. (11) takes the well-known form as given by Eq. (8).

2.4. *Formulation of BTSE in quantum well lasers*

The electron energy spectrum in a quantum well lasers can be expressed as

$$\gamma(E) = (\hbar^2 k_s^2/2m^*) + (t\pi/d_1)^2(\hbar^2/2m^*) \tag{12}$$

where $k_s^2 = k_x^2 + k_y^2$, $t = 1, 2, 3, \ldots$ and d_1 is the film thickness.

The carrier concentration can be written as

$$n_0 = -\frac{m^*}{d_1 \pi \hbar^2} \sum_{t=1}^{t_{\max}} [A_7(E_F) + A_8(E_F)] \tag{13}$$

where $A_7(E_F) = [\gamma(E_F) - (\hbar^2/2m^*)(t\pi/d_1)^2]$ and $A_8(E_F) = \sum_{r=1}^{S} \Theta_r[A_7(E_F)]$.

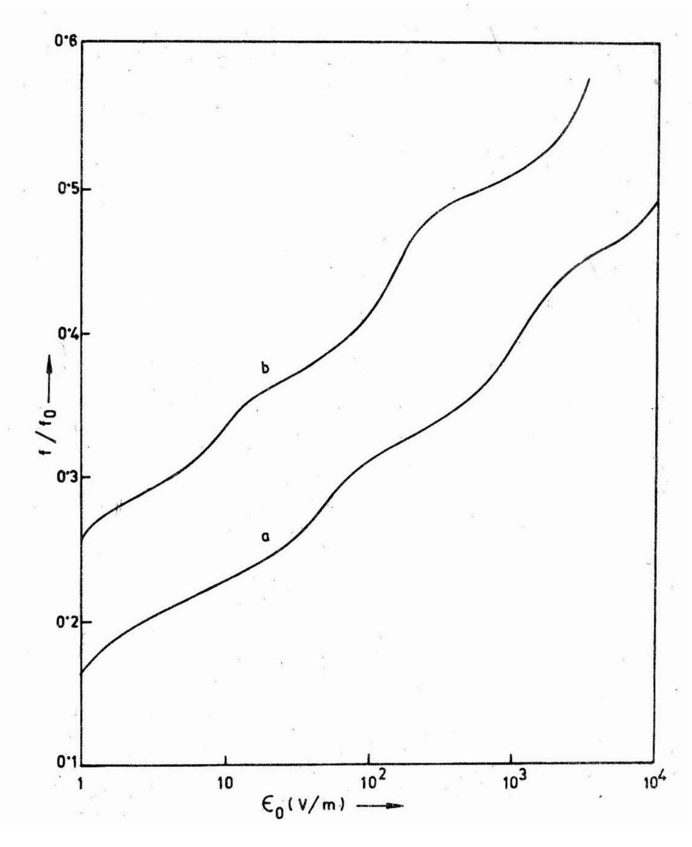

Fig. 3. Plot of normalized BTSE versus electric field under cross field configuration for the cases of Fig. 1 ($n_0 = 10^{22}\,\mathrm{m}^{-3}$, $B = 1.9$ T and $T_D = 7.2$ K, $T = 4.2$ K).

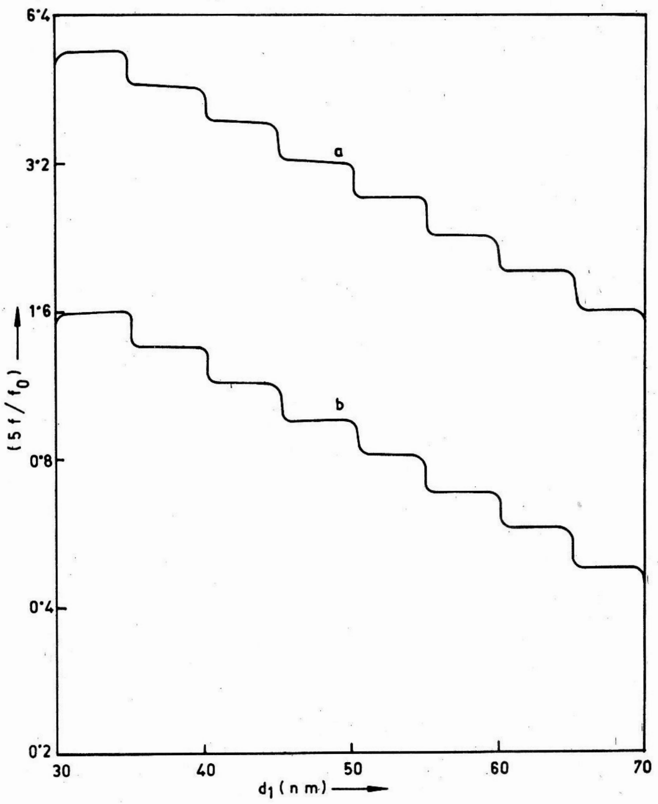

Fig. 4. Plot of normalized BTSE versus d_1 in quantum wire heavily doped InSb laser diode in accordance with the cases of Fig. 1 ($n_0 = 10^{14}\,\mathrm{m}^{-3}$). $T = 4.2$ K.

The BTSE for quantum confined laser diodes can be expressed as [13]

$$f^2 = (e^2 n_0/\varepsilon_s)\left(\frac{\mathrm{d}E_F}{\mathrm{d}n_0}\right) \tag{14}$$

where E_F and n_0 are the Fermi energy and the electron concentration under quantum confinement. Thus combining (13) and (14) we can find BTSE for quantum well lasers. For $E_g \to \infty$ and $N_i \to 0$, (13) assumes the well-known form as

$$n_0 = \frac{m^* k_B T}{d_1 \pi \hbar^2} \sum_{t=1}^{t_{\max}} F_0(\eta_4) \tag{15}$$

where $\eta_4 = (k_B T)^{-1}[E_F - (\hbar^2/2m^*)(t\pi/d_1)^2]$.

2.5. Formulation of BTSE in quantum wire lasers

The electron energy spectrum in quantum wire laser assumes the form

$$\gamma(E) = \hbar^2 k_x^2/2m^* + \frac{\hbar^2}{2m^*}\left(\frac{u\pi}{d_2}\right)^2 + \frac{\hbar^2}{2m^*}\left(\frac{t\pi}{d_1}\right)^2 \tag{16}$$

where d_2 is the film thickness along the y direction and $u = 1, 2, 3, \ldots$. The electron concentration can be expressed as

$$n_0 = \frac{2}{d_1 \pi d_2} \sum_{t,u} [A_9(E_F) + A_{10}(E_F)] \tag{17}$$

where $A_9(E_F) = [2m^* \gamma(E_F)\hbar^{-2} - (u\pi/d_2)^2 - (t\pi/d_1)^2]^{1/2}$ and $A_{10}(E_F) = = \sum_{r=1}^{s} \Theta_r [A_9(E_F)]$.

Combining (17) and (14) we can find BTSE in quantum wire lasers. For $E_g \to \infty$ and $N_i \to 0$, (17) takes the simple form

$$n_0 = \left[\sqrt{2m^* k_B T}/d_1 d_2 h\right] \sum_{t,u} F_{-\frac{1}{2}}(\eta_5) \tag{18}$$

where $\eta_5 = (k_B T)^{-1}[E_F - C_1]$, $C_1 = (\hbar^2 \pi^2)/2m^*)[(t/d_1)^2 + (u/d_2)^2]$.

2.6. Suggested experimental method of determining the BTSE in laser diodes

The thermoelectric power can, in general, be expressed as [14]

$$J = (\pi^2 k_B^2 T/3en_0)\left(\frac{\mathrm{d}n_0}{\mathrm{d}E_F}\right). \tag{19}$$

Combining (5) and (19) we get

$$f^2 = [e^2 \pi^2 k_B^2 T/3\varepsilon_s^{3/2}](J)^{-1} \tag{20}$$

and for quantum confined laser combining (19) and (14) we get

$$f^2 = \pi^2 k_B T e/3\varepsilon_s J. \tag{21}$$

Since J is a measurable experimental quantity we can measure BTSE for any laser.

3. Results and discussion

Using the appropriate equations together with the parameters [15] $E_g = 0.23$ eV, $C = 0.9$ eV, $N = 8L_0^{-3}$, L_0 is lattice constant, $\Omega = L_0^3/4$, $L_0 = 0.69$ nm, $m^* = 0.014m_0$, $\varepsilon_s = 17.5\varepsilon_0$, $n_0 \simeq N_i$ we have plotted the normalized f as a function of various physical variables as shown in Figs. 1 to 5. In all the figures we have also

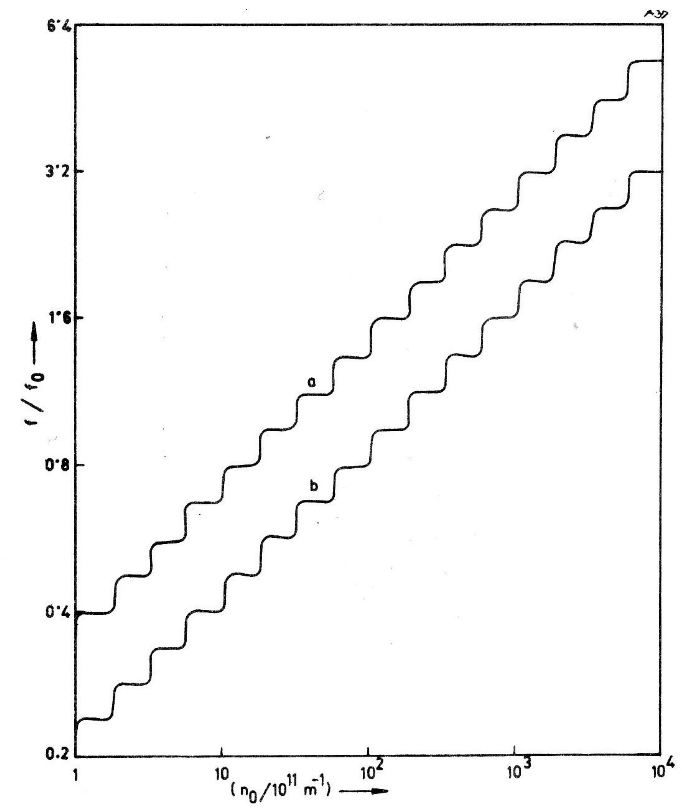

Fig. 5. Plot of normalized BTSE versus electron concentration in quantum wire heavily doped InSb *laser diode in accordance with the case of Fig. 1 ($d_1 = d_2 = 40$ nm). $T = 4.2$ K.*

plotted all cases for $N_i \to 0$ for the purpose of assesing the influence of heavy doping on BTSE for all types of quantum confined lasers. It appears from Fig. 1 that the BTSE increases with increasing n_0 in a monotonous manner as expected for heavily doped laser materials. Fig. 2 indicates that f should show regular oscillations when plotted as a function of the inverse quantizing magnetic field due to the SdH effect. From Fig. 3 it appears that the f increases with increasing electric field though the presence of damping can not seriously limit the amplitude of oscillations. The BTSE in quantum well lasers increases with decreasing film thickness as apparent from Fig. 4. The oscillations are like Heaviside step function which, in turn, reflect the influence of the density of states function on the electronic properties of quantum confined lasers. From Fig. 5 it appears that the BTSE in quantum wire lasers increases with increasing electron concentration in the form of nonideal steps due to the influence of finite temperature.

It appears from the literature that there has been considerable interest in studying the physical properties of quantum confined heavily doped laser diodes by considering various types of known density of states functions of the tail states without starting from the first principle. Besides, the influence of complex band structures of the materials as considered on the BTSE starting from the first principle has been neglected in all the analyses of doped laser diodes. We have also suggested an experimental method of determining of BTSE in quantum confined laser diodes having arbitrary energy band structure. We can not compare our theoretical formulation with our suggessted experimental method of determining f, since the experimental values of J are still not known in our case. The theoretical results of f as given here would be useful in probing the band structure of heavily doped laser diodes. Finally, it may be noted the basic purpose of our present work is not solely to investigate the BTSE in heavily doped laser diodes having various types of quantization of band states but also to formulate appropriate carrier concentration since the investigation of the various physical properties of quantum confined heavily doped laser diodes are based on the carrier statistics in such materials.

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O ENERGIJI ŠIRENJA REPOVA VRPCI U JAKO DOPIRANIM LASERSKIM DIODAMA PRI RAZLIČITIM FIZIKALNIM UVJETIMA

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U radu je razmatrana energija širenja repova vrpci u jako dopiranim laserskim diodama unutar volumena uz prisustvo magnetske kvantizacije te u ukrštenom električnom i magnetskom polju. Navedenu energiju izračunali smo i za kvantne žice s degeneriranom elektronskom gustoćom koja formira rep vrpce. Račun je napravljen na osnovi novo formuliranog zakona disperzije za laserske materijale s jako degeneriranom koncentracijom nosilaca naboja. Razmatrajući InSb laser kao primjer, nađeno je da energija širenja repova vrpci oscilira kao funkcija inverznog magnetskog polja, raste s porastom gustoće nosilaca naboja te porasta električnog polja. Uz 2D i 1D kvantizaciju, ta energija opada s porastom debljine sloja na oscilatoran način. Jako dopiranje znatno utječe na vrijednost širenja repova vrpci u svim vrstama kvantnog zasužnjenja. Također, sugerirali smo eksperimentalne metode za određivanje energije širenja repova vrpci za laserske materijale s proizvoljnom strukturom vrpce.